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RESEARCH ARTICLE

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Bridging geometries and potentials in DBI cosmologies

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Abstract We investigate the link between the warp function and the potential in Dirac-Born-Infeld (DBI) power-law cosmologies. We give a prescription to take advantage of the fact that there is always a choice of potential resulting in a constant ratio between pressure and energy density for a given a warp factor. We illustrate the method with several examples, and complete the investigation by showing how symmetries can be used to generate new DBI solutions from existing ones.

Keywords Inflation, Scalar fields, DBI model

1 Introduction

The idea that the inflaton might be an excitation of a D-brane[1] [1] in the form of an open string has attracted quite a few researchers (see [2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12; 13; 14; 15; 16; 17; 18; 19; 20; 21; 22] and references therein for regular papers and [23] for a review). Specifically, the inflaton could be a mode describing the position of a D3-brane wandering (radially) in a ten-dimensional space–time with a warped metric. Part of the interest of this interpretation of inflation is due to the fact that in these scenarios, inflation can proceed with much steeper potentials than in the standard weakly coupled slow roll inflation model.

The motion of the brane seems to admit an effective good description in terms of a Dirac-Born-Infeld action coupled to gravity [10], and it results in a scalar field theory with non-canonical kinetic terms. The usual assumption is that the metric on the brane is a flat Friedman-Robertson-Walker (FRW) one, and it is usual to

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[1] An extended object to which the end points of open strings are attached, and which represents a non-perturbative feature of some string theory flavors.

take advantage of the perfect fluid interpretation of the energy-momentum tensor. The modifications with respect to canonical scalar field models are related to the speed of the wandering brane, and investigations using inflationary parameters have been carried out to estimate how this non-canonical features in the kinetic terms of the model affect the cosmological observables [8; 9; 10; 11; 12; 13; 14; 15; 16; 17; 18; 19; 20; 24; 25; 26; 27].

Power-law solutions for the background evolution of the Universe have physical motivation; on the one hand, the perturbation equations get considerably simplified, and on the other hand, such evolutions play a preminent role as asymptotic equilibrium states [28]. In brane inflation scenarios, there is one more ingredient of interest, as brane anhiliation seems to provide a natural end to power-law inflation [29; 30; 31; 32]. In the context of DBI inflation they have received some interest [8; 21; 22] and it has been shown that they can give rise to a significant (and thus distinctive) degree of non-gaussianity in the initial conditions of primordial perturbations [19; 21]. Additionally, the equation of state parameter in the corresponding fluid picture of power-law inflation in DBI models may provide hints about high energy features, for instance, in the scenario of [8], it is associated with the mass of the inflaton.

Constructions of DBI power-law models exploit the possibilities offered by the two degrees of freedom available: one is the warp factor of the metric, which is denoted as $f(\phi)$, whereas the other is the inflaton potential $V(\phi)$. One could fix the two available degrees of freedom so as to result in the constant equation of state parameter (pressure to energy density ratio) characterizing power-law cosmological models [22]. In this paper, we provide an algorithm to exploit such correspondence towards generation of DBI solutions. The novelty and relevance of our method is that, as far as we are concerned, no exact (genuinely) DBI solutions exist in the literature, not even power-law ones. Nevertheless, some approximations and a certain degree of compromise will perhaps be required [8; 10; 16; 21; 22] if one is to enforce power-law solutions corresponding to a particular choice of either the warp factor or the potential.

Specifically, we present expressions leading to a constructive recipe for DBI power-law cosmologies which allows accommodating that sort of solutions in, for instance, exponential potential models with adequate warp factors. We then consider other cases, which lead in asymptotic regimes to the AdS throat warp factor [33], or to a generalization of the inverse power-law potential studied in connection with tachyon cosmologies [34].

In the course of the discussion we briefly comment on the conditions for the warp function f to be positive (the opposite case is not consistent from the string theory point of view, but it is admissible in the field theory spirit). Finally, we also discuss about the possibility of generating further solutions from existing ones under the use of symmetries, and how phantom DBI cosmologies could fit into this picture.

2 DBI setting

As mentioned in the Introduction, string theory provides a theoretical framework in which inflation can proceed without having to meet the tight requirement of the potential being rather flat as in conventional inflation models. According to this proposal, cosmic accelerated expansion (or inflation) can be the manifestation of the motion of an extended object (a D3-brane) through a ten-dimensional space–time with a warped metric (see for e.g. [13]):

$$ds_{10}^2 = \frac{1}{\sqrt{f(\phi)}} g_{\mu\nu} dx^\mu dx^\nu + \sqrt{f(\phi)} g_{mn} dy^m dy^n. \tag{1}$$

Here ϕ is a single radial combination of the internal coordinates y^m.

