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Study of Physical GUP-Influenced Properties of Regular Black Holes in the Context of $f(\mathcal{Q}, \mathcal{B}_{\mathcal{Q}})$ Gravity

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Abstract

This paper analyzes how the generalized uncertainty principle (GUP) affects the thermodynamic properties in a regular black hole spacetime in the context of $f(\mathcal{Q}, \mathcal{B}_{\mathcal{Q}})$ symmetric teleparallel gravity, with an arbitrary action f as a function of non-metric scalar $\mathcal{Q}$ and the boundary $\mathcal{B}_{\mathcal{Q}}$. We analyze a GUP-influenced semi-classical technique in regular black hole spacetime that incorporates the quantum tunneling mechanism. The GUP-influenced temperature results show that the GUP term reduced the vector particles' radiation in the context of $f(\mathcal{Q}, \mathcal{B}_{\mathcal{Q}})$ gravity. Moreover, we explore the GUP-influenced entropy as well as the GUP-influenced emission energy, it can help to explain the complex interactions between quantum gravity and astrophysics and highlights the important role of GUP-influenced thermodynamic properties (Hawking temperature, entropy and emission energy) in regular black hole spacetime in the context of $f(\mathcal{Q}, \mathcal{B}_{\mathcal{Q}})$ gravity. We graphically analyze the effects of different parameters on black hole geometry.

Keywords: black hole spacetime; $f(\mathcal{Q}, \mathcal{B}_{\mathcal{Q}})$ gravity; generalized uncertainty principle; tunneling mechanism in black holes; black hole thermodynamic properties

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1. Introduction

Black holes (BHs) are highly attractive and sophisticated objects that are increasingly attractive in classical and quantum gravity theories. In particular, in virtue of semi-classical quantum field theory, it was revealed that BHs are characterized by thermally distributed emission spectra. This intriguing phenomenon is known as Bekenstein–Hawking (or Hawking) radiation [1–4], now considered to be one of the primary theoretical topics of importance in fundamental physics. The GUP-influenced Bekenstein–Hawking temperature, or Hawking temperature, was significant [5–7] because it suggests that when considering quantum gravity effects, the temperature of radiation emitted by a BH Bekenstein-Hawking radiation can be modified, potentially leading to a lower temperature compared to the standard calculation. It provides insights into the nature of BHs at the quantum level; essentially, the kinetic energy of particles and quantum gravity introduced in Hawking temperature alter how particles can escape a BH, impacting its temperature. The BHs at the macroscopic level and the Heisenberg uncertainty principle at the microscopic level may be related, according to the BH uncertainty principle. The uncertainty principle can be used to analyze BHs and explain basic bounds on the accuracy with which specific pairs of physical attributes (such as momentum and position) are possible to determine simultaneously. The uncertainty principle may not just apply to minor phenomena but may also have an impact on significant, gravity-governed objects like BHs.

The theoretical analysis has been successfully demonstrated by the classified BH properties and the observation of thermodynamics [3]. One of the most significant conditions in this discipline is Hawking radiation, which characterizes the emission of particles by BHs attributed to quantum consequences. The BHs [8] are attractive special interest because of their distinct properties and their ability to shed light on basics of physics, such as the nature of spacetime and particle actions. The Heisenberg uncertainty principle implies that the results from the interaction of gravity and quantum physics are GUP-influenced [9–11]. The reliability at which some pairs of observables, such as position and momentum, may be assessed has an essential limit. The significant role of the GUP influence in BH dynamics has attracted attention [12–16]. The GUP's influence on BH dynamics changes in stability due to Hawking temperature.

In the models of GUP-influenced Hawking radiation [17–19], the temperature of the particles released drops as GUP-influenced parameters increase, indicating that a stronger GUP influence leads to an increase in Hawking temperature. This study addresses the possibility of experimentally verifying GUP-influenced astrophysics observations [20] as well as the implications of these discoveries on GUP-influenced entropy and hence the information loss of data. The Wald formula is an illustration of Bekenstein-Hawking (or Hawking) entropy, as well as the resulting solutions for BH temperature and horizon. It was concluded [21] that the theories' free parameters are used to study the heat capacities. These include a type of spherically symmetric solution that contains vacuum energy in the form of a de Sitter-like phase represented by an effective cosmological constant. A non-dynamic exterior scalar field may influence the temperature of a spacetime framework in the case of dark energy [22]. A (quasi) anti-de Sitter phase has been used [23] to study the features of two new regular spacetimes with a non-zero vacuum energy term. The points considered, this study can help to clarify the complex interactions between quantum gravity and astrophysics and highlights the crucial role of the GUP's influence in the thermal emission of non-asymptotically flat BHs. In gravity models of bumblebees [24,25], BH radiation was suggested to be the temperature of emitted particles that decreases as GUP-influenced parameters increase. A larger GUP-influence resulted in the minimization of the Hawking temperature and illustrates the significant role that the GUP plays in the thermal emission of stationary BHs that are not asymptotically flat. It may also provide insight into the complex dynamics between BH physics and quantum gravity.

In $f(\mathcal{Q}, \mathcal{B}_Q)$ gravity associated with non-linear electrodynamics, we consider spherically symmetric and static regular BHs. To review BH solutions and ensure that solutions obtained maintain Lorentz symmetry, we generated a model of BHs, allowing with the boundary term or non-metricity scalar to obtain metric functions and also describe analytical approaches for specific non-linear electrodynamics Lagrangian action.

The regular BHs are investigated for extended $f(\mathcal{Q}, \mathcal{B}_Q)$ gravity solutions. Firstly, we study the field equation of a boson from the GUP-influenced Lagrangian given by the Glashow–Weinberg–Salam model. Then we apply the Hamilton-Jacobi ansatz and Wentzel–Kramers–Brillouin (WKB) approximation to the derived equation in $f(\mathcal{Q}, \mathcal{B}_Q)$ gravity-like regular spacetime. By setting the determinant of the derived coefficient matrix to zero, a set of linear equations are shown to be solved for the radial function. Then, we compute the tunneling rate of the particles from the $f(\mathcal{Q}, \mathcal{B}_Q)$ gravity-like regular spacetime and recover the GUP-influenced Hawking temperature. This demonstrates that standard entropy to the $f(\mathcal{Q}, \mathcal{B}_Q)$ gravity BHs can be calculated from the logarithmic entropy under the impact of the GUP, which gives intriguing results in both factors of $f(\mathcal{Q}, \mathcal{B}_Q)$ gravity and GUP's influence. The results of emission energy demonstrate the effect of $f(\mathcal{Q}, \mathcal{B}_Q)$ gravity on the geometry of the BH as well as being GUP-influenced. Such gravity theories have an impact on the stability of regular BHs.

This study is organized as follows. In Section 2, we include an explanation of the $f(\mathcal{Q}, \mathcal{B}_Q)$ gravity BHs and consider some of their basic features. Section 3 is dedicated to calculating the GUP-influenced Hawking temperature of the bosonic tunneling from the $f(\mathcal{Q}, \mathcal{B}_Q)$ gravity BH. In Sections 4 and 5, we compute the GUP-influenced entropy and GUP-influenced emission energy for the $f(\mathcal{Q}, \mathcal{B}_Q)$ gravity BHs using the GUP-influenced temperature. Section 6, we summarize the findings. Natural units $G = c = \hbar = k_B = 1$ are used throughout the paper for the Newtonian gravitational constant G, the speed of light c, the reduced Planck constant $\hbar$, and the Boltzmann constant k_B.

