NUMERICAL CALCULATION OF ELECTROMAGNETIC FIELDS IN ACCELERATION CAVITIES UNDER PRECISE CONSIDERATION OF COUPLER STRUCTURES[*]

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Abstract

During the design phase of superconducting radio frequency (RF) accelerating cavities a challenging and difficult task is to determine the electromagnetic field distribution inside the structure with the help of proper computer simulations. Although dissipation due to lossy materials is neglected in the current work, in reality, because energy transfer appears due to the design of the superconducting cavities, the numerical eigenmode analysis based on real-valued variables is no longer suitable to describe the dissipative acceleration structure. Dissipation can appear with the help of dedicated higher order mode (HOM) couplers, the power coupler as well as the beam tube once the resonance frequency is above the cutoff frequency of the corresponding waveguide. At the Computational Electromagnetics Laboratory (TEMF) a robust parallel eigenmode solver based on complex-valued finite element analysis is available. The eigenmode solver has been applied to the TESLA 1.3 GHz and the third harmonic 3.9 GHz nine-cell cavities to determine the resonance frequency, the quality factor and the corresponding field distribution of eigenmodes.

INTRODUCTION

The determination of the the electromagnetic field distribution inside the superconducting (RF) accelerating cavities with the aid of computer simulations is a difficult task. So far the most efficient commercially available eigenmode solvers are based on real-valued analysis, which is sufficient to describe the entire electromagnetic field in the lossless acceleration structure. In reality, because of the dissipative components a complex-valued eigenmode solver has to be used to determine the field distribution efficiently [1]. A robust parallel complex-valued eigenmode solver has been developed at the Computational Electromagnetics Laboratory (TEMF) and is applied to the TESLA 1.3 GHz cavity and the third harmonic superconducting cavity (3.9 GHz) to determine the resonance frequency, the quality factor and the corresponding field distribution of eigenmodes [2] [3].

THEORETICAL BACKGROUND

To analyze the electromagnetic field distribution inside the elliptical RF accelerating cavities with high precision, the continuous Maxwell's formulation has been transformed to a suitable matrix equation with the help of the finite element method (FEM) [4]. The FEM discretization is based on a tetrahedral grid and higher order curvilinear elements have been be applied to satisfy the demand for high-precision modeling of the elliptical cavity [5]. In case the resonance frequency is above the cutoff frequency of the corresponding waveguide, energy transfer can occur with the help of the HOM couplers, the power coupler as well as the beam tube. For this reason port boundary conditions can be applied [4].

Implementation

The geometric modeling of the accelerating structure with the tetrahedral meshing is performed with CST MICROWAVE STUDIO (MWS) [6]. The eigenmode solver is generally computationally demanding due to the precise modeling of elliptical cavities with curved tetrahedral meshes as well as the complex-valued calculation process. To achieve a good performance with respect to simulation time, a distributed memory architecture using MPI parallelization strategy has been utilized for the implementation [4].

COMPUTATION OF EIGENMODES

Firstly, the eigenmode solver has been applied to the TESLA 1.3 GHz cavity (Fig.1) to determine the characteristic values (resonance frequency, quality factor and shunt impedance) for 192 eigenmodes up to the 5th dipole passband (3.12 GHz) [2]. The TESLA 1.3 GHz cavity is composed of a 9-cell cavity, the input coupler as well as the up- and downstream HOM couplers. Port boundary conditions are used to define the boundary conditions for the coaxial lines of the three couplers and both beam tubes.

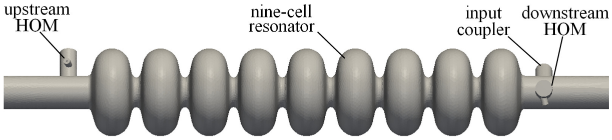

Figure 1: TDR-like TESLA 9-cell 1.3 GHz superconducting RF cavity with beam tubes as well as the input coupler and two HOM couplers.

In Fig. 2 the colored points indicate the values of frequency and quality factor for all eigenmodes up to the 5th dipole passband for a specific setup where the penetration depth of the input coupler is set to 8 mm. According to Fig. 2 the quality factor of accelerating mode TM_{010}, π is

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about $2.7{\cdot}10^6$, while the largest quality factor of the dipole modes is about $6{\cdot}10^5$. The quality factors of several HOM modes above the cutoff frequencies of the beam pipe can reach to about 10^{11}. In addition the shunt impedances (R/Q) for all 192 eigenmodes have been determined. In the near future the characteristic values together with corresponding field plots of all 192 eigenmodes up to the 5th dipole passband (3.12 GHz) will be published in a TESLA report at DESY.

