# A SMOOTHNESS CRITERIA FOR TREND CURVES USED IN ACCELERATOR ALIGNMENT.

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*Abstract*

This paper discusses physical aspects of the smooth trend curve application in accelerator alignment and how it will affect the orbit distortion. Emphasis is given to two major questions, which concern the relative alignment: "How is relative defined?" and "What is smooth?" An analogy between the orbit distortion, caused by the positioning of accelerator components not on the ideal closed orbit but on the chosen smooth curve, and forced oscillations of a mathematical pendulum, which has a movable point of suspension is drawn. A new term of spectral sensitivity of the magnetic structure is introduced. Samples of spectral sensitivity calculations for a few accelerator facilities are presented. Criteria for an estimation of the smooth trend curve, which would minimize the orbit distortion, is suggested.

## INTRODUCTION

Alignment procedures based on the use of a smooth trend curve technique are commonly applied to gain a high relative positioning of adjacent accelerator components and provide a smooth path for particle beams with a minimum adjustment for individual magnets. Different methods are applied to define the trend curve. See for instance the concepts are used at CERN [1], DESY [2], SLAC [3].

There is no clear criteria how the trend curve should be defined correctly. An alignment engineer with the use of the standard deviation of the determined parameters decides whether the chosen trend curve is smooth enough and alignment results have met the relative positioning tolerances. To minimize a number of particular elements to be adjusted he chooses the trend curve which fit the misalignment as close as possible. Any smooth trend curve, except the ideal reference line, will affect the particle motions. There is a hazard that the chosen trend curve will be not smooth enough and produce a significant orbit distortion. Finally, tracking program is applied to verify if the misalignments do not modify the beam parameters out of allowed limits.

Therefore, the best smooth trend curve should provide a closely fitting the misalignments and yield the minimal orbit distortion.

## MECHANISM OF SMOOTHING.

Why the beam path should be smooth and how its smoothness affects the orbit distortion?

Let us consider a simplest resonant oscillation system – a mathematical pendulum. Let its point of suspension will be moved along a smooth line given by Fourier harmonic with a number $k$ and an amplitude $a_k$

$$\delta y = a_k \cos kt \tag{1}$$

Transverse motions of the pendulum is given by formula

$$y^0 = \frac{\omega^2}{\omega^2 - k^2} a_k \cos kt, \tag{2}$$

$\omega$ is an own frequency of the pendulum.

Relative motions with respect to the point of suspension $y^{rel} = y^0 - \delta y$ will be

$$\begin{aligned} y^{rel} &= \frac{\omega^2}{\omega^2 - k^2} a_k \cos kt - a_k \cos kt = \\ &= \frac{k^2}{\omega^2 - k^2} a_k \cos kt \end{aligned} \tag{3}$$

The pendulum will have no relative motions if $k \to 0$ and motion with a maximum amplitude if $k=\omega$; for $k<<\omega$, $y^{rel}<<a_k$. Since any line we can consider to be a sum of Fourier harmonics, the relative amplitude of motions of the pendulum, the point of suspension of which is moved along such line, can be written as a sum of the particular amplitudes.

$$y = \sum_{k=0}^{\infty} y_k^{rel} \tag{4}$$

In order to have a minimal relative motions of the pendulum the line should include no harmonics with the number $k$ close to $\omega$ or $k>>\omega$ which have significant amplitudes $a_k$, i.e. the line should be smooth enough. It looks reasonable to restrict the number of harmonics with $k \le \omega/2$ or $y \le 1/3 a_k$ and define the line as a sum of Fourier harmonics of low orders.

## SPECTRAL SENSITIVITY.

The same considerations can be applied to the motions of particle beams. Due to a number of magnetic field errors an orbit of the particles is never be ideal. There is always the disturbed one. Misalignments result in additional oscillations of the particles with respect to this orbit. If the trend curve is smooth enough the particle orbit will follow this curve. The Hill's equation, written in a relative coordinate system (with respect to the smooth trend curve), will have no the focussing force coefficient $K(s)$ on the right hand side [4]

$$y''_{rel} + K y_{rel} = (\delta y)'' \tag{5}$$

and

$$\eta^2 = \frac{\nu^2}{4\sin^2 \pi\nu} \int_{\psi}^{\psi+2\pi} \int_{\psi}^{\psi+2\pi} f(t) f(\tau) \times \times \cos\nu(\psi + \pi - t) \cos\nu(\psi + \pi - t) dt d\tau \tag{6}$$

where $\eta = y\beta^{-\frac{1}{2}}$, $\psi = \int \frac{ds}{\nu\beta}$, $f = \beta^{\frac{3}{2}} \frac{k^2}{R^2}\delta y$, $\nu$ is a betatron tune, $R$ is an average radius of an accelerator. The misalignment $\delta y$ is weighted by $\beta$-function the specific variations of which along the particle path depend on a number of cells, a number of periods and superperiods. This fact results in additional resonance in the particle motions. Depends on an individual magnetic structure each accelerator has its own frequency of perturbations to which the particle motions are more sensitive.

