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Kylar L. Greene

Candidate

Physics and Astronomy

Department

This dissertation is approved, and it is acceptable in quality
and form for publication:

Approved by the Dissertation Committee:

Dr. Francis-Yan Cyr-Racine

Chair

Dr. Rouzbeh Allahverdi

Member

Dr. Darcy Barron

Member

Dr. Gordan Krnjaic

Member

Dr. Dinesh Loomba

Member

**Challenging Λ CDM: unraveling cosmic distances, dark sector phenomenology,
and alternative primordial B-mode sources**

BY

Kylar L. Greene

B.S., Astrophysics, University of New Mexico, 2019

M.S., Physics, University of New Mexico, 2021

DISSERTATION

Submitted in Partial Fulfillment
of the Requirements for the Degree of
Doctor of Philosophy

Physics

The University of New Mexico
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May, 2025

DEDICATION

This thesis would not have been possible without so many lovely and supportive people who have pushed and inspired me to pursue this passion, even when they might not have understood it themselves. Therefore, there are naturally many people I would like to dedicate this work to, and I want to start with the strongest and most kind person I know, my mom. You instilled in me the love of learning, and the confidence that I could become whatever I set my mind too. Your endless love and patience is reason I get to write this dedication. Thanks mom, for everything you've done to help me grow to the person I am today, and for always reading my abstracts.

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*Try not to become a man of success,
but rather try to become a man of value.*

- Albert Einstein

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*The key is to keep company only with people who uplift you,
whose presence calls forth your best.*

- Epictetus

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Challenging Λ CDM: unraveling cosmic distances, dark sector phenomenology, and alternative primordial B-mode sources

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The dominant Λ Cold Dark Matter (Λ CDM) cosmological model, while remarkably successful, increasingly shows signs that it may not fully describe our Universe, as persistent tensions in expansion rates and structure formation remain unresolved. In this thesis, I challenge the Λ CDM paradigm using novel theoretical frameworks combined with rigorous numerical analyses. I demonstrate that the expansion-rate tension fundamentally reflects underlying distance disagreements, and that the Thomson scattering rate strongly restricts higher pre-recombination expansion rates without additional physics. Further, I present a novel cosmological model using a mirror dark sector and varying fundamental constants, revealing an observational degeneracy allowing significantly higher expansion rates while remaining consistent with data. Lastly, I show that detectable B-mode polarization signals can originate from phase-transition-induced gravitational waves, challenging the long-held view of primordial B-modes as definitive evidence for inflation. Collectively, this work advances our understanding of cosmic tensions and suggests clear avenues for future research.

Contents

1	Introduction	1
1.1	A brief history of cosmology	1
1.2	Motivation and context	5
1.3	Research objectives and thesis outline	6
1.4	Contributions of this thesis	8
2	Cosmological background	10
2.1	The metric	10
2.1.1	A euclidean example	10
2.1.2	Minkowski space	12
2.1.3	The FLRW metric	14
2.2	The expanding universe	17
2.2.1	Einstein equations	17
2.2.2	The Friedmann equations	19
2.3	Distance measures and redshift	22
2.4	Local distance ladder	25
2.4.1	Anchor measurements	25
2.4.2	Calibrating standard candles	28
2.4.3	SNe Ia	30
2.5	CMB distance measurements	32
2.6	The Λ cold dark matter (Λ CDM) model	33
3	Cosmological observables	36
3.1	Big Bang nucleosynthesis (BBN)	36
3.1.1	Conditions before BBN ($t \lesssim 1$ s)	37
3.1.2	Weak freezeout ($t \sim 1 - 10$ s)	37
3.1.3	Neutrino decoupling and electron-positron annihilation ($t \sim 1 - 20$ s)	38

3.1.4	The deuterium bottleneck ($t \sim 100$ s)	40
3.1.5	The BBN reactor ($t \sim 100 - 300$ s)	42
3.1.6	Observational tests and primordial abundances	44
3.2	Cosmic microwave background	45
3.2.1	Physical origin of the CMB	46
3.2.2	Recombination	46
3.2.3	The CMB blackbody	49
3.2.4	CMB anisotropies	50
3.2.5	The TT spectrum	53
3.2.6	The EE spectrum	55
3.2.7	The TE spectrum	57
3.3	Lyman alpha forest	58
3.4	Baryon acoustic oscillations	60
4	Monte Carlo methods	62
4.1	Bayesian basics	63
4.2	MCMC algorithms	66
4.2.1	Metropolis-Hastings	66
4.2.2	Hamiltonian	68
4.2.3	MCMC in cosmology	71
5	Cosmological questions	74
5.1	The Hubble tension	74
5.1.1	Late Universe approaches	74
5.1.2	Early Universe approaches	78
5.2	The S_8 tension	80
5.3	Other tensions	81
6	Hubble distancing: Focusing on distance measurements in cosmology . .	83

6.1	Introduction	83
6.2	Implementation and usage	86
6.3	Consistency with previous results	90
6.4	A new metric to assess the tension between data sets	92
6.5	Equivalence with gaussian prior for early-time dynamics	95
6.6	Models impacting late-time cosmology: issues with gaussian H_0 prior	96
6.6.1	Test case: A sudden dark energy transition at late times	97
6.6.2	Model setup	98
6.6.3	Taking into account the distance measurements: results	99
6.7	Discussion	103
6.8	Conclusion	105
7	Thomson scattering: one rate to rule them all	107
7.1	Introduction	107
7.2	The FFAT scaling	109
7.2.1	Free Fall	110
7.2.2	Amplitude	113
7.2.3	Thomson	114
7.2.4	Cosmological anchors: breaking the scaling invariance	115
7.2.5	Avoiding symmetry-breaking with a mirror world dark sector	117
7.3	Fundamental constant variation	119
7.3.1	Thomson cross section rescaling	119
7.3.2	Fundamental constant variation constraints	120
7.3.3	Recombination with fundamental constant variation	122
7.4	Phenomenological and numerical implementation	125
7.4.1	Mimicking the MWDS in CLASS	125
7.4.2	Testing our implementation: the ionization history	127
7.4.3	Explicitly exploring the FFAT symmetry direction	130

7.5	Results	130
7.5.1	Naturally exploring the FFAT scaling	130
7.5.2	Relationship with other work	133
7.6	Conclusion	136
8	A ratio-preserving approach to cosmological concordance	139
8.1	Introduction	139
8.2	The synchronous mirror recombination model	142
8.2.1	Atomic dark matter	142
8.2.2	Thermal history of mirror sector	144
8.2.3	Phenomenological changes to $\dot{\kappa}$ using FCV	148
8.3	Methodology	149
8.4	Results and discussion	151
8.4.1	Profile likelihood with <code>Procoli</code>	157
8.4.2	Cold mirror sector volume effects	158
8.4.3	Additional mirror sector freedom	159
8.4.4	Constant binding energy	165
8.5	Conclusion	165
9	Observable CMB B-Modes from cosmological phase transitions	168
9.1	Introduction	168
9.2	B-mode polarization	169
9.3	Tensor power spectrum from bubble collisions	174
9.4	Numerical results	183
9.5	A complementary GW signal	186
9.6	Discussion	188
10	Conclusion	190
10.1	The need to question established paradigms	191

11 Appendices	193
A Exact scaling transformations for the generalized Peeble’s C-factor . . .	194
B Constraints on FCV	196
C Comments on BBN and FCV	199
D Single and double bubble spectra	201
E Scalar perturbation bounds on cosmological PTs	203
References	206

1 Introduction

Dwell on the beauty of life.

Watch the stars, and see yourself running with them.

- Marcus Aurelius

1.1 A brief history of cosmology

Humanity has always been driven by a profound curiosity to understand their origins and place in the cosmos. The oldest surviving *written* accounts of a cosmology appear in ancient Mesopotamia, where second-millennium BCE clay tablets describe the Universe emerging from a primordial watery chaos, leading to the creation of their gods, the Earth, and ultimately humankind [1]. Although such mythic narratives differ substantially from predictive modern physics-based models, they mark some of humanity’s first recorded attempts to systematically explain their place in the Universe and reflect a persistent curiosity that would, over millennia, evolve into the observational and theoretical framework now termed “modern cosmology.”

In many ways, the field of modern cosmology began with Albert Einstein’s development of the field equations describing general relativity in 1915 [2]. These equations establish a fundamental connection between energy and spacetime, succinctly summarized by John Wheeler: “Matter tells spacetime how to curve; spacetime tells matter how to move” [3]. Consistent with the prevailing scientific consensus at the time, Einstein supported the notion of a static Universe that was neither contracting or expanding. However, the field equations predicted a dynamic Universe. To align his results with the established paradigm at the time, Einstein introduced an additional term, the cosmological constant Λ , to counteract the predicted dynamics of the field equations.

However, other physicists were poised to challenge this paradigm of an unchanging cosmic order, where Alexander Friedmann [4] and Georges Lemaître [5] found solutions to the field equations indicating that the Universe could be non-static (expanding or contracting). Coincidentally, less than two decades after the inclusion of the general relativity field equations into our collective physical knowledge, observational evidence emerged supporting that the Universe was indeed expanding. That observational evidence originated with Edwin Hubble’s 1929 discovery of the relationship between galaxies recessional velocities and their distances, revealing that galaxies further away from us are receding at a greater rate [6]. In light of this discovery, Einstein referred to his inclusion of Λ as his “greatest blunder,” but as we will see, modern cosmological observations have revived Λ as a critical component of the most popular cosmological model today.