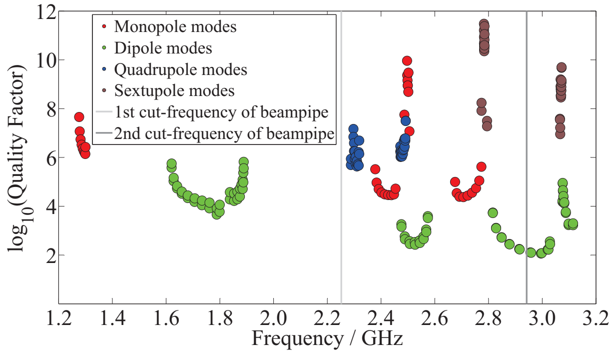

Figure 2: Quality factors versus frequencies for all eigenmodes in the TESLA 1.3 GHz cavity up to the 5th dipole passband (3.12 GHz).

Secondly, the frequencies and the corresponding R/Q for 215 eigenmodes up to the 4th monopole passband (10.61 GHz) in the third harmonic superconducting cavity (3.9 GHz) have been determined [3]. At this time the third harmonic cavity is made up of a 9-cell cavity without HOM couplers and the input coupler (Fig. 3). Port boundary conditions are used to model the infinite beam tubes. For example the shunt impedance of the results for all calculated dipole modes up to 10 GHz are presented in Fig. 4.

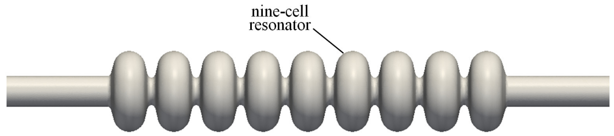

Figure 3: The third harmonic superconducting cavity (3.9 GHz) with beam tubes.

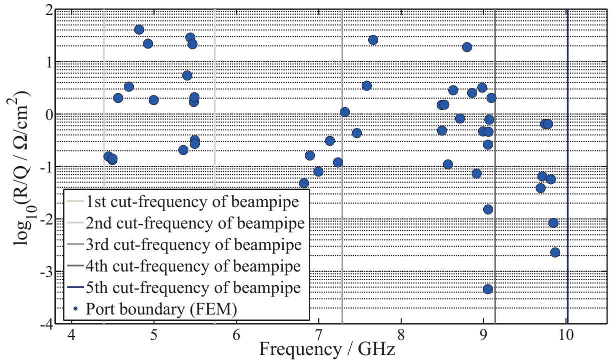

Figure 4: Shunt impedance R/Q versus frequency for dipole modes in the third harmonic superconducting cavity up to 10 GHz.

Post-processing

After determination of the resonance frequency and the corresponding field distribution of each eigenmode, there are two essential post-processing tasks to be done. The first one is to smooth the electromagnetic field using Kirchhoff's integral theorem, so that the shunt impedance of the eigenmode can be computed precisely [7]. Secondly, it is very time-consuming and inconvenient to manually identify each type of eigenmode. For this reason a dedicated algorithm has been developed to automatically identify the eigenmode type in a batch mode operation [7].

COMPARISON OF THE RESULTS

After the calculation of the eigenmodes with complex-valued FEM eigenmode solver, the results from divers eigenmode solvers have been compared.

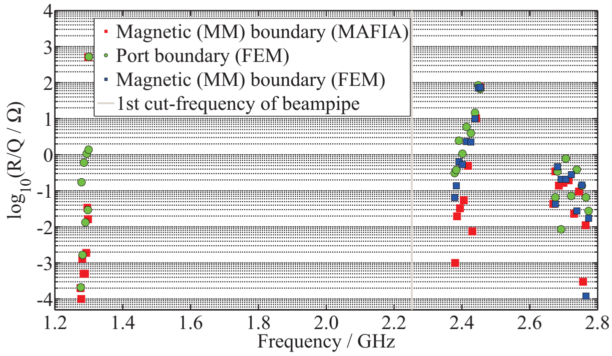

Figure 5: The shunt impedance (R/Q) of the monopole modes which have been calculated for the 9-cell TESLA 1.3 GHz cavity plotted versus the frequency of the modes. The red squares denote the modes obtained with magnetic (MM) boundary conditions from the MAFIA calculations while the blue squares correspond to several modes in 2nd and 4th monopole passband obtained with MM boundary conditions from the real-valued FEM calculations [2]. The results obtained with port boundary conditions from complex-valued FEM calculations are marked by green circles.

First of all, a graphic representation of the figure of merit R/Q for calculated monopole and dipole modes in the TESLA 1.3 GHz cavity using the electromagnetic field solver MAFIA based on real-valued finite integration technique (FIT) as well as real-valued and complex-valued FEM analysis, is given in Fig. 5 and Fig. 6 [2]. For the MAFIA calculations the 9-cell symmetric cavity (with symmetric end cells) without any couplers are applied [2], while a 9-cell cavity (with unsymmetrical end cells) with HOM couplers and input coupler is used for the FEM calculations. According to Fig. 5, a comparison between the MAFIA calculations and complex-valued FEM calculations indicates larger R/Q for most monopole modes in the 2nd and 4th monopole passbands (2.3 to 2.8 GHz). This has to be primarily attributed to different boundary conditions, mesh types and applied geometry models. Furthermore, different mesh types and geometries lead to larger R/Q of eigenmodes in 2nd and 4th monopole band obtained with MM boundary conditions from real-valued FEM calculations compared with those from MAFIA calculations. But as shown in Fig. 6, the R/Q of the most dipole modes in the 3rd dipole passband (2.4 to 2.6 GHz) obtained from real-valued FEM calculations with EE and MM boundary conditions are smaller than those from MAFIA and complex-valued FEM calculations.