Let us define an amplitude of the closed orbit distortion caused by the certain Fourier harmonics of the misalignment with an amplitude $a_k$=1 as a spectral sensitivity $\gamma_k$.

The spectral sensitivity can be calculated with the use of formula (6) utilizing the Mathcad program. Calculations of the spectral sensitivities for some accelerators in vertical direction were carried out by averaging over arbitrary phases of the misalignment harmonics $\delta y$ and over all focussing elements of the magnetic structure (fig.1÷4). The APS ring has the regular magnetic structure with 40 cells and the betatron tune $\nu$=14.3 [5]. Harmonics with number $k=\nu$ and $k=40-\nu$ are stressed. Coefficient of the spectral sensitivity $\gamma_k$ is fallen down quickly when $k$ is stepped aside from $\nu$. For $k \le \nu/2$, it is not exceed $\gamma_k<0.3$.

The same picture one can see for SPring-8 storage ring which consists of 48 cells and $\nu$=16.16 [6].

For $k=\nu/2$, $\gamma_k$=1.02 and $\gamma_k<1$ for $k<\nu/2$. LEP collider has an irregular magnetic structure divided into 4 superperiods and 7 sectors, $\nu$=98.19 [7]. Maximum of the spectral sensitivity is at $k=\nu-7-2\times4$. There are maximum at $k=\nu$, and local maximums at $k=\nu\pm n\times4$ ($n$=1,2..). The spectral sensitivity $\gamma_k<1$ for $k<34$. The High Energy Ring PEP-II has a more heterogeneous magnetic structure [8]. The spectral sensitivity increases very fast and already at $k=\nu/3$ it is equal $\gamma_k$=1.04.

## SMOOTHNESS CRITERIA.

The spectral sensitivity of an accelerator is a tool to identify the particular Fourier harmonics of the misalignment, which will result in a significant orbit distortion. The significant orbit distortion is a quite arbitrary term. We define it to be 10% of an allowed one $y_d \ge 0.1\times y_{max}$, which can be set in its turn as 1/5÷1/10 part of a minimal size of a vacuum chamber. Such harmonics and harmonics with $\gamma_k>1$ should be considered as dangerous ones. To minimize the orbit distortion due to the misalignment the trend curve should not include the dangerous harmonics. There may be some different reasons to eliminate a particular harmonic from the trend curve. A wavelength of the dangerous harmonic with a minimal number $k$ determines an area wherein the precise relative positioning of neighbor elements determined by the standard deviation should be performed.