The world of cosmology was poised for a paradigm shift after the technological advancements stemming from the end of World War II and the ongoing space race. This paradigm shift came in the form of the discovery of perhaps the most important observable available to cosmologists today. The discovery of the Cosmic Microwave Background (CMB) radiation in 1965 by Arno Penzias and Robert Wilson provided the first glimpse at the data set that would eventually grow to be the foundation of modern precision cosmology [7]. Ironically, Penzias and Wilson’s discovery was largely by accident. Penzias and Wilson were experimenting with the Holmdel Horn Antenna [8], constructed by Bell Telephone laboratories to support NASA’s “Project Echo” as a passive satellite communication device [9]. In their initial data reductions, they consistently found a continuous, low level noise in their data that was not attributable to the known systematics of the telescope system. Initially, they thought this could be due to the pigeons that had taken up residence in the horn antenna, but after disregarding squatters’ rights and evicting the unsuspecting birds, the signal remained. Discussions with Robert Dicke, Jim Peebles, and David Wilkinson revealed that they were observing a real signal that had profound implications for our understanding of

the Universe. Indeed, Penzias and Wilson had made the first confirmed observation of the radiation field left over from a much earlier time when the Universe was significantly smaller and hotter. Subsequent measurements quickly confirmed the near-perfect blackbody spectrum of the CMB, strongly supporting the hot Big Bang model and effectively ruling out competing steady-state theories [10]. The discovery and subsequent characterization of the CMB thus established one of cosmology’s most foundational observables, cementing the Big Bang paradigm as the standard cosmological framework used today.

A few decades of steady progress later, in November of 1989, the Cosmic Background Explorer (COBE) satellite was launched, carrying the Far-Infrared Absolute Spectrophotometer (FIRAS) and the Differential Microwave Radiometer (DMR) [11, 12]. These two instruments proved to be essential in the development of cosmological models, both at the time and nearly 40 years later today. FIRAS confirmed that the spectrum of the CMB precisely matched blackbody phenomenology and theoretical predictions, while improving the accuracy of the temperature of the CMB today by over a factor of 100 compared to previous tests [13, 14]. Meanwhile, DMR provided the first detection of large scale anisotropies ($\ell \lesssim 30$) in the CMB, confirming that small amplitude fluctuations exist in the otherwise nearly perfectly uniform CMB, providing the initial seeds for structure formation. These observational discoveries further supported the idea that the early Universe was thermalized and well mixed, perhaps forecasting the homogeneous and isotropic assumptions used so ubiquitously in cosmology today.

Then, in 1998 and 1999, two independent research teams, the *Supernova Cosmology Project* (SCP) lead by S. Perlmutter [15] and the *High-Z Supernova Search Team* (HZT) lead by A. Riess and B. Schmidt [16], utilized data from ground based observatories and the new, state-of-the-art Hubble space telescope to analyze the luminosity-redshift relationship of distant Type Ia supernovae (SNe Ia). SNe Ia serve as standardizable candles, meaning

their intrinsic luminosities are predictable and can be derived after calibration. Then, if the intrinsic brightness is known, observations of the same type of object give how bright it *appears* to be, and a distance can be calculated by comparing the difference. The SCP and HZT teams discovered that distant SNe Ia appeared fainter than expected when assuming a Universe dominated by matter, indicating that the cosmic expansion is in fact *accelerating*. This groundbreaking result, which eventually culminated with a trip to Stockholm, provided the first compelling observational evidence that Einstein’s cosmological constant, initially introduced and later dismissed as his “greatest blunder”, may indeed represent a real, physical component of the Universe. Remarkably, the cosmological constant found renewed significance as the most straightforward explanation for the observed acceleration of expansion in the local Universe, now representing the contribution of dark energy. Still referred to as Λ , the observational support for dark energy revolutionized modern cosmology, leading to the establishment of what some may call the “standard model of cosmology,” the Λ Cold Dark Matter (Λ CDM) model.

Subsequent satellite missions, notably BOOMERanG [17], WMAP [18], and Planck [19], have steadily improved the accuracy and precision of cosmic microwave background observations, ushering cosmology into the era of “precision cosmology.” In many ways, cosmology is now experiencing a similar transformative moment to when Einstein published the general relativity field equations, and when Penzias and Wilson recognized that their telescope was detecting a primordial signal from the early Universe. Never before has there been such a vast swath of highly precise cosmological data stemming from multiple phenomena and in vastly different time periods available to cosmologists. This gluttony of data comes at a time where computational capabilities are rapidly advancing, both in terms of raw data capacity and new analysis techniques. The confluence of the big data era with the revolutionary machine learning era has positioned the field of cosmology to explore deeper than ever before and to ask impactful, data driven, and increasingly model independent questions about the

fundamental nature of our Universe.

1.2 Motivation and context

Λ CDM has served as the standard model of cosmology for at least two decades, primarily due to its ability to explain cosmological observations from the largest scales observable in the CMB to even small scale structure formation. However, several compelling puzzles and tensions have emerged in recent years, challenging the Λ CDM model as the leading inference of our Universe. Arguably the most prominent among these is the *Hubble tension*, a (possibly) significant and persistent discrepancy between the locally inferred expansion rate of the Universe today (H_0) and the value inferred from observations of the CMB. I include the modifier “possibly” as the level of discrepancy between early- and late-time inferences of H_0 depend on what data is used to calibrate the local inference.

The late-time inference of H_0 depends on the *local distance ladder*. Distance ladder measurements depend on a series of logical connections in which objects that have geometrically derivable distances are used to calibrate standardizable candles (objects with empirically predictable luminosities) which are then used to calibrate the intrinsic luminosity of Type Ia supernova. Observations of SNe Ia in the *Hubble flow* where their apparent motion is dominated by expansion then provide us a map of how the local Universe is expanding. However, there are internal discrepancies in the distance ladder approach, which is why previously I use the word “possible” when describing the existence of the tension. There are two common intermediate steps on the ladder: Cepheid variable stars and tip of the red giant branch. I will discuss these at great length later in the thesis, but know for now that these two approaches yield different values of H_0 . But, in general the late-time observations consistently yield values of H_0 greater than those derived from Planck CMB data using the Λ CDM model, where in some cases the statistical tension exceeds 5σ [20]. Such a profound inconsistency may hint at the possibility of new physics beyond the standard cosmological paradigm, and offers

an unique stress test of Λ CDM. This thesis explores this exact scenario, questioning the accuracy of the dominant Λ CDM model and testing the foundational assumptions of the model.

In addition to exploring the Hubble tension and challenging the Λ CDM paradigm, this thesis presents a new interpretation of a possible signal originating from the CMB. Observation of B-mode polarization in the CMB has long been thought of as “smoking gun evidence” for a period of rapid and exponential expansion very early in the Universe called inflation [21–24]. The B-mode polarization which may be observed with future observation of the CMB ultimately arises from the tensor perturbations inflation induces on the background spacetime. However, inflation is not the only possible source of tensor perturbations in the early Universe. Later in this thesis, I demonstrate that alternative sources for these tensor perturbations, such as first-order phase transitions, may mimic or even overshadow inflationary signatures, greatly complicating our interpretation of future B-mode observations.

1.3 Research objectives and thesis outline

This thesis is structured, broadly, into two major sections. The initial chapters provide the theoretical foundations and observational context necessary for understanding the original research presented later in the thesis:

- Chapter 2 reviews foundational cosmological theory, including the metric, Einstein’s field equations, derivation of the Friedmann equations, and cosmological distance measurements.
- Chapter 3 introduces the key cosmological observables used extensively throughout this work, specifically discussing big bang nucleosynthesis, the cosmic microwave background, baryon acoustic oscillations, and the Lyman- α forest.
- Chapter 4 describes the statistical methodologies, particularly Monte Carlo and MCMC techniques, employed to obtain numerical results and perform Bayesian inference in

cosmological studies.

- Chapter 5 outlines the open theoretical and observational questions in cosmology that motivate this thesis.

Subsequent chapters detail my original research contributions where I, with my excellent group of collaborators, challenge the Λ CDM model in novel ways, presenting methodologies, numerical analyses, and their implications for cosmological modeling and parameter estimation:

- Chapter 6 presents the methods and findings of [2112.11567], emphasizing the importance of accurate distance measurements as the basis of expansion rate determinations, and highlighting the crucial role of distance-focused likelihoods in cosmological inference.
- Chapter 7 summarizes the key insights of [2306.06165], demonstrating how the atomic physics determined Thomson scattering rate tightly constrains higher expansion rates in the early Universe.
- Chapter 8 explores the symmetry of cosmological observables presented in [2403.05619], providing a thorough numerical analysis using MCMC methods to investigate the nature of the Hubble tension in the context of early-Universe physics and its implications for the standard Λ CDM model.
- Chapter 9 describes research from [2410.23348], challenging the prevailing notion that detection of B-mode polarization in the CMB necessarily signals inflation. This work demonstrates that first-order cosmological phase transitions could mimic, or even dominate, the expected B-mode polarization signal predicted from inflation.

