UNPHYSICAL RIEMANN SHEETS

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There has been considerable interest of late in the analyticity properties of the scattering amplitude on unphysical sheets of the energy Riemann surface. This analyticity has been used in discussions of unstable particles and the resulting scattering resonances,[1] and anomalous thresholds [2, 3, 4]. A general approach to the analytic properties on the second Riemann sheet will be discussed first. Then some practical applications of these properties will be presented.

1. THE UNPHYSICAL SHEETS

This work was carried out in collaboration with M. Goldberger, S. MacDowell, and S. Trieman. Let us first consider the case of individual partial waves, because the application of unitarity is so simple in this case. We will further restrict our attention to the scattering of scalar particles of mass μ. It is a " simple " matter to extend the discussion to more interesting cases.

It is possible to discuss the many-channel problem by utilizing the matrix formulation of Bjorken [5]. We will discuss the case of only one channel, but most of the equations we will develop are true in the many channel case if they are looked upon as matrix equations.

The analytic properties of the partial wave amplitudes have been well discussed [6]. The result is that the function

$$f_l^{(1)}(\nu+i\varepsilon)=\left(\frac{\nu+\mu^2}{\nu}\right)^{\frac{1}{2}}e^{i\delta_l}\sin\delta_l=e^{i\delta_l}\sin\delta_l/\rho(\nu)$$

is an analytic function of ν (the square of the relative momentum) with a cut along the entire positive real axis and along the negative real axis from $(-\infty)$ to $(-M^2/4)$, where $1/M$ is the range of the effective potential. The superscript one is to emphasize that this equation is defined on the physical sheet.

Below the onset of inelastic channels, the phase shift is real along the physical cut and the unitarity relation takes the usual form. We next remark that by trivial manipulation,

$$f_l^{(1)}(\nu-i\varepsilon)=f_l^{(1)}(\nu+i\varepsilon)[1+2i\rho f_l^{(1)}(\nu+i\varepsilon)]^{-1}$$

$$=f_l^{(1)}(\nu+i\varepsilon)S_l^{-1}(\nu+i\varepsilon)\,,$$

where S_l is the S-matrix. The scattering amplitude on the second sheet is introduced as the continuation across the positive axis in a counter-clockwise direction :

$$f_l^{(2)}(\nu+i\varepsilon)=f_l^{(1)}(\nu-i\varepsilon)=f_l^{(1)}(\nu+i\varepsilon)S_l^{-1}(\nu+i\varepsilon)\,.$$

It is immediately obvious that $f_l^{(2)}(\nu)$ has the same analyticity region as $f_l^{(1)}(\nu)$ plus any additional poles coming from the zeroes of the S-matrix, and the kinematic cuts coming from ρ. The l^{th} partial cross section is easily seen to enjoy analyticity in the ν-plane cut along the negative axis. The same statement holds for the functions $\rho\,\mathrm{Im} f_l$ and $\mathrm{Re} f_l$ An interesting fact, which can be readily demonstrated, is that the $\mathrm{Re} f_l$ has exactly one-half the residue that f_l has at its negative dynamic singularities.

Let us now see whether or not S_l has zeroes close to the physical region. The complex zeroes of S_l which might lead to scattering resonances are strongly dependent upon dynamics and hence are difficult to discuss. We will show that in general, S_l has at least one zero between $\nu=0$ and $(-M^2/4)$ for every other l. If there is no zero energy resonance or anomalous threshold the S-matrix is unity at zero kinetic energy. As ν approaches $(-M^2/4)$, the singular part of a Yukawa-type Born term approaches

$$f_l^{(1)}(\nu) \sim -\frac{\lambda}{2\nu} P_l\left(1+\frac{M^2}{2\nu}\right) \ln\left(1+\frac{4\nu}{M^2}\right)$$
$$\sim -\frac{2\lambda}{M^2} P_l(-1) \ln(\infty) .$$

Thus if l is odd, the Born term has the sign of the potential, λ, and approaches infinity. On the other hand, if l is even, the sign is reversed. Since f_l is bounded for $(-M^2/4)<\nu<0$ if there is no bound state, then the function $S_l = 1+2i\rho f_l^{(1)}$ must have at least one zero for every other l in this range of ν.

One final remark is worth making. If one attempts to continue to the third Riemann sheet by avoiding the negative cuts associated with the points $\nu = -\mu^2$, and $-M^2/4$, then $f^{(3)} = f^{(1)}$. Thus there are only two sheets associated with the elastic part of the physical cut.

