universe

IMPACT FACTOR 2.5

CITESCORE 4.3

Article

Topological Susceptibility of the Gluon Plasma in the Stochastic-Vacuum Approach

Dmitry Antonov

Special Issue
Quantum Field Theory, 2nd Edition

Edited by
Dr. Ralf Hofmann

MDPI

https://doi.org/10.3390/universe10090377 universe

MDPI

Article

Topological Susceptibility of the Gluon Plasma in the Stochastic-Vacuum Approach

Dmitry Antonov

Formerly at Departamento de Física and CFIF, Instituto Superior Técnico, ULisboa, Av. Rovisco Pais, 1049-001 Lisbon, Portugal; dr.dmitry.antonov@gmail.com

Abstract: Topological susceptibility of the SU(3) gluon plasma is calculated by accounting for both factorized and non-factorized contributions to the two-point correlation function of topological-charge densities. It turns out that, while the factorized contribution keeps this correlation function non-positive away from the origin, the non-factorized contribution makes it positive at the origin, in accordance with the reflection positivity condition. Matching the obtained result for topological susceptibility to its lattice value at the deconfinement critical temperature, we fix the parameters of the quartic cumulant of gluonic field strengths, and calculate the contribution of that cumulant to the string tension. This contribution reduces the otherwise too large value of the string tension, which stems from the quadratic cumulant, making it much closer to the standard phenomenological value.

Keywords: Yang–Mills vacuum; topological susceptibility; Stochastic Vacuum Model; string tension; Wilson loop; finite-temperature effects in quantum field theory

check for updates

Citation: Antonov, D. Topological Susceptibility of the Gluon Plasma in the Stochastic-Vacuum Approach. *Universe* **2024**, *10*, 377. https://doi.org/10.3390/universe10090377

Academic Editors: Ralf Hofmann and Douglas Singleton

Received: 9 August 2024
Revised: 16 September 2024
Accepted: 18 September 2024
Published: 23 September 2024

1. Introduction

As is known, QCD is plagued with the problem of CP-violation, whose essence is that the Yang–Mills Langrangian can be extended by the term which violates P- and CP-symmetries. In Minkowski space (for simplicity), that term has the form $\Delta\mathcal{L}_{\rm YM} = \frac{\alpha_s}{8\pi}\cdot\theta_0\cdot F^a_{\mu\nu}\tilde{F}^a_{\mu\nu}$, where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$ is the non-Abelian field-strength tensor, $\tilde{F}^a_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\rho}F^a_{\lambda\rho}$ is a tensor dual to $F^a_{\mu\nu}$, θ_0 is an arbitrary dimensionless parameter, and $\alpha_s = g^2/(4\pi)$ is the strong coupling constant. The term $\Delta\mathcal{L}_{\rm YM}$ can be represented as the following divergence of a vector, which is built up from the gluon fields: $\Delta\mathcal{L}_{\rm YM} = \frac{\alpha_s}{4\pi}\cdot\theta_0\cdot\partial_\mu K_\mu$, where

$$K_\mu = \varepsilon_{\mu\nu\lambda\rho}\left(A^a_\nu\partial_\lambda A^a_\rho + \frac{1}{3}f^{abc}A^a_\nu A^b_\lambda A^c_\rho\right).$$

This fact means that the contribution produced by $\Delta\mathcal{L}_{\rm YM}$ to the Yang–Mills action, vanishes for perturbative configurations of gluon fields, while it does not vanish for the non-perturbative configurations, such as instantons. Hence, the CP-symmetry of QCD is broken at the non-perturbative level. Furthermore, by means of the axial anomaly, quarks yield an additional contribution, which has the same functional form as $\Delta\mathcal{L}_{\rm YM}$, being proportional to the phase of the determinant of the quark mass matrix, M_q. The full term, thus, has the form $\Delta\mathcal{L}_\theta = \frac{\alpha_s}{8\pi}\cdot\theta\cdot F^a_{\mu\nu}\tilde{F}^a_{\mu\nu}$, where $\theta = \theta_0 + {\rm Arg}\,(\det M_q)$. The θ-term $\Delta\mathcal{L}_\theta$ gives rise to a non-vanishing electric dipole moment of the neutron. Although the latter is not yet experimentally discovered, the existing data provide an upper limit for its possible values, which, in turn, yields the following upper limit for the absolute value of θ: $|\theta| \lesssim 0.3\cdot 10^{-9}$. The necessity of explaining this smallness of θ is the essence of the CP-problem of strong interactions.

Had the quark Lagrangian, at the classical level, been invariant under the so-called Peccei–Quinn axial $\mathrm{U}(1)_{\rm PQ}$ symmetry [1,2], $q_L \to {\rm e}^{i\beta}q_L$, $q_R \to {\rm e}^{-i\beta}q_R$, the θ-term could be nullified by means of phase rotations of the quark fields. However, this symmetry is broken by the quark mass terms, which yields a massless (at the classical level) Goldstone field $a(x)$, called an axion. Under the $\mathrm{U}(1)_{\mathrm{PQ}}$-transformations, this field is being transformed as $a \to a + \beta \cdot f_{\mathrm{PQ}}$, where the parameter f_{PQ}, of the dimensionality of mass, characterizes the scale of the $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry breaking. The aforementioned transformation laws, for q_L, q_R, and a, yield the following quark mass term modified by the axion field: $m_q \bar{q}_R \mathrm{e}^{-2ia/f_{\mathrm{PQ}}} q_L +$ H.c. Accordingly, at the quantum level, Arg $(\det M_q)$ yields the low-energy Lagrangian $C \cdot \frac{\alpha_s}{8\pi} \cdot \frac{a}{f_{\mathrm{PQ}}} \cdot F^a_{\mu\nu} \tilde{F}^a_{\mu\nu}$, where the constant $C = \mathcal{O}(1)$ is determined by the charges of quarks with respect to the $\mathrm{U}(1)_{\mathrm{PQ}}$ group. Hence, at the quantum level, the $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry is explicitly broken, with the axion being the corresponding pseudo-Goldstone boson, and the θ-parameter becomes a field as $\theta \to \theta(x) = \theta + Ca(x)/f_{\mathrm{PQ}}$. The CP-invariance in QCD would be restored once the v.e.v. $\langle a(x) \rangle$ were such as to make $\langle \theta(x) \rangle$ vanishing. Fortunately, this is indeed happening, owing to the chiral symmetry breaking, which leads to the effective potential $\simeq m_\pi^2 f_\pi^2 \theta^2(x)/8$, where m_π is the pion mass and f_π is the pion decay constant. Accordingly, the axion obtains the following mass: $m_a \simeq C m_\pi f_\pi / (2 f_{\mathrm{PQ}})$. The breaking of the $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry leads to the non-conservation of the corresponding $\mathrm{U}(1)_{\mathrm{PQ}}$ current, $\partial_\mu J^{\mathrm{PQ}}_\mu = a_{\mathrm{PQ}} q$, where $q(x)$ is the local density of topological charge, and a_{PQ} is a constant. As was further shown in [3–6], the following expression for m_a, analogous to the Veneziano–Witten formula for $m_{\eta'}$, takes place: $m_a^2 \simeq \frac{a^2_{\mathrm{PQ}}}{f^2_{\mathrm{PQ}}} \chi$. Here, $\chi = \int \mathrm{d}^4x \langle q(x) q(0) \rangle$ is the topological susceptibility of the Yang–Mills vacuum, and we henceforth work in the Euclidean space-time. In this paper, we address the temperature dependence of χ, so that $\int \mathrm{d}^4x \to \int \mathrm{d}^3x \int_0^{1/T} \mathrm{d}x_4$, in the deconfinement phase, i.e., at $T > T_c$. To this end, we model the aforementioned non-perturbative field configurations by the stochastic background Yang–Mills fields, which are characterized by the finite vacuum correlation length and the vacuum condensates within the Stochastic Vacuum Model [7,8]. Lattice data [9] indicate that, at $T = T_c$, the chromo-electric condensate $\langle (gE^a_i)^2 \rangle$ vanishes, which leads to the deconfinement phase transition, while the chromo-magnetic condensate $\langle (gB^a_i)^2 \rangle$ does not vanish, which leads to the so-called spatial confinement, quantified by the area law of large spatial Wilson loops, in the deconfinement phase [10].