1.4 Contributions of this thesis

The research presented in this thesis contributes significantly to the broader fields of cosmology and physics by:

1. Establishing a more robust methodological framework for analyzing cosmological distances with a publicly available likelihood package *distanceladder* and clearly demonstrating how distance measures underpin observational tensions, particularly for cosmological models that modify physics in the late Universe.
2. Identifying and characterizing the fundamental connection between the Hubble tension and the Thomson scattering rate at recombination, highlighting the crucial role of well-understood atomic physics in limiting Λ CDM based models at resolving the Hubble tension.
3. Demonstrating that cosmological observations exhibit a previously underappreciated scaling symmetry within the Λ CDM framework. Exploiting this symmetry allows significantly higher inferred values of H_0 without compromising observational consistency, providing a novel lens through which to interpret cosmological tensions.
4. Challenging the widely-held assumption that detection of B-mode polarization in the CMB would be definitive evidence of inflation. I demonstrate that first-order phase transitions can produce comparable, or even stronger, B-mode polarization signals, thus broadening the theoretical landscape and motivating future observational strategies aimed at distinguishing between these scenarios.
5. Providing a detailed exploration of the interplay between local astrophysical environments and the inference of global cosmological parameters. This exploratory research highlights the importance of accurately modeling local structures to avoid biased conclusions about fundamental cosmological quantities.

Together, these contributions advance our understanding of how fundamental physical processes shape cosmic observables, clarify key tensions within modern cosmology, and identify promising new theoretical and observational directions beyond the current standard cosmological paradigm.

2 Cosmological background

He who does not know what the world is does not know where he is.

- Marcus Aurelius

This chapter presents much of the background information utilized extensively in this thesis. To begin, I establish the foundations of spacetime and of general relativity, and how these fundamental concepts lead to an expanding Universe. I then review distance measurements in cosmology, and how they underpin all observations of the expansion rate of the Universe. This section is not meant to be a comprehensive review of cosmology, and for those seeking such a review I highly recommend my three favorite textbooks discussing the topic: *Physical Foundations of Cosmology* by Mukhanov [25], *Modern Cosmology, 2nd Edition* by Dodelson [26], and *Particle Cosmology & Astrophysics* by Hooper [27].

2.1 The metric

Before introducing field equations or discussing cosmic tensions, we must first clarify what is meant by a “point” in spacetime and how to measure the “distance” between two such points. While this may seem trivial, it quickly becomes nontrivial once the background spacetime is expanding. In this endeavor, we will also reveal how causality is preserved and how the assumptions of homogeneity and isotropy of the Universe are baked into the metrics used in the standard model of cosmology.

2.1.1 A euclidean example

To begin, consider a two-dimensional Euclidean geometry defined by orthogonal coordinates x and y . The distance between two points in this plane is given by the familiar Pythagorean theorem;

$$s^2 = x^2 + y^2. \tag{1}$$

For infinitesimal changes in these coordinates, the differential distance is

$$ds^2 = dx^2 + dy^2. \quad (2)$$

The term ds^2 is called the line element (also sometimes called the invariant interval, for reasons which will become apparent soon) and in matrix form becomes

$$ds^2 = \begin{pmatrix} dx & dy \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix} \quad (3)$$

where the *metric tensor* becomes apparent and, in this case, is the identity matrix in two dimensions:

$$\delta_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (4)$$

The line element can be written in the compact form

$$ds^2 = \sum_{i=1}^2 \sum_{j=1}^2 \delta_{ij} dx^i dx^j \quad (5)$$

where $dx^i = \{dx, dy\}$. There is a common notational simplification that is made throughout the literature, known as the “Einstein summation rule.” When discussing metrics and tensor calculus, indices will be summed over frequently, and most equations will typically require more than a couple summation signs. So, rather than write every Σ explicitly, it is assumed that if an index variable (say i) appears in both as a superscript and subscript in a single product, it is *assumed* to be summed over such that

$$\sum_i A^i B_i = A^i B_i. \quad (6)$$

Therefore, the final, most compact form of the line element is

$$ds^2 = \delta_{ij} dx^i dx^j. \quad (7)$$

where the relationship between the x and y coordinate is apparent from the metric tensor δ_{ij} . Extending this notion to higher dimensions is straightforward. In the most general case, consider a Euclidean space of n dimension. The line element of this space is simply

$$ds^2 = \delta_{ij}^{(n)} dx^i dx^j \quad (8)$$

where the metric tensor is an n rank identity matrix. Unfortunately, the geometry of the spacetime that our Universe exists in is not strictly Euclidean. Instead, it has a *pseudo* - *Euclidean* structure that takes into account the role of time. For that, we will require different and more complex metric tensors.

2.1.2 Minkowski space

The Cartesian coordinate system discussed earlier is inherently static, lacking an intrinsic notion of temporal evolution. To incorporate dynamics, we extend the system by introducing time as an additional coordinate. This extension leads naturally to the concept of a spacetime, the simplest example of which is Minkowski space [28]. Minkowski space unifies time and the three spatial dimensions into a four-dimensional continuum. This space is central to the theory of special relativity, providing a geometric framework in which events are represented as points in spacetime and the structure naturally encodes the invariance of the speed of light and the relativity of simultaneity.

An event in Minkowski space can be labeled by a set of four coordinates. In this work, we adopt the convention

$$p = \{t, \vec{x}\}, \quad (9)$$

where p is a point in spacetime, t denotes the time coordinate and

$$\vec{x} = (x, y, z) \tag{10}$$

represents the spatial position as a three-dimensional vector. It is important to note that other texts (especially older ones) might reverse the order of time and space, but the choice here is consistent throughout this work.

In Minkowski space, the line element between two infinitesimally close events is written as

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2 \tag{11}$$

where c is the speed of light. The factor of $-c^2$ coupled to dt^2 ensures that time is properly incorporated with the spatial coordinates, preserves dimensional consistence, and ensures causality is respected¹. This form reflects the pseudo-Euclidean signature of the metric and underpins the structure of special relativity.

The corresponding Minkowski metric tensor $\eta_{\mu\nu}$ in the coordinate basis $\{t, x, y, z\}$ is then expressed as

$$\eta_{\mu\nu} = \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{12}$$

Thus, the line element can be compactly written as

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu \tag{13}$$

¹Indeed, this definition underscores the difficulty of theories with a varying speed of light: if c itself changes, then the fundamental conversion factor between space and time cannot be fixed, making it unclear how to maintain a consistent notion of distance and time intervals.

where I have introduced a subtle but powerful notational tool used in general relativity, in the form of the differentiation between Greek indices and Roman indices. Greek indices (μ , ν , etc) run over all time and spatial coordinates, while Roman indices (i , j , etc) run only over the spatial coordinates.

An important feature of the Minkowski metric is how it classifies the separation between events based on the sign of ds^2 :

- When $ds^2 < 0$ the interval is **Timelike**, meaning the temporal separation between events is greater than the spatial separation. In this case, a causal signal (traveling slower than or at the speed of light) can connect the events.
- When $ds^2 = 0$ the interval is **Null**, meaning that the events are connected precisely by a light signal.
- When $ds^2 > 0$ the interval is **Spacelike**, meaning the spatial separation exceeds the temporal separation. Here, no causal influence can propagate between the events without exceeding the speed of light.

These distinctions define the causal structure inherent in Minkowski space (and indeed many physically relevant spacetime). Crucially, ds^2 (or the invariant interval) remains the same for all inertial observers, ensuring that while different observers may disagree on how much of the separation between two events is “time” versus “space,” they always agree on whether those events can causally influence one another and on the ordering of events. This is critical, as when considering very large distances in which the speed of light is slow comparatively, it will be important that our metrics inherently preserve casual structure.

2.1.3 The FLRW metric

We now seek a metric capable of describing distances in the *physical* Universe. Observations of Type Ia supernova indicate that the Universe is expanding. Large scale galaxy surveys

support that on sufficiently large scales (above roughly 100 Mpc) it appears homogeneous and isotropic. Of course, this does not hold at smaller scales, where we observe galaxies, voids, and physicists. Nonetheless, modern cosmology adopts these assumptions of homogeneity and isotropy at large scales along with the fact of expansion and the possibility of global spatial curvature.

Up to this point, we have focused on metrics in flat spacetimes, where parallel light rays remain parallel and the interior angles of a triangle sum to 180 degrees. However, the physical Universe need not be globally flat; spacetime can have nonzero curvature. In this cosmological context, we refer to global curvature, distinct from the local curvature that can be induced by localized sources of energy and matter. Under our simplifying assumptions, no single region breaks the overall homogeneity or isotropy, so we do not consider localized curvature effects here. Instead, we set out to write the most general metric, often called the Friedmann–Lemaître–Robertson–Walker (FLRW) metric—that accommodates (1) large-scale homogeneity, (2) large-scale isotropy, (3) an expanding coordinate system, and (4) the possibility of global curvature [4, 5, 29, 30]. Later in this thesis, we will revisit how local inhomogeneities might affect the measurement of global parameters that describe the Universe.

As a brief review, recall that homogeneity means there is no preferred location in space, and isotropy means there is no preferred direction at any point. Together, these conditions imply that every point in space looks identical in all directions. In mathematical terms, such a space is called *maximally symmetric*, meaning it admits the largest possible group of continuous symmetries. For a spacetime to respect these requirements, any curvature must be constant and can be flat, open, or closed:

- Flat (Euclidean): Global space has zero curvature ($k = 0$) and follows standard Euclidean geometry.
- Closed (spherical): Global space has positive curvature ($k = +1$), analogous to the

surface of a sphere but in three dimensions.