Effectively, our scenario is that of a four-dimensional spatially flat FRW space-time filled with a non-canonical scalar field. Using the customary perfect fluid interpretation we set

$$\rho = \frac{\gamma^2}{1+\gamma}\, \dot{\phi}^2 + V(\phi), \tag{2}$$

$$p = \frac{\gamma}{1+\gamma}\, \dot{\phi}^2 - V(\phi), \tag{3}$$

with

$$\gamma = \frac{1}{\sqrt{1 - f(\phi)\dot{\phi}^2}}, \tag{4}$$

where, in principle, f and V are arbitrary functions. Usage of the symbol γ was originally motivated by its analogy to the Lorentz factor of Special Relativity, given that $\sqrt{f(\phi)}\dot{\phi}$ is interpreted as the proper velocity of the brane [8]. According to this, the scalar field cannot roll down arbitrarily fast as the speed threshold leading to nonanalytic behavior of γ would be trespassed. The slow roll limit of the model corresponds to $f(\phi)\dot{\phi}^2 \ll 1$, and in this nonrelativistic motion regime the familiar expressions for the energy the energy density and pressure of the canonical scalar field are recovered.

Assuming for the above fluid a barotropic equation of state of the form $p = (\Gamma - 1)\rho$, we get

$$\Gamma = -\frac{2\dot{H}}{3H^2} = \frac{\gamma\dot{\phi}^2}{\rho}, \tag{5}$$

and the Einstein equations read

$$3H^2 = \frac{\gamma\dot{\phi}^2}{\Gamma}, \tag{6}$$

$$-2\dot{H} = \rho + p \equiv \gamma\dot{\phi}^2. \tag{7}$$

The last two equations provide all the basic information needed to elaborate our very general description of the realization of power-law inflation in this theoretical framework.

3 Power-law solutions

Using Eqs. (5)–(7), the energy conservation equation can be written as

$$\left(\frac{\gamma\dot{\phi}^2}{\Gamma}\right)^{\cdot} + 3H\gamma\dot{\phi}^2 = 0. \tag{8}$$

For $\Gamma = \Gamma_0$, upon integration we will obtain from Eq. (5) the power-law scale factor $a \propto t^{2/3\Gamma_0}$. This means in this case the conservation equation gets readily integrated resulting in $\gamma\dot{\phi}^2 = c/a^{3\Gamma_0}$, where c is an integration constant which gets fixed upon replacement into the Friedmann equation, so that finally

$$a = t^{2/3\Gamma_0}, \quad \gamma\dot{\phi}^2 = \frac{4}{3\Gamma_0 t^2}. \tag{9}$$

The idea behind the procedure for reconstructing the warp factor f and the potential V is giving the scalar field as an invertible function of time, $\phi = \phi(t)$, so that $t = t(\phi)$ can be obtained. Obviously, one can also compute the derivative of the field with respect to time and then reexpress it in terms of the field, $\dot{\phi}(t) = \dot{\phi}(t(\phi))$. This means that, if we are able to express both f and V in terms of time and the derivative of the field, then we will have solved the problem of finding f and V as functions of ϕ.

From Eq. (4) one can calculate the function f

$$f = \frac{1}{\dot{\phi}^2}\left[1 - \frac{9\Gamma_0^2}{16}t^4\dot{\phi}^4\right], \tag{10}$$

whereas, in the case $\Gamma = \Gamma_0$, and from Eqs. (2), (5) and (10), one can easily solve for the potential:

$$V = \frac{4}{3\Gamma_0 t^2}\left[\frac{1}{\Gamma_0} - \frac{4}{4 + 3\Gamma_0 t^2\dot{\phi}^2}\right]. \tag{11}$$

Once again, the known result that for any throat geometry there is a potential which leads to power-law inflation for some range of parameters has become manifest. This may be viewed as a generalization to the well-known facts that for $f = 0$ (canonical scalar field models) power-law inflation is possible if the potential is exponential [35] or the analogous result for the case $f = 1$ (tachyon cosmologies) in which such kind of inflation is obtained with inverse square potentials [36; 37; 38].

