Charged black hole in the presence of dark energy

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We discuss exact solutions of Einstein's field equations describing the charged Schwarzschild black hole in two different dark energy backgrounds. These are regarded as embedded solutions that the charged Schwarzschild black hole is embedded into the dark energy spaces producing charged Schwarzschild-dark energy black holes, as Reissner-Nordstrom-dark energy black holes. Here we consider the dark energy solutions having the equation of state parameter $w = -1/2$. It is found that the space-time geometry of solution is non-vacuum Petrov type D in the classification of space-times. We study the strong energy conditions for the energy-momentum tensors of the Reissner-Nordstrom-dark energy solution, which can be able to explain the different between the repulsive gravitational field for *dark energy* as well as the attractive gravitational field for *normal matter* like electromagnetic field. It is also shown that the metric tensor for the Reissner-Nordstrom-dark energy solution can be expressed in two different Kerr-Schild ansatze establishing the fact that the embedded dark energy solution is an exact solution of Einstein's field equations.

1. Introduction

The standard general relativistic interpretation of dark energy is based on the cosmological constant [1], which has the simplest model for a fluid with the equation of state parameter $w = p/\rho = -1$, $\rho = -p = \Lambda/K$ with $K = 8\pi G/c^4$ [2]. It turns out the cosmological constant to be the de Sitter solution with cosmological constant Λ representing a relativistic dark energy with the non-perfect fluid energy-momentum tensor $T_{ab} = 2\,\rho\,\ell_{(a}\,n_{b)} + 2\,p\,m_{(a}\bar{m}_{b)}$, whose trace is $T = 2(\rho - p)$ [3], different from the one, $T^{(\mathrm{pf})} = \rho - 3p$ of the perfect fluid energy-momentum tensor $T^{(\mathrm{pf})}_{ab} = (\rho^* + p)u_a\,u_b - p\,g_{ab}$. This suggests that a relativistic dark energy must have a line-element describing a space-time geometry having gravitational field in the form of energy-momentum tensor possessing a negative pressure with a minus sign in the equation of state parameter. It is also true that a vacuum space-time with $T_{ab} = 0$ cannot have negative pressure to determine the equation of state parameter value with minus sign. Hence, the cosmological constant in *vacuum* Einstein's field equations cannot describe the negative pressure to possess a minus sign in the equation of state parameter. The criteria of minus sign in the equation of state is to indicate the matter distribution in the space-time to be a *dark energy*, otherwise the plus sign in the equation of state is for the *normal matter*. According to the properties of de Sitter solution as relativistic dark energy, there is another relativistic dark energy solution admitting an energy-momentum tensor with the equation of state parameter $w = p/\rho = -1/2$, $\rho = 4m/Kr$, $p = -2m/Kr$, where m is a non-zero constant considered to be the mass of dark energy [4]. It is also to emphasize that the equation of state parameter $w = -1/2$ for the dark energy belongs to the range $-1 < w < 0$ focussed for the best fit with cosmological observations [5] and references there in. Here we shall refer this solution simply as dark energy solution without giving any extra prefix. It is also to note that the energy-momentum tensor with negative pressure for the de Sitter dark energy model violates the strong energy condition, showing the repulsive gravitational field of the matter, whereas the energy-momentum tensor of a normal matter field with positive pressure satisfies the strong energy condition, showing the attractive gravitational field of the matter.

In general relativity the Schwarzschild solution is regarded as a black hole in an asymptotically flat space. Its generalization is the Reissner-Nordstrom black hole. The Reissner-Nordstrom-de Sitter solution is the charged extension of the Schwarzschild-de Sitter solution which is interpreted as a black hole in an asymptotically de Sitter space with the cosmological constant Λ [6]. The Reissner-Nordstrom-de Sitter solution is also considered as an embedded black hole that the Reissner-Nordstrom solution is embedded into the de Sitter space to produce the Reissner-Nordstrom-de Sitter black hole [7]. Here we are looking for an exact solution to describe the Reissner-Nordstrom black hole in the dark energy with the parameter $w = p/\rho = -1/2$ as Reissner-Nordstrom-dark energy black hole. The embedded dark energy solution will be the generalization of Schwarzschild-dark energy black hole [8] .