2. Introductory Review of Black Hole Spacetime in the Context of $f(\mathcal{Q}, \mathcal{B}_Q)$ Gravity

In this study, we present solutions predicated on a recent proposal by expanding the $f(\mathcal{Q})$ gravity, specifically, $f(\mathcal{Q}, \mathcal{B}_Q)$ gravity [26,27], with $\mathcal{B}_Q$ being the boundary component obtained in Ref. [28]. In this strategy, the action is stated as

$$I_{f(\mathcal{Q},\mathcal{B}_Q)} = \int \sqrt{-g}\mathrm{d}^4x[f(\mathcal{Q}, \mathcal{B}_Q) + 2k^2\mathbb{L}_m], \tag{1}$$

where g is the determinant of the 4-dimensional metric tensor, x is the 4-coordinate, and $\mathbb{L}_m$ represents the matter field Lagrangian. We examine $f(\mathcal{Q}, \mathcal{B}_Q)$ gravity combined with non-linear electrodynamics, the action is performed by

$$I = \int \sqrt{-g}\mathrm{d}^4x[f(\mathcal{Q}, \mathcal{B}_Q) + 2k\mathbb{L}(\mathcal{F})]. \tag{2}$$

with the Lagrangian density illustrated in the non-linear electrodynamics represented by $\mathbb{L}(\mathcal{F})$, which depends on the electromagnetic scalar $\mathcal{F}$:

$$\mathcal{F} = \frac{1}{4}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu}, \tag{3}$$

with the electromagnetic field $\mathcal{F}_{\mu\nu}$, which is described as

$$\mathcal{F}_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu, \tag{4}$$

with Greek letter indices for temporal (0) and space (1,2, and 3) coordinates. This constitutes the antisymmetric Maxwell-Faraday tensor, where $\mathcal{A}_\beta$ represents the magnetic vector potential. The static and spherically symmetric spacetime used to obtain the results that we explore as a result in the framework of regular BHs by the line element by

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{1}{f(r)}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\chi^2), \tag{5}$$

in the spherical coordinates with $f(r)$ the space-time function. There are numerous approaches to simplify the complicated problem of solving motion equations. As a starting point, let us model the boundary factor considered in Ref. [28] and imply that it is a regular quantity to solve the differential equation. That is, take this specific boundary term by

$$\mathcal{B}_Q(r) = -\frac{r^2\acute{\acute{f}}(r) + 6r\acute{f}(r) + 4f(r) - 2}{r^2}, \tag{6}$$

where the prime symbol on the top denotes the r-derivative. The non-metricity scalar has the form

$$\mathcal{Q}(r) = -\frac{2\left(r\acute{f}(r) + f(r)\right)}{r^2}. \tag{7}$$

The solution's regularity is determined by the combination $\mathcal{Q} - \mathcal{B}_Q = \mathcal{R}$, where $\mathcal{R}$ is the Ricci scalar and the non-metricity scalar $\mathcal{Q}$ has been studied in $f(\mathcal{Q})$ theory [29]. For the BH solution and to maintain Lorentz symmetry, we consider the following relation as

$$f_{\mathcal{Q}} = -f_{\mathcal{B}}. \tag{8}$$

The symmetric teleparallel with the special case, $f(\mathcal{Q}, \mathcal{B}_Q) = \mathcal{Q} - \mathcal{B}_Q$, is contained in Equation (8), which is identically satisfied even if $f_{\mathcal{Q}} = 1$ and $f_{\mathcal{B}} = -1$. Consistently, we impose scenario (8) in the coincident gauge. The boundary term can be defined as

$$\mathcal{B}_{\mathcal{Q}}(r) = b_0/r^2. \tag{9}$$

The BH solution can be determined via Equation (6) as

$$\frac{b_0}{r^2} = -\frac{r^2\acute{\acute{f}}(r) + 6r\acute{f}(r) + 4f(r) - 2}{r^2}. \tag{10}$$

In Ref. [28], the spacetime function is given as

$$f(r) = \frac{2 - b_0}{4} - \frac{2\mathcal{M}}{r} + \frac{b_1}{r^4}, \tag{11}$$

where $\mathcal{M}$ denotes the BH mass and b_0 and b_1 are the constants. By inserting $b_0 = -2$ and $b_1 = 0$ in Equation (11), one is able to obtain the Schwarzschild spacetime function. The spacetime function is asymptotically flat in the limit $r \to \infty$ and diverges in the limit $r \to 0$. The values of parameters b_0 and b_1 have significant effects on BH electromagnetic and gravity features. This reflects a deviation from basic metric invariance, with b_0 being the Lorentz symmetry-breaking factor. Changes in dispersion relations and particle interactions, which significantly affect BH generation and evaporation, are indicated by a non-zero b_0. Thermal properties and Hawking radiation are both impacted by the parameters b_0 and b_1. Magnetic charge and electromagnetic interactions are also associated with b_1 affecting the stability, magnetic field, and BH charge. Non-linear electromagnetic effects, magnetic monopoles, and topological features are also impacted by the parameter b_1.

The concept of Hawking temperature with surface gravity expressed as [30]

$$T_H = \frac{\mathcal{M}}{2\pi r_+^2} - \frac{b_1}{\pi r_+^5}. \tag{12}$$

Thus, the Hawking temperature is associated with b_1, magnetic charge and electromagnetic properties are associated with b_1), the mass $\mathcal{M}$, and the horizon radius r_+. Moreover, if $b_1 = 0$, Hawking temperature (12) changes into the Schwarzschild temperature (i.e., $T_{Sc} = \frac{1}{8\pi\mathcal{M}}$ when $r_+ = 2\mathcal{M}$).

Figure 1 shows Hawking temperature T_H against horizon radius r_+ for varying values of mass $\mathcal{M}$ (Figure 1a) and parameter b_1 (Figure 1b) and explores how T_H changes when its parameters are varied the mass M and the parameter b_1, which can influence the properties of the BH or corrections to the standard model. All curves start at zero or near zero when r_+ is relatively small, then rise sharply to the peak temperature, and then gradually decrease as r_+ increases. By increasing the mass $\mathcal{M}$, the peak temperature shifts downward, and the peak itself shits towards a larger radius r_+ of the event horizon. The Schwarzschild curve lies above all the correction curves, indicating that the correction or parameter b_1 reduce the peak temperature. A significant characteristic that is associated with the BHs maximum thermal emission is the T_H peak. Larger BHs are colder, as seen by the temperature dropping after the T_H peak as the event horizon's radius expands. The phenomenon of temperature decreasing as r_+ increases is consistent with classical BH thermodynamics: the lower the temperature, the larger and heavier the black hole, that is, the large black holes emit less radiation and have a tendency to become thermodynamically stable. The peak temperature and its variation with mass $\mathcal{M}$ indicate that mass influences thermal stability and evaporation characteristics. The greater the mass of a BH, the lower its maximum temperature and the lower its radiation intensity. The parameter b_1 modifies the standard behavior (Schwarzschild law) and can represent quantum corrections, thus reducing the overall temperature.

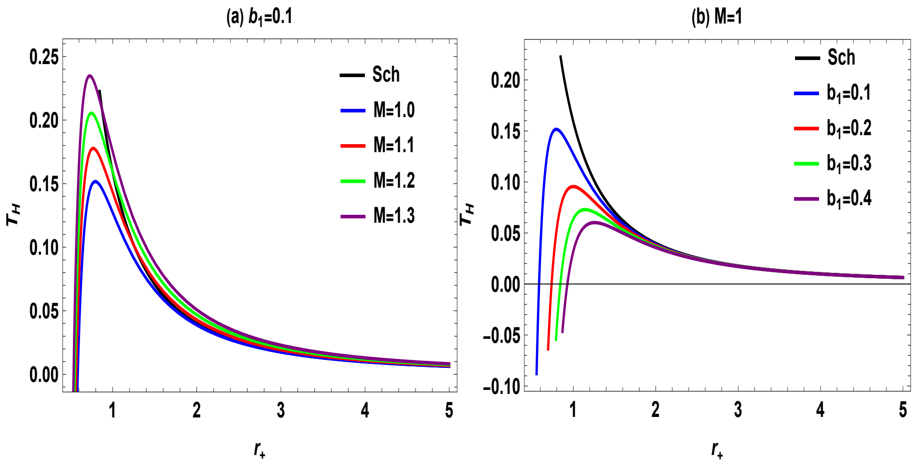

Figure 1. Hawking temperature T_H (12) against horizon radius r_+ for (**a**) fixed value of b_1 and varying values of mass $\mathcal{M}$, and (**b**) fixed value of $\mathcal{M}$ and varying parameter b_1.