Note that the topological susceptibility of the high-temperature instanton-based Yang–Mills vacuum, which is missing spatial confinement, is given by the following integral over instanton sizes ρ (cf. Ref. [11]):

$$\chi \sim \int \frac{\mathrm{d}\rho}{\rho^5} \, \mathrm{e}^{-\frac{2N}{3}(\pi\rho T)^2 - \frac{8\pi^2}{g^2(\rho)}}, \tag{1}$$

where

$$\mathrm{e}^{-\frac{8\pi^2}{g^2(\rho)}} = (\rho\Lambda)^b. \tag{2}$$

(In this expression, $b = \frac{11}{3}N$ is the absolute value of the leading coefficient of the Yang–Mills β-finction, and Λ is the UV cutoff.) In particular, for $N = 3$, the square root of the variance of the Gaussian distribution in Equation (1), yields $\rho \lesssim \frac{1}{2\pi T}$, so that, already for $T = T_c \simeq 270\,\mathrm{MeV}$ [10], one has $\rho \lesssim 0.12\,\mathrm{fm}$. As these values of ρ are significantly smaller than the typical instanton size of 0.33 fm in the instanton-liquid model of the Yang–Mills vacuum [12], the Boltzmann factor (2) indicates that instantons' contribution to χ, given by Equation (1), is suppressed at $T > T_c$. Consequently, instead of the $\mathcal{O}(1/T^7)$-behavior of χ, suggested by Equations (1) and (2) for $N = 3$ (cf. the corresponding lattice data [13]), one can expect the $\mathcal{O}(T^4)$-behavior, suggested by Equation (1) on purely dimensional grounds. In what follows, we will obtain the $\mathcal{O}(T^4)$-behavior of χ, along with the corresponding proportionality coefficient, in the aforementioned Stochastic Model of the Yang–Mills vacuum [7,8].

2. Calculation of $\chi(T)$

Let us consider the expression $\varepsilon_{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}$, where one of the indices can be equal to 4, and $F_{\mu\nu}$ is some antisymmetric tensor. One can readily see that this expression is equal to $4\varepsilon_{4ijk}F_{4i}F_{jk}$. Further, by using the reduction of the product $\varepsilon_{4ijk}\varepsilon_{4lmn}$ to the determinant of the 3×3 matrix of Kronecker deltas, one has

$$\varepsilon_{4ijk}\varepsilon_{4lmn}F_{4i}(x)F_{jk}(x)F_{4l}(0)F_{mn}(0) = 2F_{4i}(x)F_{jk}(x)\Big[F_{4i}(0)F_{jk}(0) + 2F_{4k}(0)F_{ij}(0)\Big].$$

Let us now consider the local density of topological charge,

$$q(x) = \frac{g^2}{32\pi^2}\varepsilon_{\mu\nu\lambda\rho}\,\mathrm{tr}(F_{\mu\nu}(x)F_{\lambda\rho}(x)),$$

where, henceforth, $F_{\mu\nu} = F^a_{\mu\nu}T^a$, with T^a's being the SU(N)-generators in the fundamental representation, $a = 1,\ldots,N^2-1$. By using the standard normalization condition, $\mathrm{tr}\,T^aT^b = \delta^{ab}/2$, and the formulae above, one has

$$\langle q(x)q(0)\rangle = \frac{g^4}{128\pi^4}\Big[\langle F^a_{4i}(x)F^a_{jk}(x)F^b_{4i}(0)F^b_{jk}(0)\rangle + 2\langle F^a_{4i}(x)F^a_{jk}(x)F^b_{4k}(0)F^b_{ij}(0)\rangle\Big]. \quad (3)$$

Let us start with the factorized part of Equation (3), which amounts to considering six pairwise products of the two-point correlation functions of the field strengths (cf. Ref. [14]). Noticing that $\langle F^a_{4i}(x)F^b_{4j}(y)\rangle = 0$ in the stochastic Yang–Mills vacuum at $T > T_c$, one has

$$\langle q(x)q(0)\rangle_{\text{factorized}} = \frac{1}{128\pi^4}\Big[\langle g^2F^a_{4i}(0)F^a_{jk}(0)\rangle^2 + \langle g^2F^a_{4i}(x)F^b_{jk}(0)\rangle^2 +$$
$$2\langle g^2F^a_{4i}(0)F^a_{jk}(0)\rangle\langle g^2F^b_{4k}(0)F^b_{ij}(0)\rangle + 2\langle g^2F^a_{4i}(x)F^b_{ij}(0)\rangle\langle g^2F^a_{4k}(x)F^b_{jk}(0)\rangle\Big]. \quad (4)$$

We see that $\langle q(x)q(0)\rangle_{\text{factorized}}$ is fully expressed in terms of the correlation function $\langle g^2E^a_i(x)B^b_k(0)\rangle$, which can be parameterized through some function $f(x)$ as

$$\langle g^2E^a_i(x)B^b_k(0)\rangle = \delta^{ab}\varepsilon_{ikn}x_nf(x). \quad (5)$$

