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Yang–Mills Instantons in the Dual-Superconductor Vacuum Can Become Confining

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Abstract: As known, the realistic, exponential, fall-off of the rate of production of light mesons in the chromo-electric field of a quark–antiquark string, as a function of the meson mass, can be obtained from the Schwinger-formula Gaussian fall-off within a phenomenological approach which assumes a certain distribution of the string tension. This approach gets a clear meaning in the London limit of the dual superconductor, where the logarithmic increase of the chromo-electric field towards the core of the string leads precisely to the change of the Gaussian fall-off to the exponential one, thus allowing for an identification of the phenomenological distribution of the string tension. In this paper, we demonstrate that, for this distribution of the string tension, the distribution of large-size Yang–Mills instantons, which are interacting with the confining monopole background, becomes $\mathcal{O}(1/\rho^3)$, where ρ is the size of an instanton. Since such a distribution of large-size instantons is known to yield confinement, we conclude that, in the London limit of the dual-superconductor vacuum, instantons can form a confining medium, and we evaluate their contribution to the total string tension.

Keywords: dual-superconductor scenario of quark confinement; London limit; QCD flux tubes; Yang–Mills instantons; pair production in the chromo-electric field

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1. Introduction

With the increase of separation between two heavy quarks, the QCD string interconnecting them is expected to break by way of the light-mesons production [1]. Due to the heaviness of the quarks, the QCD string in this process can be treated as a flux tube, so that the string breaking can be considered as the decay of such a tube, caused by the creation of a light meson in the tube's chromo-electric field. The associated rate of production of light quark–antiquark pairs can be most straightforwardly evaluated by approximating that chromo-electric field by its mean value, averaged over the tube's cross-section [2,3]: $g\langle E\rangle \propto \sigma_0$, where σ_0 is the string tension. Constant chromo-electric field $\langle E\rangle$ yields then the Schwinger formula for the pair-production rate, so that the masses of produced light mesons have a Gaussian distribution [4,5]

$$w \simeq N_c N_{\rm f} \frac{(g\langle E\rangle)^2}{(2\pi)^3} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \exp\left(-\frac{\pi n m^2}{g\langle E\rangle}\right).$$

Experimental data, however, indicate that the distribution of light-meson masses corresponding to the flux-tube decay, should rather be exponential [1]. In order to comply with these experimental data, a phenomenological model has been suggested in Ref. [6], which treats the string tension as a fluctuating quantity, whose distribution is falling off exponentially: $p(\sigma) \sim {\rm e}^{-a\sigma/\sigma_0}$. Then, the Schwinger formula, averaged with such a distribution, yields the observed exponential distribution of light-meson masses: $\int_0^\infty d\sigma\, {\rm e}^{-m^2/(b\sigma)}\, p(\sigma) \sim {\rm e}^{-2m\sqrt{a/(b\sigma_0)}}$. It turns out that this idea can be given a field-theoretical status within the dual-superconductor scenario of confinement, notably in the

London limit of the dual superconductor. This scenario assumes that the confining properties of the Yang–Mills vacuum are based on the condensate of magnetic monopoles, which can be described by the dual Higgs field within the dual Abelian Higgs model, while the quark–antiquark string is realized as a dual Abrikosov–Nielsen–Olesen string in that model [7–10].

In the London limit, where the dual Higgs field is much heavier than the dual vector boson, the electric field of a static flux tube can be found analytically, in terms of a Macdonald function, and reads [11–13]

$$E(r) = \frac{M^2}{g_m} K_0(Mr). \tag{1}$$

Here, r is the transverse distance to the tube's center line, and $M = \sqrt{2} g_m \eta$ is the mass of the dual vector boson, with η being the v.e.v. of the dual Higgs field, and g_m being the magnetic coupling constant, which is related to the electric coupling constant g via the Dirac quantization condition $g_m = 2\pi/g$. Due to the exponentially fast fall-off of the Macdonald function, the field averaged over the flux-tube cross section, whose area is $S = \pi/M^2$, can be defined as $\langle E \rangle = \frac{1}{S} \int \mathrm{d}^2 r E(r)$. This averaged field obeys the relation $g\langle E \rangle = 4\sigma_0/L$, where $\sigma_0 = 2\pi\eta^2 L$ is the string tension in the London limit, and $L \equiv \ln \frac{M_H}{M}$ (with M_H standing for the mass of the dual Higgs field) is the logarithm of the Ginzburg–Landau parameter; $L \gg 1$ in the London limit.

