Gravitational lensing by wormholes in binary systems

S. Pietroni

Dipartimento di Fisica "E.R. Caianiello", Università di Salerno, Via Giovanni Paolo II 132, I-84084 Fisciano, Italy

Istituto Nazionale di Fisica Nucleare, Sezione di Napoli, Via Cintia, 80126, Napoli, Italy

E-mail: spietroni@unisa.it

We investigate binary lenses with $1/r^n$ potentials in the asymmetric case with two lenses with different indexes n and m. These kinds of potentials have been widely used in several contexts, ranging from galaxies with halos described by different power laws to lensing by wormholes or exotic matter.

Keywords: Gravitational lensing; black holes; wormholes; galaxies

1. Introduction

Space-time is curved by the presence of massive bodies and this curvature influences the motion of the bodies themselves: this leads to a geometry in constant evolution. One of the consequences is that even light, supposed to be massless, bends its trajectory while passing close to a massive body.[1]

Gravitational lensing is an important tool in astrophysics and in cosmology widely used to study both populations of compact objects (including exoplanets, black holes and other stellar remnants) , and extended objects, such as galaxies, clusters of galaxies and large-scale structures . Since most of the mysteries of our Universe do not show up in observations based on electromagnetic interactions, gravitational lensing is more and more employed to study the dark side of the Universe, including dark matter, dark energy, and any kind of exotic matter (such as wormholes) conjectured by theorists.[2]

2. Lensing by Wormholes

The metric of the Ellis wormhole falls down asymptotically as $1/r^2$ and its deflection angle goes as the inverse square of the impact parameter $1/u^2$.

Metrics falling as $1/r^n$ were investigated also by Kitamura et al.[3] who found out that the deflection angle falls down with the same exponent as the metric, $\hat{\alpha} \sim 1/u^n$ with $n > 1$, and that a demagnification of the total lensed images could appear at $\beta > \frac{2}{n+1}$ (in units of θ_E, the Einstein radius, and under a large-n approximation, β is the source position). This demagnification effect may be evidence of an Ellis wormhole and it might be used for hunting the search for exotic matter. Particular attention was posed on the study of caustics of $1/r^n$ binary lenses by Bozza and Melchiorre in Ref.[4]

The implications for the energy-momentum tensor supporting this kind of metrics falling as $1/r^n$ were investigated by Bozza and Postiglione,[5] we remind that $0 < n < 1$ may describe galactic halos, $n > 1$ would be the signature of a violation of the weak energy condition and so the existence of exotic matter, $n = 2$ correspond to the Ellis wormhole; n is the ratio between tangential and radial pressure, $n = -2p_t/p_r$.

3. The Lens Equation, Critical Curves and Caustics

In a binary system composed by two lenses (binary lenses, A and B), the lens equation can be written as follows

$$\vec{\beta} = \vec{\theta} - \frac{\vec{\theta} - \vec{\theta_A}}{|\vec{\theta} - \vec{\theta_A}|^{n+1}} - \gamma^{m+1} \frac{\vec{\theta} - \vec{\theta_B}}{|\vec{\theta} - \vec{\theta_B}|^{m+1}}, \tag{1}$$

where $\gamma = \theta_{E,B}/\theta_{E,A}$ is the “strength ratio”.

The lens equation allows us to find images, given the source position and a lens configuration.

The number of images formed at a given source β depends on the source position and these regions with a different number of images are delimited by caustics.

The condition $J(z) = 0$ (the Jacobian determinant of the lens map) defines the critical curves on the lens plane; and by applying the lens map on critical points we find the corresponding points on the source plane, which form the caustics. When a source crosses a caustic, a new pair of images is created on the corresponding point in the critical curve.

4. Cases and Topology Regimes

We investigated 3 cases: the equal-strength binary with $\gamma = 1$, the unequal-strength binary with $\gamma = \sqrt{0.1}$ (the bigger lens is the standard one with fixed $n = 1$), and the reversed unequal-strength binary with $\gamma = \sqrt{0.1}$ (the standard lens is the smaller one).

For the standard binary Schwarzschild lens in the equal-strength case, we know that three topologies exist:

- close separation, for $s < s_{CI}$;
- intermediate separation, for $s_{CI} < s < s_{IW}$;
- wide separation, for $s > s_{IW}$;

and the two transitions are $s_{CI} = 1$ and $s_{IW} = 2\sqrt{2}$ in our units.

The three topologies exist for any value of m and n.

Our model contains 4 parameters: the indexes of the two potentials n, m, the separation between the two lenses s, and the ratio of the two Einstein radii γ.