Multiplying this equation by T^aT^b and taking the trace, one has

$$\mathrm{tr}\,\langle g^2E^a_i(x)T^aB^b_k(0)T^b\rangle = \frac{N^2-1}{2}\,\varepsilon_{ikn}x_nf(x).$$

On the other hand, the same quantity can be expressed by means of Equation (2.9) from Ref. [9]:

$$\mathrm{tr}\,\langle g^2E^a_i(x)T^aB^b_k(0)T^b\rangle = -\frac{1}{2}\varepsilon_{ikn}x_n\frac{\partial D_1^{\mathrm{BE}}}{\partial x_4}.$$

That yields the following expression for the function $f(x)$ in terms of the function $D_1^{\mathrm{BE}}(x)$, which was measured on the lattice:

$$f(x) = -\frac{1}{N^2-1}\frac{\partial D_1^{\mathrm{BE}}}{\partial x_4}. \quad (6)$$

Noticing further that $E^a_i = F^a_{i4}$ and $B^a_k = \frac{1}{2}\varepsilon_{kij}F^a_{ij}$, we readily obtain

$$\langle g^2F^a_{4i}(x)F^b_{jk}(0)\rangle = \delta^{ab}\big(\delta_{ij}\delta_{kn} - \delta_{ik}\delta_{jn}\big)x_nf(x). \quad (7)$$

Equations (4) and (7) yield

$$\langle q(x)q(0)\rangle_{\text{factorized}} = \frac{1}{128\pi^4}\Big[\langle g^2F^a_{4i}(x)F^b_{jk}(0)\rangle^2 + 2\langle g^2F^a_{4i}(x)F^b_{ij}(0)\rangle\langle g^2F^a_{4k}(x)F^b_{jk}(0)\rangle\Big] =$$

$$-\frac{N^2-1}{32\pi^4}\mathbf{x}^2 f^2. \tag{8}$$

For the non-perturbative ansatz used in Ref. [9], $D_1^{\mathrm{BE}}(x) = A\,\mathrm{e}^{-m|x|}$, Equation (6) thus yields

$$\langle q(x)q(0)\rangle_{\text{factorized, non-pert.}} = -\frac{(Am)^2}{32\pi^4}\frac{1}{N^2-1}\frac{\mathbf{x}^2 x_4^2}{|x|^2}\,\mathrm{e}^{-2m|x|}. \tag{9}$$

Note that this expression is negative, and vanishes at the origin. Here, $A = A(T)$ is the amplitude of the function $D_1^{\mathrm{BE}}(x)$, and $m = m(T)$ is the inverse correlation length of the chromo-magnetic vacuum. The temperature dependence of these quantities will be discussed below, in Equation (24).

In a similar way, one can calculate the perturbative contribution, $\langle q(x)q(0)\rangle_{\text{factorized, pert.}}$. To this end, we use the perturbative part of the field-strength tensor, $f_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a$, to write

$$\langle g^2 E_i^a(x) B_k^b(y)\rangle_{\text{pert}} = \frac{1}{2}\varepsilon_{klm}\langle g^2 f_{i4}^a(x) f_{lm}^b(y)\rangle = \varepsilon_{klm}\partial_4^x\partial_m^y\langle g^2 A_i^a(x) A_l^b(y)\rangle,$$

where $\langle g^2 A_i^a(x) A_l^b(y)\rangle = \frac{g^2}{4\pi^2}\frac{\delta^{ab}\delta_{il}}{(x-y)^2}$ is the gluon propagator in the Feynman gauge. Hence,

$$\langle g^2 E_i^a(x) B_k^b(0)\rangle_{\text{pert}} = g^2\delta^{ab}\varepsilon_{ikn}\partial_4\partial_n\frac{1}{4\pi^2 x^2}. \tag{10}$$

At finite temperature $T \equiv 1/\beta$, the Euclidean propagator $\frac{1}{4\pi^2 x^2}$ takes the form

$$\int_0^\infty \frac{ds}{(4\pi s)^2}\sum_{n=-\infty}^{+\infty}\exp\left[-\frac{\mathbf{x}^2+(x_4+\beta n)^2}{4s}\right]. \tag{11}$$

The Poisson resummation yields

$$\sum_{n=-\infty}^{+\infty}\exp\left[-\frac{(x_4+\beta n)^2}{4s}\right] = 2T\sqrt{\pi s}\sum_{k=-\infty}^{+\infty}\exp(-\omega_k^2 s + i\omega_k x_4), \tag{12}$$

where $\omega_k = 2\pi T k$ is the k-th Matsubara frequency. As $\omega_0 = 0$, it does not contribute to Equation (10) upon the differentiation over x_4, so that one can approximate the sum by the terms with $k = \pm 1$. That yields the following approximation for the finite-temperature counterpart of $\partial_4\partial_n\frac{1}{4\pi^2 x^2}$:

$$\frac{T^2 x_n}{4\sqrt{\pi}}\int_0^\infty\frac{ds}{s^{5/2}}\,\mathrm{e}^{-(2\pi T)^2 s - \frac{\mathbf{x}^2}{4s}}\sin(2\pi T x_4).$$

Performing the s-integration in this expression, and recalling Equation (5), we have

$$f_{\text{pert}}(x) \simeq \frac{2\pi g^2 T^3}{\mathbf{x}^2}\left(1+\frac{1}{2\pi T|\mathbf{x}|}\right)\mathrm{e}^{-2\pi T|\mathbf{x}|}\sin(2\pi T x_4).$$

Thus, by using Equation (8), we obtain

$$\langle q(x)q(0)\rangle_{\text{factorized, pert.}} \simeq -\frac{N^2-1}{8\pi^2}\frac{(g^2T^3)^2}{\mathbf{x}^2}\left(1+\frac{1}{2\pi T|\mathbf{x}|}\right)^2\mathrm{e}^{-4\pi T|\mathbf{x}|}\sin^2(2\pi T x_4). \tag{13}$$