Note that, in general, while the London limit implies an *extreme* type-II dual superconductor, a stable quantum vacuum containing closed dual Abrikosov–Nielsen–Olesen strings is only possible for the type-II dual superconductor. Indeed, away from the London limit, in addition to the repulsive force, which is caused by the gauge-boson interactions, an attractive force, caused by the Higgs-boson interactions, starts acting onto two same-oriented strings. For sufficiently large separations d between the strings, the strengths of both interactions fall off exponentially, at $d = 1/M$ and $d = 1/M_H$, respectively, [11,14]. Once the Bogomolny limit [15] of $M = M_H$ has been reached, the two forces completely cancel each other, while further in the type-I dual superconductor (i.e., for $M > M_H$), the interaction is getting attractive, thereby destroying the grand canonical ensemble of closed strings. Another argument in favor of the type-II dual superconductor is Casimir scaling, which was reproduced on the lattice for $\frac{M_H}{M} = 5 \div 9$ [16] and proved analytically in the London limit [17]. Still, lattice data for the chromo-electric field of an open flux tube in the Yang–Mills theory can be fitted by Equation (1) with Mr replaced by $\sqrt{(Mr)^2 + (M/M_H)^2}$, with this correction getting sizeable in the type-I dual superconductor [18]. Nevertheless, as discussed, e.g., in Ref. [19], this fit only works if the values of $\frac{M_H}{M}$ vary with the quark–antiquark separation, while the only parameter M can be sufficient to fit the same lattice data for any quark–antiquark separation. Moreover, even if the mentioned fit of Ref. [18] can, in some sense, be regarded as a good approximation to the lattice data, it does not appear as a solution to the Ginzburg–Landau equations, which only provide stable vortex-type solutions in the type-II superconductors or in the Bogomolny limit [11–13,15].

In the one-loop effective action of a created light quark, whose imaginary part yields the pair-production rate, the electric field (1) appears in the form [2,3]

$$\int \mathrm{d}^2 r \exp\left(\frac{ig}{2} E(r) \varepsilon_{ij} \Sigma_{ij}\right) = \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{ig}{2} \varepsilon_{ij} \Sigma_{ij}\right)^k \left(\frac{M^2}{g_m}\right)^k \int \mathrm{d}^2 r \, (K_0(Mr))^k. \tag{2}$$

In this expression, $\Sigma_{ij} \equiv \int_0^s d\tau \dot{x}_i x_j$, where s is the Schwinger proper time of the quark, and indices i and j take the values 3 and 4, for a flux tube directed along the 3rd axis. For the latter integral, one has:

$$\int \mathrm{d}^2 r \, (K_0(Mr))^k \simeq \int_{r < \frac{2}{M}} \mathrm{d}^2 r \left(\ln \frac{2}{Mr}\right)^k = \frac{\pi}{M^2} 2^{2-k} k!.$$

The thus occurring cancellation of the factor $k!$ yields for Equation (2) a geometric series, $\frac{4\pi/M^2}{1-\frac{ig^2M^2}{8\pi}\varepsilon_{ij}\Sigma_{ij}}$, which can be represented in the form $\frac{4\pi}{M^2}\int_0^\infty \mathrm{d}t\, \mathrm{e}^{-t\left(1-\frac{ig^2M^2}{8\pi}\varepsilon_{ij}\Sigma_{ij}\right)}$. This representation introduces an effective t-dependent electric field $\mathcal{E} \equiv \frac{tgM^2}{4\pi}$, leading further to the following pair-production rate, which indeed turns out to be exponentially falling off with m:

$$w \simeq \frac{N_c N_{\mathrm{f}}}{2\pi^3}\left(\frac{\sigma_0}{L}\right)^2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \int_0^\infty \mathrm{d}t\, t^2\, \mathrm{e}^{-t-\frac{\pi L m^2 n}{\sigma_0 t}} =$$

$$= \frac{N_c N_{\mathrm{f}}}{\pi^{3/2}} \sqrt{\frac{\sigma_0}{L}}\, m^3 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}} K_3\left(2m\sqrt{\frac{\pi L n}{\sigma_0}}\right). \quad (3)$$