From Figure 1b, one sees that the temperature of a BH (Schwarzschild) is initially quite high when r_+ is comparably small and decreases monotonically without assuming getting negative values. As b_1 increases, the T_H value drops sharply and forms an apparent negative region when r_+ is relatively small, which is physically unusual. The temperature eventually increases from quite large negative values of r_+ and approaches zero from below. Increasing b_1 shifts the curve downward, indicating a greater deviation from standard Schwarzschild behavior. Negative temperatures are not neglectable: they suggest instability thermodynamic behavior, which may indicate that at smaller event horizon radii the system is not thermodynamically stable or that the model fails. By Increasing b_1, lowers the Hawking temperature and introduces the anomalous negative region, which may be related to some physical effects or quantum gravity.

Exceptionally massive BHs stay cold when r_+ is relatively high because the temperature goes to zero, which is in line with classical predictions. The amplitude and qualitative behavior of T_H are altered by b_1, which implies that this parameter regulates essential characteristics such as coupling constants or corrections that transcend classical general relativity.

The Hawking temperature associated with the magnetic charge and electromagnetic interactions in spacetime is the physical phenomenon that is connected with the mathematical observation of $b_1 > 0$. As well, the third law of thermodynamics is violated by the negative temperature. The earlier studeis have already investigated negative temperatures in BHs [31–34].

3. GUP's Influence on Tunneling Radiation

In this paper, we investigate the Hawking radiation generated by mass particles in the $f(\mathcal{Q}, \mathcal{B}_{\mathcal{Q}})$ gravity theory arising from the BH configuration. Spin-1 particles such as $W^{\pm}$ bosons have a significant impact on the Glashow-Weinberg-Salam Model [35]. The GUP-influenced Lagrangian equation depicts the motion of boson field in the $f(\mathcal{Q}, \mathcal{B}_{\mathcal{Q}})$ gravity BH geometry, taking into account the massive bosons. This has been accomplished by studying the tunneling technique of Hawking radiation and using the Hamilton–Jacobi ansatz to investigate the Hawking temperature close to the horizon of BH and its modified form. To calculate the entropy and emission energy of the $f(\mathcal{Q}, \mathcal{B}_{\mathcal{Q}})$ gravity BH, we take into account GUP-influenced temperature, which suggests highly attractive results and BH evaluation.

In loop quantum gravity and string theory, for non-commutative geometry, among the theories of quantum gravity that all possess the smallest detectable length are studied in Refs. [36–39]. This smallest length can be achieved with the GUP's influence [40–42]. A suitable successful GUP-influenced description, which involves the central point of significant additional dimensions in one-dimensional quantum theory, has been shown [43] as

$$L_f k_i(P) = \tanh\left(\frac{P}{\mathcal{M}_f}\right), \quad (13)$$

$$L_f w_1(\mathbb{E}) = \tanh\left(\frac{\mathbb{E}}{\mathcal{M}_f}\right), \quad (14)$$

for momentum P and energy $\mathbb{E}$ with L_f, $\mathcal{M}_f$, and κ_i denoting the smallest detectable length of higher-dimensional space, Planck mass, and wave vector, respectively. The changes in frequency in space and time are related to the w_1 operator. Then the equality $L_f\mathcal{M}_f = \hbar$ is satisfied for L_f and $\mathcal{M}_f$. Position $\hat{s} = s$ approximates a real value can be derived as

$$w_1 = i\partial_t, \quad \kappa_i = -i\partial_s, \quad (15)$$

where $\partial_a \equiv \partial/\partial a$. The relatively small energy limitation $P \ll \mathcal{M}_f$ allows the consideration on the order of $\left(P/\mathcal{M}_f\right)^3$ as

$$\mathbb{E} \simeq i\hbar\partial_t\left(1 - \sigma\hbar\partial_t^2\right), \quad (16)$$

$$P \simeq -i\hbar\partial_s\left(1 - \sigma\hbar\partial_s^2\right), \quad (17)$$

with $\sigma = 1/(3\mathcal{M}_f^2)$. Thus, the updated commutation connection is represented by

$$[s, P] = i\hbar\left(1 + \sigma P^2\right), \quad (18)$$

and the expanded the uncertainty connection relation reads

$$\Delta s \Delta P \geq \frac{\hbar}{2}\left[1 + \sigma(P^2)\right]. \quad (19)$$

with $\sigma = \sigma_0 l_p^2/\mathcal{M}_f$, where $\sigma_0 < 10^5$ represents the dimensionless parameter [44,45], $l_p = G\hbar/c^2$ is the Plank length, and the Planck mass $\mathcal{M}_f = \sqrt{\hbar c/G}$. Equations (18) and (19) indicate in a straightforward way that GUP's influence by the Heisenberg uncertainty concept raises the corresponding particle's momentum. Note that Equations (16)–(19) are limited to the case explored in that investigation, such as the minimal energy limitation $P \gg \mathcal{M}_f$. To demonstrate the uncertainty concept, simulations like those proposed in

Refs. [46,47] must restrict the value of α to the minimal energy space. Several generalized uncertainty interactions have been identified in earlier analyses [36,41]. Considering the Hamilton–Jacobi methodology, which takes into consideration the smallest length factor via Equations (16) and (17), we consider the GUP-influenced tunneling of immense spin-1 particles via BH horizons. Based on the algorithms used here, the mass of the BH and the mass and angular momentum of the boson particles that are emitted by BH are found to be related to the GUP-influenced modification. The quantum correction eliminates the standard ability for a temperature increase at a certain stage throughout evaporation, leading to the appearance of remnants and delaying the rate at which the temperature expands through the BH evaporation mechanism.

We explore GUP-influenced tunneling beyond a spacetime horizon via the GUP-influenced Lagrangian equation. Due to the real and imaginary computation, the mass of the particle affects the GUP-influenced Hawking temperature for the GUP-influenced tunneling particle. Considering a minor term in the roots of an exponential equation with physical significance suggests that the GUP-influenced Hawking temperature of spacetime is affected by the mass and kinetic energy of the discharged particle. The realistic analysis illustrates that the $f(\mathcal{Q},\mathcal{B}_{\mathcal{Q}})$ gravity of the emitted particle determines the GUP-influenced Hawking temperature for the vector that is GUP-influenced tunneling over the space-time structure. The kinetic factor of the uncharged vector boson field in flat spacetime corresponds the GUP-influenced concept is continued to. The $\frac{1}{2}\mathbb{B}_{\mu\nu}\mathbb{B}^{\mu\nu}$ is the modified field strength tensor [48], which is generated by $\mathbb{B}^{\mu\nu}$ defined as

$$\mathbb{B}_{\mu\nu} = \left(1-\sigma\hbar^2\partial_\mu^2\right)\partial_\mu\mathbb{B}_\nu - \left(1-\sigma\hbar^2\partial_\nu^2\right)\partial_\nu\mathbb{B}_\mu\,, \tag{20}$$

with D_μ a covariant derivatives in the gauge principle, where the derivatives have implications of the local unitary transformation operator $U(s)$ [49]. It must be stated that there are further derivative terminologies. The context of a vector field $(\mathtt{W}^{\pm})$ in BH spacetime is generalized as

$$\left(1-\sigma\hbar^2\partial_0^2\right)\partial_0 \quad\rightarrow\quad \left(1+\sigma\hbar^2 f^{00}D_0^{\pm 2}\right)D_0^{\pm}, \tag{21}$$

$$\left(1-\sigma\hbar^2\partial_i^2\right)\partial_i \quad\rightarrow\quad \left(1-\sigma\hbar^2 f^{ii}D_i^{\pm 2}\right)D_i^{\pm}. \tag{22}$$