Let us further evaluate $\langle q(x)q(0)\rangle_{\text{non-factorized, non-pert.}}$. To this end, we consider the non-perturbative contribution to the quartic cumulant in the form of the two following tensor structures [15]:

$$\langle g^4 F_{\mu_1\nu_1}^{a_1}(x_1) F_{\mu_2\nu_2}^{a_2}(x_2) F_{\mu_3\nu_3}^{a_3}(x_3) F_{\mu_4\nu_4}^{a_4}(x_4)\rangle_{\text{non-factorized, non-pert.}} =$$

$$\langle g^4 F^a_{\alpha\beta}(0) F^a_{\alpha\beta}(0) F^b_{\lambda\rho}(0) F^b_{\lambda\rho}(0)\rangle \times$$
$$\Big\{ \Big[\delta^{a_1a_2}\delta^{a_3a_4}(\delta_{\mu_1\mu_2}\delta_{\nu_1\nu_2} - \delta_{\mu_1\nu_2}\delta_{\mu_2\nu_1})(\delta_{\mu_3\mu_4}\delta_{\nu_3\nu_4} - \delta_{\mu_3\nu_4}\delta_{\mu_4\nu_3}) +$$
$$\delta^{a_1a_3}\delta^{a_2a_4}(\delta_{\mu_1\mu_3}\delta_{\nu_1\nu_3} - \delta_{\mu_1\nu_3}\delta_{\mu_3\nu_1})(\delta_{\mu_2\mu_4}\delta_{\nu_2\nu_4} - \delta_{\mu_2\nu_4}\delta_{\mu_4\nu_2}) +$$
$$\delta^{a_1a_4}\delta^{a_2a_3}(\delta_{\mu_1\mu_4}\delta_{\nu_1\nu_4} - \delta_{\mu_1\nu_4}\delta_{\mu_4\nu_1})(\delta_{\mu_2\mu_3}\delta_{\nu_2\nu_3} - \delta_{\mu_2\nu_3}\delta_{\mu_3\nu_2}) \Big] G(z_1,\ldots,z_6) +$$
$$\Big[f^{a_1a_2c} f^{a_3a_4c}(\varepsilon_{\mu_1\nu_1\mu_3\nu_3}\varepsilon_{\mu_2\nu_2\mu_4\nu_4} - \varepsilon_{\mu_1\nu_1\mu_4\nu_4}\varepsilon_{\mu_2\nu_2\mu_3\nu_3}) +$$
$$f^{a_1a_3c} f^{a_2a_4c}(\varepsilon_{\mu_1\nu_1\mu_2\nu_2}\varepsilon_{\mu_3\nu_3\mu_4\nu_4} - \varepsilon_{\mu_1\nu_1\mu_4\nu_4}\varepsilon_{\mu_3\nu_3\mu_2\nu_2}) +$$
$$f^{a_1a_4c} f^{a_2a_3c}(\varepsilon_{\mu_1\nu_1\mu_2\nu_2}\varepsilon_{\mu_4\nu_4\mu_3\nu_3} - \varepsilon_{\mu_1\nu_1\mu_3\nu_3}\varepsilon_{\mu_4\nu_4\mu_2\nu_2}) \Big] G_1(z_1,\ldots,z_6) \Big\}. \quad (14)$$

The structure with Kronecker deltas in this formula contributes to the effective action of the quark–antiquark string, and it can even provide a fully quantum quark–antiquark string in 4D [16], while the structure with Levi-Civita symbols does not contribute to the string effective action. Also, $z_1 = x_1 - x_2, \ldots, z_6 = x_3 - x_4$, and the notations G and G_1 were invented by analogy with the notations for functions $D(x)$ and $D_1(x)$, which were used for the parameterization of the confining and the non-confining contributions to the correlation function $\langle g^2 F^a_{\mu\nu}(x) F^b_{\lambda\rho}(0)\rangle$ [7,8]. Furthermore, setting in Equation (14) $x_1 = \cdots = x_4$, $\mu_1 = \mu_2$, $\nu_1 = \nu_2$, $\mu_3 = \mu_4$, $\nu_3 = \nu_4$, $a_1 = a_2$, $a_3 = a_4$, and noticing that $f^{abc} f^{abc} = N(N^2-1)$ and $\varepsilon_{\mu\nu\lambda\rho}\varepsilon_{\mu\nu\lambda\rho} = D!$, one obtains the following normalization condition (cf. Refs. [15,16]):

$$(N^2-1)\{(D^2-D)[(N^2-1)(D^2-D)+4]\, G(0,\ldots,0) - 2ND!\, G_1(0,\ldots,0)\} = 1. \quad (15)$$

Henceforth, we set $D = 4$. Finally, we use the approximation

$$\langle g^4 F^a_{\alpha\beta}(0) F^a_{\alpha\beta}(0) F^b_{\lambda\rho}(0) F^b_{\lambda\rho}(0)\rangle \simeq \langle (gF^a_{\alpha\beta})^2\rangle^2, \quad (16)$$

known as the Vacuum Dominance Hypothesis, which states that the dominant contribution to even-order condensates is the factorized one [17].

As mentioned in the Introduction, the chromo-electric condensate $\langle (gE^a_i)^2\rangle$ vanishes at $T = T_c$, so that, at $T > T_c$, $\langle (gF^a_{\alpha\beta})^2\rangle$ goes over to $2\langle (gB^a_i)^2\rangle$. Accordingly, we obtain

$$\langle g^4 F^a_{4i}(x) F^a_{jk}(x) F^b_{4i}(0) F^b_{jk}(0)\rangle = 24(N^2-1)\langle (gB^a_i)^2\rangle^2\,(3G + NG_1),$$
$$\langle g^4 F^a_{4i}(x) F^a_{jk}(x) F^b_{4k}(0) F^b_{ij}(0)\rangle = 24(N^2-1)\langle (gB^a_i)^2\rangle^2\,(-G + 2NG_1),$$

where $G \equiv G(0,x,x,x,x,0)$, $G_1 \equiv G_1(0,x,x,x,x,0)$, and we have used the fact that $\varepsilon_{4ijk}\varepsilon_{4ijk} = 6$. Plugging these two expressions into Equation (3), we obtain

$$\langle q(x)q(0)\rangle_{\rm non-factorized,\, non-pert.} = \frac{3(N^2-1)}{16\pi^4}\langle (gB^a_i)^2\rangle^2\,(G + 5NG_1). \quad (17)$$

Let us now proceed to the calculation of various contributions to the topological susceptibility, $\chi = \int d^3x \int_0^\beta dx_4 \langle q(x)q(0)\rangle$. Such a calculation is mostly simple in the case of $\chi_{\rm non-factorized,\, non-pert.}$, which corresponds to Equation (17) and amounts to calculating the integral $I \equiv \int d^3x \int_0^\beta dx_4\, {\rm e}^{-M\sqrt{\mathbf{x}^2+x_4^2}}$. Here, $M = 4m$, due to the four arguments "x" in G and G_1, and we henceforth restrict ourselves to the zeroth winding mode on the left-hand side of Equation (12), as the contribution of other winding modes is exponentially smaller than that one. The integral I can be calculated by using the representation ${\rm e}^{-M\sqrt{\mathbf{x}^2+x_4^2}} = \int_0^\infty \frac{d\lambda}{\sqrt{\pi\lambda}}\, {\rm e}^{-\lambda - \frac{M^2(\mathbf{x}^2+x_4^2)}{4\lambda}}$ and first performing the so-emerging Gaussian $\mathbf{x}$-integration, which yields $I = \frac{8\pi}{M^3}\int_0^\infty d\lambda\, \lambda\, {\rm e}^{-\lambda} \int_0^\beta dx_4\, {\rm e}^{-\frac{M^2 x_4^2}{4\lambda}}$. Performing further the