- Open (hyperbolic): Global space has negative curvature ($k = -1$), similar to the shape of a saddle but in three dimensions.

The FLRW invariant interval is

$$ds^2 = -c^2 dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right] \quad (14)$$

where a is the scale factor encoding the expansion (or contraction) of the Universe, r is the *co-moving* radial coordinate, $k \in \{-1/R^2, 0, +1/R^2\}$ is the curvature parameter, R is the radius of curvature, and $d\Omega^2$ is the angular component of the metric given by $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, where θ and ϕ are the usual angular coordinates. Pulling this all together, the FLRW metric describes a Universe which is homogeneous and isotropic at each moment in time, yet can evolve via global expansion or contraction through the scale factor $a(t)$.

From this point forward, we adopt the convention of natural units, standard in particle physics and cosmology, by setting the speed of light and the reduced Planck constant to unity such that $c = \hbar = 1$. Although we have thus far shown c explicitly to highlight its role in pseudo-Euclidean metrics, this unit system greatly simplifies many expressions and the comparison of different physical quantities. In natural units, all kinematic quantities can be expressed in energy units, where mass and momentum are said to have dimensionality of $[+1]$ and length and time have dimensionality of $[-1]$. In natural units, the corresponding metric tensor for the FLRW invariant interval, or typically just referred as the metric, is given by;

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^2(t) & 0 & 0 \\ 0 & 0 & a^2(t) & 0 \\ 0 & 0 & 0 & a^2(t) \end{pmatrix}. \quad (15)$$

Before proceeding to applications of the FLRW metric, there are a few comments on the scale factor I would like to make. In cosmology, $a(t)$ normalizes the overall size of the Universe relative to its value at the present time, t_0 , where $a(t_0) = 1$. This convention means that when $a(t) = 0.25$ at some earlier time, the physical distances in the Universe then were one-quarter of their present size. This simple normalization allows for comparisons of the Universe at different epochs and ties directly into cosmological observables. However, note that under these assumptions, the scale factor $a(t)$ is indeed a *global* scale factor, implying that the Universe uniformly expands at all points.

2.2 The expanding universe

The Friedmann equations dictate the evolution of the scale factor present in the FLRW metric in Eq. (15). This requires application of the Einstein equations which will be presented here, but not derived. Those seeking a derivation of the various tensors required to construct the Einstein equations can consult any standard general relativity textbook, such as *Spacetime and Geometry* by Carroll [31], or *Gravitation* by Misner, Thorne, and Wheeler [32].

2.2.1 Einstein equations

The Christoffel connections Γ define the *metric connection* between vectors in tangent spaces of nearby points. In other words, it is the Christoffel connections themselves which enable distance measurements in general relativity. The Christoffel connections are given by;

$$\Gamma_{\mu\nu}^{\lambda} = \frac{1}{2}g^{\lambda\sigma}(\partial_{\mu}g_{\nu\sigma} + \partial_{\nu}g_{\sigma\mu} - \partial_{\sigma}g_{\mu\nu}) \quad (16)$$

where $g_{\mu\nu}$ is the metric describing the spacetime in question. Critically, the Christoffel connections are *only* a function of the metric, further highlighting the influence our choice of metric has on our physical understanding of the Universe.

The connections appear in a few places critical to general relativity, but for the sake of this derivation lets focus on the partial derivative. In curved spacetimes the partial derivative requires a generalization to account for the mixing of space and time. The *covariant derivative* ∇_μ of a vector field V^ν is given by

$$\nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma_{\mu\sigma}^\nu V^\sigma. \quad (17)$$

Once the Christoffel connections and covariant derivative are defined, the Riemann tensor which describes curvature can be constructed such that

$$R_{\sigma\mu\nu}^\rho = \partial_\mu \Gamma_{\nu\sigma}^\rho - \partial_\nu \Gamma_{\mu\sigma}^\rho + \Gamma_{\mu\lambda}^\rho \Gamma_{\nu\sigma}^\lambda - \Gamma_{\nu\lambda}^\rho \Gamma_{\mu\sigma}^\lambda. \quad (18)$$

More precisely, the Riemann tensor describes how two nearby geodesics (the paths of freely falling particles) diverge/converge during parallel transport, revealing the presence of curvature. The Ricci scalar $\mathcal{R}$ is given by contracting the Riemann tensor with the inverse metric such that

$$\mathcal{R} = g^{\mu\nu} R_{\mu\nu} \quad (19)$$

and represents how much space is curved at each point, averaged over all possible directions. Once we have the metric, the Riemann tensor, and the Ricci scalar, we can construct the Einstein tensor such that

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = \frac{8\pi T_{\mu\nu}}{M_{\text{Pl}}^2} \quad (20)$$

where $T_{\mu\nu}$ is the stress energy tensor representing the energy content of the Universe and in the last equality, I have made the unit choice to give G in units of the Planck mass such that $M_{\text{Pl}} \equiv G^{-1/2}$. Eq. (20) is precisely what the famous quote from Wheeler is referencing. On the left hand side of the equation, there is only information regarding the metric of choice, and on the right hand side there is only the notion of the stress energy density of the Universe. Indeed, spacetime tells matter how to move, and matter tells spacetime how

to curve.

2.2.2 The Friedmann equations

The Friedmann equations can be gotten from the Einstein tensor by considering the time-time and space-space components of $G_{\mu\nu}$. The last piece we need before we construct the Friedmann equations is the stress-energy tensor. Thankfully, this is very straightforward in the perfectly homogeneous and isotropic FLRW metric, such that

$$T_{\nu}^{\mu} = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \quad (21)$$

where ρ is the energy density of the fluid, p is the pressure, and we make the further assumption of the energy content of the Universe behaving as a perfect fluid. Before we move onto the Friedmann equations themselves, it is useful to consider a unique form of the stress energy tensor in which the conservation of energy becomes clear. Consider the zero component

$$0 = \nabla_{\mu} T_0^{\mu} = -\partial_t \rho - 3 \frac{\dot{a}}{a} (p + \rho) \quad (22)$$

where overdots represent derivatives with respect to physical time and I have dropped the explicit time dependence on the global scale factor such that $a(t) = a$. Therefore, the conservation of energy conservation equation becomes

$$\frac{\dot{\rho}}{\rho} = -3(1 + \omega) \frac{\dot{a}}{a} \quad (23)$$

where I have defined the *equation of state* ω that describes the relationship between pressure and density for a perfect fluid as

$$\omega = \frac{p}{\rho}. \quad (24)$$

Assuming that the equation of state parameter ω is constant, Eq. (23) can be integrated to obtain

$$\rho \propto a^{-3(1+\omega)} \quad (25)$$

which allows us to determine how different types of energy density (radiation, matter, etc) scale with the global scale factor as a function of their equation of state. For example, matter is defined as any nonrelativistic particles with zero pressure such that its equation of state is $\omega = 0$. This immediately implies that matter energy density goes as

$$\rho_{\text{m}} \propto a^{-3}. \quad (26)$$

Similarly, the equation of state for radiation is $\omega = 1/3$ such that radiation like energy density decays as

$$\rho_{\text{r}} \propto a^{-4}. \quad (27)$$

From this, it is clear that the Universe evolves differently when different types of energy dominate, which is critical to understanding current cosmological models.

Coming back to the Friedmann equations, lets first consider the time-time (00) component of Eq. (20) given by

$$R_{00} - \frac{1}{2}g_{00}\mathcal{R} = \frac{8\pi T_{00}}{M_{\text{Pl}}^2}. \quad (28)$$

After a bit of algebra, we arrive at a form of the previous equation only dependent on the global scale factor, the curvature κ , and the energy density:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} = \frac{8\pi\rho}{3M_{\text{Pl}}^2}. \quad (29)$$

In the FLRW Universe, the first term of Eq. (29) has profound physical meaning, as it represents how the scale factor changes with time. In other words, it *is* the expansion rate (!) given by

$$H(a) = \frac{\dot{a}}{a} \quad (30)$$

where $H(a)$ is the Hubble parameter describing the expansion rate of the Universe at a given time. With this final definition, the first Friedmann equation takes it familiar form of

$$H^2 = \frac{8\pi\rho}{3M_{\text{Pl}}^2} - \frac{k}{a^2} + \frac{\Lambda}{3} \quad (31)$$

where it is clear that the expansion of the Universe is dependent on the energy content within it, and I have explicitly indicated the dependence on the cosmological constant. Note that Λ has no parametric dependence on a , just as we would expect. Now let's consider the spatial components of Eq. (20)

$$R_{ij} - \frac{1}{2}g_{ij}\mathcal{R} = \frac{8\pi T_{ij}}{M_{\text{Pl}}^2}. \quad (32)$$

After a good bit of algebra we arrive at the second Friedmann equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3M_{\text{Pl}}^2}(\rho + 3p) + \frac{\Lambda}{3}. \quad (33)$$

Eq. 33 tell us about the *acceleration* of the Universe, where notably it clearly shows that matter ($p = 0$) and radiation ($p = \rho/3$) slow down the expansion of the Universe and a positive Λ ($p = -\rho$) speeds it up.