The distinction in signatures of the $O(\sigma)$ factor in Equations (21) and (22) can be verified through the statement that the f^{00} and f^{ii} typically have distinct signatures, which can be significant and are defined as

$$D_0^{\pm} = \left(1+\sigma\hbar^2 f^{00}D_0^{\pm 2}\right)D_0^{\pm} \;\text{ and }\; D_i^{\pm} = \left(1-\sigma\hbar^2 f^{ii}D_i^{\pm 2}\right)D_i^{\pm}.$$

In the boson field $(\mathtt{W})$, the Lagrangian GUP-influenced action is defined as

$$\mathcal{I}^{\text{GUP}} = -\frac{1}{2}\left(D_\mu^{+}\mathtt{W}_\nu^{+} - D_\nu^{+}\mathtt{W}_\mu^{+}\right)\left(D^{-\mu}\mathtt{W}^{-\nu} - D^{-\nu}\mathtt{W}^{-\mu}\right) - \frac{m_{\mathtt{W}}}{\hbar}\mathtt{W}_\mu^{+}\mathtt{W}^{-\mu}, \tag{23}$$

where $m_{\mathtt{W}}$ denotes the $\mathtt{W}$ mass. Therefore, the proper extensive action that is required to be implemented is

$$\mathcal{S}^{\text{GUP}} = \int d^4s\sqrt{-g}\,\mathcal{I}^{\text{GUP}}\left(\mathtt{W}_\mu^{\pm},\, \partial_\mu\mathtt{W}_\nu^{\pm},\, \partial_\rho\partial_\mu\mathtt{W}_\nu^{\pm},\, \partial_\rho\partial_\mu\partial_\lambda\mathtt{W}_\nu^{\pm}\right). \tag{24}$$

We find that the regular BH radiation is significant within the framework of $f(\mathcal{Q},\mathcal{B}_{\mathcal{Q}})$ theory of gravity. We presented the thermodynamic properties of the BH. There are meaningful differences in studying the Hawking temperature of regular BH within the $f(\mathcal{Q},\mathcal{B}_{\mathcal{Q}})$

gravity compared to other BHs. We compute the GUP-influenced tunneling rate of the boson particles, we characterize the GUP-influenced temperature utilizing a semi-classical approach. The GUP-influenced tunneling method for symmergent BH and Kiselev-like AdS $f(R,\ T)$ gravity spacetime has been studied by Riasat Ali and colleagues [50,51], and the GUP-influenced temperature for the corresponding geometry of BHs has been obtained. The BH's stability feature and the physical significance of the GUP-influenced Lagrangian equation are examined. The GUP-influenced parameter is the field equation without a singularity that is extended as a standard model. To investigate vector tunneling radiation, we apply Lagrangian action via the boson field $\mathtt{W}_\mu$. The Lagrangian GUP-influenced equation is examined in terms of its physical relevance. The field equation is extended as a Lagrangian GUP-influenced field equation without a singularity. We analyze the boson radiation phenomena using the Lagrangian GUP-influenced equation of actions, incorporating the boson field as

$$\partial_\mu(\sqrt{-f}\,\mathtt{W}^{\nu\mu}) + \sqrt{-f}\frac{m^2}{\hbar^2}\mathtt{W}^\nu + \sigma\hbar^2\partial_0\partial_0\partial_0(\sqrt{-f}f^{00}\mathtt{W}^{0\nu}) - \sigma\hbar^2\partial_i\partial_i\partial_i(\sqrt{-f}f^{ii}\mathtt{W}^{i\nu}) = 0, \quad (25)$$

with the determinant of the coefficient matrix, antisymmetric tensor, and particle mass denoted by f, m, and $\mathtt{W}$. The antisymmetric tensor then is recognized as

$$\mathtt{W}_{\nu\mu} = (1 - \sigma\hbar^2\partial_\nu^2)\partial_\nu\mathtt{W}_\mu - (1 - \sigma\hbar^2\partial_\mu^2)\partial_\mu\mathtt{W}_\nu. \quad (26)$$

The constituents of $\mathtt{W}^\mu$ and $\mathtt{W}^{\mu\nu}$ to be measured as

$$\mathtt{W}^0 = \frac{1}{f(r)}\mathtt{W}_0, \quad \mathtt{W}^1 = f(r)\mathtt{W}_1, \quad \mathtt{W}^2 = \frac{1}{r^2}\mathtt{W}_2, \quad \mathtt{W}^3 = \frac{1}{r^2\sin^2\theta}\mathtt{W}_3, \quad \mathtt{W}^{01} = -\mathtt{W}_{01},$$
$$\mathtt{W}^{02} = \frac{1}{f(r)r^2}\mathtt{W}_{02}, \mathtt{W}^{03} = \frac{1}{f(r)r^2\sin^2\theta}\mathtt{W}_{03}, \quad \mathtt{W}^{12} = \frac{f(r)}{r^2}\mathtt{W}_{12}, \quad \mathtt{W}^{13} = \frac{f(r)}{r^2\sin^2\theta}\mathtt{W}_{13},$$
$$\mathtt{W}^{23} = \frac{1}{r^4\sin^2\theta}\mathtt{W}_{23}.$$

The investigation of BHs is crucial for the concept of quantum gravity implications, so a number of BH experiments have implemented the use of the GUP's influence. The thermodynamic structure of BH has been looked at in the structure of the GUP's influence [48]. Considering the incorporation of the GUP-influenced tunneling algorithm, the GUP-influenced tunneling probability of a Schwarzschild BH was explored in Ref. [52]. The GUP-influenced Hamilton–Jacobi model for bosons in the curved shape of spacetime is included in [48], associated with the GUP-influenced Hawking temperatures estimated for many different spacetime structures. Despite this attribute naturally resulting in a regained mass in the BH evaporated state, the implications of quantum gravity were demonstrated to minimize the growth in GUP-influenced Hawking temperatures via boson GUP-influenced tunneling analysis. The WKB technique is stated by [53]

$$\mathtt{W}_\nu = a_\nu \exp\left[\frac{i}{\hbar}A_0(t,r,\theta,\chi) + \Sigma\hbar^n A_n(t,r,\theta,\chi)\right], \quad (27)$$

in this instance, a_ν is the constant term, and $(A_0,\ A_n)$ reptesent arbitrary functions. Then, the system of equatons obtained by omitting the higher orders in the GUP-influenced Lagrangian (25), while only up to the first-order in $\hbar$ in the WKB approximation (27) reads

$$
\begin{aligned}
& f(r)\left[a_1(\partial_0 A_0)(\partial_1 A_0)+\sigma a_1(\partial_0 A_0)^3(\partial_1 A_0)-a_0(\partial_1 A_0)^2-\sigma a_0(\partial_1 A_0)^4\right] \\
& +\frac{1}{r^2}\left[a_2(\partial_0 A_0)(\partial_2 A_0)+\sigma a_2(\partial_0 A_0)^3(\partial_2 A_0)-a_0(\partial_2 A_0)^2-\sigma a_0(\partial_2 A_0)^4\right] \\
& +\frac{1}{r^2 \sin^2\theta}[a_3(\partial_0 A_0)(\partial_3 A_0)+\sigma a_3(\partial_0 A_0)^3(\partial_3 A_0)-a_0(\partial_3 A_0)^2-\sigma a_0(\partial_3 A_0)^4] \\
& -a_0 m^2=0,
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
& -\frac{1}{f(r)}\left[a_0(\partial_0 A_0)(\partial_1 A_0)+\sigma a_0(\partial_0 A_0)(\partial_1 A_0)^3-a_1(\partial_0 A_0)^2-\sigma a_1(\partial_0 A_0)^4\right] \\
& +\frac{1}{r^2}\left[a_2(\partial_1 A_0)(\partial_2 A_0)+\sigma a_2(\partial_1 A_0)^3(\partial_2 A_0)-a_1(\partial_2 A_0)^2-\sigma a_1(\partial_2 A_0)^4\right] \\
& +\frac{1}{r^2 \sin^2\theta}\left[a_3(\partial_1 A_0)(\partial_3 A_0)+\sigma a_3(\partial_1 A_0)^3(\partial_3 A_0)-a_1(\partial_3 A_0)^2-\sigma a_1(\partial_3 A_0)^4\right] \\
& -a_1 m^2=0,
\end{aligned} \tag{29}
$$