Furthermore, the comparison of $\mathcal{E}$ with $\langle E \rangle$ suggests the following definition of a fluctuating t-dependent string tension: $g\mathcal{E} = 4\sigma/L$, so that we get $t = 4\sigma/\sigma_0$. Together with Equation (3), this expression yields for the string tension the following distribution function:

$$p(\sigma) \sim \sigma^2\, \mathrm{e}^{-4\sigma/\sigma_0}. \quad (4)$$

This way, the London limit of the dual superconductor provides an explicit realization of the idea of a fluctuating string tension, yielding a concrete exponentially falling off distribution function $p(\sigma)$. In the next section, we will see that this distribution of the string tension can be associated with a change in the Yang–Mills vacuum structure.

2. Yang–Mills Instantons in the London Limit of the Dual Superconductor

Let us now consider the Yang–Mills vacuum consisting not only of the confining medium, emerging through the condensation of magnetic monopoles, but also of the Yang–Mills instantons, which are known to be important for chiral symmetry breaking in QCD [20]. Such a superposition of the monopole- and the instanton-based models of the vacuum was studied, e.g., in Ref. [21]. We will be interested in the distribution of large-size instantons in the London limit of the dual superconductor, by modeling the effects produced on instantons by the monopole background with the aid of the above-discussed fluctuating string tension. To this end, we will use an expression for the differential instanton density, $\mathrm{d}n$, in terms of the instanton effective action S_{eff} in the stochastic confining background [22]: $\frac{\mathrm{d}n}{\mathrm{d}^4z\,\mathrm{d}\rho} \propto \frac{1}{\rho^5}\,\mathrm{e}^{-S_{\mathrm{eff}}}$, where z is the coordinate of the instanton center, and ρ is the instanton size. The effective action S_{eff} is given by the sum of the perturbative and the non-perturbative parts, with the confining background appearing in terms of the string tension and the vacuum correlation length λ, where the latter is equal to $1/M$ in the London limit of the dual superconductor. The perturbative part of the effective action, initially given just by the expression for the classical instanton action, albeit with the one-loop Yang–Mills running coupling g defined at the instanton size, i.e., $S_{\mathrm{pert}} = \frac{8\pi^2}{g^2(\rho)} = b\,\ln\frac{1}{\rho\Lambda}$ (where $b = \frac{11}{3}N_c$, and $\Lambda \simeq 200\,\mathrm{MeV}$ in the Pauli–Villars regularization scheme), gets modified by the effects of the confining background, which appear through the infra-red "freezing" of the Yang–Mills coupling, as $S_{\mathrm{pert}} = \frac{b}{2}\ln\frac{1/\rho^2+c_1\sigma_0}{\Lambda^2}$, where $c_1 = \mathcal{O}(1)$ is some positive constant. The non-perturbative part of the effective action, for instanton sizes $\rho \gtrsim \lambda$ of interest, is dominated by the color-diamagnetic interaction of the instanton field with the confining background, and has the form [22] $S_{\mathrm{non-pert}} = c_2\sigma_0\rho^3/\lambda$, where $c_2 = \mathcal{O}(1)$ is some other positive constant. Replacing, in these expressions, σ_0 by the fluctuating string tension σ, and setting $N_c = 3$, we obtain for the differential instanton density:

$$\frac{\mathrm{d}n}{\mathrm{d}^4z\,\mathrm{d}\rho} \propto \frac{\Lambda^{11}}{\rho^5} \cdot \int_0^\infty \mathrm{d}\sigma\, \frac{p(\sigma)}{\left(\frac{1}{\rho^2}+c_1\sigma\right)^{11/2}}\, \mathrm{e}^{-c_2\sigma\rho^3/\lambda}.$$