Let us now turn to a discussion of the scattering amplitude without expanding in partial waves.

In order to discuss the analyticity of the scattering amplitude at fixed angle we will assume that a Mandelstam representation holds in the physical sheet. Thus, we write

$$F^{(1)}(\nu, z) = \frac{1}{\pi}\int_{M^2}^{\infty} \frac{dt' A_3(t', \nu)}{t'+2\nu(1-z)} + \frac{1}{\pi}\int_{M^2}^{\infty} \frac{du' A_2(u', \nu)}{u'+2\nu(1+z)} .$$

The only property of the weight functions A_2 and A_3 that is needed, is that they are analytic functions of ν, with a cut along the positive real axis. Subtractions will not affect our discussion, and are therefore suppressed. The unitarity relation is

$$\mathrm{Im}\, F^{(1)}(\nu, z) = \rho \int \frac{d\Omega'}{4\pi} F^{(1)}(\nu-i\varepsilon, x) F^{(2)}(\nu+i\varepsilon, z') ,$$

where $x = zz'-[(1-z^2)(1-z'^2)]^{\frac{1}{2}} \cos\phi'$. In the same manner as before, the scattering amplitude on the second sheet is introduced as

$$F^{(2)}(\nu, z) = F^{(1)}(\nu, z) - 2i\rho \int \frac{d\Omega'}{4\pi} F^{(1)}(\nu, x) F^{(2)}(\nu, z') .$$

It is a simple matter to transform this into a non-singular integral equation of the form

$$F^{(2)}(\nu, z) = F^{(1)}(\nu, z) - 2i \int_{-1}^{+1} dz' K(z, z'; \nu) F^{(2)}(\nu, z') ,$$

where

$$K(z, z'; \nu) = \frac{\rho}{4\pi} \int d\phi' F^{(1)}(\nu, x) .$$

This azimuthal integration is readily carried out and the result is

$$K(z, z'; \nu) = \frac{\rho}{4\pi} \int dt' A_3(t', \nu) \left[(1+\frac{t'}{2\nu}-zz')^2 - (1-z^2)(1-z'^2) \right]^{-\frac{1}{2}}$$
$$+\frac{\rho}{4\pi} \int du' A_2(u', \nu) \left[(1+\frac{u'}{2\nu}+zz')^2 - (1-z^2)(1-z'^2) \right]^{-\frac{1}{2}} .$$

The Fredholm solution to this integral equation can be readily examined. Since the regions of integrations are finite, it follows from standard arguments that the analyticity domain of $F^{(2)}(\nu,z)$ for physical z is at least as large as that of $F^{(1)}(\nu,z')$ for all physical z'. In addition, however, there is the possibility of zeroes of the Fredholm determinant $D(\nu)$, and the kinematic branch point $\nu = -\mu^2$ coming from the phase space factor $\rho(\nu)$. The poles due to the vanishing of $D(\nu)$ arise from the same source as in the partial wave case. By using properties of the eigenfunctions of the kernel, K, it can be shown that

$$D(\nu) = \prod_l (1+2i\rho f_l^{(1)}(\nu)) .$$

When the preceding discussion is carried out for fixed momentum transfer $t<0$, it is found that $F^{(2)}(\nu,t)$ has complex singularities in ν. The source of these singularities is to be found in the kernel K. If one iterates the equation for $F^{(2)}$, the first iteration does not have complex singularities but the higher order iterations do.

From these equations it is possible to discuss the analyticity in z for fixed complex ν. In particular, we are interested in the possibility of making a partial wave expansion of $F^{(2)}(\nu,z)$. If the nearest singularities in z are complex or real and a finite distance outside the interval $(-1, 1)$, it is possible to pass an ellipse through these points enclosing the physical region. Then an expansion in a Legendre series is possible within this region.

From the expression for the kernel $K(z, z'; \nu)$ it is seen that for fixed ν, there is analyticity in z except when

$$z^2 + z'^2 + (1+\frac{t'}{2\nu})^2 \pm 2zz'(1+\frac{t'}{2\nu}) = 1\,.$$

The condition that this singularity lies in the interval $(-1, 1)$ is that $\nu \leq -\mu^2/4$. Thus as long as ν is *not* on the negative cut, a Legendre expansion is possible for $F^{(2)}$. Similar results hold for $\operatorname{Im} F(\nu, z)$ as well.