The novelty here is that we find analytical correspondences between the warp factor and the potential for power-law cases, without needing to resort to the ultra-relativistic regime simplification. Once a given time dependence of ϕ is chosen, the geometry of the warped metric can be recovered, the existence of inflation depends on the value of Γ_0 entering f as a free parameter, and only values of Γ_0 giving a large enough f will lead to inflationary behavior.

On the other hand, if one is to stick to an interpretation based on a string theory setting, there will be a restriction in the sign of the warp factor f, as it could be non strictly positive for arbitrary choices of ϕ. This would be problematic because the warped metric depends on f through its square root, so in the case of a negative warp factor the entries of the metric would become imaginary and one would need to consider some sort of analytic continuation, but we are not aware of results showing how one can proceed in this direction.

There is however, the possibility to consider DBI actions from a field theory point of view, in which case a negative f does not represent a problem. This is perfectly consistent and work along these lines has been carried out in [39], but one could argue the interest of providing a string theory interpretation of this negative f models.

Having made these remarks, let us return to the main line we pursue here. In the next section we are going to exploit the above presented algorithm to elaborate on three examples.

3.1 Example 1

Let us begin with the simple choice leading to the exponential (or Liouville) potential. It stems from the input

$$\phi = \frac{2}{A} \ln |t|. \tag{12}$$

with A a constant. Incidentally, it also leads to an f expressed in terms of a single exponential. We find it also convenient to rearrange the parameters so that the relation with the canonical scalar field becomes clearer. From the second bit of Eq. (9) we find the relation $A^2 = 3\Gamma_0\gamma_0$, with γ_0 a constant. Then, combining Eqs. (10), (11) and (12) we get f and V; explicitly:

$$f = \frac{A^2}{4}\left[1 - \frac{1}{\gamma_0^2}\right] e^{A\phi}, \tag{13}$$

$$V = \frac{4\gamma_0^2}{A^2}\left[\frac{3}{A^2} - \frac{1}{1+\gamma_0}\right] e^{-A\phi}. \tag{14}$$

Finally, the scale factor gets reexpressed in the following fashion:

$$a = t^{2\gamma_0/A^2}. \tag{15}$$

This way of formulating the solution is very interesting, as in the $\gamma_0 = 1$ limit, that is, for $f = 0$, the expressions just above go over to their simplified form for the canonical scalar field. This is concordant with the novel way of writing the energy density and pressure of the DBI fluid we have put forward: the new parametrizations (2, 3) recover their conventional scalar field form for $\gamma = \gamma_0 = 1$. Note that, as in the $f = 0$ limit a blow-up of the warped metric occurs, the aboved discussed string theory inspired interpretation is not admitted in such case; but the field theory interpretation remains of course valid.

3.2 Example 2

This example we consider now corresponds to

$$\phi = \phi_0 t^n, \quad \phi_0 = cons, \tag{16}$$

and it follows that

$$\gamma = \frac{4}{3\Gamma_0 n^2 \phi^2}. \tag{17}$$

Once more, on using the latter one gets,

$$f = \frac{\phi^{2/n-2}}{n^2 \phi_0^{2/n}} \left[1 - \frac{9\Gamma_0^2 n^4}{16}\phi^4\right], \tag{18}$$

and

$$V = \frac{4\phi_0^{2/n}}{3\Gamma_0 \phi^{2/n}} \left[\frac{1}{\Gamma_0} - \frac{4}{4+3\Gamma_0 n^2\phi^2}\right]. \tag{19}$$

In the large ϕ regime f goes like a negative power of ϕ for $-1<n<0$. Actually, the large ϕ regime for this example is subject to the interpretational restrictions mentioned above as it characterized by $f<0$, and so the scheme is only valid in the field theory interpretation. In contrast, in the small ϕ regime f goes like a negative power of ϕ for $n>1$. Interestingly, the AdS throat often explored in the literature, i.e. $f \sim \phi^{-4}$, can be realized in our model either in a large ϕ regime if $n=-1/3$, or in a small ϕ regime if we rather consider $n=-1$.

The asymptotic behavior of the potential with respect to the scalar field ϕ is simpler, as both in the large and small ϕ regimes we have $V \sim \phi^{-2/n}$.

