Nuclear clustering over the nuclear chart from a covariant EDF approach

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Abstract. Nuclear clustering has been studied with covariant Energy Density Functional approaches for more than a decade. It recently allowed to bridge microscopically the description of cluster states in light nuclei with the one of cluster and alpha decays in heavy ones. The proper use of theoretical tools, such as the density and the nucleonic localization function, allows to shed light on the mechanisms of formation and identification of clusters in nuclei. Finally, a global analysis of nuclear clustering is discussed, in order to identify control parameters for a nuclear cluster phase.

1 Introduction

The covariant Energy Density Functional (EDF) has been proven to describe in the same framework quantum liquid and cluster states [1–4]. It allowed to show, in a single framework, that clusterization can be induced not only by the vicinity of the alpha emission threshold as inferred by the famous Ikeda conjecture [5], but also by the depth of the potential [2], the excitation energy [6], the increase of the deformation [6], the decrease of the density [7, 8] and the one of the nucleon number [3, 4]. This led to a more universal picture of nuclear states, between quantum liquid and clusterization, as validated by a good description of the data, namely the excitation spectrum built on various bands in ^{20}Ne [9] and ^{12}C [10]. More recently, the covariant EDF approach has also been able to describe dynamical processes, such as alpha emission [11], as discussed in section 2. In section 3, the recent several one-body functions [12, 13], used to pinpoint the presence of clusters in nuclei, raise a necessary discussion. Finally, in section 4, we also discuss the identification of control parameters for the cluster phase in nuclei, such as the density and the number of nucleons, to provide an overview of this phenomenon.

2 The covariant EDF approach and comparison with the data

An obvious difference between an alpha-emitting nucleus and an alpha-clusterized one is that the corresponding Q value is negative in the latter. However, it is expected that beyond these differences, similarities remain, such as in the formation of the alpha particle. As discussed in the introduction, covariant EDF approaches have been shown to describe cluster phenomena in light nuclei, and are also known to describe fission from a least action path [14]. Therefore, one can naturally wonder if this approach could describe alpha decay on the same ground. Indeed, several studies have successfully shown that covariant EDF approaches are able to predict alpha decay with a good description of the measured alpha emission half-life, within typically one order of magnitude [11, 15, 16]. More precisely, in such an approach, the least action path is followed on a potential energy surface calculation. The key point here is the deformation degrees of freedom to be considered. For instance, in the alpha emission of average mass nuclei, it has been shown that both the quadrupole and octupole degrees of freedom have to be considered. However, in the case of alpha emission by heavy nuclei, such as ^{212}Po, or Ra and Rn isotopes, the hexadecapole degree of freedom also has to be considered for the heavy system to form a neck, useful for the alpha emission.

In the same spirit, it has also been shown that cluster radioactivity could be described with covariant EDFs [11]. It is interesting to note that cluster emissions were first described by Gogny EDF [17], which is non-relativistic. However, it seems that for now, only the covariant EDF is able to describe alpha emission. This may be due to its good capability of describing alpha cluster states in light nuclei. It should be noted that cluster emission can be more difficult to describe than the alpha one for covariant approaches, because the cluster, such as ^{14}C can bring some internal degrees of freedom, which would require a

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more advanced description than the so-called ATDHF approach [11], and also an improve of the inertial mass description. However, it should be noted that calculations of alpha and cluster emissions are at the limit of what can achieved by today's nuclear structure models, in terms of computing calculations.

Concomitantly to the alpha decay description, another decay path has appeared in the calculations, which corresponds to the emission of two alpha particles at once [11, 15]. This double alpha decay is discussed in the contribution of L. Heitz to the present conference.