$$
\begin{aligned}
& \frac{1}{f(r)}\left[a_0(\partial_0 A_0)(\partial_2 A_0)+\sigma a_0(\partial_0 A_0)(\partial_2 A_0)^3-a_2(\partial_0 A_0)^2-\sigma a_2(\partial_0 A_0)^4\right] \\
& -f(r)\left[a_2(\partial_1 A_0)^2+\sigma a_2(\partial_1 A_0)^4-a_1(\partial_1 A_0)(\partial_2 A_0)-\sigma a_1(\partial_1 A_0)(\partial_2 A_0)^3\right] \\
& +\frac{1}{r^2 \sin^2\theta}\left[a_3(\partial_2 A_0)(\partial_3 A_0)+\sigma a_3(\partial_2 A_0)^3(\partial_3 A_0)-a_2(\partial_3 A_0)^2-\sigma a_2(\partial_3 A_0)^4\right] \\
& -m^2 a_2=0,
\end{aligned} \tag{30}
$$

$$
\begin{aligned}
& \frac{1}{f(r)}\left[a_0(\partial_0 A_0)(\partial_3 A_0)+\sigma a_0(\partial_0 A_0)(\partial_3 A_0)^3-a_3(\partial_0 A_0)^2-a_3\sigma(\partial_0 A_0)^4\right] \\
& +f(r)\left[a_3(\partial_1 A_0)^2+\sigma a_3(\partial_1 A_0)^4-a_1(\partial_1 A_0)(\partial_3 A_0)-\sigma a_1(\partial_1 A_0)(\partial_3 A_0)^3\right] \\
& +\frac{1}{r^2}\left[a_3(\partial_2 A_0)^2+\sigma a_3(\partial_2 A_0)^4-a_2(\partial_2 A_0)(\partial_3 A_0)-\sigma a_2(\partial_3 A_0)^3(\partial_2 A_0)\right] \\
& -m^2 a_3=0.
\end{aligned} \tag{31}
$$

The concept of variable split is explored by

$$
A_0=-\Omega_E t+\mathtt{W}(r,\theta)+\Theta\chi, \tag{32}
$$

where Θ and E relation show the particles' angular momentum and angle χ interrelation with energy, if one considers $\Omega_E=(E-\Theta\Omega)$. The technique that is applied follows insertion of Equation (32) into Equations (28)–(31) to arrive at a matrix of the order 4×4 by

$$
H(a_0,a_1,a_2,a_3)^T=0. \tag{33}
$$

It appears that the matrix H is not trivial with the components as follows:

$$\begin{aligned}
H_{00} &= \mathtt{W}_r^2 f(r) + \sigma \mathtt{W}_r^4 f(r) - \frac{1}{r^2}(\Theta^2 + \sigma\Theta^4) + \frac{1}{r^2\sin^2\theta}(\mathtt{W}_\theta^3 + \sigma \mathtt{W}_\theta^4) - m^2,\\
H_{01} &= -f(r)(\Omega_E + \sigma\Omega_E^3)\mathtt{W}_r,\\
H_{02} &= -\frac{1}{r^2}(\Omega_E + \sigma\Omega_E)\Theta,\\
H_{03} &= -\frac{1}{r^2\sin^2\theta}(\Omega_E + \sigma\Omega_E^3)\mathtt{W}_\Theta,\\
H_{10} &= \frac{\Omega_E \mathtt{W}_r + \sigma\Omega_E \mathtt{W}_r^3}{f(r)},\\
H_{11} &= \frac{\Omega_E^2 + \sigma\Omega_E^4}{f(r)} - \frac{1}{r^2}(\Theta^2 - \sigma\Theta^4) - \frac{1}{r^2\sin^2\theta}(\mathtt{W}_\theta^2 - \sigma \mathtt{W}_\theta^4) - m^2,\\
H_{12} &= \frac{1}{r^2}(\mathtt{W}_r + \sigma \mathtt{W}_r^3)\Theta, \;\; H_{13} = \frac{1}{r^2\sin^2\theta}(\mathtt{W}_r + \sigma \mathtt{W}_r^3)\mathtt{W}_\theta,\\
H_{20} &= -\frac{1}{f(r)}(\Omega_E\Theta + \sigma\Omega_E\Theta^3)\\
H_{21} &= f(r)(\mathtt{W}_r\Theta + \sigma \mathtt{W}_r\Theta^3),\\
H_{22} &= \frac{1}{f(r)}(\Omega_E^2 + \sigma\Omega_E^4) - f(r)(\mathtt{W}_r^2 + \sigma \mathtt{W}_r^4) - \frac{1}{r^2\sin^2\theta}(\mathtt{W}_\theta^2 + \sigma \mathtt{W}_\theta^4) - m^2,\\
H_{23} &= \frac{1}{r^2\sin^2\theta}(\Theta + \sigma\Theta^3)\mathtt{W}_\theta,\\
H_{30} &= -\frac{1}{f(r)}(\mathtt{W}_\theta + \sigma \mathtt{W}_\theta^3)\Omega_E,\\
H_{31} &= -\; f(r)(\mathtt{W}_\theta + \sigma \mathtt{W}_\theta^3)\mathtt{W}_r, \;\; H_{32} = -\frac{1}{r^2}(\Theta + \sigma\Theta^3)\mathtt{W}_\theta,\\
H_{33} &= -\frac{1}{f(r)}(\Omega_E^2 + \sigma\Omega_E^4) + f(r)(\mathtt{W}_r^2 + \sigma \mathtt{W}_r^4) - \frac{1}{r^2}(\Theta^2 + \sigma\Theta^4) - m^2,
\end{aligned}$$

where $\Theta = \partial_\chi A_0$, $\mathtt{W}_r = \partial_r A_0$, and $\mathtt{W}_\theta = \partial_\theta A_0$. Setting H to null leads to the imaginary part obtained, the determination of H is a nontrivial matrix outcome determined by

$$\mathrm{Im}\mathtt{W}^+ = \int \sqrt{\frac{\Omega_E^2 + \mathcal{Z}_1\left[1 + \sigma\frac{\mathcal{Z}_2}{\mathcal{Z}_1}\right]}{f^{-1}(r)}}dr, \tag{34}$$

with the BH emitted and BH absorbed by particles designated by positive and negative signs, respectively. The $\mathcal{Z}_1$ and $\mathcal{Z}_2$ terms are given as follows:

$$\mathcal{Z}_1 = \frac{\Theta^2}{r^2}, \quad \text{and} \quad \mathcal{Z}_2 = \frac{\sigma\Omega_E^4}{f(r)} - \sigma \mathtt{W}_r^4 f(r) + \frac{\sigma\Theta^4}{r^2} + m^2,$$

govern the angular velocity at the regular BH horizon in the framework of $f(\mathcal{Q}, \mathcal{B}_\mathcal{Q})$ gravity. Equation (34) implies

$$\mathrm{Im}\mathtt{W}^+ = \pi\frac{\Omega_E}{2\kappa(r_+)}(1 + \sigma\Xi), \tag{35}$$

where $\kappa(r_+)$ is the surface gravity of the regular BH outer horizon in the context of $f(\mathcal{Q}, \mathcal{B}_\mathcal{Q})$ gravity, and the kinetic energy component at the radiation-producing position at the tangent path of the horizon region is denoted by Ξ. Pair formation is suggested as a mathematical explanation for tunneling GUP-influenced radiation in regular BH with $f(\mathcal{Q}, \mathcal{B}_\mathcal{Q})$ gravity. The two particles in a pair may be separated in a region with strong gravitational pulls, in which they can annihilate at the regular BH horizon in the context of $f(\mathcal{Q}, \mathcal{B}_\mathcal{Q})$ gravity. According to quantum theory, annihilation happens when a negative particle tunnels within the BH and a tunnel of positive particles in the outside regular BH horizon in the context of $f(\mathcal{Q}, \mathcal{B}_{\mathcal{Q}})$ gravity as a form of radiation. Then, the tunneling probability for boson particles in GUP-influenced BH reads

$$T(\mathtt{W}^{+}) = \exp(-4\mathrm{Im}\mathtt{W}^{+}) = \exp\left(-2\pi\frac{\Omega_E}{\kappa(r_+)}\right)(1 + \Xi\sigma). \tag{36}$$

