The Critical Behavior of QED in any dimension

HAJIME NAKATANI

Department of Physics, Nagoya University,
Nagoya 464-01, Japan

ABSTRACT

We consider the critical behavior of QED in 3 $\sim$ 6 dimensional space-time. We obtain the chiral-symmetry-breaking solutions of the Schwinger-Dyson equation in d-dimensional QED in the quenched ladder approximation and show that for $d > 4$ the sclaing law is the mean-field type. We also study QED_3 beyond the quenched ladder approximation and show that the scaling law is dependent of the value of the infrared cutoff.

In this report, we consider the critical behavior of the d-dimensional Quantum Electromagnenic Dynamics (QED_d).[1~3] Especially we study QED_d where d is not equal to 4*. In QED_4, it is well-known that in the quenched ladder approximation, the scaling behavior is the singurality-type or so called "Miransky scaling".[4] Is this scaling common irrespective of the space-time dimensions? Is this scaling also correct even if there is vacuum polarization included? Here we try to answer these questions in the framework of the Schwinger-Dyson equation.

* For QED_4, see Kondo's reprot in this proceedings.

First, we consier the scaling law of the dynamical mass and the coupling in QED_d in the quenched ladder approximation, that is, the vertex $\Gamma_\mu(p,k)$ is bare and the vacuum polarization function $\Pi(k)$ is zero;

$$\Gamma_\mu(p,k) = \gamma_\mu \qquad \text{and} \qquad \Pi(k) = 0 \ . \tag{1}$$

We write the fermion propagator $S(p)$ as

$$S(p)^{-1} = A(p^2)\, \not{p} + B(p^2) \ . \tag{2}$$

Then the Schwinger-Dyson equation for $A(p^2)$ and $B(p^2)$ is written by

$$A(x) = 1 + g_d \int_\epsilon^{\Lambda^2} dy \frac{y^{(d-2)/2} A(y)}{yA^2(y) + B^2(y)} L_d(x,y) \tag{3}$$

$$B(x) = g_d \int_\epsilon^{\Lambda^2} dy \frac{y^{(d-2)/2} A(y)}{yA^2(y) + B^2(y)} K_d(x,y) \tag{4}$$

where $x := p^2, y := k^2$, ϵ is the infrared cutoff, Λ the ultraviolet cutoff and

$$g_d^2 := \frac{e^2}{2^d \pi^{(d+1)/2} \Gamma(\frac{d-1}{2})} \ . \tag{5}$$

$L_d(x,y)$ and $K_d(x,y)$ are kernels. As proved in ref.[2], in quenched planar approximationan and in the Landau gauge $L_d(x,y)$ is simply zero so that the wave renormalization function $A(x) = 1$. Note that this is consisitent with the Ward-Takahashi identity. $K_d(x,y)$'s in $3 \sim 6$ dimensions are the following,

$$K_3(x,y) = \frac{2}{\sqrt{xy}} \ln \frac{\sqrt{x} + \sqrt{y}}{|\sqrt{x} - \sqrt{y}|} \ , \tag{6}$$

$$K_4(x,y) = \frac{3\pi}{x + y + |x - y|} \ , \tag{7}$$

$$K_5(x,y) = \frac{2(x+y)}{xy} - \frac{(x-y)^2}{(xy)^{3/2}} \ln \frac{\sqrt{x} + \sqrt{y}}{|\sqrt{x} - \sqrt{y}|} \ , \tag{8}$$

$$K_6(x,y) = \frac{5\pi}{8}\left[\frac{3x-y}{x^2}\theta(x-y) + \theta(y-x)\right] . \tag{9}$$

We solve numerically the above SD equation with kernels (6) $\sim$ (9) to obtain the scaling laws against the dimensionless coupling β_d,

$$\beta_d := \frac{\Lambda^{4-d}}{(d-1)g_d^2} . \tag{10}$$

The dynamical mass can be writen using β_d as

$$m = \Lambda f(\beta_d) , \tag{11}$$

where $f(\beta_d)$ is defined as the scaling function.