3 Criteria and indicators to pinpoint an alpha cluster

With the understanding that the Ikeda conjecture was not the only lever acting on the appearance of alpha cluster states, such states have also been searched for in heavy nuclei. Experimentally, this has become a hot topic with the use of (p,pα) reactions on various nuclei over the nuclear chart [18, 19]. On the theoretical side, some indicators have been designed to pinpoint the presence of alpha particles. It should be noted that the mere nucleonic density, showing localization, is already a signal, as obtained with covariant EDF in clusterized nuclei [2, 20]. However, since other approaches, such as Skyrme EDF, usually do not predict alpha clusterization as frequently as what is observed [2], some alternative indicators have been considered. For instance, the nucleonic localization function (NLF) has recently been used in several works [21, 22]. Also, the alpha particle density, in a covariant framework that explicitly includes alphas in the Lagrangian, is another indicator [23]. More recently, the so-called local alpha removal strength has also been introduced [13]. These 2 last indicators are discussed in the contribution of the present conference of S. Typel and T. Nakatsukasa, respectively. Finally, one could also consider the localization parameter [2, 3, 24], which can be calculated microscopically or in the HO approximation, to pinpoint the presence of cluster states in nuclei.

In a recent study, we have shown that the NLF is not pinpointing the presence of clusters, but rather indicates the level of purity of the nucleonic wave functions [20]. This has also been discussed in [22, 25] in the case of fission. However, the NLF could be helpful to enlighten the alpha formation process, as it is complementaty to the covariant density. More precisely, in the alpha decay process, the nucleonic wave function first gets pure as an alpha one, and then in a second step, gets localized [20]. This sheds a microscopic light on the alpha preformation mechanism at work before alpha emission: an alpha cluster is not preformed in the heart of the daughter nucleus at once, but rather on its surface and with the above-mentioned two-step mechanism.

4 Is there a cluster phase in nuclei ?

With the emergence of a global understanding of clusterization over the nuclear chart within covariant EDF approaches, one could consider cluster as a transitional phase between the less delocalized quantum liquid phase and the more localized crystal one. Such a transitional state of localization is known to happen in different fields of physics [26]. Dealing with phase transition, one could look for the control parameters that drive the transition of the system. In the case of nuclei, the number of nucleons has been identified as a control parameter [3, 4, 27], since the localization parameter showed how alpha clusters are more likely in light than in heavy nuclei. In these studies, it was also realized that the crystal phase in nuclei would require too few nucleons to exist.

Another control parameter is the nuclear density, related to the Mott transition. Here, it was shown that in the low-density regime, which can be reached either by diluting a nucleus [7, 8], as for instance in the last stages of heavy ion collisions, or by analyzing the occurrence of alpha presence in the very surface of nuclei [23], a transition from a quantum liquid to a cluster phase could be obtained. Hence, the density shall also be a control parameter for the occurrence of a cluster phase in nuclei. Figure 1 displays the corresponding phase diagram in nuclei obtained by a dimensionless analysis, and confirmed by both HO and microscopic EDF calculations.

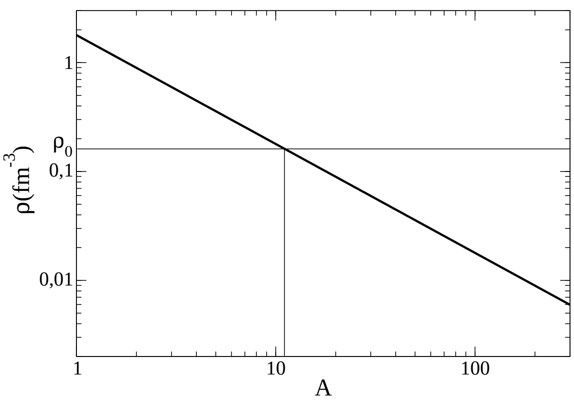

Figure 1. Density vs nucleon number phase diagram in nuclei, obtained in a dimensionless analysis [27]. The solid line represents the clusterization condition: nuclear state are more likely to be clusterized below this line, and more likely to behave as delocalized quantum liquid state above this line.

Finally, a recent work analyzed the role of the temperature on nuclear clusterization, showing the disappearance of the cluster phase in hot nuclei [28]: interestingly, pairing plays an important role, by synchronizing the critical temperatures of cluster and shape phase transitions.