By incorporating the Boltzmann factor $T_{\mathtt{W}} = \exp(-\Omega_E/T'_H)$, one assesses the Hawking GUP-influenced temperature for regular BHs under the influence of $f(\mathcal{Q}, \mathcal{B}_{\mathcal{Q}})$ gravity as

$$T'_H = \left(\frac{\mathcal{M}}{2\pi r_+^2} - \frac{b_1}{\pi r_+^5}\right)(1 - \sigma\Xi). \tag{37}$$

Further, we recognize the Hawking GUP-influenced temperature (37) is affected by regular BHs with $f(\mathcal{Q}, \mathcal{B}_{\mathcal{Q}})$ gravity as well as boson particle kinetic energy. The first-order GUP-influenced correction in the case has to be lower than the standard (zero-order) term, whereas maintaining the GUP-influenced correction in the case is the same as the semi-classical initial Bekenstein–Hawking term. The $f(\mathcal{Q}, \mathcal{B}_{\mathcal{Q}})$ gravity parameter (b_1), arbitrary parameter (Ξ), GUP-influenced parameter (σ), regular BH mass $\mathcal{M}$, and regular BH outer radius (r_+) all affect the T'_H. Furthermore, we recover the initial temperature of the regular BH in the framework of $f(\mathcal{Q}, \mathcal{B}_{\mathcal{Q}})$ gravity while neglecting the GUP-influenced parameter ($\sigma = 0$) in Equation (37). The GUP-influenced tunneling radiation is specifically referred to as GUP-influenced Hawking radiation in the context of regular BHs. It is significant that BHs can emit particles due to quantum mechanical effects near the event horizon to tunnel out of the BH, which leads to a gradual loss of mass and potential evaporation of the BH over time. This phenomenon is crucial for understanding the quantum nature of gravity and the thermodynamic properties of BHs.

In the case of $f(\mathcal{Q}, \mathcal{B}_{\mathcal{Q}})$ gravity-like regular spacetime, the GUP-influenced action makes the T'_H of a BH unstable. In an expression of boson particle mass, Θ is particle's angular momentum and θ is displayed by the X, so that

$$\Xi = \left(\frac{6\Theta_\chi^2 \csc^2\theta + 6\Theta_\theta^2}{r_+^2} + 6m^2\right) > 0. \tag{38}$$

The GUP-influenced temperature is generated by incorporating Equation (38) into Equation (37) as

$$T'_H = \left(\frac{\mathcal{M}}{2\pi r_+^2} - \frac{b_1}{\pi r_+^5}\right)\left(1 - \frac{6\sigma\Theta_\chi^2 \csc^2\theta + 6\sigma\Theta_\theta^2}{r_+^2} + 6\sigma m^2\right). \tag{39}$$

The GUP-influenced temperature (39) provides us with more details about the thermodynamic nature of regular BHs. If the GUP's influence is omitted, then temperature (39) rises during evaporation, and the GUP's influence gradually brings the temperature back to equilibrium. Since the GUP-influenced tunneling technique provides us with a more generalized dynamic model of the regular BH radiation, back-reaction implications are not included while considering the regular BH evaporation process. Despite various strategies of predicting the thermodynamic features of regular BHs being realized, it is crucial to recognize the dynamic nature of the emission and interactions. Numerous of these considerations include the Hawking temperature, which is the basic regular structure parameter. The first of the GUP-influenced tunneling technique's most significant aspects is the ability to be used to assess GUP-influenced regular BH radiation in the framework of $f(\mathcal{Q}, \mathcal{B}_{\mathcal{Q}})$ gravity. Considering the boson field specifies the temperatures for $2 \leq r_+ \leq 5$ Table 1, which shows that the temperature in the GUP-influenced example and the temperature without the GUP's influence support the first principle of thermodynamics. Thanks to the constancy of relative particle kinetic energy, the standard temperature exceeds the GUP-influenced temperature. The data in Table 1 was used to study the reduction in GUP-influenced temperature. The temperature in a regular BH with $f(\mathcal{Q},\mathcal{B}_{\mathcal{Q}})$ gravity thermodynamics refers to a transition within exclusive thermodynamic phases that are often connected with the GUP-influenced factor.

Table 1. The BH temperatures T_H (12) and T'_H (37) for distinct GUP-influenced levels and the $f(\mathcal{Q},\mathcal{B}_{\mathcal{Q}})$ gravity parameters with $\Xi = 1$. The outer horizon is set to $2 \leq r_+ \leq 5$ despite the regular BH horizons are in distinct regions.

r_+	$\mathcal{M}$	b_1	T_H	σ	T'_H
2 $\mathcal{M}_{\text{Sc}}$	1	0	≈0.03979	0.5	≈0.01989
3	2	0.1	≈0.03523	0.2	≈0.02819
4	2.5	0.5	≈0.02472	0.3	≈0.017304
5	3	1	≈0.01899	0.4	≈0.01139

Figure 2 depicts the corrected temperature T'_H (37) against horizon radius r_+ for varying values of the GUP gravity parameter σ (Figure 2a) and parameter b_1 (Figure 2b). Figure 2a exhibits that the peak temperature decreases with increasing σ. For larger values of r_+, the temperature curve approaches zero more quickly as r_+ increases. As σ increases, the peak shifts to some degree to larger r_+ values. The role of the GUP parameter σ is to suppress the Hawking temperature correction. Stronger effects of quantum gravity are associated with larger values of σ, which reduces the temperature at smaller event horizon radii. Due to quantum gravitational adjustments, the temperature of the BH is lower, since the effective temperature is always lower than the Schwarzschild BH temperature. The increase in temperature indicates that quantum processes are starting to contribute and that a critical radius is reached, which may indicate a slowdown in evaporation or the formation of a BH remnant.

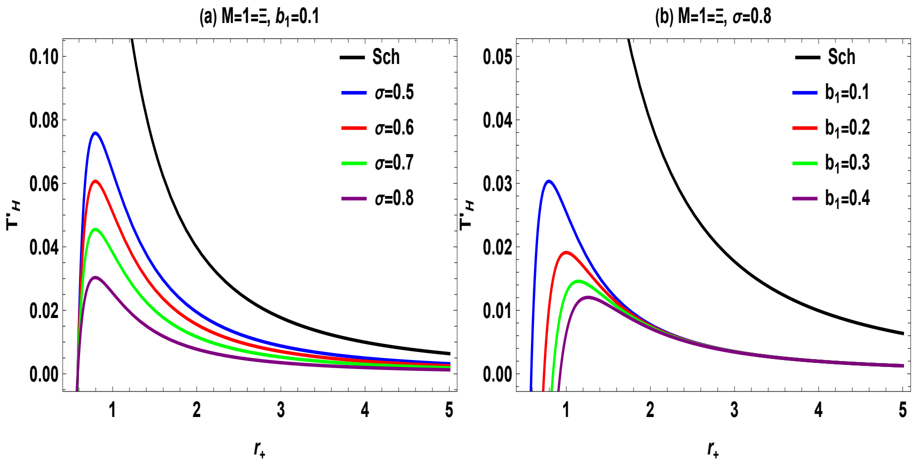

Figure 2. Corrected temperature T'_H (37) against horizon radius r_+ for (**a**) fixed values of mass $\mathcal{M}$, kinetic energy Ξ and parameter b_1, and varying GUP gravity parameter σ, and (**b**) fixed values of mass $\mathcal{M}$, kinetic energy Ξ and GUP gravity parameter σ, and varying parameter b_1, as indicated. See text for more details.