3.3 Example 3

We now start off from the choice

$$\phi = \frac{2}{A}\ln|B+t^n|. \tag{20}$$

with A a constant. Some straightforward steps involving use of Eqs. (10,11) allow obtaining f and V as functions of ϕ. Our field choice leads to

$$f = \frac{A^2 e^{A\phi}}{4n^2\left(e^{\frac{A\phi}{2}}-B\right)^{2-\frac{2}{n}}}\left(1-\frac{9n^4\Gamma_0^2}{A^4 e^{2A\phi}}\left(e^{\frac{A\phi}{2}}-B\right)^4\right) \tag{21}$$

and

$$V = \frac{4}{3\Gamma_0}\left(\frac{1}{\Gamma_0}-\frac{A^2e^{A\phi}}{e^{A\phi}A^2+3\left(B-e^{\frac{A\phi}{2}}\right)^2 n^2\Gamma_0}\right)\times\left(e^{\frac{A\phi}{2}}-B\right)^{-\frac{2}{n}} \tag{22}$$

If ϕ is non-negative, then strict positiveness of the warp factor is guaranteed if
$3n^2\Gamma_0$
$\max(1,B^2) < A^2$.

3.3.1 Case $B = 1$

For this particular case, and in the regime $1 \gg t^n$, one approximately has

$$f \sim \phi^{-2+2/n}. \tag{23}$$

$$V \sim \phi^{-2/n}, \tag{24}$$

which on the other hand are the same asymptotic expressions one has for the above example in its small field regime; this is consistent, as the $\phi(t)$ expressions of the last two examples coincide at first order in the low field ($1 \gg t^n$) regime.

If we make in this regime the extra restriction that $n \geq 1$ in either Example 2 or 3, then the form of the potential becomes approximately of inverse square type. It can, thus, be viewed as a generalization of the $1/\phi^2$ for k-essence and tachyon cosmologies (Figs. 1, 2).

3.3.2 Case $B \neq 1$

For an arbitrary model the speed limit $\dot{\phi}^2 \leq f^{-1}(\phi)$ applies. In the $B = 1$ cases with $n > 1$, this restriction makes it impossible for the scalar field to reach the origin in finite time. This problem is absent, however, from their $B \neq 1$ counterparts. This is advantageous in connection with reheating, as provided the potential has a minimum at the origin, then the field will be able to oscillate around it and and reheating will proceed. One can check that the necessary condition for the potentials discussed in this example to have a minimum at that location is

$$\Gamma = \frac{2A^2}{A\left(A \pm \sqrt{12(B-1)^2 B n^3 + A^2}\right) - 6(B-1)^2 n^2} \tag{25}$$

Of course, the power-law solutions one can obtain with the warp factor and potential presented in this example are not a representation of this oscillatory behavior, we just want to bring about some nice properties of this model which make it interesting beyond its mere ability to accommodate power-law expansionary behavior.

4 Duality

The interest of form-invariance transformations as a method to obtain new exact solutions to the Einstein equations from already existing ones has been shown before [40; 41; 42; 43]. In the context of spatially flat perfect fluid FRW cosmologies, such transformations can be viewed as a prescription relating the quantities a, H, ρ and p in a given initial scenario to quantities $\bar{a}$, $\bar{H}$, $\bar{\rho}$ and $\bar{p}$ corresponding to a new cosmological model. As our investigation of power-law DBI cosmologies are concerned, there is a class of form-invariance transformations which stands out, it is the one

Fig. 1 Warp factor corresponding to Example 3 with $\Gamma_0 = 0.1$, $A = 3$, and respectively $n = 3$ and $B = 0.2$ (*upper figure*) and $n = 4$ and $B = 0.7$ and $n = 4$ (*lower figure*)

Fig. 2 Warp factor corresponding to Example 3 with $\Gamma_0 = 0.9$, $A = 3$, and respectively $n = -1$ and $B = 1$

given by $H \to \bar{H} = \eta H$, $\rho \to \bar{\rho} = \eta^2 \rho$, $p + \rho \to \bar{p} + \bar{\rho} = \eta(p + \rho)$ for an arbitrary real constant η. As will be shown below it allows generating new power-law DBI cosmologies from existing ones, and if we particularize it further to the case $\eta = -1$ we can consider it as a duality transformation, which provides a method for phantomization (the process of transforming a conventional cosmological model into a phantom one by performing a form-invariance transformation). Back to the general constant η case, we would like to stress that our method provides the means to generate inflationary power-law DBI cosmologies from non-inflationary ones.