Here we consider the dark energy solution possessing a non-perfect fluid energy-momentum tensor having an equation of state parameter $\omega = p/\rho = -1/2$ with negative pressure derived in [4]. For deriving the Reissner-Nordstrom-dark energy solution we adopt the mass function expressed in a power series expansion of the radial coordinate [9] as

$$\hat{M}(u,r) = \sum_{n=-\infty}^{+\infty} q_n(u)\, r^n, \tag{1}$$

where $q_n(u)$ are arbitrary functions of retarded time coordinate $u = t - r$. The mass function $\hat{M}(u,r)$ has a powerful role in generating new exact solutions of Einstein's field equations [10]. Wang and Wu [9] have utilized the mass function in deriving *non-rotating* embedded Vaidya solution into other spaces by choosing the function $q_n(u)$ corresponding to the index number n. Further utilizations of the mass function $\hat{M}(u,r)$ have been extended in rotating system and found the role of the number n in generating rotating embedded solutions of the field equations [10]. Here we shall consider the cases of the index number n as $n = 0, -1, 2$. That the value $n = 0$ corresponds to the Schwarzschild solution, $n = -1$ for the charge term and $n = 2$ for the dark energy solution possessing the equation of state parameter $\omega = -1/2$ [4]. These values of n will conveniently combine in order to obtain embedded charged Schwarzschild-dark energy black hole or Reissner-Nordstrom-dark energy black hole. The resulting solution will be an extension of the non-charged Schwarzschild-dark energy black hole discussed in [8], which is again a further extension of the work of [4] with the dark energy when $n = 2$. Here we recall conveniently that the Reissner-Nordstrom-de Sitter black hole is the combination of two solutions corresponding to the index number $n = 0, -1$ (Reissner-Nordstrom) and $n = 3$ (de Sitter), different from the Reissner-Nordstrom-dark energy solution (to be discussed here) with $n = 0, -1, 2$ in the power series expansion of mass function $\hat{M}(u,r)$.

2. Reissner-Nordstrom black hole in dark energy

In this section we shall show the derivation of an embedded Reissner-Nordstrom-dark energy solution to Einstein's field equations. This solution will describe charged Schwarzschild black hole in asymptotically dark energy background as the Reissner-Nordstrom-de Sitter black hole is regarded as a black hole in asymptotically de Sitter space [12]. For deriving an embedded Reissner-Nordstrom-dark energy solution, we choose the Wang-Wu function $q_n(u)$ in the expansion series of the mass function $\hat{M}(u,r)$ as

$$q_n(u) = \begin{cases} M, & \text{when } n = 0 \\ -e^2/2, & \text{when } n = -1 \\ m, & \text{when } n = 2 \\ 0, & \text{when } n \neq 0, -1, 2, \end{cases} \tag{2}$$

where M and e are constants. Then, the mass function takes the form

$$\hat{M}(u,r) = \sum_{n=-\infty}^{+\infty} q_n(u)\, r^n = M + r^2 m - \frac{e^2}{2r}. \tag{3}$$

Then using this mass function in general canonical metric in Eddington-Finkestein coordinate system (u, r, θ, ϕ),

$$ds^2 = \Big\{1 - \frac{2\hat{M}(u,r)}{r}\Big\} du^2 + 2du\, dr - r^2 d\Omega^2,$$

with $d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$, we find a line element

$$ds^2 \;=\; \Big[1 - r^{-2}\Big\{2r(M + mr^2) - e^2\Big\}\Big]\, du^2 + 2du\, dr - r^2 d\Omega^2, \tag{4}$$

where m is a constant regarded as the mass of the dark energy; and M and e denote the mass and the charge of Reissner-Nordstrom black hole. The line-element will reduce to that of Schwarzschild black hole when $m = e = 0$ with singularity at $r = 2M$, and also it will be that of dark energy when $M = e = 0$ having singularity at $r = (2m)^{-1}$ [4]. The line-element (4) will have a singularity when $g_{uu} = 0$, which has three roots $r = r_i$, $i = 1, 2, 3$. The Reissner-Nordstrom solution has two roots $r = r_\pm$ for $g_{uu} = 0$ when $m = 0$. It is noted that the mass m should not be considered here to be zero for the existence of the dark energy.