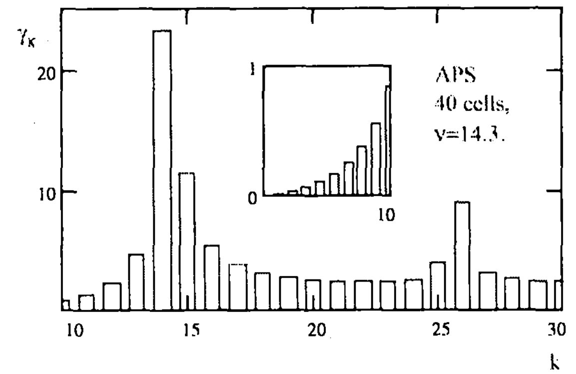


Fig.1 Spectral sensitivity of APS storage ring

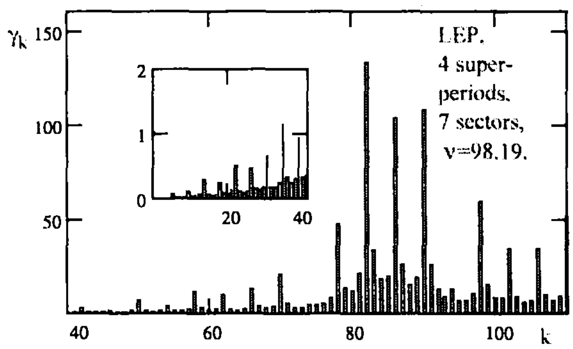


Fig.3 Spectral sensitivity of LEP main ring

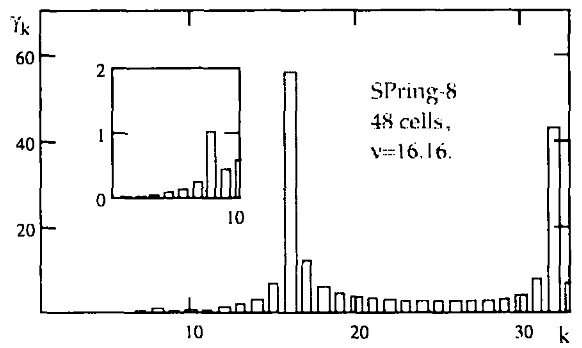


Fig.2 Spectral sensitivity of SPring-8 storage ring

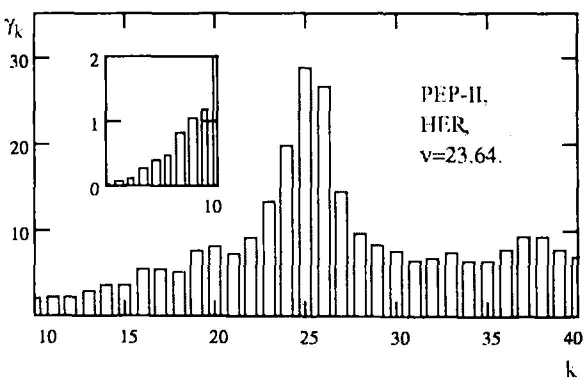


Fig.4 Spectral sensitivity of PEP-II, High Energy Ring

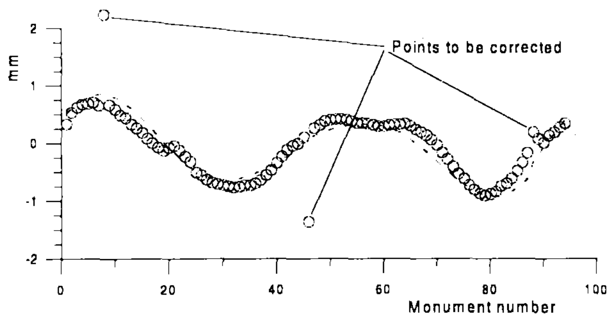


Fig. 5 Radial position of VEPP-4m alignment monuments. (Trend curve is shown by dashed line).

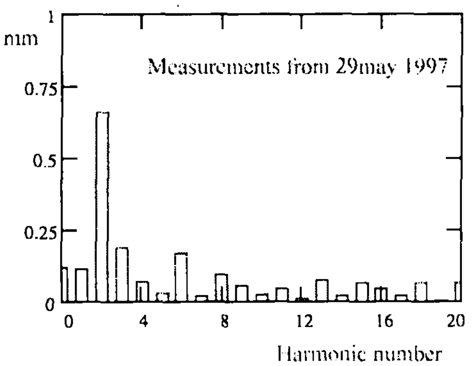


Fig. 6 Spectral content of the magnet position deviations in radial direction.

In realignment procedures of VEPP-4m storage ring, BINP, Novosibirsk [9], we utilize the spectral sensitivity for the smooth trend curve definition. Results of a control survey of the magnet positions are expanded into Fourier series. Then a quantity

$$y = \sum a_k \times \gamma_k \tag{8}$$

is calculated. It is summed up, started from a smallest number $k$, till $y$ will be equal or exceed the value of $y_{max}$. A sum of the appropriated Fourier harmonics defines a smooth trend curve. The alignment engineer compares values of $y$ and the allowed one $y_{max}$ and decides which harmonic should be included in the trend curve, but anyway the dangerous harmonics should be eliminated from summing up. In fig.5,6,7 a survey results of a radial position of VEPP-4m magnets, their Fourier decomposition and the spectral sensitivity are shown. The second harmonic of the misalignment has a very high amplitude $a_2$=0.7mm but the magnetic structure of VEPP-4m is almost not sensitive to it, $\gamma_2$=0.2 and $a_2 \times \gamma_2$=0.14mm<0.1$\times y_{max}$=0.2mm. The trend curve (dashed line in fig.5) is a sum of three lowest harmonics ($\gamma_{1+3}$<1). The standard deviation of the monuments from the smooth line, without three marked points, is $\sigma$<0.1mm. Since the 6$^{th}$ harmonic is dangerous one, $a_6 \times \gamma_6 = 0.4mm$, the area of the high relative positioning of the elements is specified to be 2$\times\lambda_6\approx$20 meters.

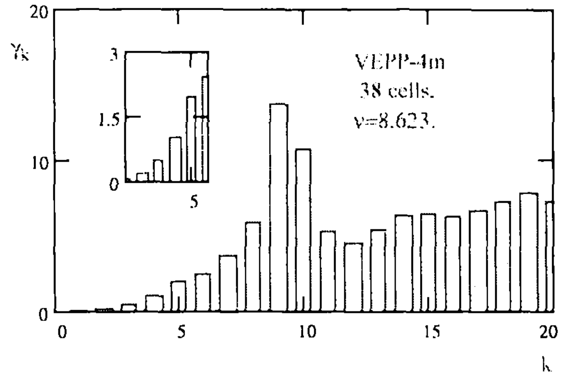


Fig.7 Spectral sensitivity of VEPP-4m storage ring.

## SUMMARY

It is offered to define the smooth trend curve as a sum of Fourier harmonics of low orders. It should not include the dangerous harmonics determined with the use of the spectral sensitivity of the magnetic structure. The value $\sum \gamma_k a_k$ is a measure of the trend curve smoothness. The wavelength of the dangerous harmonic with a lowest number determines the region of the precise relative alignment. This methodic is successfully applied in the realignment process of VEPP-4m storage ring, BINP, Novosibirsk.

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