Let us review the method from the very general perspective of a flat FRW spacetime filled with a perfect fluid. The Einstein equations read

$$3H^2 = \rho, \tag{26}$$

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{27}$$

where ρ is the energy density, p the pressure and $H = \dot{a}/a$ are invariant form under the symmetry transformations

$$\bar{\rho} = \bar{\rho}(\rho) \tag{28}$$

$$\bar{H} = \left(\frac{\bar{\rho}}{\rho}\right)^{1/2} H \tag{29}$$

$$\bar{p} = -\bar{\rho} + \left(\frac{\rho}{\bar{\rho}}\right)^{1/2} (\rho + p)\frac{d\bar{\rho}}{d\rho}. \tag{30}$$

Here $\bar{\rho} = \bar{\rho}(\rho)$ is assumed to be an invertible function. The above result allows to conclude the FRW equations for a perfect fluid have a form-invariance symmetry. The symmetry transformations (28)–(30) define a continuous Lie group which can be used to solve the FRW equations and get accelerated expansion scenarios, as will be seen below.

The symmetries we will exploit are of three different kinds: those with $|\eta| > 1$ make the energy density of the universe bigger, those with $\eta = \pm 1$ do not alter it, and those with $|\eta| < 1$ do decrease. According with this taxonomy, and assuming the seed cosmological model is an expanding one ($H > 0$), the cosmological model produced by the transformation will also be an expanding one if $\eta > 0$, but on the contrary will be a contracting one for $\eta < 0$. We call dual symmetry to the special transformation obtained with $\eta = -1$, and it ensures the existence of a duality between a contracting universe filled with an ordinary fluid and an expanding universe driven by phantom energy.

Now, it is interesting to investigate the transformation properties of the relevant physical parameters under the symmetry transformations (28)–(30). For instance, the deceleration parameter $q(t) = -H^{-2}(\ddot{a}/a)$, transforms as

$$\bar{q} + 1 = \left(\frac{\rho}{\bar{\rho}}\right)^{3/2} \frac{d\bar{\rho}}{d\rho}(q + 1). \tag{31}$$

In addition, if we consider perfect fluids with equations of state $p = (\Gamma - 1)\rho$ and $\bar{p} = \left(\bar{\Gamma} - 1\right)\bar{\rho}$ respectively, then the barotropic indices Γ and $\bar{\Gamma}$ transform as

$$\bar{\Gamma} = \left(\frac{\rho}{\bar{\rho}}\right)^{3/2} \frac{d\bar{\rho}}{d\rho} \Gamma. \tag{32}$$

Besides, using (31) and (32) we get a form invariant relation $(\bar{q}+1)/\bar{\Gamma} = (q+1)/\Gamma$ between the deceleration parameter and the barotropic index. As these results are readily applicable to any flat FRW power-law cosmological model (regardless of the theoretical framework it fits in), they can be indeed exploited in connection with the DBI cosmologies considered in the previous sections.

In general, inflationary solutions occur when $\ddot{a} > 0$; this means that the expansion is dominated by a gravitationally repulsive stress that violates the strong energy condition, so that $\rho + 3p < 0$. Imposing this condition on (31) we obtain $d\bar{\rho}^{(-1/2)}/d\rho^{-1/2} < 1/(q+1)$, which for a non-accelerated cosmological model with $q \approx \text{const} > 0$, gives $\bar{\rho} > (q+1)^2\rho$. Such model, with $\bar{q} < 0$, is accelerated. This can be understood in terms of assisted inflation, as one achieves inflation by enhancing the energy density of the field. Recall we are considering the possibility of having DBI cosmologies displaying inflation, it should be clear that our transformations can be used to generate new combination of geometry (throat) and potential with inflation starting perhaps from others without that sort of expansionary behavior, so the solution generation ability gets significantly enlarged.

So far we have progressed in a formal and rather general way, but it is convenient to provide further insight into the details of the transformation in the case we are concerned with. In order to avoid that the energy density and pressure turn into complex quantities we are going to impose the condition that the function γ remains invariant a given duality transformation; this implies the product $f\dot{\phi}^2$ must transform into itself. It turns out that the form of the corresponding Einstein equations does not change under the application of the following transformation:

$$\dot{\bar{\phi}}^2 = \eta\dot{\phi}^2 \tag{33}$$

$$\bar{f} = \eta f \tag{34}$$

$$\bar{V} = (\eta^2 - \eta)\rho + V = (\eta^2 - \eta)\frac{\gamma^2}{1+\gamma}\dot{\phi}^2 + \eta^2 V \tag{35}$$

Consistently, it follows that the form-invariance of Eqs. (7) and (26) under the latter transformation requires $\Gamma = \Gamma/\eta$, so one goes from one power-law model into another.