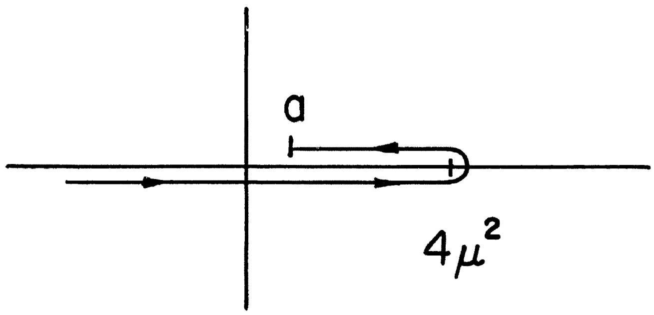

Fig. 2 Anomalous threshold behaviour.

2. ANOMALOUS THRESHOLDS

As a first application of the previous results, consider the form factor for a scalar particle of mass M_a which interacts with a scalar photon through a pair of particles of mass μ as illustrated in Fig. 1. For this discussion it will prove convenient to follow the procedure developed by Mandelstam [2] instead of the equivalent method described by Blankenbecler and Nambu [3].

In these methods, one is forced to continue certain functions to the second Riemann sheet, and it is this aspect of the problem which is of interest here. Following Frazer and Fulco [7], the form factor in the normal case (M small) can be written in the form

$$F(s) = \frac{1}{\pi}\int_{4\mu^2}^{\infty} ds' \frac{A(s')}{s'-s}$$

$$A(s) = \rho(s)\exp\left[\Delta^* + \Delta\right] \times \int_{-\infty}^{a} dt \frac{e^{-\Delta(t)}\alpha(t)}{t-s}\,.$$

Here $\alpha(t)$ is the discontinuity across the negative cut in the partial wave amplitude for the annihilation process and $\Delta(s)$ is the usual line integral over the relevant phase shift of μ-μ scattering. If one now continues in the external mass M_a, by giving it a small negative imaginary part, the point a moves in a path illustrated in Fig. 2. The line integral from $(4\mu^2)$ to (∞) in $F(s)$ must be deformed to avoid this protruding branch cut. In order to perform these continuations one must write $e\Delta^* = e^{\Delta} S^{-1}(s)$ and also continue the factor $\exp\left[-\Delta(t)\right]$ onto its second sheet as M_a reaches its anomalous value, or a reaches $4\mu^2$. The result after this continuation is

$$A(s+i\varepsilon) = \rho(s) e^{2\Delta(s)} S^{-1}(s) \left[\int_{-\infty}^{4\mu^2 - i\eta} dt \frac{e^{-\Delta(t)}\alpha(t)}{t-s} - \int_{4\mu^2}^{a-i\eta} dt \frac{e^{-\Delta(t)}\alpha(t)}{t-s} S(t+i\eta) \right].$$

One might superficially expect that when the anomalous threshold a reaches the point where S has a zero, the continuation would break down in a manner that could not be discerned from perturbation theory. This is *not* the case, since from the integral over $\alpha(t)$, a factor of S miraculously appears to cancel any such pole. The final result is that

$$F(s) = \frac{1}{\pi}\int_{a}^{\infty} ds' \frac{B(s')}{s'-s}\,,$$

where

$$B(s) = 2\pi\alpha(s)(i\rho)\exp\left[\Delta(s)\right] \quad \text{for} \quad a < s < 4\mu^2\,,$$

and for $s > 4\mu^2$,

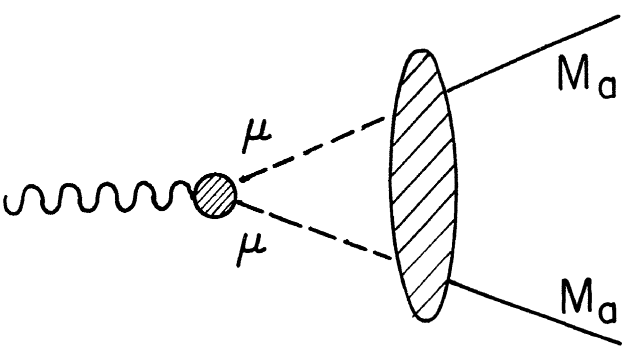

Fig. 1 Form factor graph.