Finally, as a brief aside, it is common when discussing densities in cosmology to refer the density parameter given by

$$\Omega_i = \frac{\rho_i}{\rho_{\text{crit}}} = \frac{8\pi}{3M_{\text{Pl}}^2 H^2} \rho_i \quad (34)$$

where I have defined the critical density as

$$\rho_{\text{crit}} = \frac{3H^2 M_{\text{Pl}}^2}{8\pi}. \quad (35)$$

2.3 Distance measures and redshift

Measuring distances in astronomy and cosmology is not a trivial task for two reasons. First and foremost, we observe the sky as a two dimension object. We have no natural sense of depth perception here because the distances are so great, making distance measurements to most of the Universe depend on physics other than geometric measurements. The second is that we fundamentally exist in an expanding spacetime, further complicating what distance is even relevant. Do we discuss distances in terms of proper distance, which represents the actual physical separation between two points, or do we consider the comoving distance, where we track the distance photons have had to travel to get to us from that point? For the most part, we deal with comoving distances in cosmology, but this does not settle the issue with actually *measuring* distances.

Before we begin the dive into distance measurements, why do we even need to measure distances in the first place? Underlying every single measurement of the expansion rate of our Universe are distance measures. There is no other way to infer the expansion rate than by mapping out the relative movement of objects away from us due to the expansion, where their rate of recession tells us about the expansion rate at that redshift. However, as will be soon clear, this is not enough information to actually compute a distance. Redshift is a dimensionless quantity and does not solely provide a physical distance. A metric must be chosen to relate redshift to distance, and the standard paradigm metric to do this is the FLRW metric covered previously.

All distance measurement first depends on establishing an absolute dimensionful anchor

that defines the problem’s overall scale. Such anchors do not necessarily need to be absolute distance scales, but can also be a time interval, the linear size of an object, an acceleration, or a temperature, among others. In the following, I briefly review different distance measurement techniques in cosmology, highlighting the absolute *dimensionful scale* anchoring each approach, and describe the logical flow from that anchor to the inferred cosmological distances. Our goal here is to provide a concise end-to-end description of the distance measurements that are instrumental to the existence of the Hubble Tension. As such, we focus below on the local distance ladder and the cosmic microwave background, as these are the distance measurements driving the tension. Other important techniques include cosmic chronometers [33–39], gravitational wave standard sirens [40–44], surface brightness fluctuations [45–50], Tully-Fisher relation [51–57], Type II supernovae [58–63], and HII galaxies [64–67].

Subtly underlying nearly² all cosmological distance measurements is the concept of cosmological redshift. Cosmological redshift arises due to cosmic expansion, where photons emitted by distant objects lose energy as the Universe expands, observationally resulting in a stretching or “reddening” of their wavelengths. This phenomenon is fundamentally distinct from other types of redshift, such as those arising from the relative proper motion of objects (Doppler redshift) or gravitational fields (gravitational redshift). While these effects can be significant in certain astrophysical contexts, cosmological redshift dominates at large distances relevant for most cosmological studies.

Redshift is crucial to cosmological distance measurements because it provides a direct relationship to distance, *but only once* a cosmological model or metric is specified. Redshift itself is inherently a dimensionless quantity, defined by the fractional difference between

²Geometric measurements of distance are direct measurements and do not rely on redshift.

emitted and observed wavelengths:

$$z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} - 1. \quad (36)$$

As it is dimensionless, redshift alone cannot directly yield distance measurements. Therefore, *any* cosmological distance determination based on redshift involves inference from a chosen cosmological model. This model dependence is clearly seen in the expression for the comoving distance $\chi(z)$,

$$\chi(z) = \int_0^z \frac{dz'}{H(z')}, \quad (37)$$

which explicitly shows that the distance calculation relies on integrating over the expansion history of the Universe, encapsulated by the Hubble parameter $H(z')$, which in Sec. 2.2, is demonstrated to be a function solely of the metric and energy density.

The two most prominent distance measurements in cosmology are the *angular diameter distance*, D_A , and the *luminosity distance*, D_L . In the case of a perfectly homogeneous and isotropic FLRW Universe, D_A is given by

$$D_A(z) \equiv \frac{l}{\theta} = \frac{\chi(z)}{1+z}, \quad (38)$$

where l is the physical size of the object, and θ is the angle subtended by the object. This measure is particularly useful when the intrinsic physical size of a phenomenon is known, allowing its angular size on the sky to serve as a distance probe. These are referred to as *standard rulers*.

Under the same assumptions, D_L is defined as

$$D_L(z) \equiv \sqrt{\frac{L}{4\pi F}} = (1+z) \chi(z), \quad (39)$$

where L is the intrinsic luminosity of the object, and F is the observed flux. The luminosity distance is particularly useful when the intrinsic luminosity (or absolute magnitude) of the observed object is independently known or well calibrated. These are referred to as *standard candles*.

2.4 Local distance ladder

Every cosmological distance measurement relies on establishing an absolute *dimensionful* length scale that other distance measurements are then compared to. Fortunately, we live in a Universe where multiple such absolute distances are available: photon-baryon sound horizon [68–72], parallax distances to nearby stars [73–75], distances to eclipsing binaries [76–78], distances to water masers [79–81], and time-delay distances to strong gravitational lenses [82–84].

2.4.1 Anchor measurements

In cosmology, distance ladders are constructed by establishing absolute distances to nearby stars or galaxies. At these relatively short distances, the expansion of the Universe is negligible, as measurements occur within gravitationally bound structures where local dynamics dominate over cosmic expansion³. Such distances measurements, usually referred to as anchors, are typically founded on direct geometric measurements that establish the overall scale of the problem. Common methods of measuring the distance to these anchors include stellar parallax [85–91], detached eclipsing binary stars [76, 92–95], and MASER emission from the accretion disk of super-massive black holes [79, 80, 96–100]. I briefly describe these below,

³It is important to highlight the apparent tension in the foundational assumptions here. Namely, that bound systems do not exhibit cosmic expansion but the fundamental assumptions underpinning the FLRW metric and Λ CDM cosmology, which presume all-scale homogeneity and isotropy, would assume that they do expand. While rigorous studies have consistently demonstrated that local inhomogeneities have subdominant effects at large scales, it remains intriguing and somewhat paradoxical that local distance measurements in this nearby and intermediate scales, which will later be shown to explicitly depend on the global metric, inherently arise from regions where these cosmological assumptions do not strictly hold.

noting the key dimensionful scales entering the problem in each case.

Parallax The parallax method measures the apparent movement of a stellar object with respect to the background stars on the sky as the Earth revolves around the sun. Because the star is very far away compared to the perceived motion of the star on the night sky, the parallax equation is simply

$$d = \frac{1}{p}, \quad (40)$$

where d is the distance in parsecs, and p is the parallax angle measured in arcseconds. The absolute *dimensionful scale* for the modern parallax method is the baseline distance established between two observations, typically the diameter of Earth’s orbit. This is a precisely measured value known as an astronomical unit (AU) equivalent to $149,597,870,700 \pm 3$ meters [101, 102]. While parallax measurements are highly precise, they are severely limited in the measurable distance, with the furthest measurements reaching the Milky Way’s galactic center using the Gaia space telescope [74, 103–105].

Detached eclipsing binaries The Detached Eclipsing Binary (DEB) method utilizes binary stars whose orbit takes the pair within the observer’s line of sight, resulting in primary and secondary eclipses [76, 77, 94, 106–108], similar to exoplanet detection where the planet passes in front of the host star. Here, the pair’s “detached” nature means that the binary stars’ separation is much larger than their individual radii, which avoids complications related to mass transfer and accretion disks. The absolute *dimensionful scale* entering the problem is the orbital period of the binary pair, which can be measured both from the light curve and from radial velocity spectroscopic data. These observations can also be used to measure the inclination and eccentricity of the DEB system’s orbit, which, when combined with the orbital period measurement, allows one to determine the orbital radius from Kepler’s laws. Then, the shape of the primary and secondary eclipses can be used to calculate the radius of each star. Finally, spectroscopic and color information is used to estimate the surface

brightness of each star, which can be used to compute the distance to the pair

$$d = \left(\frac{F_1}{F_{1,\text{tel}}} \right)^{1/2} R_1 = \left(\frac{F_2}{F_{2,\text{tel}}} \right)^{1/2} R_2 \quad (41)$$

where F_i is the flux from each star, $F_{i,\text{tel}}$ is the flux received at the telescope, and R_i is the radius of that star determined using the method outlined above. The two estimates for d in Eq. (41) provide a vital consistency check for the distance measurement. This procedure can also be carried in multiple photometric bands, providing further cross-checks of the distance estimate. The assumptions surrounding the surface brightness estimate of each star represent the most significant potential systematics of the DEB technique. However, previous work has demonstrated consistent models for surface brightness with negligible dependence on metallicity effects [109, 110]. Additionally, these systems are difficult to detect due to their detached nature, causing transits to be very short, which limits the number of available distance measurements [76].