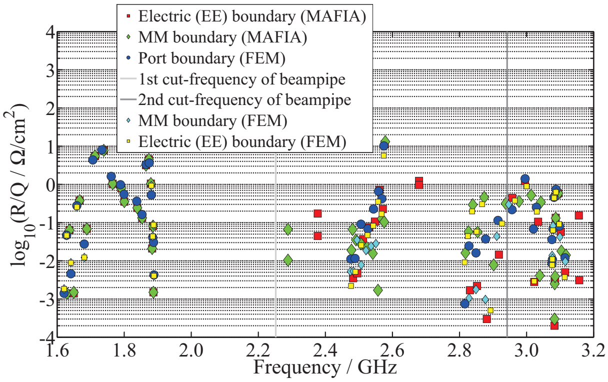

Figure 6: The parameter (R/Q) of the dipole modes calculated for the 9-cell TESLA 1.3 GHz cavity plotted versus the frequency of the modes. Red squares: electric (EE) boundary conditions, MAFIA [2]. Yellow squares: EE boundary conditions, real-valued FEM. Green diamonds: MM boundary conditions, MAFIA[2]. Cyan diamonds: MM boundary conditions, real-valued FEM. Blue circles: Port boundary conditions, complex-valued FEM.

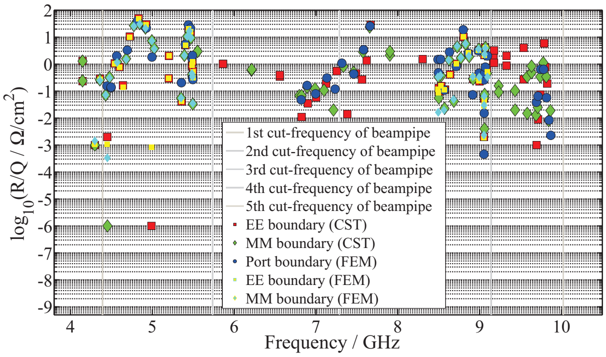

Figure 7: The parameter (R/Q) of the dipole modes calculated for the third harmonic superconducting cavity plotted versus the frequency of the modes. Red squares: electric (EE) boundary conditions, CST MWS [3]. Yellow squares: EE boundary conditions, real-valued FEM. Green diamonds: MM boundary conditions, CST MWS [3]. Cyan diamonds: MM boundary conditions, real-valued FEM. Blue circles: Port boundary conditions, complex-valued FEM.

Secondly, the R/Q for calculated dipole modes in the third harmonic superconducting cavity, by using CST MWS based on real-valued FIT analysis as well as real-valued and complex-valued FEM analysis, is graphically presented in Fig. 7 [3]. At this time, a 9-cell cavity without couplers is applied for the three numerical analyses [3]. The results in Fig. 7 show that, despite the different mesh types the R/Q of the most dipole modes in 4.3 to 5.5 GHz and 8.4 to 9.2 GHz frequency band obtained with MM and EE boundary conditions from real-valued FEM calculations are close to those calculated with MM and EE boundary conditions from CST MWS calculations. But the R/Q of dipole modes obtained with port boundary conditions from complex-valued FEM calculations differ from the R/Q calculated with CST MWS and real-valued FEM calculations due to the different boundary conditions. For example in the band 8.4 to 9.2 GHz the R/Q of the most modes obtained from complex-valued FEM calculations are larger than those from MAFIA and CST MWS calculations. In addition some beam-pipe modes (for example, the modes in frequency range between 5.8 and 6.6 GHz) obtained from CST MWS calculations can not be found by complex-valued FEM analysis anymore.

CONCLUSION

A robust parallel eigenmode solver on the basis of complex-valued finite element analysis, which utilizes basis function up to the second order on curved tetrahedral elements and port boundary conditions, has been successfully applied to analyze the electromagnetic fields inside the TESLA 1.3 GHz and the third harmonic (3.9 GHz) nine-cell cavities. In order to calculate the shunt impedance of the eigenmodes with high precision, the electromagnetic field inside the cavity can be smoothed on a physically motivated basis using Kirchhoff's integral theorem. Lastly, the comparisons between the different eigenmode solvers indicate that the complex-valued eigenmode solver can be used to determine the field distribution inside the acceleration structure efficiently.

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