The numerical results show the following scaling functions,

$$f_3(\beta_d) \propto \frac{1}{\beta_d} , \tag{12}$$

$$f_4(\beta_d) \propto \exp(-\frac{\pi}{\sqrt{\beta_d/\beta_d^c - 1}}) , \tag{13}$$

$$f_5(\beta_d) \propto \sqrt{\beta_d^c - \beta_d} , \tag{14}$$

$$f_6(\beta_d) \propto \sqrt{\beta_d^c - \beta_d} . \tag{15}$$

where β_d^c is the critical coupling. In QED_4, the scaling law is singularity type. On the other hand, in QED_3 there is no phase transition, that means that only symmery-breaking phase survives. This is also confirmed by the analytical solution in the bifurcation method. And in $QED_d(d > 4)$ the scaling law is the mean-field type. Although we have no analytical proof, in any higher dimensions than 4 the scaling would be the mean-field type. So these results imply that the singularity-type scaling is rather spacial one in QED, and to confirm the scaling type we should study more details in QED beyond the quenched ladder approximation.

As one of examples beyond the quenched ladder approximation, we consider the vacuum polarization effect in QED_3.

QED_3 is an interesting model which is superrenormalizable and have a similarity with QED_4. And also it seems to be related to the recent study for high T_c superconductor, the quark confinement and so on. Furthermore, the calculability of the angular integration in SD equation without any approximation makes easy to analyze the flavor dependence of the model.

So far many pepole has been discussing this model, in the framework of the Schwinger-Dyson(SD) equation combined with the $1/N$ expansion for the vacuum polarization.[5~8] There are two claims about the question, whether or not there exists the critical point of the fermion number N_c in QED_3. Appelquist, Nash and Wijewardhana[6] (ANW) pointed out that there exists the finite critical point, $N_c = 32/\pi^2$, using Appelquist et al.'s assumption[5] that the wave function renormalization would be negligible in the large N limit. Matsuki et al.[9] also obtained the same result from the viewpoint of the effective potential. The existence of the critical point is also supported by the Monte Carlo(MC) calculation by Dagotto et al.[10] On the other hand, Pennington and Webb[7] (PW) and Atkinson, Jhonson and Pennington[8] (AJP) claimed that if one takes into account the $1/N$ correction to the wave-function renormalization, the critical point N_c in the infinite cutoff limit goes away to infinity against ANW's result. This means that only the symmetry-breaking-phase survives in QED_3.

Generally, in QED the wave-function renormalization is unavoidable if the vacuum polarization in photon propagator is included. This is in sharp contrast with the quenched planar QED in the Landau gauge.[2] In fact, the one-loop correction to the photon propagator leads to the non-trivial wave-function renormalization even in the Landau gauge. Therefore Appelquist et al.'s assumption is not justified *a priori*, if the effect of the fermion loop is included.

We solve the SD gap equation in QED_3 combined with the $1/N$ expansion for the vacuum polarization without using the Appelquist et al.'s assumption.[5,6] Actual calculation have been done with the approximately equivalent differntial equation.[3] We consider the leading correction in the $1/N$ expansion, i.e. the one-loop correction in the photon propagator for massless fermion,[11]

$$\Pi(p) = \frac{\tilde{\alpha}}{p}\,, \tag{16}$$

where

$$\tilde{\alpha} := \frac{e^2 N}{8}\,. \tag{17}$$

The SD equation for the fermion propagator in Landau gauge is written by

$$\begin{aligned} A(p) &= 1-\frac{\tilde{\alpha}}{\pi^2 N}\frac{1}{p^3}\int_{\epsilon}^{\infty} dk \frac{kA(k)G(p^2,k^2)}{k^2A(k)^2+B(k)^2} \\ &\quad\times\Bigg[\tilde{\alpha}^2 \ln\frac{p+k+\tilde{\alpha}}{|p-k|+\tilde{\alpha}} - \tilde{\alpha}(p+k-|p-k|)+2pk \\ &\quad -\frac{1}{\tilde{\alpha}}|p^2-k^2|(p+k-|p-k|) \\ &\quad -\frac{1}{\tilde{\alpha}^2}(p^2-k^2)^2\{\ln\frac{p+k+\tilde{\alpha}}{|p-k|+\tilde{\alpha}} - \ln\frac{p+k}{|p-k|}\}\Bigg]\,, \end{aligned} \tag{18}$$

$$B(p) = \frac{4\tilde{\alpha}}{\pi^2 N}\frac{1}{p}\int_{\epsilon}^{\infty} dk \frac{kB(k)G(p^2,k^2)}{k^2A(k)^2+B(k)^2}\ln\frac{p+k+\tilde{\alpha}}{|p-k|+\tilde{\alpha}}\,. \tag{19}$$