$$B(s)=\rho(s)\exp\left[\Delta^{*}+\Delta\right]\times\left\{\int_{-\infty}^{a} dt\frac{e^{-\Delta(t)}\alpha(t)}{t-s}+\int_{a}^{4\mu^2}\frac{dte^{-\Delta(t)}\alpha(t)}{t-s}(1+S(t)\right\}.$$

The effect of a narrow low energy resonance in μ-μ scattering on the absorptive part of the form factor is illustrated in Fig. 3. For $\alpha(t)$, the first order Born contribution with masses corresponding to the Λ intermediate state in the Σ form factor has been assumed. The solid line is the lowest order contribution ($\Delta=0$) and the dotted line includes the rescattering. R. Marr, L. Landovitz and myself are looking into the effect of a pion-pion resonance on the perturbation theoretic calculations of the vector anomalous moment of the Σ particle. The effect does not seem to be negligible.

Results similar to the anomalous form factor discussion hold also for the scattering situation depicted in Fig. 4. The absorptive part of the scattering matrix G has the form

$$\operatorname{Im} G_l(s)=\rho H_b{}^{*}(s)H_a(s)\,,$$

where

$$H_{[a,b]}(s)=e^{\Delta(s)}\int_{-\infty}^{[a,b]} dt\frac{e^{-\Delta(t)}}{t-s}\left[\alpha(t),\beta(t)\right].$$

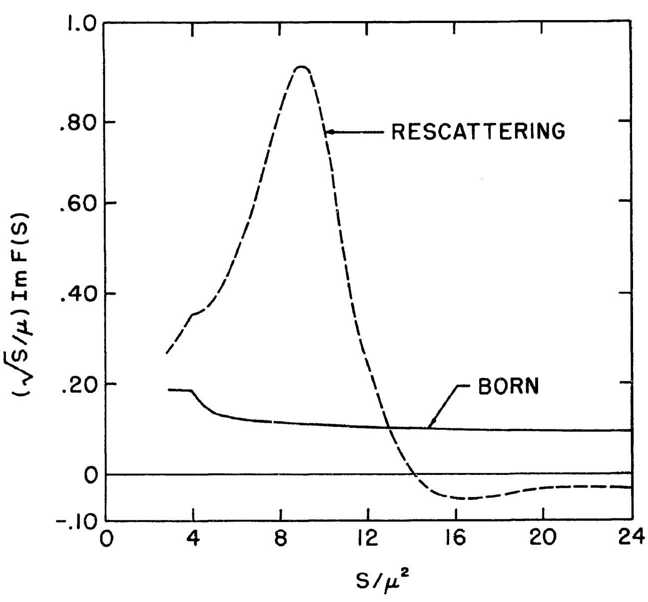

Fig. 3 Imaginary part of form factor.

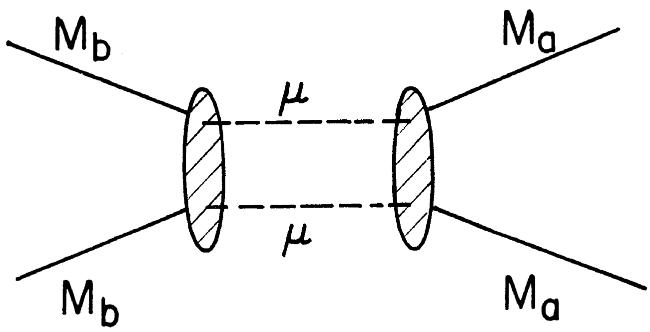

Fig. 4 Scattering graph.

if M_a and M_b are small enough. Let us first consider the case where only M_a is large enough for an anomalous threshold. By standard procedures the scattering amplitude G is found to be

$$G_l(s)=\frac{1}{\pi}\int_{a}^{\infty} ds'\frac{I(s')}{s'-s}\,,$$

where for $a<s<4\mu^2$, the imaginary part of G is

$$I(s)=2\pi\alpha(s)(i\rho)H_b(s)\,.$$

Now if the mass M_b is increased until $b=a$, which can obviously occur even if $H_b(s)$ has a normal threshold, then G becomes complex. The condition $b=a$ is just the condition found in perturbation theory by Karplus, Sommerfield and Wichmann[8] for the "super"-anomalous case. The essential point here for our purposes is that the superficially dangerous factor of S^{-1} cancels.