We further use for $p(\sigma)$ a normalized distribution function, which is given by Equation (4) divided by the normalization integral $\int_0^\infty d\sigma\,\sigma^2\,e^{-4\sigma/\sigma_0} = \sigma_0^3/32$. Introducing a dimensionless integration variable, $x = \sigma\rho^3/\lambda$, we have

$$\frac{dn}{d^4z\,d\rho} \propto \frac{\Lambda^{11}\lambda^3}{\sigma_0^3} \cdot \frac{1}{\rho^3} \cdot \int_0^\infty dx\, \frac{x^2}{\left(1 + c_1\frac{\lambda}{\rho}\,x\right)^{11/2}}\, e^{-x\left(\frac{4\lambda}{\sigma_0\rho^3}+c_2\right)}.$$

Here, the term $c_1\frac{\lambda}{\rho}\,x$ is non-negligible in comparison with 1 only for $x \gtrsim \frac{\rho}{c_1\lambda}$, i.e., x's for which the integrand is already exponentially suppressed. Hence, neglecting this term in comparison with 1, we obtain in the large-ρ limit of interest:

$$\frac{dn}{d^4z\,d\rho} \propto \frac{\Lambda^{11}\lambda^3}{\sigma_0^3} \cdot \frac{1}{\rho^3} \quad \text{at} \quad \rho \gtrsim (\lambda/\sigma_0)^{1/3}.$$

Note that the scale $(\lambda/\sigma_0)^{1/3}$ exceeds the vacuum correlation length λ by a factor of $(\sigma_0\lambda^2)^{-1/3}$, where the parametric smallness $\sigma_0\lambda^2 \ll 1$ ensures the convergence of the cumulant expansion in the Yang–Mills vacuum [23]. Integrating the differential instanton density $\frac{dn}{d^4z\,d\rho}$ over the positions of instanton centers z, we obtain the coefficient D entering Equation (A1) from Appendix A, $D \propto \left(\frac{N}{V}\right)^{-1}\frac{\Lambda^{11}\lambda^3}{\sigma_0^3}$, so that Equation (A2) from Appendix A finally yields instantons' contribution to the string tension:

$$\sigma_{\text{inst}} \propto \frac{\Lambda^{11}\lambda^3}{\sigma_0^3}.$$

Finally, noticing that $\Lambda = \mathcal{O}(\sqrt{\sigma_0})$, we have $\frac{\sigma_{\text{inst}}}{\sigma_0} \propto (\sigma_0\lambda^2)^{3/2} \ll 1$, which determines the parametric smallness of the obtained instantons' contribution to the total string tension, $\sigma_{\text{tot}} = \sigma_0 + \sigma_{\text{inst}}$.

3. Summary

In this paper, we considered a model of the QCD vacuum consisting of a confining medium formed by magnetic monopoles and a chiral-symmetry-breaking medium formed by the Yang–Mills instantons. In the London limit of the dual superconductor, monopoles produce only an UV-divergent contribution to the gluon condensate, which should thus be subtracted alongside with other perturbative contributions [24,25]. Accordingly, due to the proportionality which is expected to take place between the chiral and the gluon condensates [26,27], monopoles do not produce non-perturbative contributions to the chiral condensate, either. That makes the considered monopole–instanton vacuum model a good approximation for the description of both confinement (produced primarily by monopoles) and the spontaneous chiral symmetry breaking (produced entirely by instantons) in QCD [21]. We modeled the interaction of instantons with the dual-superconductor background by using instanton's effective action in the stochastic confining field [22], where the respective string tension was considered fluctuating, with the distribution corresponding to the London limit of the dual superconductor. Averaging the differential instanton density with respect to the fluctuating string tension, we obtained the $1/\rho^3$-distribution for instantons of large sizes ρ. Such a distribution means that the ensemble of instantons is becoming confining, yielding a contribution to the string tension. Even though that contribution turns out to be parametrically small in comparison with the standard string tension, we find the so-emerging confining property of instantons, induced by their interaction with monopoles, remarkable in itself.