The null tetrad components for metric line element are obtained as follows

$$\ell_a = \delta^1_a,$$
$$n_a = \frac{1}{2}\Big[1 - \frac{1}{r^2}\Big\{2r(M + mr^2) - e^2\Big\}\Big]\,\delta^1_a + \delta^2_a,$$
$$m_a = -\frac{r}{\sqrt{2}}\left\{\delta^3_a + i\sin\theta\,\delta^4_a\right\} \tag{5}$$

with the normalization conditions $\ell_a n^a = 1 = -m_a\bar{m}^a$ and other inner products being zero. The above stationary space-time (4) possesses an energy-momentum tensor describing the interaction of dark energy with the electromagnetic field as the source of gravitational field:

$$T_{ab} \;=\; 2\,\rho\,\ell_{(a}\,n_{b)} + 2\,p\,m_{(a}\bar{m}_{b)}, \tag{6}$$

where the quantities are found as

$$\rho = \frac{4}{Kr}m + \frac{e^2}{Kr^4},$$
$$p = -\frac{2}{Kr}m + \frac{e^2}{Kr^4}. \tag{7}$$

These ρ and p are the density and pressure respectively for dark energy interacting with the electromagnetic field. Here K denotes the universal constant $K = 8\pi G/c^4$. The equation (7) indicates that the contribution of the gravitational field to T_{ab} is measured by dark energy mass m and the electric charge e of Reissner-Nordstrom solution. The energy-momentum tensor (6) is calculated from Einstein's field equations $R_{ab} - (1/2)Rg_{ab} = -KT_{ab}$ of gravitational field for the space-time metric (4) as shown in the reference [4].

As in general relativity the physical properties of a space-time geometry are determined by the nature of the matter distribution in the space, it is convenient to express the energy-momentum tensor (6) in such a way that one must be able to understand it easily in order to study the physical properties of the embedded solution. Thus, the total energy-momentum tensor (EMT) for the solution (4) may, without loss of generality, be decomposed in the following form as:

$$T_{ab} = T^{(\mathrm{e})}_{ab} + T^{(\mathrm{DE})}_{ab}, \tag{8}$$

where the EMTs for the electromagnetic field $T^{(\mathrm{e})}_{ab}$ and the dark energy $T^{(\mathrm{DE})}_{ab}$ are respectively given as:

$$T^{(\mathrm{e})}_{ab} = 2\,\rho^{(\mathrm{e})}\ell_{(a}\,n_{b)} + 2\,p^{(\mathrm{e})}m_{(a}\bar{m}_{b)}$$
$$T^{(\mathrm{DE})}_{ab} = 2\rho^{(\mathrm{DE})}\,\ell_{(a}\,n_{b)} + 2p^{(\mathrm{DE})}\,m_{(a}\bar{m}_{b)},$$

where the coefficients are as

$$\rho^{(\mathrm{e})} = p^{(\mathrm{e})} = \frac{1}{Kr^4}e^2, \tag{9}$$

$$\rho^{(\mathrm{DE})} = \frac{4}{Kr}m, \quad p^{(\mathrm{DE})} = -\frac{2}{Kr}m. \tag{10}$$

Thus, the equation of state parameters for the dark energy and the electromagnetic field are found as

$$\omega^{(\mathrm{DE})} = \frac{p^{(\mathrm{DE})}}{\rho^{(\mathrm{DE})}} = -\frac{1}{2} \tag{11}$$

$$\omega^{(\mathrm{e})} = \frac{p^{(\mathrm{e})}}{\rho^{(\mathrm{e})}} = 1. \tag{12}$$

These two equations show that the equation of state parameter for dark energy has a minus sign in (11), whereas the electromagnetic field (normal matter) has a plus sign (12) indicating the difference between dark energy and the normal matter.