3. BOUND STATES

Another interesting application of the analytic properties of the scattering amplitude on the second sheet is to be found in the problem of bound states. One may entertain the question of whether the Mandelstam program is complete enough to yield the masses and "coupling constants" of bound states, or whether one must put them in by hand. A physical example is to be found in nucleon-nucleon scattering in the triplet S state. If one applies the standard N/D procedure without explicitly putting in the deuteron pole, then it is reasonable to expect that D will develop a zero at the appropriate energy. This can even be rigorously demonstrated in the case of potential scattering. However a new problem arises in field theory. For example, the contribution to the physical cut from the $N+P+\pi$ intermediate state must somehow extend its threshold from $(2M+\mu)^2$ to $(M_d+\mu)^2$. By what mechanism is this cut extended? We would like to show that by making very plausible assumptions about the scattering amplitude on the second sheet, the bound state problem can be handled in principle within the Mandelstam program.

As an extreme example consider a Fermi-Yang [9] model of nucleon-nucleon scattering in which both the pions and deuterons are assumed to be bound S states. For example, the fundamental interaction could be considered to be of a four field nature, described by some coupling constant which will be allowed to vary. The process nucleon-nucleon scattering will be labeled process one, with energy s. The crossed processes, nucleon-antinucleon scattering with energies u and t, are labeled two and three. The Mandelstam representation, without subtractions and with spin labels suppressed, has the form

$$G(s,u,t)=\frac{1}{\pi}\int_{4M^2}^{\infty} ds'\,\frac{\operatorname{Im} f_1(s')}{s'-s}+\frac{1}{\pi}\int_{4M^2}^{\infty} du'\,\frac{\operatorname{Im} f_2(u')}{u'-u}+$$

$$+\frac{1}{\pi}\int_{4M^2}^{\infty} dt'\,\frac{\operatorname{Im} f_3(t')}{t'-t}+G'(s,u,t)\ ,$$

where G' contains the double dispersion integrals, subtracted in such a way that the absorptive part of the S wave in all three channels is given by the single dispersion integrals alone. As we have seen, these absorptive parts are given by

$$\operatorname{Im} f_i(x)=\rho(x)f_i^{(1)}(x)f_i^{(1)}(x)[1+2i\rho(x)f_i^{(1)}(x)]^{-1}$$

in the elastic region.

The essential point is to recall now that if there are no bound states and the effective potential is attractive, then $S(x)$ has a zero for x between zero and $4M^2$. As the coupling constant increases, this point would seem to move towards $4M^2$. Guided by what does occur in potential scattering, we will assume that this zero moves in a path illustrated in Fig. 5, and that the scattering amplitude is an analytic function of the position of this zero. Thus when the zero

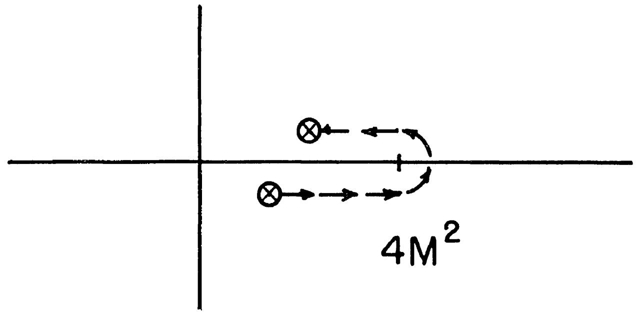

Fig. 5 Path of the zero of S.

passes around the point $4M^2$, the line integrals over x must be deformed to avoid this pole in $\operatorname{Im} f_i(x)$. This deformed path can be shrunk to a small circle about the pole plus the contribution from $4M^2$ to infinity. The small circles yield poles in the scattering amplitude of the form

$$\frac{\Gamma^2}{s-M_d^2}+\frac{g^2}{u-\mu^2}+\frac{g^2}{t-\mu^2}\,,$$

where the pole in $\operatorname{Im} f_1$ has been placed at M_d^2 and those in $\operatorname{Im} f_{2,3}$ have been placed at μ^2. This is the mechanism by which the poles move from the second to the first sheet of the scattering amplitude as true bound states are formed.