A particular case of the latter transformation on which we are going to concentrate now is $\eta = -1$, i.e. the phantomization of the model, but before giving further details the remark is in order that the new solution of the dynamical equations corresponds to an imaginary field $\bar{\phi} = i\phi$ driven by a real potential $\bar{V} = 2\rho - V$ [41; 42]

The phantomization process leads to a universe with $\dot{\rho} > 0$ so the weak energy condition $(\rho + p) < 0$ is violated. It follows that two cases can be distinguished: (a) ρ has a constant asintote for $t \to \infty$ or (b) ρ grows unboundedly. In the (a) case the scale factor ends up becoming that of the de Sitter solution; whereas in the (b) case, if we admit the asymptotic energy density is $\rho \to \rho_0 a^{\eta}$ with $\eta > 0$, then the asymptotic solution to the Friedman equation is

$$a^- \to \left[\frac{2}{\eta\sqrt{\rho_0}\,(t_0-t)}\right]^{2/\eta}, \quad t<t_0, \tag{36}$$

$$a^+ \to \left[\frac{2}{\eta\sqrt{\rho_0}\,(t-t_0)}\right]^{2/n}, \quad t>t_0. \tag{37}$$

Clearly the expanding solution a^- is defined for $t<t_0$ and displays a big rip at $t=t_0$ because the scale factor diverges at the finite time t_0 and a future singularity occurs. On the other hand the contracting solution a^+ begins at a past singularity at $t=t_0$. Summarizing, the solution a^- arises by phantomization of the solution $1/a^-$ which ends with a big crunch at $t=t_0$. In terms of form-invariance transformations the phantomization is originated by the $1/a^- \to a^-$ duality existing between those two solutions to the Einstein equations.

In order to get a phantomization with a real potential and a real field [43] one must introduce a DBI$^-$ model with a sign reversal in the kinetic term entering the energy density and pressure (2) and (3), so that

$$\rho^- = -\frac{\gamma^2}{1+\gamma}\,\dot\phi^2+V(\phi) \tag{38}$$

$$p^- = -\frac{\gamma}{1+\gamma}\,\dot\phi^2-V(\phi) \tag{39}$$

This allows enlarging the form invariance symmetry group, because now we can exchange not only solutions to the original equations (6), (7) among them, but also solutions to both equation sets among them. In order to investigate this new enlarged symmetry group, let us rewrite Einstein equations in two form-invariant scenarios:

$$3H^2=s\frac{\gamma\dot\phi^2}{\Gamma}, \quad -2\dot H=s\gamma\dot\phi^2, \tag{40}$$

$$3\bar H^2=\bar s\frac{\bar\gamma\dot{\bar\phi}^2}{\bar\Gamma}, \quad -2\dot{\bar H}=\bar s\bar\gamma\dot{\bar\phi}^2, \tag{41}$$

where $s=\pm1$ y $\bar s=\pm1$. In this case the transformations (33)–(35) become

$$\dot{\bar\phi}^2 = -\frac{s}{\bar s}\dot\phi^2 \tag{42}$$

$$\bar f = -\frac{\bar s}{s}f \tag{43}$$

$$\bar V = 2\rho - V \tag{44}$$

This way it is possible now to achieve a phantomization with a real scalar field if we admit the existence of the theory DBI$^-$.

5 Conclusions

The so called DBI cosmologies are scalar field models with one additional functional degree of freedom as compared to conventional scalar field configurations.

This extra function has a geometrical meaning when DBI actions are interpreted as describing the motion of a brane in a warped space time, the warp factor being indeed the additional input required to specify the model. In this setup inflation is interpreted as the consequence of the motion of the brane in a background with extra dimensions, the compactification of which gives rise to a potential for the scalar field.

These scenarios have been profusely studied, and particular attention has been paid to the possibility they admit power-law solutions, as they seem to be favored to play the role of equilibrium asymptotic states. In this paper we have shown that it is possible to find such solutions in an exact way. The method relies on providing parametrizations of the scalar field in terms of time which can be used to reconstruct the warp factor and the potential upon the sole requirement that the equation of state parameter be constant (in the fluid interpretation of the model). Our main result is illustrated by resorting to some examples with interesting asymptotic limits coinciding with some models studied in the literature: the AdS throat, cut-off throats, and the inverse square potential.

Finally, in the last section, we show how to obtain new solutions from existing ones using symmetries, specifically we give transformation rules for the scalar field, warp factor and potential. This method has interesting applications as it offers the possibility of realizing assisted inflation in a DBI context, but also permits playing with the idea of phantom DBI cosmologies (to be generated from non phantom ones).

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