5. Critical Curves and Caustics: Wide, Intermediate and Close Separations

Here we show the equal-strength case with $\gamma = 1$, the full research can be found in: *Bozza V., Pietroni S., Melchiorre C., Universe 2020, 6(8), 106*.[6]

The red curve is for the standard Schwarzschild case $n = m = 1$, we keep $n = 1$ for the first lens and we see what happens when m varies in the second lens

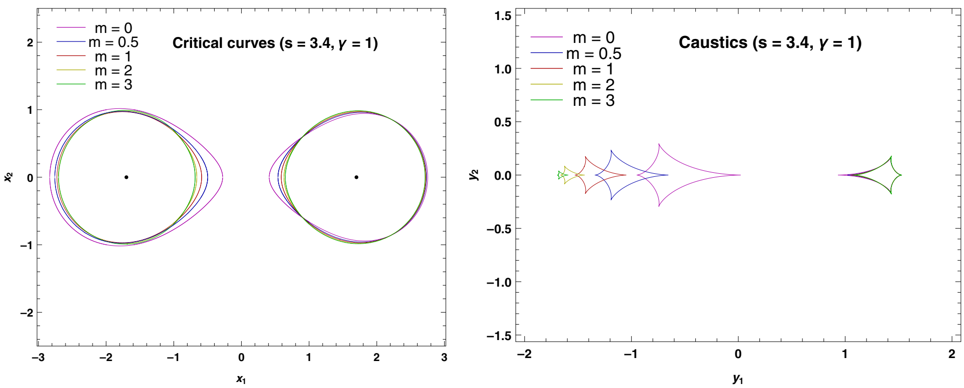

Fig. 1. Critical curves and caustics in the equal-strength binary, wide separation with fixed $n = 1$ and variable m: $m = 0$ is the singular isothermal sphere already investigated by Shin and Evans, $m = 0.5$ is the galactic halo, $m = 2$ is the Ellis wormhole and $m = 3$ is for exotic matter.

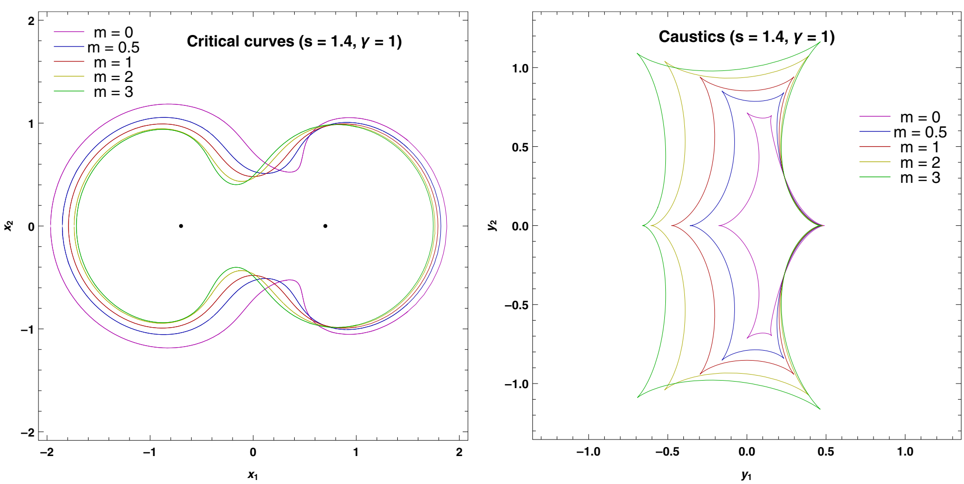

Fig. 2. Critical curves and caustics in the equal-strength binary, intermediate separation with fixed $n = 1$ and variable m. The red curve is for the standard Schwarzschild case $n = m = 1$.

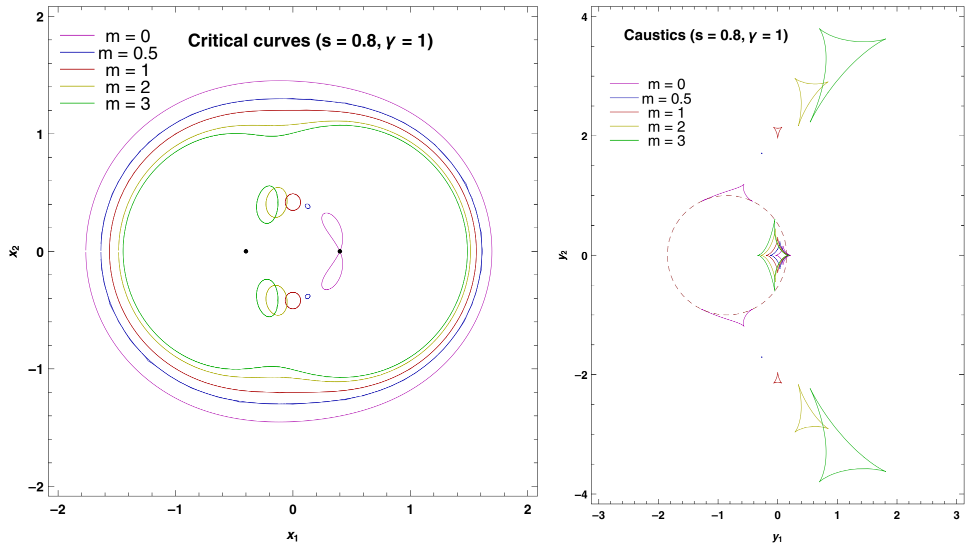

Fig. 3. Critical curves and caustics in the equal-strength binary, close separation with fixed $n = 1$ and variable m. The red curve is for the standard Schwarzschild case $n = m = 1$. Dashed magenta circle indicates the pseudocaustic for $m = 0$.