We have paid special attention to the critical value N_c of the fermion flavor and the scaling law in the neighborhood of N_c. We showed that the scaling behavior of the dynamical mass is restricted by the inequality and discussed the relation between the scaling law and the "generalized vertex ansatz",

$$\Gamma_\mu(p,k) = \gamma_\mu A(k)^n\,. \tag{20}$$

Our numerical results show that the scaling law depends on the infrared cutoff. Actually in the limit of the infrared cutoff $\epsilon \to 0$, we have three types of the scaling law depending on the vertex ansatz, i.e. the exponential type, the essential-singularity type and the power-law type,

$$f(N) \propto \exp(-CN)\,, \qquad \text{for } n < 2\,, \tag{21}$$

$$f(N) \propto \exp(-2\pi/\sqrt{N_c/N - 1}) , \qquad \text{for } n = 2 , \tag{22}$$

$$f(N) \propto (N_c - N)^\lambda \qquad \text{for } n > 2 . \tag{23}$$

We can give an explanation on this difference based on the concept of the effective coupling. The result of the exponential type would be the physical one and agrees with the previous result obtained by PW and AJP. Recent works by Atkinson, Johnson and Maris[14] proposed the $n = 2$ case as the physical one from the analysis of the anomalous dimensions.

On the other hand, in the presence of the finite infrared cutoff, the scaling obeys the mean-field type independent of the vertex ansatz,

$$f(N) \propto (N_c - N)^{1/2} , \tag{24}$$

and N_c has a finite value which depends on the infrared cutoff. According to MC results, there exists a finite critical value for fermion flavor. It is, however, still an open question what the scaling type really is in QED_3. It should be remarked that infrared cutoff introduced in our framework may correspond to the lattice size in MC simulation, while the ultraviolet cutoff corresponds to the lattice spacing. It appears that our framework provides us with a possibility, which enables us to explain apparently conflicting results based on the SD equation[7,8] and the MC simulation.[10] It is quite interesting that resent MC simulation by DESY group[13] is fit with the analysis of the mena-field method.

Our investigation have been restiricted in the Landau gauge and quenched ladder apporximation or atmost including the one-loop correction in the vacuum polarigation. Beyond these restriction we plan to perform the numerical calculation of the SD equation beyond one-loop corrction[12] to the vacuum plarization including the improvement of the vertex.[15]

REFERENCES

1. K.-I.Kondo and H.Nakatani, Nagoya University preprint, DPNU-89-04, to be published in Mod. Phys. Lett. A.
2. K.-I.Kondo and H.Nakatani, Mod. Phys. Lett. A,4(1989) 2155.
3. K.-I.Kondo and H.Nakatani, Nagoya University preprint, DPNU-89-36.
4. V.A.Miransky, Nuovo Cimento 90A (1985) 149
5. T.Appelquist, M.Bowick, D.Karabali and L.C.R.Wijewardhana, Phys. Rev. D33, (1986) 3704.
6. T.Appelquist, D.Nash and L.C.R.Wijewardhana, Phys. Rev. Lett. 60, (1988) 2575.
7. M.R.Pennington and S.P.Webb, BNL preprint-40886 (January,1988).
8. D.Atkinson, P.W.Johnson and M.R.Pennington, BNL preprint-41615 (August,1988).
9. T.Matsuki, L.Miao and K.S.Viswanathan, Simon Fraser Univ. preprint, June 1987(revised version: May 1988).
10. E.Dagotto, J.B.Kogut and A.Kocić, Phys. Rev. Lett. 62, (1989) 1083; ILL-(TH)-89-#40, 1989.
11. R.Pisarski, Phys. Rev. D29, (1984) 2423
12. For the higher order correction, see D.Nash, Phys. Rev. Lett. 62, (1989) 3024.
13. M.Göckeler, R.Horsley, E.Laermann, P.Rakow, G.Schierholz, R.Sommer, and U.-J.Wiese, DESY 89-124/HLRZ 89-69, October 1989.
14. D.Atkinson, P.W.Johnson and P.Maris, private communication.
15. K.-I. Kondo, H.Nakatani, in preparation