We have not yet achieved the correct representation if μ^2 is less than M^2, since there must be cuts in u and t beginning at $4\mu^2$. A qualitative understanding of the origin of these cuts can be achieved by examining the four nucleon intermediate state depicted in Fig. 6. The absorptive part can be considered as a sum over the angular momentum of each nucleon pair. From this double sum we will restrict our attention to the configuration where both of the pairs are in relative S states. It is clear that when rescattering is taken into account, the matrix element will have

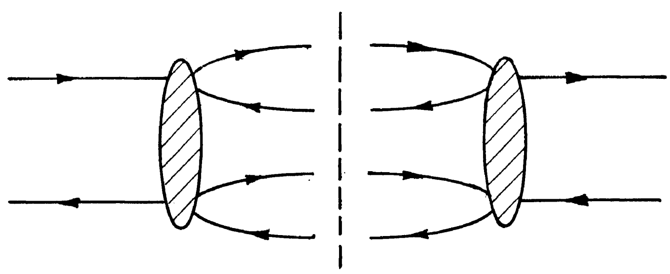

Fig. 6 Four-nucleon contribution to N—$\overline{\text{N}}$ scattering.

factors such as exp $[\Delta(m_i^2)]$, where m_i^2 is the (variable) mass of the i^{th} pair. The absorptive part will be a finite integral over the masses m_i^2 of a function which will contain factors of $S^{-1}(m_i^2)$.

When the strength of the coupling is increased, the integrals over the m_i^2 (which start at $4M^2$) must be deformed to avoid the wandering pole of S^{-1}. The absorptive part then picks up a cut in t starting at $4\mu^2$ and passing through the point $16M^2$ before reversing direction. The dispersion integral over the absorptive part starts at $16M^2$ and it must be deformed to avoid this protruding out. Finally, one achieves a dispersion integral over t with a threshold at $4\mu^2$. This type of continuation is highly reminiscent of the anomalous threshold case.

In discussing the partial waves for N-N scattering, this type of continuation is necessary in order to get the one pion exchange cuts, and there is no difficulty in reproducing our previous results using the full representation.

We can now see that if the pole of S^{-1} crosses into the physical sheet, it must move as indicated. If it moved a finite distance away from the real axis, then from the higher order inelastic intermediate states complex spikes would develop. These would be in obvious contradiction to the unitarity condition in the physical region. It would be amusing if this type of continuation, which uses unitarity so strongly, could be used to prove dispersion relations for nucleon-nucleon scattering. That is, one would not continue to an imaginary nucleon mass but instead put the pion mass on the second sheet.

It thus seems clear that if the S-matrix has the required behavior as a function of the coupling strength, the bound state problem is solvable in terms of the double dispersion relations and unitarity.

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DISCUSSION

EDEN : I would like to ask whether the assumption that the zero of the S-matrix moves around into the upper sheet has been proved for potential scattering.

BLANKENBECLER : Yes.

OPPENHEIMER : Are the zeros of high l from virtual states protected by the centrifugal barrier?

BLANKENBECLER : Probably, but the exact connection is not clear.

OEHME : I would like to add a remark on the continuation into the first unphysical sheet of a partial wave amplitude. It is a simple consequence of the unitarity condition that the branch line from the threshold of the elastic process to the threshold of the first inelastic process is a cut which connects just two Riemann sheets. So, if one wants, one can write a dispersion relation in both sheets and eliminate completely the integral over the elastic cut. Instead, one gets contributions from the second sheet. These are the pole terms, an integral over the left hand cut and an integral over a branch line which is related to inelastic processes.

OPPENHEIMER : For potential scattering Peierls had done this.

OEHME : That is right, but it is true in general. It also holds for the vertex function and the propagator.

BREIT: In some respects, it is trivial but I think not altogether trivial that there is a connection between this and the old Gamow theory of radioactive disintegration which led one to consider the scattering amplitude as a function of a complex variable. I think the remark the chairman made is very close to the situation of potential barriers in the Gamow theory, but one can also follow this out in a more systematic fashion by just considering what happens to the scattering amplitudes as a function of energy. That has been done. If one assumes that the imaginary part is small, then one gets a factorization for the constants which is common in nuclear reaction theory in approximate formulae. Somehow I fail to see in this presentation any evidence of this factorization, although it probably is there. The factorization comes out rather naturally in the other treatment. It follows very readily by bringing in the time.

OPPENHEIMER: I think this was brought out in Taylor's remarks where the energy dependence has been explicitly exhibited for the case of a sharp resonance.

BREIT: I had this same question in my mind then as to whether the factorization was present, and I did not understand whether it was present or not.

J. G. TAYLOR: Yes, it is present here.

BREIT: So somehow it must degenerate into the case which is known without these formal manipulations in the complex plane.

J. G. TAYLOR: It is difficult in this formalism to discuss the time dependence of the amplitude.

BREIT: That's right, but if you do, then the case of a small damping is very easily treated.