6. The Elliptic Umbilic and the Extremely Unequal-Strength Ratio Limit

In the range $0 \leq m < 1$ an *elliptic umbilic catastrophe* exists in the close separation. The value of s at which the catastrophe happens is

$$s_{euc} = \left(\frac{1-mn}{m+1}\right)^{\frac{1}{n+1}} \sqrt{1 + \frac{\gamma^2 (m+1)^{\frac{2}{n+1}}}{(n+1)^{\frac{2}{m+1}} (1-mn)^{\frac{2(m-n)}{(m+1)(n+1)}}}}. \qquad (2)$$

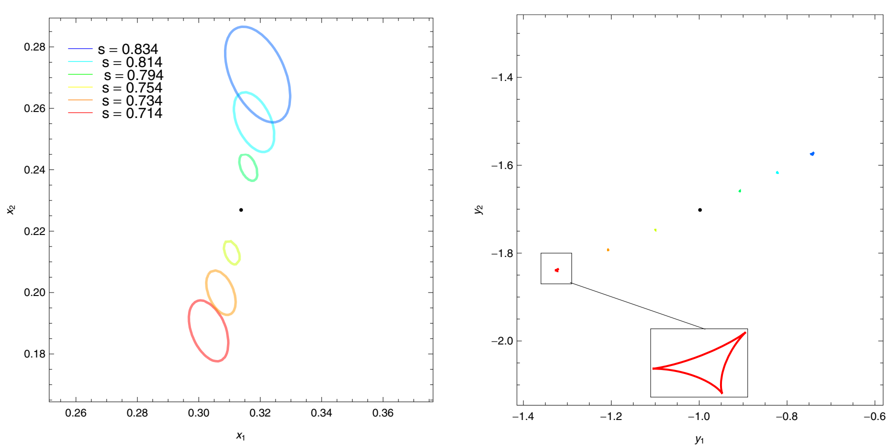

Fig. 4. The elliptic umbilic catastrophe for $n = 2$, $m = 0.25$. The separation at which the catastrophe occurs is $s_{euc} = 0.774$.

For any n, m, γ we found the boundaries for s_{CI}, given only numerically, and the analytical expression for s_{IW}.

The caustic evolution in the extreme limit $\theta_{E,B} << \theta_{E,A}$, in the case of two Schwarzschild objects ($n = m = 1$), is the so-called "planetary" limit. For the caustics of the perturbing object in the wide case we have an extension of the caustics in the parallel and in the vertical direction:

$$\Delta\zeta_{||,wide} = 2(n+1)\frac{\gamma}{s^{\frac{m(n+1)}{m+1}}\left(s^{n+1}-1\right)^{\frac{1}{m+1}}} \tag{3}$$

$$\Delta\zeta_{\perp,wide} = 2(n+1)\frac{\gamma}{s^{\frac{m(n+1)}{m+1}}\left(s^{n+1}+n\right)^{\frac{1}{m+1}}}. \tag{4}$$

7. Conclusions

These mixed binary lenses are important from the astrophysical point of view in the investigation of pairs of galaxies with different halos ($n, m < 1$), in the opening of a new channel in the search for wormholes when they appear in a non-isolated environment ($n = 1$, $m = 2$), in the case in which one object is made up of exotic matter and the other one is a normal star ($n \geq 2$, $m = 1$).

References

1. Einstein, *Entwurf einer verallgemeinerten Relativitätstheorie und eine Theorie der Gravitation* (Leipzig, Berlin: Teubner, 1913).
2. V. Bozza, *Gravitational Lensing by Black Holes* (Gen. Rel. and Grav., 2010).
3. K. A. H. Kitamura, T.; Nakajima, *Demagnifying gravitational lenses toward hunting a clue of exotic matter and energyr* (Phys. Rev. D, 2013).
4. C. M. Bozza, V., *Gravitational Lensing by Black Holes Caustics of* $1/r^n$ *binary gravitational lenses: from galactic haloes to exotic matter* (JCAP, 2016).
5. P. A. Bozza, V., *Gravitational Lensing by Black Holes Alternatives to Schwarzschild in the weak field limit of General Relativity.* (JCAP, 2015).
6. M. C. Bozza V., Pietroni S., *Caustics in gravitational lensing by mixed binary systems* (Universe, 2020).