λ-integration, we have $I = \frac{4\pi}{M}\int_0^\beta \mathrm{d}x_4\, x_4^2 K_2(Mx_4)$, where $K_\nu(x)$ henceforth stands for the Macdonald functions. We use now the parametrization $m = cg^2T$, where $c \simeq 1$ [18], as well as the approximation $g \simeq 1$, which is known to be valid throughout the range of temperatures $T_c < T \lesssim 10T_c$ of interest (cf. e.g., Ref. [10]). That yields $I \simeq \frac{4\pi}{M^4}\int_0^4 \mathrm{d}y y^2 K_2(y)$, where the numerical value of the latter integral is 4.35, being quite close to the value of $\int_0^\infty \mathrm{d}y y^2 K_2(y) = \frac{3\pi}{2} \simeq 4.71$. Thus,

$$\chi_{\rm non-factorized,\, non-pert.} \simeq$$

$$\frac{3\cdot 4.35\cdot (N^2-1)}{4^5\pi^3}\,\frac{\langle (gB_i^a)^2\rangle^2}{m^4}\,[G(0,\dots,0) + 5NG_1(0,\dots,0)]. \tag{18}$$

In particular, by using Equation (15) with $N = 3$, which reads

$$9600\, G(0,\dots,0) - 1152\, G_1(0,\dots,0) = 1, \tag{19}$$

we have

$$G(0,\dots,0) + 5NG_1(0,\dots,0)\Big|_{N=3} = 126\, G(0,\dots,0) - \frac{5}{384}. \tag{20}$$

In a similar way, one can calculate $\chi_{\rm factorized,\, non-pert.}$. With reference to Equation (9), let us start with the function $J \equiv \frac{\mathrm{e}^{-M\sqrt{\mathbf{x}^2+x_4^2}}}{\mathbf{x}^2+x_4^2}$, where now $M = 2m$, and differentiate it over the parameter M, which yields

$$-\frac{\partial J}{\partial M} = \frac{\mathrm{e}^{-M\sqrt{\mathbf{x}^2+x_4^2}}}{\sqrt{\mathbf{x}^2+x_4^2}} = \frac{M}{2\sqrt{\pi}}\int_0^\infty \frac{\mathrm{d}\lambda}{\lambda^{3/2}}\,\mathrm{e}^{-\lambda-\frac{M^2(\mathbf{x}^2+x_4^2)}{4\lambda}}.$$

By using this representation, we can again perform the emerging Gaussian $\mathbf{x}$-integration, and obtain

$$-\int \mathrm{d}^3x\,\mathbf{x}^2\frac{\partial J}{\partial M} = \frac{24\pi}{M^4}\int_0^\infty \mathrm{d}\lambda\lambda\,\mathrm{e}^{-\lambda-\frac{M^2x_4^2}{4\lambda}} = \frac{12\pi x_4^2}{M^2}K_2(Mx_4).$$

Recalling Equation (9), we further obtain

$$-\int \mathrm{d}^3x\,\mathbf{x}^2\int_0^\beta \mathrm{d}x_4 x_4^2\,\frac{\partial J}{\partial M} \simeq \frac{12\pi}{M^7}\int_0^2 \mathrm{d}y y^4 K_2(y) \equiv \frac{B}{M^7}, \tag{21}$$

where $\int_0^2 \mathrm{d}y y^4 K_2(y) \simeq 3.50$ is significantly smaller than $\int_0^\infty \mathrm{d}y y^4 K_2(y) = \frac{15\pi}{2} \simeq 23.56$. Integrating Equation (21) over M and noticing that $\int \mathrm{d}^3x\,\mathbf{x}^2\int_0^\beta \mathrm{d}x_4 x_4^2\, J \to 0$ at $M \to \infty$, we obtain $\int \mathrm{d}^3x\,\mathbf{x}^2\int_0^\beta \mathrm{d}x_4 x_4^2\, J = \frac{B}{6M^6}$. Thus,

$$\chi_{\rm factorized,\, non-pert.} \simeq -\frac{3.5}{32^2\pi^3(N^2-1)}\cdot\frac{A^2}{m^4}, \tag{22}$$

where the amplitude parameter A was discussed after Equation (9).

Finally, by using Equation (13), we calculate $\chi_{\rm factorized,\, pert.}$, given by

$$\int \mathrm{d}^3x\int_0^\beta \mathrm{d}x_4\langle q(x)q(0)\rangle_{\rm factorized,\, pert.} =$$

$$-\frac{N^2-1}{8\pi^2}(g^2T^3)^2\frac{1}{2T}\cdot 4\pi\int_{1/\Lambda}^\infty \mathrm{d}x\left(1+\frac{1}{2\pi Tx}\right)^2\mathrm{e}^{-4\pi Tx},$$

where the prefactor of $\frac{1}{2T}$ stems from $\int_0^\beta dx_4 \sin^2(2\pi T x_4)$. Within the same approximation under which Equation (13) was obtained, we set the UV cut-off Λ equal to $\omega_1 = 2\pi T$. This yields

$$\chi_{\text{factorized, pert.}} \simeq -\frac{N^2-1}{16\pi^2 e^2}(gT)^4. \tag{23}$$

Note also that the perturbative $\times$ non-perturbative factorized contribution to χ, which could stem from the terms

$$\langle g^2 F_{4i}^a(x) F_{jk}^b(0)\rangle_{\text{pert}} \langle g^2 F_{4i}^a(x) F_{jk}^b(0)\rangle_{\text{non-pert}} \text{ and } \langle g^2 F_{4i}^a(x) F_{ij}^b(0)\rangle_{\text{pert}} \langle g^2 F_{4k}^a(x) F_{jk}^b(0)\rangle_{\text{non-pert}}$$

in Equation (4), vanishes, since $\int_0^\beta dx_4 \sin(2\pi T x_4) = 0$.