5 Conclusion

Covariant EDFs provide a general and broad understanding of the cluster phenomenon and its links to the quantum liquid state in nuclei. Several upgrades could be made within such an approach. For instance, including more degree of freedom such as triaxiality, in the case of cluster decay. Using four body indicators for alpha cluster identification would also be relevant. Finally, several open questions remain: the community should agree on the role and limitations of the various identification functions for clusters in nuclei. Also, the identification of possible order parameters of the cluster phase transition in nuclei should be more precisely investigated.

References

[1] P. Arumugam, B.K. Sharma, S.K. Patra, R.K. Gupta, Relativistic mean field study of clustering in light nuclei, Phys. Rev. C **71**, 064308 (2005). 10.1103/PhysRevC.71.064308

[2] J.P. Ebran, E. Khan, T. Nikšić, D. Vretenar, How atomic nuclei cluster, Nature **487**, 341 (2012). 10.1038/nature11246

[3] J.P. Ebran, E. Khan, T. Nikšić, D. Vretenar, Localization and clustering in the nuclear fermi liquid, Phys. Rev. C **87**, 044307 (2013). 10.1103/PhysRevC.87.044307

[4] J.P. Ebran, E. Khan, T. Nikšić, D. Vretenar, Cluster-liquid transition in finite, saturated fermionic systems, Phys. Rev. C **89**, 031303 (2014). 10.1103/PhysRevC.89.031303

[5] K. Ikeda, N. Takigawa, H. Horiuchi, The Systematic Structure-Change into the Molecule-like Structures in the Self-Conjugate 4n Nuclei, Progress of Theoretical Physics Supplement **E68**, 464 (1968). 10.1143/PTPS.E68.464

[6] J.P. Ebran, E. Khan, T. Nikšić, D. Vretenar, Density functional theory studies of cluster states in nuclei, Phys. Rev. C **90**, 054329 (2014). 10.1103/PhysRevC.90.054329

[7] M. Girod, P. Schuck, α-particle clustering from expanding self-conjugate nuclei within the hartree-fock-bogoliubov approach, Phys. Rev. Lett. **111**, 132503 (2013). 10.1103/PhysRevLett.111.132503

[8] J.P. Ebran, M. Girod, E. Khan, R.D. Lasseri, P. Schuck, α-particle condensation: A nuclear quantum phase transition, Phys. Rev. C **102**, 014305 (2020). 10.1103/PhysRevC.102.014305

[9] P. Marević, J.P. Ebran, E. Khan, T. Nikšić, D. Vretenar, Quadrupole and octupole collectivity and cluster structures in neon isotopes, Phys. Rev. C **97**, 024334 (2018). 10.1103/PhysRevC.97.024334

[10] P. Marević, J.P. Ebran, E. Khan, T. Nikšić, D. Vretenar, Cluster structures in ^{12}C from global energy density functionals, Phys. Rev. C **99**, 034317 (2019). 10.1103/PhysRevC.99.034317

[11] J. Zhao, J.P. Ebran, L. Heitz, E. Khan, F. Mercier, T. Nikšić, D. Vretenar, Microscopic description of $\alpha, 2\alpha$, and cluster decays of $^{216-220}$Rn and $^{220-224}$Ra, Phys. Rev. C **107**, 034311 (2023). 10.1103/PhysRevC.107.034311

[12] P.G. Reinhard, J.A. Maruhn, A.S. Umar, V.E. Oberacker, Localization in light nuclei, Phys. Rev. C **83**, 034312 (2011). 10.1103/PhysRevC.83.034312

[13] T. Nakatsukasa, N. Hinohara, Local α-removal strength in the mean-field approximation, Phys. Rev. C **108**, 014318 (2023). 10.1103/PhysRevC.108.014318