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Appendix A. Heavy-Quark Potential for the $1/\rho^3$-Distribution of Large-Size Instantons

For review-type purposes, we present here a brief summary of the main results of Ref. [28]. The qualitative argument as to why the instanton distribution

$$\nu(\rho) = \frac{D}{\rho^3}, \text{ with constant } D \text{ at } \rho \to \infty, \tag{A1}$$

leads to the linearly rising large-distance heavy-quark potential, is based on the evaluation of the renormalization ΔM_Q of the heavy-quark mass M_Q due to instantons. As ΔM_Q is proportional to the instanton density, $\Delta M_Q \propto \frac{N}{V}$, it should be proportional to the mean cube of the instanton size, $\Delta M_Q \sim \frac{N}{V}\langle\rho^3\rangle$, where $\langle\rho^3\rangle = \int_0^\infty \mathrm{d}\rho\rho^3\nu(\rho)$, thereby becoming linearly divergent. Accordingly, if one additionally considers an antiquark at a large distance R from the quark, the divergent integral is getting cut off at $\rho_{\max} = R$, and one arrives at a linear potential: $V_{Q\bar{Q}} \sim \frac{N}{V} DR$ at $R \to \infty$.

Quantitatively, by calculating the quark–antiquark P-exponent in the instanton field, one obtains, e.g., in the simplest SU(2)-case [28,29]:

$$V_{Q\bar{Q}} = \frac{4\pi}{d(r)} \frac{N}{V} \int_0^\infty \mathrm{d}\rho\rho^3\nu(\rho) \sum_{J\in r}(2J+1)F_J(x),$$

where

$$F_J(x) = \int_0^\infty \mathrm{d}y y^2 \int_0^1 \mathrm{d}t \left(1 - \cos\phi_+ \cos\phi_- + \frac{x^2 - y^2}{\sqrt{(x^2+y^2)^2 - (2xyt)^2}} \sin\phi_+ \sin\phi_-\right)$$

and

$$\phi_\pm = 2\pi\sqrt{\frac{J(J+1)}{3}}\left(\sqrt{\frac{x^2+y^2\pm 2xyt}{x^2+y^2\pm 2xyt+1}} - 1\right).$$

In these formulae, $x = R/(2\rho)$, $y = |\mathbf{z}|/\rho$, with $\mathbf{z}$ being the shortest 3D vector between the center of an instanton and the $Q\bar{Q}$ line, and t denotes the cosine of the angle between $\mathbf{z}$ and $\mathbf{R}$. Furthermore, $d(r) = \sum_{J\in r}(2J+1)$ is the dimension of representation r, under which Q and $\bar{Q}$ transform.

The numerical analysis shows that the function $F_J(x)$ behaves as $\mathcal{O}(x^2)$ at $x \lesssim 1$, while going over to a J-dependent constant at $x \gg 1$ and interpolating smoothly between these two limiting behaviors at $x \sim 1$. For $\nu(\rho) = D/\rho^3$, one thus obtains the linear potential

$$V_{Q\bar{Q}} = \frac{2\pi D}{d(r)} \frac{N}{V} \cdot R \cdot \sum_{J\in r}(2J+1)C(J), \tag{A2}$$

with the convergent integral $C(J) = \int_0^\infty \mathrm{d}\left(\frac{1}{x}\right)F_J(x)$ acquiring its main contribution at $x \simeq 1$, i.e., at $\rho \simeq R/2$, which is by a factor of $1 \div 2$ larger than the average instanton size in the ensemble. The full instanton distribution $\nu(\rho)$ (as opposed to its large-ρ limit (A1)) was argued to be peaked around approximately the same value, $\rho_0 \simeq 0.44$ fm (cf. Ref. [28]). The known instantons' contributions to the chiral condensate and to the constituent quark mass are dominated by the instanton sizes $\rho \simeq \rho_0$, being therefore insensitive to the particular form of the large-ρ asymptotic behavior of $\nu(\rho)$.

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