Let us further discuss the important sign property (called reflection positivity condition), which should be respected by the full $\langle q(x)q(0)\rangle$: it should be non-positive for all $x \neq 0$, while yielding a positive χ at the same time [19]. Comparing Equations (9) and (13) with each other, we see that Equation (9) is parametrically larger, as its exponential suppression is weaker and its pre-exponent is increasing with the increase of $|x|$. For this reason, Equation (13) can be neglected in comparison to Equation (9). Now, comparing Equation (9) with Equation (17), we first notice that Equation (17) stays constant at the origin, whereas Equation (9) vanishes, due to its pre-exponential factor. Rather, at $x \neq 0$, Equation (9) is parametrically larger than Equation (17), not only due to the same pre-exponential factor, but foremost due to the stronger exponential suppression of Equation (17), with $M = 4m$. Thus, parametrically, the desired sign property of the full $\langle q(x)q(0)\rangle$ is respected.

The full topological susceptibility is given by the sum of the three calculated contributions, given by Equations (18), (22) and (23). To evaluate it numerically, we notice that, already at $T \gtrsim 1.3\, T_c$, the temperature dependence of dimensionful quantities entering these equations can be parameterized as follows [14]:

$$A(T) = A_c \cdot \left(\frac{g^2 T}{g_c^2 T_c}\right)^4, \;\; m(T) = m_c \cdot \frac{g^2 T}{g_c^2 T_c}, \;\; \langle (gB_i^a)^2\rangle = \langle (gB_i^a)^2\rangle_c \cdot \left(\frac{g^2 T}{g_c^2 T_c}\right)^4, \tag{24}$$

where the subscript "c" means "at $T = T_c$". We further adopt approximation (cf. Ref. [9])

$$A_c \simeq \langle (gB_i^a)^2\rangle_c, \tag{25}$$

as well as approximations $m_c \simeq g_c^2 T_c$ (cf. $c \simeq 1$ above) and $g \simeq g_c \simeq 1$. Using Equation (20) along with parameterizations (24) and (25), we obtain

$$\chi \simeq$$

$$\frac{1}{1024\,\pi^3}\left[104.4\left(126\,G(0,\ldots,0) - \frac{5}{384}\right) - 0.5\right]\frac{\langle (gB_i^a)^2\rangle_c^2}{m_c^4}\left(\frac{T}{T_c}\right)^4 \text{ at } T \gtrsim 1.3\,T_c. \tag{26}$$

Thus, for

$$G(0,\ldots,0) \gtrsim \frac{\frac{0.5}{104.4} + \frac{5}{384}}{126} \simeq 1.4 \cdot 10^{-4}, \tag{27}$$

the obtained χ appears to be positive. The inequality (27) can be viewed as a lower bound for the possible values of $G(0,\ldots,0)$. Let us extrapolate Equation (26) down to $T = T_c$, and approximate the gluon condensate and the vacuum correlation length by their zero-temperature values in the SU(3) Yang–Mills theory [20],

$$\langle (gB_i^a)^2\rangle_c \simeq \frac{1}{2}\langle (gF_{\mu\nu}^a)^2\rangle_{T=0} \simeq 2.84\,\text{GeV}^4 \text{ and } \frac{1}{m_c} \simeq 1.1\,\text{GeV}^{-1}. \tag{28}$$

Using then for χ its zero-temperature lattice value [21], $\chi \simeq (193\,\text{MeV})^4$, we obtain from Equation (26):

$$G_{T=0}(0,\ldots,0) \simeq 4.2 \cdot 10^{-4}, \tag{29}$$

which respects inequality (27).

With this value of $G_{T=0}(0,\ldots,0)$ at hand, we can readily calculate the correction to the zero-temperature string tension, which is produced by the quartic cumulant. To this end, we make use of the cumulant expansion for the Wilson loop, which yields

$$\langle W(C)\rangle \simeq$$
$$\frac{1}{N}\,\mathrm{tr}\,\exp\Bigg[-\frac{1}{2^2\,2!}\int_{S_{\min}} d\sigma_{\mu_1\nu_1}(x_1)\int_{S_{\min}} d\sigma_{\mu_2\nu_2}(x_2)\,\langle g^2 F_{\mu_1\nu_1}(x_1)F_{\mu_2\nu_2}(x_2)\rangle_{\mathrm{non-pert.}}+$$
$$\frac{1}{2^4\,4!}\int_{S_{\min}} d\sigma_{\mu_1\nu_1}(x_1)\int_{S_{\min}} d\sigma_{\mu_2\nu_2}(x_2)\int_{S_{\min}} d\sigma_{\mu_3\nu_3}(x_3)\int_{S_{\min}} d\sigma_{\mu_4\nu_4}(x_4)\times$$
$$\langle g^4 F_{\mu_1\nu_1}(x_1)F_{\mu_2\nu_2}(x_2)F_{\mu_3\nu_3}(x_3)F_{\mu_4\nu_4}(x_4)\rangle_{\mathrm{non-factorized,\,non-pert.}}\Bigg]. \quad (30)$$

Using further Equations (14) and (16), we have the following contribution to the non-local string action, produced by the quartic cumulant:

$$\mathcal{A}_{\mathrm{quartic}} = -\frac{\langle (gF^a_{\alpha\beta})^2\rangle^2}{2^4\,4!}\,G_{T=0}(0,\ldots,0)\times$$
$$\frac{1}{N}\,\mathrm{tr}\int_{S_{\min}} d\sigma_{\mu_1\nu_1}(x_1)\int_{S_{\min}} d\sigma_{\mu_2\nu_2}(x_2)\int_{S_{\min}} d\sigma_{\mu_3\nu_3}(x_3)\int_{S_{\min}} d\sigma_{\mu_4\nu_4}(x_4)\,T^{a_1}T^{a_2}T^{a_3}T^{a_4}\times$$
$$\Big[\delta^{a_1a_2}\delta^{a_3a_4}(\delta_{\mu_1\mu_2}\delta_{\nu_1\nu_2}-\delta_{\mu_1\nu_2}\delta_{\mu_2\nu_1})(\delta_{\mu_3\mu_4}\delta_{\nu_3\nu_4}-\delta_{\mu_3\nu_4}\delta_{\mu_4\nu_3})+$$
$$\delta^{a_1a_3}\delta^{a_2a_4}(\delta_{\mu_1\mu_3}\delta_{\nu_1\nu_3}-\delta_{\mu_1\nu_3}\delta_{\mu_3\nu_1})(\delta_{\mu_2\mu_4}\delta_{\nu_2\nu_4}-\delta_{\mu_2\nu_4}\delta_{\mu_4\nu_2})+$$
$$\delta^{a_1a_4}\delta^{a_2a_3}(\delta_{\mu_1\mu_4}\delta_{\nu_1\nu_4}-\delta_{\mu_1\nu_4}\delta_{\mu_4\nu_1})(\delta_{\mu_2\mu_3}\delta_{\nu_2\nu_3}-\delta_{\mu_2\nu_3}\delta_{\mu_3\nu_2})\Big]\,e^{-m(|z_1|+\cdots+|z_6|)}.$$