In Figure 2b, increasing b_1 lowers the peak temperature and causes the curve to flatten more rapidly towards zero. As b_1 increases, the position of the temperature peak shifts to some degree towards larger event horizon radii. The parameter b_1 adapts the strength of the quantum correction to the event horizon radius. Larger values of b_1 increase the temperature suppression, which is consistent with stronger quantum effects or GUP effects. Similar to σ, the parameter b_1 also indicates a mechanism that prevents the temperature from decaying at small enough radii, reinforcing the concept of a minimum length scale or BH remnant. At larger values of b_1, the tendency of the temperature towards zero at smaller values of r_+ implies that the BH evaporation has a stable endpoint, which is a key feature of quantum gravity.

4. GUP-Influenced Entropy

For modern thermodynamic and statistical physics studies, particularly in BH physics and quantum gravity, logarithmic entropy modifications are crucial. These corrections, which provide details about the microscopic states influencing the system's entropy, emerge spontaneously in systems with several quantum effects. In the entropy–area relation, those effects often appear as minor components represented as a logarithmic function of a system characteristic like area or volume. In quantum field theory and string theory, where variations around classical configurations result in modifications to the conventional entropy formula, the logarithmic entropy modification has theoretical support. According to research on BHs, these modifications may reveal the basic quantum structure of spacetime [54]. In statistical mechanics, the logarithmic correction improves the classical Boltzmann entropy [55] by taking into account finite-dimensional effects and quantum fluctuations. Entropy corrections have been examined in Refs. [56–58] using the null geodesic technique. We calculate the modified entropy of a BH with $f(\mathcal{Q}, \mathcal{B}_\mathcal{Q})$ gravity using the Bekenstein–Hawking entropy formula for a first-order correction [59]. We obtain the logarithmic corrected entropy of a BH with $f(\mathcal{Q}, \mathcal{B}_\mathcal{Q})$ gravity using the corrected formula for the temperature T'_H and the standard entropy $\mathbb{S}_0$ as follows:

$$\mathbb{S}_f = \mathbb{S}_0 - \frac{1}{2}\ln\left|T'^2_H \mathbb{S}_0\right| + \dots . \tag{40}$$

The first term in Equation (40) represents the standard entropy for a BH with $f(\mathcal{Q}, \mathcal{B}_\mathcal{Q})$ gravity:

$$\mathbb{S}_0 = \frac{A_f}{4}, \tag{41}$$

where

$$\begin{aligned} A_f &= \int_0^{2\pi}\int_0^{\pi} \sqrt{g_{\theta\theta}g_{\phi\phi}}\, d\theta d\phi, \\ &= 4\pi r^2. \end{aligned} \tag{42}$$

That is, the standard entropy for a BH with $f(\mathcal{Q}, \mathcal{B}_\mathcal{Q})$ gravity is obtained as

$$\mathbb{S}_0 = \pi r^2. \tag{43}$$

Upon inserting the expressions (43) and (37) into Equation (40), one obtains the corrected entropy as follows:

$$\mathbb{S}_f = \pi r^2 - \frac{1}{2}\ln\left|\frac{1}{\pi r_+^2}\left(\frac{\mathcal{M}}{2} - \frac{b_1}{r_+^3}\right)^2 (1-\sigma\Xi)^2\right| + \dots \tag{44}$$

The expression (44) gives the corrected entropy for a BH with $f(\mathcal{Q}, \mathcal{B}_\mathcal{Q})$ gravity depending on mass $\mathcal{M}$, GUP gravity parameter σ, arbitrary parameter Ξ, and constant parameter b_1.

Figure 3 represents corrected entropy S_f (40) against horizon radius r_+ for varying values of the GUP gravity parameter σ Figure 3a and parameter b_1 Figure 3b. Figure 3a shows that the Schwarzschild entropy increases uniformly and monotonically with r_+, in accordance with the classical Bekenstein–Hawking relation. As σ increases, the corrected entropy starts to deviate significantly from the Schwarzschild entropy: when r_+ is comparably small, the entropy S_f is initially negative or considerably low, then increases sharply. For all non-zero values of σ, the entropy shows a significant peak or divergence at $r_+ \approx 1.0$. After this peak, the entropy stabilizes at a value higher than the Schwarzschild entropy. The entropy curve typically swings higher as σ increases, indicating an increase in the entropy value. The peak indicates a thermodynamic phase transition or critical behavior of the BH caused by the gravitational parameter σ. This behavior indicates a change in stability. For exceptionally small values of r_+, negative entropy is unexpected, indicating nonphysical states or that quantum corrections may cause the classical entropy system to break down. As σ increases, extra corrections are brought that may cause the entropy of the BH to exceed the value predicted through the classical theory. This behaviour may also indicate modifications in the gravity and the importance of a more complicated thermodynamic framework.

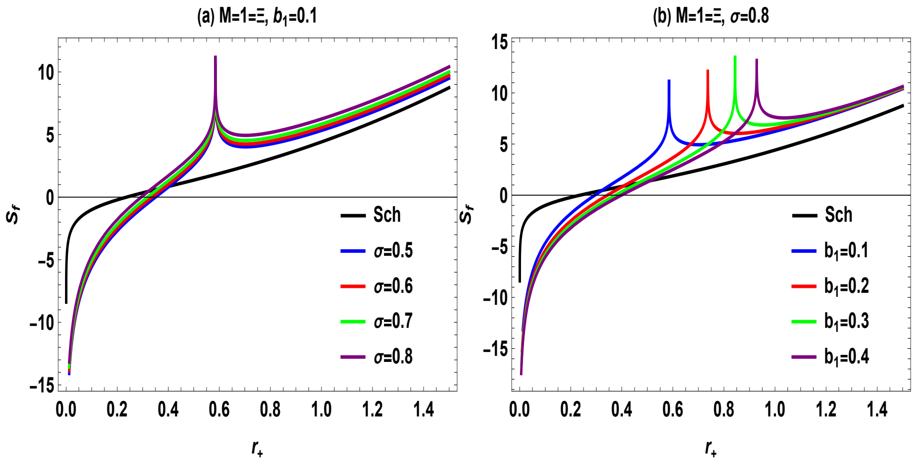

Figure 3. Corrected entropy S_f (44) against horizon radius r_+ for (**a**) fixed values of mass $\mathcal{M}$, kinetic energy Ξ and b_1 parameter and varying GUP gravity parameter σ and (**b**) fixed mass $\mathcal{M}$, kinetic energy Ξ and GUP gravity parameter σ, and varying parameter b_1 as indicated.

In Figure 3b, as b_1 increases, the entropy curve fluctuates to the right and shows a visible peak or divergence when the value of r_+ is quite large. The increase in degrees of freedom manifests itself as a sharp increase in entropy above the Schwarzschild entropy after the peak. The entropy value before the peak is smaller than the Schwarzschild entropy value, indicating more complex thermodynamic behavior. As b_1 increases, the peak becomes more pronounced and shifts to higher values of r_+. The steep peak indicates that b_1 has a significant effect on the phase transition or instability, indicating that this parameter controls the prevailing behavior. The increase in b_1 delays the transition to a higher r_+, suggesting that b_1 controls the timing of the BH undergoing this thermodynamic change. The overall increase in entropy after the transition implies an improvement in microstructure or a correction to quantum gravity. Lower entropy before the peak indicates that the microstate is suppressed, which may indicate a non-equilibrium state.

5. GUP-Influenced Emission Energy

The rate at which a BH generates energy to Hawking radiation is in direct relation to the GUP-influenced Hawking temperature, which is the region where a distant observer's light becomes dim by the BH's gravity. A $f(\mathcal{Q}, \mathcal{B}_{\mathcal{Q}})$ gravity may suggest a change rate of energy from regular spacetime. This gravity arises from the fact that a regular spacetime radius corresponds to an emission event horizon, which influences the Hawking temperature and rate of radiation.