The energy-momentum tensor (8) satisfies the energy conservation law [3] expressed in Newman-Penrose (NP) formalism [13]

$$T^{ab}{}_{;b} = T^{(\mathrm{e})ab}{}_{;b} + T^{(\mathrm{DE})ab}{}_{;b} = 0. \tag{13}$$

Here T^{ab} itself satisfies the conservation law. On the other hand, we also find that both $T^{(\mathrm{e})ab}$ and $T^{(\mathrm{DE})ab}$ are separately satisfied the same (vide [8]). The above equation (13) shows the fact that the metric of the line element (4) describing embedded Reissner-Nordstrom-dark energy is a solution of Einstein's field equations. We find the trace of the energy momentum tensor T_{ab} (6) as

$$T = 2(\rho - p) = \frac{12}{Kr}m. \tag{14}$$

Here we observe that $\rho - p$ must be always greater than zero for the existence of the dark energy in embedded Reissner-Nordstrom-dark energy solution (4) with $m \neq 0$, (if $\rho = p$ implies that m will vanish). It is found that the charge e does not appear in (14) showing the fact that the trace of the energy-momentum tensor for electromagnetic field always vanishes. The decomposition of energy-momentum tensor (8) indicates the interaction of electromagnetic field $T^{(\mathrm{e})}_{ab}$ with the dark energy $T^{(\mathrm{DE})}_{ab}$ in the Reissner-Nordstrom-dark energy space-time (4).

It is also convenient to write the energy-momentum tensor (6) in terms of time-like u_a $(u^a u_a = 1)$ and space-like v_a $(v^a v_a = -1)$ vector fields as

$$T_{ab} = (\rho + p)(u_a u_b - v_a v_b) - p g_{ab} \tag{15}$$

where $u_a = (1/\sqrt{2})(\ell_a + n_a)$ and $v_a = (1/\sqrt{2})(\ell_a - n_a)$. This is certainly different from the energy-momentum tensor of the perfect fluid $T^{(\mathrm{pf})}_{ab} = (\rho + p)u_a\, u_b - p\, g_{ab}$ with the trace $T^{(\mathrm{pf})} = \rho - 3p$. We observe from the energy densities and the pressures given in (9) and (10) that the energy-momentum tensors for dark energy and electromagnetic field obey the weak energy and the dominant energy conditions given in [8]. However, $T^{(\mathrm{DE})}_{ab}$ violates the strong energy condition

$$p^{(\mathrm{DE})} \geq 0, \quad \rho^{(\mathrm{DE})} + p^{(\mathrm{DE})} \geq 0 \tag{16}$$

due to the negative pressure (10), and implies that the gravitational force of the dark energy is repulsive which may cause the acceleration of the model, like the cosmological constant leads to the acceleration of the expansion of the Universe. But, the energy-momentum tensor for electromagnetic field $T^{(\mathrm{e})}_{ab}$ possessing the positive pressure (9) obeys the strong energy condition leading the attractive gravitational field

$$p^{(\mathrm{e})} \geq 0, \quad \rho^{(\mathrm{e})} + p^{(\mathrm{e})} \geq 0. \tag{17}$$

Here we establish the different between the repulsive gravitational field for *dark energy* as well as the attractive gravitational field for *normal matter* like electromagnetic field.

The non-vanishing tetrad components of Weyl tensor C^a_{bcd} of the embedded Reissner-Nordstrom-dark energy black hole (4) is found as

$$\psi_2 = \frac{1}{r^4}\Big\{ - rM + e^2\Big\}. \tag{18}$$

The Weyl scalar ψ_2 indicates that the space-time of the embedded solution (4) is Petrov type D in the classification of space-times. The mass m of the dark energy does not appear in (18) which shows the intrinsic property of the conformally flatness of the dark energy, even embedded into the Reissner-Nordstrom black hole.