Contracting the indices, we obtain

$$\mathcal{A}_{\mathrm{quartic}} = -\frac{3\,\langle (gF^a_{\alpha\beta})^2\rangle^2}{4\cdot 4!}\,G_{T=0}(0,\ldots,0)\,(T^aT^a)^2\times$$
$$\int_{S_{\min}} d\sigma_{\mu\nu}(x_1)\int_{S_{\min}} d\sigma_{\mu\nu}(x_2)\,e^{-m|x_1-x_2|}\int_{S_{\min}} d\sigma_{\lambda\rho}(x_3)\int_{S_{\min}} d\sigma_{\lambda\rho}(x_4)\,e^{-m|x_3-x_4|}\times$$
$$e^{-m(|x_1-x_3|+|x_1-x_4|+|x_2-x_3|+|x_2-x_4|)}. \quad (31)$$

The leading terms in the derivative expansion of the latter integrals read [8] (for a review, see [22])

$$\int_{S_{\min}} d\sigma_{\mu\nu}(x_1)\int_{S_{\min}} d\sigma_{\mu\nu}(x_2)\,e^{-m|x_1-x_2|}\simeq\frac{2}{m^2}\int d^2z\,e^{-|\mathbf{z}|}\int d^2x_1\,\sqrt{g(x_1)}=$$
$$\frac{4\pi}{m^2}\int d^2x_1\,\sqrt{g(x_1)} \quad (32)$$

and

$$\int_{S_{\min}} d\sigma_{\lambda\rho}(x_3)\int_{S_{\min}} d\sigma_{\lambda\rho}(x_4)\,e^{-m|x_3-x_4|}\simeq\frac{4\pi}{m^2}\int d^2x_3\,\sqrt{g(x_3)}. \quad (33)$$

Here, $g=\det g_{ab}$ is the determinant of the induced-metric tensor $g_{ab}=\partial_a x_\mu\cdot\partial_b x_\mu$ corresponding to the vector-function $x_\mu = x_\mu(\boldsymbol{\xi})$, which parameterizes the surface $S_{\min}$. Furthermore, indices a and b take the values 1 and 2, and $\boldsymbol{\xi}=(\xi^1,\xi^2)$ is a 2D-vector, for which we adopt the Gauss' map [23,24], i.e., $\boldsymbol{\xi}=(x^1,x^2)$, so that the differentials in Equations (32) and (33) read $d^2x_1 = dx_1{}^1\,dx_1{}^2$ and $d^2x_3 = dx_3{}^1\,dx_3{}^2$. Next, due to the proximity of x_2 to x_1 and of x_4 to x_3, ensured by the factors $e^{-m|x_1-x_2|}$ and $e^{-m|x_3-x_4|}$ in

Equation (31), we can approximate there $\mathrm{e}^{-m(|x_1-x_3|+|x_1-x_4|+|x_2-x_3|+|x_2-x_4|)} \simeq \mathrm{e}^{-4m|x_1-x_3|}$. The so-emerging integral has the form

$$\int \mathrm{d}^2x_1 \sqrt{g(x_1)} \int \mathrm{d}^2x_3 \sqrt{g(x_3)}\, \mathrm{e}^{-4m|x_1-x_3|},$$

thereby differing from the integral of the form of Equation (32) by the absence of the product $t_{\mu\nu}(x_1)t_{\mu\nu}(x_3)$, where $t_{\mu\nu} = \frac{\varepsilon^{ab}}{\sqrt{g}}\partial_a x_\mu \cdot \partial_b x_\nu$ is the so-called extrinsic-curvature tensor. Fortunately, the leading, Nambu–Goto, term in the derivative expansion (32) stems from the local approximation, where both $t_{\mu\nu}$'s are considered at the same point, so that $t_{\mu\nu}^2 = 2$. Within the same approximation, we thus have

$$\int \mathrm{d}^2x_1 \sqrt{g(x_1)} \int \mathrm{d}^2x_3 \sqrt{g(x_3)}\, \mathrm{e}^{-4m|x_1-x_3|} \simeq \frac{1}{(4m)^2} \int \mathrm{d}^2z\, \mathrm{e}^{-|\mathbf{z}|} \int \mathrm{d}^2x_1 \sqrt{g(x_1)} = \frac{\pi}{8m^2} \int \mathrm{d}^2x_1 \sqrt{g(x_1)}.$$

Bringing all of the factors together, and noticing that $T^aT^a = \frac{N^2-1}{2N}\hat{1}$, we obtain, for $N = 3$, the following correction to the string tension, stemming from the action (31):

$$\Delta\sigma = -\frac{\pi^3}{9}\, G_{T=0}(0, \ldots, 0)\, \frac{\langle (gF^a_{\alpha\beta})^2\rangle^2}{m^6}. \tag{34}$$

It can be compared with the leading contribution to the string tension, which stems from the quadratic-cumulant contribution to Equation (30),

$$\langle W(C)\rangle \simeq$$

$$\frac{1}{N}\,\mathrm{tr}\,\exp\left[-\frac{1}{2^2\,2!}\int_{S_{\min}} \mathrm{d}\sigma_{\mu_1\nu_1}(x_1) \int_{S_{\min}} \mathrm{d}\sigma_{\mu_2\nu_2}(x_2)\, \langle g^2 F^a_{\mu_1\nu_1}(x_1) F^b_{\mu_2\nu_2}(x_2)\rangle\, T^aT^b\right] =$$

$$\exp\left[-\frac{\kappa\,\langle (gF^a_{\mu\nu})^2\rangle}{8N(D^2-D)} \int_{S_{\min}} \mathrm{d}\sigma_{\mu\nu}(x_1) \int_{S_{\min}} \mathrm{d}\sigma_{\mu\nu}(x_2)\, \mathrm{e}^{-m|x_1-x_2|}\right],$$

where we have used the parametrization

$$\langle g^2 F^a_{\mu_1\nu_1}(x_1) F^b_{\mu_2\nu_2}(x_2)\rangle = \kappa\, \langle (gF^a_{\mu\nu})^2\rangle\, \frac{\delta^{ab}}{(N^2-1)(D^2-D)} \left(\delta_{\mu_1\mu_2}\delta_{\nu_1\nu_2} - \delta_{\mu_1\nu_2}\delta_{\mu_2\nu_1}\right) \mathrm{e}^{-m|x_1-x_2|}$$

with $\kappa \simeq 0.83$ [20] being the parameter which determines the relative weight of *confining* self-interactions of the stochastic background fields (cf. also Ref. [14]). Setting here $D = 4$ and $N = 3$, and using Equation (32), one obtains

$$\sigma = \frac{\pi\,\kappa}{72}\, \frac{\langle (gF^a_{\mu\nu})^2\rangle}{m^2} \simeq 0.25\,\mathrm{GeV}^2 \tag{35}$$

so that

$$\frac{|\Delta\sigma|}{\sigma} = \frac{8\pi^2}{\kappa}\, G_{T=0}(0, \ldots, 0)\, \frac{\langle (gF^a_{\mu\nu})^2\rangle}{m^4}.$$