MASERS Super-massive black holes (SMBH) heat the surrounding gas in their accretion disk to very high temperatures, producing x-ray emission. This strong radiation field can stimulate various molecules present within the disk, particularly that of water, resulting in localized MASER emission [111–115]. Such MASERS are high intensity and point-like, allowing for line-of-sight (LOS) velocities to be measured in the accretion disk within thousands of Schwarzschild radii of the SMBH. Furthermore, the orbital speed of the MASERS around the central engine is large enough that their LOS acceleration can also be measured over a few years. Once the LOS acceleration, LOS velocity, and velocity gradient are measured, a distance can be determined according to

$$d = \frac{\partial_\theta v_{\text{LOS}}}{a_{\text{LOS}}} v_{\text{LOS}} \quad (42)$$

where v_{LOS} is the LOS velocity, θ is the angle on the sky according to the observer, $\partial_{\theta}v_{\text{LOS}}$ is the LOS velocity gradient on the sky, and a_{LOS} is the LOS acceleration [116]. In this case, the absolute *dimensionful scale* established by the MASER method is the LOS acceleration. The MASER method additionally can directly measure distances into the Hubble flow, notably by the Megamaser Cosmology Project who directly measure from distances to be $H_0 = 73.9 \pm 3.0$ km/s/Mpc [80]. However, current systematic errors associated with radio phase calibration make the MASER method challenging to perform [117]. Additionally, the small number of known MASERS with nearby phase calibrators limits the method’s applicability.

2.4.2 Calibrating standard candles

Once a distance anchor measurement is chosen, we must find a way to extend these nearby distances to distances further into the Hubble flow to hope to measure the expansion rate. The second step of the distance ladder depends on calibrating standard candles. These calibrators must satisfy the following criteria; (i) have anchor measurements of their distance from us using one of the techniques outlined in section 2.4.1, and (ii) exist in a host galaxy with other, brighter standard candles (such as SNe Ia) that are observable in the Hubble flow. While we focus below on Cepheids and the tip of the red giant branch, note that Mira variable and RR Lyrae stars have also been used as calibrating standard candles [118–122].

Cepheid variable stars Leavitt’s law gives a relationship between the period and apparent magnitude of a Cepheid variable star [123]. Once anchor measurements determine the distance to a subsample of Cepheids, Leavitt’s Law can be calibrated. Then, the apparent magnitude of a hypothetical Cepheid star with a one day-long period is calculated using linear regression. The absolute magnitude of Cepheids, M_{ceph} , is determined by the distance modulus equation using the anchor distance measurement of a Cepheid and the apparent magnitude of the hypothetical one day period Cepheid. In terms of magnitude, the distance

modulus μ is given by

$$\mu = m - M \quad (43)$$

where m is the apparent magnitude and M is the absolute magnitude. Then, the apparent magnitude of a hypothetical one day period Cepheid in a nearby host galaxy is calculated using observations of Cepheid populations. Finally, the distance to the host galaxy is calculated from Eq. (43) using M_{ceph} and the calculated apparent magnitude of a one day period Cepheid in the same host galaxy. Recently, the nature of Leavitt’s Law has come under question as the relationship between period and magnitude for Cepheids may not be wholly linear but have a ‘break’ where the slope changes value [124–126]. Additionally, metallicity effects on the star may change the period to luminosity relationship, although previous studies have found the overall effect from metallicity to be small [127, 128].

Tip of the red giant branch Red giant branch stars are old, evolved stars with an inert helium core. Once the inert helium core reaches conditions to allow for fusion, it ignites in a highly energetic event and expels a large amount of matter from the star’s atmosphere, leading to an immediate decrease in luminosity. This behavior leads to the ‘tip of the red giant branch’ (TRGB) as a standard candle, as the helium flash will occur at similar mass conditions for all red giant branch stars [129]. The peak absolute magnitude of the tip, M_{TRGB} , is calculated by CCHP (Carnegie-Chicago Hubble Program) and Freedman et al. in a three-step process [130–138]. First, a large sample of potential red giant branch stars from the same galaxy undergoes a selection cut based on color to remove other evolved stars. Second, the luminosity function is determined from the cut sample. Third, a Sobel edge detection filter is used to determine the edge of the luminosity function, which corresponds to the TRGB. Once M_{TRGB} is calculated and calibrated with an anchor measurement (using one of the techniques described in section 2.4.1), the TRGB can be used to determine the distance to SNe Ia host galaxies much in the same way that other calibrating standard candles do. However, only observations of the halo’s of galaxies should be used to calculate M_{TRGB}

as the inner regions of galaxies will have asymptotic giant branch stars which contaminate the sample [136].

2.4.3 SNe Ia

The final rung of the distance ladder involves Type Ia supernova (SNe Ia). SNe Ia occurs in binary systems in which a white dwarf is actively accreting material from its binary companion. Eventually, the electron degeneracy pressure can no longer support the mass of the white dwarf, and the Chandrasekhar mass limit [139] is reached (approximately equal to $1.44 M_{\odot}$), at which point the white dwarf collapses, and a supernova occurs. These are excellent standard candles for measuring distances well into the Hubble flow because the mass at which these supernovae occur is consistent and the event’s magnitude is very bright.

Local SNe Ia Measuring the absolute magnitude of SNe Ia, M_{sn} , requires measurements of SNe Ia in the same host galaxy as calibrating standard candles [140, 141], such as Cepheids or TRGB stars. Because the distance to the host galaxy is much greater than the host galaxy’s size, it is assumed that the calculated distance modulus of the calibrating standard candle is the same as the SNe Ia. Ref. [136] describes 18 recorded SNe Ia occurring in galaxies in which TRGB measurements are available, while Ref. [20] describes 42 recorded SNe Ia occurring in galaxies in which Cepheid data are available.

Hubble flow SNe Ia The distance modulus to distant SNe Ia well into the Hubble flow is then measured using M_{sn} and apparent magnitude measurements. These distant SNe Ia also have redshift measurements, enabling a distance modulus-redshift relationship to be defined, which in terms of luminosity distance is given by

$$\mu = 5\log(d_L) + 25, \tag{44}$$

where d_L is the luminosity distance in Mpc which is given by

$$d_L = (1 + z) \int_0^z \frac{dz'}{H(z')}. \quad (45)$$

Therefore, we can solve eq. (43) for H_0 in terms of only observable quantities to find

$$\mu \propto \log_{10} \left(\frac{1}{H_0} \int_0^z \frac{dz'}{E(z')} \right) \quad (46)$$

where $E(z) = H(z)/H_0$ is the dimensionless Hubble rate. Now, a careful reader may have noticed that I call the distance ladder method of determining H_0 an *inference*, which is in stark contrast to the commonly parroted statement that the distance ladder is a *direct* measurement of the expansion rate. The distance ladder approach is fundamentally an inference because it depends on the metric of choice, as clearly seen in Eqs. (44) and (45) which explicitly depend on the Hubble parameter. It might be a pretty good estimation for a direct measurement, but it still requires assumptions about the background nature of spacetime that is commonly overlooked in the literature, and may be of importance as the field seeks to achieve sub 1% precision in its measurements.

Eq. (46) shows that distance measurements determine the value of H_0 . Indeed, calculating H_0 at $z = 0$ from the local distance ladder typically relies on distance measurements of SNe Ia in the redshift range $0.02 \lesssim z \lesssim 2$, with the lower bound set to avoid the coherent flow of local SNe Ia [142] and the upper bound set from data availability. Additionally, the overall shape of the redshift-luminosity relationship is relevant due to the redshift integral in Eq. (45) allowing distance measurements deep in the Hubble flow to be sensitive to the timing of matter-dark energy equality [143].

2.5 CMB distance measurements

The most precise measurements of the Hubble constant obtained so far arise from detailed observations of the cosmic microwave background (CMB) [144]. Light from the CMB last-scattering surface forms a two-dimensional projected map on the sky whose primary observables are temperature and polarization fluctuations on different angular scales. These perturbations tell us about critical physical processes in the pre-recombination era, which we will cover at length in the next chapter. By themselves, fractional temperature and polarization fluctuations seen projected on the sky do not set an absolute distance scale from which the Hubble constant can be inferred [145]. What fundamentally allows us to determine H_0 from CMB observations in the Λ CDM model is the knowledge of the CMB temperature today T_0 , which provides an *absolute scale* on which cosmological distances important to the CMB can be calibrated [146].

The most prominent CMB angle on the sky is the angular size of the sound horizon at last scattering ($z_\star$), which is given by

$$\theta_\star = \frac{r_s}{D_A(z_\star)}, \quad (47)$$

where r_s is the comoving baryon-photon sound horizon, and $D_A(z_\star)$ is the comoving angular diameter distance to the CMB. Generally, r_s and $D_A(z_\star)$ are given by

$$r_s = \int_{z_\star}^{\infty} \frac{c_s dz}{H(z)} \quad \text{and} \quad D_A(z_\star) = \int_0^{z_\star} \frac{dz}{H(z)}, \quad (48)$$

where c_s is the sound speed of the primeval plasma, and $H(z)$ is the Hubble expansion rate. On dimensional grounds, the comoving sound horizon is approximately given in Λ CDM by $r_s \simeq 10^{-4} M_{\text{pl}}/T_0^2$, where the prefactor has mild dependence on the redshift of matter-radiation equality, the photon-to-baryon ratio, and $z_\star$. On the other hand, the comoving

Parameter	Description	Planck 2018 Value
$\Omega_b h^2$	physical baryon density	0.02237 ± 0.00015
$\Omega_c h^2$	physical cold dark matter density	0.1200 ± 0.0012
$100\theta_*$	angular size of sound horizon	1.04092 ± 0.00031
τ_{reio}	optical depth to reionization	0.0544 ± 0.0073
$\ln(10^{10} A_s)$	primordial power spectrum amplitude	3.044 ± 0.014
n_s	primordial power spectrum tilt	0.9649 ± 0.0042

Table 1: This table lists the standard six free parameters describing the Λ CDM model of cosmology. The parameter symbol is given in the first column, the second column gives a brief description of the parameter, and the third reports the best fit value from Ref. [144].

angular diameter distance scales as $D_A(z_*) \simeq (3.1H_0)^{-1}$, which implies that the Hubble constant inferred in Λ CDM by the CMB is approximately

$$H_0 \sim 3.1 \times 10^4 \theta_* \frac{T_0^2}{M_{\text{pl}}}. \quad (49)$$

Note that the only observed dimensional quantity entering this approximation is the CMB temperature. While precise measurements of the anisotropies on the CMB give a measurement of θ_* to an accuracy of 0.03% [144], uncertainties in other (dimensionless) cosmological parameters entering the prefactor in eq. (49) result in the CMB measuring H_0 to a 0.8% precision within Λ CDM.