The Reissner-Nordstrom-dark energy metric can be expressed in Kerr-Schild ansatz on the dark energy background

$$g^{(\mathrm{RNDE})}_{ab} = g^{(\mathrm{DE})}_{ab} + 2Q(r)\ell_a\ell_b \tag{19}$$

where $Q(r) = -Mr^{-1} + e^2 r^{-2}/2$. Here, $g^{(\mathrm{DE})}_{ab}$ is the dark energy metric and ℓ_a is geodesic, shear free, expanding and zero twist null vector for both $g^{(\mathrm{DE})}_{ab}$ as well as $g^{(\mathrm{RNDE})}_{ab}$. The above Kerr-Schild form can also be recast on the Reissner-Nordstrom background as

$$g^{(\mathrm{RNDE})}_{ab} = g^{(\mathrm{RN})}_{ab} + 2\hat{Q}(r)\ell_a\ell_b \tag{20}$$

where $\hat{Q}(r) = -mr$. These two Kerr-Schild forms (19) and (20) show the fact that the Reissner-Nordstrom-dark energy space-time (4) with the mass m of the dark energy is a solution of Einstein's field equations. It is to emphasize the fact that the two metrics $g^{(\mathrm{RN})}_{ab}$ for Reissner-Nordstrom solution and $g^{(\mathrm{DE})}_{ab}$ for dark energy cannot be added in order to obtain $g^{(\mathrm{RNDE})}_{ab}$ as $g^{(\mathrm{RNDE})}_{ab} \neq \frac{1}{2}\Big\{g^{(\mathrm{RN})}_{ab} + g^{(\mathrm{DE})}_{ab}\Big\}$. It is the fact that in general relativity two physically known solutions cannot be added to derive a new embedded solution.

3. Conclusion

In this paper we proposed an exact solution of Einstein's field equations describing the Reissner-Nordstrom black hole embedded into the dark energy space having negative pressure as Reissner-Nordstrom-dark energy black hole. This embedded solution is the straightforward generalization of Schwarzschild-dark energy solution [8]. Here we have followed the method of generating embedded solutions of Wang and Wu [9] by considering the power index n as $n = 0, -1$ and 2 in the derivation of the solution. Then we calculate all the NP quantities for the line element and find that the embedded space-time possesses an energy-momentum tensor of the electromagnetic field interacting with the dark energy having negative pressure. We have shown the difference between the dark energy and the normal matter (like electromagnetic field) that dark energy has the equation of state parameter with minus sign, whereas the normal matter has the parameter with plus sign. The energy-momentum tensor of the dark energy distribution in the embedded space-time (4) violates the strong energy condition leading to a repulsive gravitational force, whereas that of the electromagnetic field satisfies the strong energy condition producing attractive gravitation field. The metric tensor of Reissner-Nordstrom-dark energy solution is able to express in Kerr-Schild ansatze on different backgrounds (19) and (20) establishing the fact that the Reissner-Nordstrom-dark energy space-time (4) with the mass m of the dark energy is a solution of Einstein's field equations.

The decomposition (8) of energy-momentum tensor (6) indicates the interaction of electromagnetic field with the dark energy. This is one of the remarkable properties of the Reissner-Nordstrom-dark energy that two different matters of distinct physical properties are present in one energy-momentum tensor (6) as the source of gravitational field. It is also seen that the trace of T_{ab} is $T = 2(\rho - p) = 2(\rho^{(\mathrm{DE})} - p^{(\mathrm{DE})})$ which is different from that of perfect fluid $T^{(\mathrm{pf})} = \rho - 3p$. The energy-momentum tensor for the dark energy with negative pressure violates the strong energy condition while that for the electromagnetic field with positive pressure obeys the condition showing the difference between the dark energy and the normal matter (electromagnetic field). In fact the embedded solution (4) possessing a non-perfect fluid energy-momentum tensor may be an example of space-times which are enable to explain how the dark energy is different from the normal matter.

4. Acknowledgement

One of the authors (Ibohal) is thankful to the Inter-University Centre for Astronomy and Astrophysics (IUCAA), Pune for hospitalities during his visit.

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