Numerically, by using Equations (28) and (29), we obtain $\frac{|\Delta\sigma|}{\sigma} \simeq 0.34$. Accordingly, the decrease of σ, due to the quartic cumulant, reads $\sigma + \Delta\sigma \simeq 0.17\,\mathrm{GeV}^2$. This corrected value of σ turns out to be closer to the standard phenomenological value of 0.19 GeV^2 than the value provided by Equation (35). This finding demonstrates the consistency of our analysis.

3. Summary

In conclusion, we have used the Stochastic Model of the Yang–Mills vacuum to explicitly obtain the leading $\mathcal{O}(T^4)$-term (26) in the high-temperature expression for the topological susceptibility of the SU(3) gluon plasma. This approach turns out to respect the general reflection positivity condition [cf. the discussion in the paragraph between Equations (23) and (24)]. Extrapolation of Equation (26) down to $T = T_c$ yields the parameter of the quartic cumulant, Equation (29). The total value of the string tension, accounting for the negative correction (34) produced by the quartic cumulant, turns out to be much closer to the standard phenomenological value than its counterpart (35) corresponding to the quadratic cumulant alone.

Funding: This research received no external funding.

Acknowledgments: The author thanks Ralf Hofmann for valuable discussions.

Conflicts of Interest: The author declares no conflict of interest.

References

1. Peccei, R.D.; Quinn, H.R. CP conservation in the presence of pseudoparticles. *Phys. Rev. Lett.* **1977**, *38*, 1440. [CrossRef]
2. Peccei, R.D.; Quinn, H.R. Constraints imposed by CP conservation in the presence of pseudoparticles. *Phys. Rev. D* **1977**, *16*, 1791. [CrossRef]
3. Shifman, M.A.; Vainshtein, A.I.; Zakharov, V.I. Can confinement ensure natural CP invariance of strong interactions? *Nucl. Phys. B* **1980**, *166*, 493. [CrossRef]
4. Preskill, J.; Wise, M.B.; Wilczek, F. Cosmology of the invisible axion. *Phys. Lett. B* **1983**, *120*, 127. [CrossRef]
5. Abbott, L.F.; Sikivie, P. A cosmological bound on the invisible axion. *Phys. Lett. B* **1983**, *120*, 133. [CrossRef]
6. Dine, M.; Fischler, W. The not-so-harmless axion. *Phys. Lett. B* **1983**, *120*, 137. [CrossRef]
7. Dosch, H.G. Gluon condensate and effective linear potential. *Phys. Lett. B* **1987**, *190*, 177. [CrossRef]
8. Di Giacomo, A.; Dosch, H.G.; Shevchenko, V.I.; Simonov, Y.A. Field correlators in QCD. Theory and applications. *Phys. Rept.* **2002**, *372*, 319. [CrossRef]
9. D'Elia, M.; Di Giacomo, A.; Meggiolaro, E. Gauge-invariant field-strength correlators in pure Yang–Mills theory and full QCD at finite temperature. *Phys. Rev. D* **2003**, *67*, 114504. [CrossRef]
10. Boyd, G.; Engels, J.; Karsch, F.; Laermann, E.; Legeland, C.; Luetgemeier, M.; Petersson, B. Thermodynamics of SU(3) lattice gauge theory. *Nucl. Phys. B* **1996**, *469*, 419. [CrossRef]
11. Gross, D.J.; Pisarski, R.D.; Yaffe, L.G. QCD and instantons at finite temperature. *Rev. Mod. Phys.* **1981**, *53*, 43. [CrossRef]
12. Shuryak, E.V. The role of instantons in quantum chromodynamics: (I). Physical vacuum. *Nucl. Phys. B* **1982**, *203*, 93. [CrossRef]
13. D'Angelo, F.; Athenodorou, A.; Bonanno, C.; Bonati, C.; Clemente, G.; D'Elia, M.; Maio, L.; Martinelli, G.; Sanfilippo, F.; Todaro, A. Topological susceptibility in high temperature full QCD via staggered spectral projectors. *Proc. Sci. (LATTICE2022)* **2023**, 393.
14. Antonov, D. Contributions of stochastic background fields to the shear and bulk viscosities of the gluon plasma. *Ann. Phys.* **2010**, *325*, 1304. [CrossRef]
15. Kornelis, W. Higher cumulants in the cluster expansion in QCD. *Nucl. Phys. Proc. Suppl. B* **2001**, *96*, 426. [CrossRef]
16. Antonov, D. Dual superconductor model of confinement: Quantum-string representation of the 4D Yang–Mills theory on a torus and the correlation length away from the London limit. *Universe* **2022**, *8*, 7. [CrossRef]
17. Shifman, M.A.; Vainshtein, A.I.; Zakharov, V.I. QCD and resonance physics. Theoretical foundations. *Nucl. Phys. B* **1979**, *147*, 385. [CrossRef]
18. Petreczky, P. Screening in hot non-Abelian plasma. *arXiv* **1999**, arXiv:hep-ph/9907247.
19. Vicari, E.; Panagopoulos, H. θ dependence of SU(N) gauge theories in the presence of a topological term. *Phys. Rept.* **2009**, *470*, 93. [CrossRef]
20. Meggiolaro, E. Field strength correlators in QCD: New fits to the lattice data. *Phys. Lett. B* **1999**, *451*, 414. [CrossRef]
21. Dürr, S.; Fodor, Z.; Hoelbling, C.; Kurth, T. Precision study of the SU(3) topological susceptibility in the continuum. *JHEP* **2007**, *4*, 55. [CrossRef]
22. Antonov, D. String nature of confinement in (non-)Abelian gauge theories. *Surveys High Energ. Phys.* **2000**, *14*, 265. [CrossRef]
23. Diamantini, M.C.; Trugenberger, C.A. Geometric aspects of confining strings. *Nucl. Phys. B* **1998**, *531*, 151. [CrossRef]
24. Antonov, D.; Diamantini, M.C. 3D Georgi–Glashow model and confining strings at zero and finite temperatures. In *From Fields to Strings: Circumnavigating Theoretical Physics*; World Scientific: Singapore, 2005; pp. 188–265.