We study how $f(\mathcal{Q}, \mathcal{B}_{\mathcal{Q}})$ gravity affects BH GUP-influenced emission energy. The absorb cross-section changes at a limiting constant (C_l) and an applicable boson particle radius by [60–64]

$$C_l \approx \pi r_+^2, \tag{45}$$

with r_+ depicts the regular BH outer radius in the context of $\mathcal{B}_{\mathcal{Q}}$ gravity and GUP-influenced energy emission rate as.

$$\frac{\mathrm{d}^2 E}{\mathrm{d}w\mathrm{d}t} = \frac{2\pi C_l w^3}{\exp(w/T'_H) - 1}. \tag{46}$$

Here, $E = \frac{d^2E}{\mathrm{d}w\mathrm{d}t}$ and w are the GUP-influenced emission energy and boson particle frequency, respectively. By combining Equations (45) and (46), one constructs a GUP-influenced emission energy formulation by

$$E = \frac{2\pi^3 r_+^2 w^3}{\exp\left(w/T'_H\right) - 1}. \tag{47}$$

We predicted the GUP-influenced emission energy in expression (47) in the $f(\mathcal{Q}, \mathcal{B}_{\mathcal{Q}})$ gravity parameter of the regular BH function that corresponds to a boson particle propagating under the influence of $f(\mathcal{Q}, \mathcal{B}_{\mathcal{Q}})$ gravity and boson particle frequency. Equation (47) estimates the boson particle content at infinity of a boson field distributed in the regular BH spacetime.

Figure 4 shows energy emission E (47) against photon frequency ω for varying values of the GUP gravity parameter σ Figure 4a and parameter b_1 ω (Figure 4b) with fixed radius $r_+ = 1.2$, mass, and arbitrary parameter $\mathcal{M} = \Xi = 1$. Figure 4a shows for each value of σ, the energy emission rate E peaks at a certain photon frequency ω. As σ increases from 0.1 to 0.5, the peak value of E continues to decrease. As σ increases, the peak frequency shifts to some degree to lower values of ω. For lower values of σ, the overall ejection rate curve becomes flatter, with the tail extending to higher frequencies. The parameter σ may represent a correction to the gravitational field or an additional parameter related to gravity (possibly related to quantum corrections or alternative theories of gravity). A larger value of σ reduces the maximum energy emission rate, indicating that the effect of stronger gravitational parameters suppresses the BH's photon emission. As σ increases, the peak frequency decreases to lower values, indicating that the BH's photons are relatively soft as the gravitational influence increases. A higher value of σ may indicate that the BH is more "stable" in terms of radiation loss, which is a behavior that may reflect how gravitational variations affect the BH Hawking radiation spectrum.

Figure 4b shows that, similar to σ, when b_1 increases from 0.1 to 0.5, the peak energy emission E decreases. The peak frequency ω shifts to lower values as b_1 increases. The shape of the curve is similar to Figure 4a. Increasing b_1 leads to reduced emission and a more uniform energy distribution. In extended gravitational theories, the parameter b_1 may represent the coupling constant, spin correction, charge, or other characteristics of the BH. As b_1 increases, the intensity of Hawking radiation decreases due to the decrease in emissivity. The BHs that encounter b_1 may have a lower effective temperature or altered event horizon characteristics, which affect the emission of photons, as can be seen from the shift in peak frequency and the emission reduction. This means that b_1 stabilizes the BH from a thermodynamic perspective, reducing the energy produced and possibly extending the lifetime of the BH.

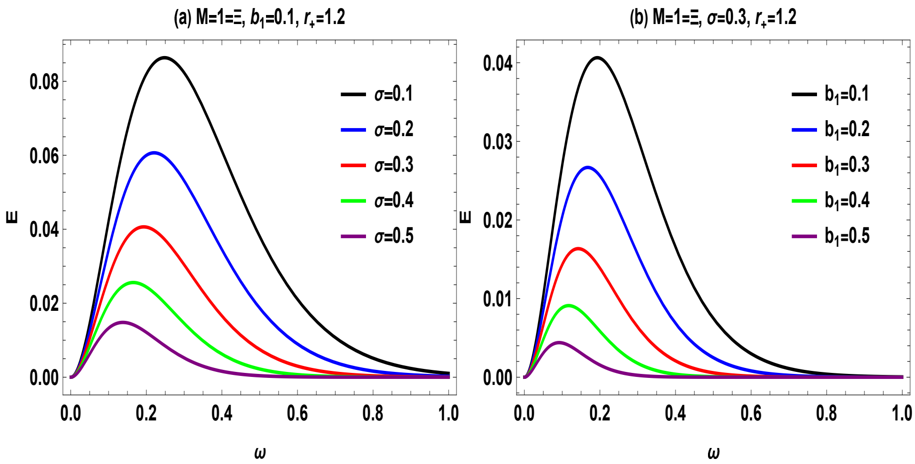

Figure 4. Energy emission rate E (47) against photon frequency ω for (**a**) for fixed values of mass $\mathcal{M}$, horizon radius r_+, kinetic energy Ξ and b_1 parameter and varying values of GUP gravity parameter σ and (**b**) for fixed values of mass $\mathcal{M}$, kinetic energy Ξ, GUP parameter σ and horizon radius r_+ and varying parameter b_1.

6. Conclusions

In the regular BH spacetime in the context of $f(\mathcal{Q}, \mathcal{B}_\mathcal{Q})$ gravity, the theory of BH thermodynamics has been studied by implementing semi-classical methodology. The key objective of this study was to calculate the GUP-influenced temperature, GUP-influenced entropy, and GUP-influenced emission energy of the regular BH including $f(\mathcal{Q}, \mathcal{B}_\mathcal{Q})$ gravity. First, we considered the GUP-influenced Lagrangian equation in which we implemented the WKB approximation for standard spacetime coordinates.

For the smallest order, WKB has been implemented to make a set of GUP-influenced field equations, and also the concept of a variable split to obtain a nontrivial matrix equation. We have considered components of the matrix to observe the imagined boson particle action and tunneling in the boson field. We have incorporated the Boltzmann factor to obtain the GUP-influenced Hawking temperature. The GUP-influenced temperature was shown to be consistent with the standard BH universality. It should be emphasized that the back-reaction effects of the emitted boson on the BH geometry, as well as self-gravitational interactions, were reasonably neglected [65]. The derived GUP-influenced temperature was calculated only to leading order terms. A more complete analysis can compute the boson tunneling probability and the GUP-influenced temperature by fully incorporating the conservation of charge and energy. There is no need to solve the semi-classical Einstein field equations for the structure of the context BH spacetime in equilibrium due to its Hawking radiation [66,67]. Moreover, we have analyzed the effects of various parameters on the BH geometry and temperatures in the presence and absence of the quantum gravity parameter.

Finally, we used GUP-influenced temperature to study the GUP-influenced entropy and GUP-influenced emission energy for regular BH spacetime in the context of $f(\mathcal{Q}, \mathcal{B}_\mathcal{Q})$ gravity. Furthermore, we have studied the effects of the GUP gravity parameter and BH geometry constant parameter b_1 on corrected entropy and GUP-influenced emission energy and discussed the stable conditions of $f(\mathcal{Q}, \mathcal{B}_\mathcal{Q})$ gravity BHs.

The Hawking temperature is a well highly-motivated feature of BH; specifically, the semi-classical model has been studied to provide the uncertainty principle by taking the Hawking radiation to $M87^*$. These intriguing issues are left for further study. It is the first to give precise constraints on new physics in the semi-classical model for BH radiation using the Event Horizon Telescope perspective of $M87^*$. More specifically, by combining theory with observation, our method is a little but essential step towards the analysis of gravity theory in the vector-field regime.

Author Contributions: Conceptualization, R.A.; methodology, R.A.; software, R.B.; validation, R.A.; investigation, T.X. and R.B.; resources, R.B.; data curation, T.X.; writing—original draft, R.A. and R.B.; visualization, R.A. and R.B.; project administration, T.X.; funding acquisition, T.X. All authors have read and agreed to the published version of the manuscript.

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