[14] J. Zhao, J. Xiang, Z.P. Li, T. Nikšić, D. Vretenar, S.G. Zhou, Time-dependent generator-coordinate-method study of mass-asymmetric fission of actinides, Phys. Rev. C **99**, 054613 (2019). 10.1103/PhysRevC.99.054613

[15] F. Mercier, J. Zhao, J.P. Ebran, E. Khan, T. Nikšić, D. Vretenar, Microscopic description of 2α decay in ^{212}Po and ^{224}Ra isotopes, Phys. Rev. Lett. **127**, 012501 (2021). 10.1103/PhysRevLett.127.012501

[16] F. Mercier, J. Zhao, R.D. Lasseri, J.P. Ebran, E. Khan, T. Nikšić, D. Vretenar, Microscopic description of the self-conjugate ^{108}Xe and ^{104}Teα-decay chain, Phys. Rev. C **102**, 011301 (2020). 10.1103/PhysRevC.102.011301

[17] M. Warda, L.M. Robledo, Microscopic description of cluster radioactivity in actinide nuclei, Phys. Rev. C **84**, 044608 (2011). 10.1103/PhysRevC.84.044608

[18] J. Tanaka, Z. Yang, S. Typel, S. Adachi, S. Bai, P. van Beek, D. Beaumel, Y. Fujikawa, J. Han, S. Heil et al., Formation of α clusters in dilute neutron-rich matter, Science **371**, 260 (2021). 10.1126/science.abe4688

[19] P.J. Li, D. Beaumel, J. Lee, M. Assié, S. Chen, S. Franchoo, J. Gibelin, F. Hammache, T. Harada, Y. Kanada-En'yo et al., Validation of the ^{10}Be ground-state molecular structure using ^{10}Be$(p, p\alpha)^{6}$He triple differential reaction cross-section measurements, Phys. Rev. Lett. **131**, 212501 (2023). 10.1103/PhysRevLett.131.212501

[20] E. Khan, L. Heitz, F. Mercier, J.P. Ebran, α-particle formation and clustering in nuclei, Phys. Rev. C **106**, 064330 (2022). 10.1103/PhysRevC.106.064330

[21] P. Jerabek, B. Schuetrumpf, P. Schwerdtfeger, W. Nazarewicz, Electron and nucleon localization functions of oganesson: Approaching the thomas-fermi limit, Phys. Rev. Lett. **120**, 053001 (2018). 10.1103/PhysRevLett.120.053001

[22] Z.X. Ren, D. Vretenar, T. Nikšić, P.W. Zhao, J. Zhao, J. Meng, Dynamical synthesis of ^{4}He in the scission phase of nuclear fission, Phys. Rev. Lett. **128**, 172501 (2022). 10.1103/PhysRevLett.128.172501

[23] S. Typel, Neutron skin thickness of heavy nuclei with $\alpha-$particle correlations and the slope of the nuclear symmetry energy, Phys. Rev. C **89**, 064321 (2014). 10.1103/PhysRevC.89.064321

[24] J.P. Ebran, E. Khan, R.D. Lasseri, D. Vretenar, Single-particle spatial dispersion and clusters in nuclei, Phys. Rev. C **97**, 061301 (2018). 10.1103/PhysRevC.97.061301

[25] A. Bulgac, Examining the justification for the introduction of a fermion localization function, Phys. Rev. C **108**, L051303 (2023). 10.1103/PhysRevC.108.L051303

[26] H. Falakshahi, X. Waintal, Hybrid phase at the quantum melting of the wigner crystal, Phys. Rev. Lett. **94**, 046801 (2005). 10.1103/PhysRevLett.94.046801

[27] J.P. Ebran, L. Heitz, E. Khan, Use of quantality in nuclei and many-body systems (2024), `2406.03378`, `https://arxiv.org/abs/2406.03378`

https://doi.org/10.1051/epjconf/202431100011

[28] E. Yüksel, F. Mercier, J.P. Ebran, E. Khan, Clustering in nuclei at finite temperature, Phys. Rev. C **106**, 054309 (2022). 10.1103/PhysRevC.106.054309