2.6 The Λ cold dark matter (Λ CDM) model

Now that we understand how the Universe expands under a FLRW metric and how distances are measured to probe the expansion rate, let’s briefly review the most popular cosmological model in use today, known as the Λ CDM model. Λ CDM provides a succinct yet comprehensive description of a wide variety of cosmic phenomena, including the cosmic microwave background, large-scale structure formation, and the Universe’s evolutionary history. Remarkably, the model is characterized by only six free parameters as described in Tab. 1.

The Λ CDM model rests upon four core assumptions:

1. **Homogeneity and isotropy:** Perhaps the most fundamental assumption behind Λ CDM is that the Universe is accurately described by the Friedmann–Lemaître–Robertson–Walker (FLRW) metric, governed by Einstein’s equations. This implies that the Universe is homogeneous and isotropic, and its expansion can be characterized by a single global scale factor $a(t)$, evolving according to the Friedmann equations.
2. **Cosmological constant:** The observational evidence for accelerated expansion motivates the inclusion of a cosmological constant, or dark energy, into Einstein’s equations. This component represents a uniform and constant energy density field, often associated with vacuum energy, which dominates the current energy budget of the Universe.
3. **Cold dark matter:** Most of the matter in the Universe is assumed to be composed of collisionless, non-relativistic particles termed “dark matter,” which do not interact electromagnetically and enable hierarchical structure formation. The cold nature of these particles ensures that structures form first at small scales, well before baryons in the visible sector can, subsequently merging into larger structures over cosmic time. It is these dark matter seeds which eventually lead to the observable structure today.
4. **Inflation and gaussianity:** Initial density perturbations are assumed to originate from quantum fluctuations amplified to cosmological scales during an early inflationary epoch, generating nearly scale-invariant Gaussian fluctuations which evolve adiabatically.

While Λ CDM explains a wide array of observables, which I discuss in detail in the next chapter, I briefly summarize a few of the most critical predictions here. Λ CDM accurately predicts: the distinct acoustic oscillations in temperature and polarization observed by WMAP and Planck, the clustering and distribution of galaxies as observed by all-sky surveys, the number and distribution of dwarf galaxies orbiting their host galaxies, the observed accelerated expansion at late times, primordial elemental abundances (hydrogen, helium, deuterium, and lithium), and the current energy content of the Universe. The latter

including dark energy, comprising approximately 69% of the total energy density, cold dark matter at about 26%, baryonic matter at roughly 5%, and a negligible fraction of radiation.

Despite its success in predicting such a diverse range of cosmic observables, Λ CDM has increasingly come under scrutiny as observational precision improves. Perhaps most notably, there is a significant disagreement concerning the local expansion rate, H_0 , known as the “Hubble tension,” which I cover extensively in Ch. 5. Additionally, there appear to be potential anomalies related to large-scale structure formation. These open questions constitute a significant motivation for the research presented in this thesis, signaling the need to perform necessary stress tests on the Λ CDM model.

3 Cosmological observables

Observe constantly that all things take place by change, and accustom yourself to consider that the nature of the Universe loves nothing so much as to change the things which are, and to make new things like them.

- Marcus Aurelius

3.1 Big Bang nucleosynthesis (BBN)

The theory of Big Bang Nucleosynthesis offers the earliest probe of the Universe today, where observations of the abundances of hydrogen and helium in the early Universe allow constraints to be placed on the expansion rate and the number of relativistic species present a mere few minutes after the Big Bang. It forms one of the foundational observational pillars of modern cosmology, providing compelling evidence that the early Universe was once much more hot and dense than it is today. BBN describes how primordial nuclear abundances of the lightest elements like hydrogen, helium, deuterium, and lithium were created through a series of nuclear fusion reactions. Its remarkable precision in predicting these primordial abundances, subsequently confirmed by astronomical observations, offers stringent constraints on cosmological models and the physics governing the early Universe, particularly those related to the number of relativistic species at the time and the free electron number density in the later Universe. In this section, I will provide a detailed and time-ordered narrative of the physical conditions and processes the Universe undergoes during the BBN epoch, much of which is based on Refs. [147–149]. The primary aim is to illustrate clearly how the standard BBN model robustly predicts the resulting light elemental abundances.

3.1.1 Conditions before BBN ($t \lesssim 1$ s)

Just one second after the Big Bang, the Universe was filled with a hot, dense, radiation dominated plasma with temperature of order $T \sim \mathcal{O}(\text{MeV})$. This plasma is a mixture of electrons and positrons ($e^\pm$), neutrinos (ν) of three flavors, and nucleons (protons p and neutrons n) kept in thermal equilibrium through collisional processes. The baryon-to-photon ratio $\eta \equiv n_b/n_\gamma$ is very small ($\eta \sim 10^{-10}$), meaning photons dominated the dynamics of the plasma. At this time, protons and neutrons were able to freely converted back and forth via weak interactions such that

- β^- decay: $n \leftrightarrow p + e^- + \bar{\nu}_e$
- electron/neutrino induced inverse β decay: $p + e^- \leftrightarrow n + \nu_e$

Because these weak reactions were very rapid at $T \gtrsim 2$ MeV, neutrons and protons maintained equilibrium abundances, with the ratio of neutrons to protons following a Boltzmann distribution, given by

$$\frac{n}{p} = \exp\left(-\frac{Q}{T}\right) \quad (50)$$

where T is temperature and Q is the neutron-proton mass difference $Q = m_n - m_p \approx 1.293$ MeV. At early times, when $T \gg Q$, neutrons and protons stayed in equilibrium ($\frac{n}{p} \approx 1$).

3.1.2 Weak freezeout ($t \sim 1 - 10$ s)

However, equilibrium did not last. As the Universe expanded and cooled below an MeV, the rate of the weak interactions dropped faster than the Hubble expansion rate, allowing the neutron-proton ratio to fall out of equilibrium. At this point, known as the *weak freeze-out*, the neutron-to-proton ratio becomes effectively “frozen in” as the proton and neutron number changing processes become inefficient and cease to influence the ratio. To estimate the proton-to-neutron ratio at weak freeze out, consider the temperature at freeze-out, T_{fr} ,

which can be estimated by equating the weak reaction rate $\Gamma_{n\leftrightarrow p}(T)$ with the expansion rate $H(T)$:

$$\Gamma_{n\leftrightarrow p}(T_{\text{fr}}) \sim H(T_{\text{fr}}). \quad (51)$$

Given that the weak reaction rates scale approximately as $\Gamma_{n\leftrightarrow p} \sim G_F^2 T^5$ (with G_F being the Fermi coupling constant), and the Hubble rate during radiation domination is $H \sim 1.66\sqrt{g_*} T^2/m_{\text{Pl}}$, we find:

$$T_{\text{fr}} \sim \left(\frac{\sqrt{g_*}}{G_F^2 M_{\text{Pl}}} \right)^{1/3} \approx 0.85 \text{ MeV} \quad (52)$$

where g_* is the number of relativistic degrees of freedom in the system. Inserting the freeze-out temperature ($T_{\text{fr}} \approx 0.8 \text{ MeV}$) into the equilibrium neutron-to-proton ratio equation yields:

$$\left. \frac{n}{p} \right|_{T_{\text{fr}}} \approx e^{-Q/T_{\text{fr}}} \approx e^{-1.293/0.8} \approx \frac{1}{6}, \quad (53)$$

indicating that there is roughly one neutron for every six protons at freeze-out. This ratio provides a crucial initial condition for BBN, as it sets the neutron and proton number density which will be of critical importance for the later stages of BBN.

3.1.3 Neutrino decoupling and electron-positron annihilation ($t \sim 1 - 20 \text{ s}$)

Shortly before nucleosynthesis begins, another crucial process unfolds: the decoupling of neutrinos from the primordial plasma. At early times, neutrinos are maintained in thermal equilibrium with the electrons in the plasma through weak interactions such as

$$e^- + e^+ \leftrightarrow \nu + \bar{\nu}, \quad (54)$$

$$e^\pm + \nu \leftrightarrow e^\pm + \nu. \quad (55)$$

This keeps the neutrinos in thermal equilibrium with the electrons, who are of course still tightly coupled to the photon bath. However, once the weak interaction freezes out, neutrinos cease to interact frequently enough to remain in thermal equilibrium and decouple from

the photon-electron plasma.