NEWTON: First, I want to make a remark about what Oppenheimer said. The zeros in the other sheet of the Riemann surface are not necessarily connected with the virtual states in the sense of a centrifugal barrier.

OPPENHEIMER: The zeros on the real axis for high l?

NEWTON: Yes, but they can occur even for l equals zero.

OPPENHEIMER: Yes, but you will have them even for weak attractive forces for high l.

BLANKENBECLER: That is right.

NEWTON: In the case of potential scattering, if the potential is cut off there must be infinitely many zeros in the wrong sheet, not all on the real axis, of course. I also wanted to ask Taylor a question. I had the impression that when you wrote things down for analytic continuation you acted as though the complex conjugate of the function was analytic. That must be a wrong impression.

J. G. TAYLOR: I think that when you look at the absorptive part as the product of, say, MM^* where M^* is the boundary value on the lower half of the cut, and M is the boundary value on the upper half, then M is the one that you want to continue across the cut in order to go into the unphysical sheet. M^* is automatically continuable into the lower half plane because it is already a boundary value in the lower half plane.

OMNES: I should like to remark that it is possible to prove this property of poles which go from the real axis in the lower sheet to the other sheet by using the integral representation of the Wigner R-functions. In the case of potential scattering or in the Castillejo-Dalitz-Dyson cases, you can show that you have these properties. I have tried to prove it more generally but I did not succeed. Great difficulties arise when you have non-trivial crossing relations. It is an interesting question to know if these properties of poles remain in more general cases.

EDEN: Could I just make a small comment on the question of the Mandelstam representation in the first unphysical sheet? Taylor obtained the result that the representation does not apply because there are complex singularities on the physical sheet. If there are curves of singularities on the boundary of the physical sheet, we know that they do not connect the surfaces which run into the physical sheet. Therefore, the surfaces must go into the first non-physical sheet. The fact that there is a spectral function at all means that you will not have the Mandelstam representation holding in the first unphysical sheet.

J. G. TAYLOR: One might be able to write down a more complicated representation since we have located the singularities in $\cos\theta$ explicitly. It might be possible to write down something which is more complicated but which still may have some value.

It should be a rather compact representation of the singularities in other sheets.

BLANKENBECLER : One might also hope that if these curves of singularities actually lie in the physical sheet, there does not exist a sheet in which a Mandelstam representation holds.

EDEN : On that point, if there are super anomalous thresholds so that the Mandelstam representation does not apply in the physical sheet, then it is possible in fourth order that your remark is correct.

BLANKENBECLER : I mean the generalized sheets in which you have ignored only those parts of the singularities which have wound their way around into the physical sheet.

EDEN : I was going to say that in the higher order diagrams you do not have the same conditions as in fourth order. By removing the anomaly from fourth order, you do not remove the anomaly from sixth order, so that at best you would have to have a very multiply-connected sheet.

PERTURBATION THEORY WITHOUT HAMILTONIAN (*)

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Various model approaches to the theory of elementary particles have so far been extensively discussed. In such an approach one specifies the fundamental fields as well as the fundamental interactions, and then derives the mass levels of certain states which are conventionally referred to as elementary particles.

We shall take an extreme opposite approach to this problem here, namely we do not distinguish between elementary and composite particles, nor do we specify the basic interactions whatsoever. Such a theory may be called a " phenomenological field theory."

This is an S matrix formulation of field theory and shares many features with dispersion theory, nevertheless there are also many important differences between the two approaches. Though the present approach is a kind of S matrix formulation of renormalized field theories, we do not confine ourselves to the S matrix elements on the mass shell and take explicitly those matrix elements which are off the mass shell.

The basic idea of formulating field theories in this manner may be stated in the following way : suppose that the conventional renormalizable field theories offer, at least approximately, the correct description of the properties of fundamental particles; then there should be an alternative approach to such a theory which explicitly avoids the occurrence of divergences in the course of calculation.

This idea would be implemented by exhausting all possible relationships among renormalized finite expressions, and in what follows we try to carry out this program.

For this purpose we postulate two kinds of universal relationships in this formalism, namely the generalized unitarity condition and the parametric dispersion relations for Green's functions. After studying field theories satisfying both conditions we find that what determines the dynamics of such a system are the assumed number of subtractions in the parametric dispersion relations. Through this work it seems to

(*) The essential part of this talk appeared in the author's article, Phys. Rev. **119**, p. 485 (1960).