Once neutrinos decouple from the plasma, they freely stream away and the electrons and positrons remain in equilibrium with photons down to temperatures approaching the electron mass scale, $T \sim 0.5 \text{ MeV}$. When the photon temperature falls below this threshold, electrons and positrons annihilate efficiently into photon pairs:

$$e^+ + e^- \rightarrow \gamma + \gamma. \quad (56)$$

This annihilation event transfers entropy exclusively into the photon bath, reheating it relative to the decoupled neutrinos, whose temperature evolves independently as radiation. Before annihilation, photons and electron-positron pairs collectively have an effective number of relativistic degrees of freedom of $g_* = 2 + \frac{7}{8}(2 + 2) = 11/2$, accounting for photons ($g = 2$) and electron-positron pairs (4 degrees of freedom total, each counting $7/8$ as fermions). After annihilation, only photons remain, leaving $g_* = 2$. The decrease in g_* implies an increase in photon temperature relative to neutrinos. Conservation of entropy dictates that the photon temperature after electron-positron annihilation is higher relative to the neutrino temperature. In the instantaneous neutrino decoupling limit, one finds the temperature relationship

$$\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11} \right)^{1/3} \approx 0.714 \quad (57)$$

where T_ν is the neutrino temperature, and T_γ is the photon temperature. After electron-positron annihilation, neutrinos continue as free-streaming relics, forming the cosmic neutrino background (CνB). Their primary impact on BBN arises through their contribution to the Universe's total radiation density, affecting the expansion rate of the Universe. Specifically, the radiation density after annihilation can be expressed as

$$\rho_{\text{rad}} = \rho_\gamma + \rho_\nu = \frac{\pi^2}{15} \left[2 + \frac{7}{8} N_\nu \left(\frac{T_\nu}{T_\gamma} \right)^4 \right] T_\gamma^4, \quad (58)$$

where $N_\nu = 3$ is the number of neutrino species in Λ CDM. Since neutrinos decouple well before nucleosynthesis begins, their influence during BBN is indirect, affecting only the expansion rate and therefore the timing and efficiency of nuclear reactions, rather than directly participating in nuclear reactions themselves. This allows us to place strict constraints on the number of relativistic species at the time of BBN, as because the Universe is in deep radiation domination, any changes to the available amount of radiation at this time strongly influences the expansion rate, and thus the later nuclear reaction rates. It is the background expansion rate which sets the time scale for BBN. Changing the expansion rate directly results in changes to the final yields of light elemental production in BBN.

3.1.4 The deuterium bottleneck ($t \sim 100$ s)

After neutrinos have decoupled, the primordial plasma consists predominantly of free protons, neutrons, electrons, and photons, and the conditions are approaching those needed for nucleosynthesis. However, there is a key component missing still: Deuterium. The nuclear reaction chains which eventually form the elements heavier than hydrogen strongly depend on the presence of deuterium, so before nucleosynthesis can proceed, deuterium must be abundant. Also, keep in mind that a free neutron is not stable, with a half-life of about 10 minutes. While the Universe is waiting for deuterium, neutrons are constantly decaying in the background, changing the neutron-proton ratio.

In principle, nuclear fusion between neutrons and protons forming deuterium (D) could begin as soon as the temperature falls below the deuterium binding energy, $B_D = 2.224$ MeV. However, despite this energetic possibility, the actual synthesis of deuterium is significantly delayed. This delay is known as the *deuterium bottleneck*, and it arises because the photon density at these high temperatures is exceedingly large (recall that $\eta \sim 10^{-10}$), and high-

energy photons can efficiently dissociate any newly formed deuterium via the reaction

$$\gamma + D \rightarrow p + n. \quad (59)$$

The criterion for overcoming the deuterium bottleneck can be understood in terms of the balance between deuterium production and destruction rates. Initially, the rate of photodissociation greatly exceeds the formation rate of deuterium. To quantify the conditions under which deuterium becomes stable against photodissociation, we require that the number of photons capable of dissociating deuterium per baryon falls below unity. This can be roughly expressed by the condition

$$\eta^{-1} e^{-B_D/T} \sim 1, \quad (60)$$

where η is the baryon-to-photon ratio and T is the plasma temperature. Given the observed baryon-to-photon ratio, $\eta \approx 6 \times 10^{-10}$, this condition is met only once the Universe cools to approximately $T \sim 0.1$ MeV, corresponding to roughly 100 seconds after the Big Bang.

Physically, this temperature marks the point where the exponential suppression factor, $e^{-B_D/T}$, sufficiently reduces the number of energetic photons capable of dissociating deuterium, allowing the newly synthesized deuterium nuclei to survive for significant periods. Only after the Universe cools below this temperature can the deuterium abundance begin to rise appreciably.

The deuterium bottleneck is therefore not simply a delay; it represents a critical turning point in the early Universe. Prior to this moment, the baryonic matter content is predominantly free neutrons and protons. After the bottleneck is broken, however, the elemental abundances rapidly evolve, transitioning towards a mixture of stable, heavier nuclei. Thus, overcoming the deuterium bottleneck signifies the true onset of primordial nucleosynthesis.

3.1.5 The BBN reactor ($t \sim 100 - 300$ s)

After the deuterium bottleneck breaks, nucleosynthesis proceeds rapidly. The main BBN nuclear reaction network includes a series of reactions that efficiently convert free neutrons and protons into primarily helium-4 nuclei (${}^4\text{He}$), accompanied by trace amounts of deuterium (D), helium-3 (${}^3\text{He}$), and lithium-7 (${}^7\text{Li}$).

The first critical step following deuterium formation involves the rapid fusion of deuterium into helium isotopes via reactions such as:



These reactions create the nuclei necessary for subsequent helium-4 production. Tritium (${}^3\text{H}$) produced through the second reaction is unstable, with a half-life of approximately 12 years, and ultimately decays into helium-3 via beta decay (${}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e$). However, during the short BBN timescale ($\sim$ minutes), this decay is negligible, and tritium primarily participates in further nuclear reactions.

Once helium-3 and tritium are present, the formation of helium-4 occurs rapidly through reactions such as:



These reactions are highly efficient and strongly exothermic, rapidly funneling nearly all

available neutrons into helium-4. This efficiency results from helium-4 being the most tightly bound nucleus among the light elements, with a total binding energy of approximately 28.3 MeV. Consequently, helium-4 production dominates the nuclear outcome of BBN.

A critical factor preventing the synthesis of heavier elements beyond helium-4 during BBN is the existence of nuclear stability gaps at atomic mass numbers $A = 5$ and $A = 8$. Nuclei such as ${}^5\text{He}$ and ${}^5\text{Li}$, if formed, are highly unstable and immediately decay back into lighter nuclei. This gap effectively halts the formation of heavier nuclei because overcoming these instability barriers would require highly suppressed multi-particle or rare, high-energy reactions that are negligible under BBN conditions. As a result, the BBN nuclear cascade rapidly stalls at helium-4, with only small quantities of heavier nuclei produced.

Despite this difficulty, small quantities of lithium-7 are synthesized through more complicated reaction pathways, primarily:



Beryllium-7 (${}^7\text{Be}$) produced initially is unstable, decaying via electron capture to lithium-7 on timescales much longer than the duration of BBN ($t_{1/2} \sim 53$ days). Thus, the primordial lithium abundance observed today is largely the result of initially formed ${}^7\text{Be}$ that later decayed into lithium-7 after BBN concluded.

Finally, although deuterium is rapidly consumed in helium-4 formation, a small fraction remains unburned due to declining temperatures and decreasing nuclear reaction rates. This residual deuterium abundance serves as a sensitive probe of the baryon-to-photon ratio η , as

the precise fraction remaining depends critically on the competition between the expansion rate of the Universe and nuclear fusion reaction rates.

3.1.6 Observational tests and primordial abundances

The robust predictions of BBN for primordial elemental abundances provide critical observational tests of the standard cosmological model. Precise measurements of these primordial abundances rely on astrophysical observations of environments minimally contaminated by subsequent stellar nucleosynthesis, typically found dwarf galaxies where stellar formation is ongoing, but very slow.

The primordial abundance of helium-4 (Y_p) is typically determined by spectroscopic analysis of metal-poor, extragalactic HII (ionized hydrogen) regions. These regions, found in dwarf galaxies with very low metallicity, are selected because their elemental composition has undergone minimal stellar evolution since their initial formation. Observations of recombination emission lines of helium and hydrogen yield accurate estimates of the helium-to-hydrogen ratio, from which Y_p is inferred. Current observations consistently yield a helium mass fraction $Y_p \approx 0.245\text{--}0.250$ [150, 151], in excellent agreement with standard BBN predictions of $Y_p \approx 0.246$.

The primordial deuterium abundance (D/H) is best measured through absorption-line spectroscopy of distant quasars. Intervening gas clouds along the line of sight absorb quasar emission at characteristic wavelengths associated with hydrogen and deuterium, enabling precise determination of their relative abundances. Deuterium is particularly useful as a cosmological probe because it is only created in significant amounts during BBN and is subsequently destroyed in stellar interiors. Thus, observed deuterium abundances represent a firm lower limit to the primordial abundance. Current observational determinations yield

$(D/H)_p \approx 2.5 \times 10^{-5}$ [152], again in remarkable agreement with standard BBN predictions.

The primordial abundance of helium-3 is inferred primarily from observations within our Galaxy, such as planetary nebulae and HII regions, or from solar system measurements. However, interpreting these abundances is complicated by significant stellar processing and production uncertainties. Thus, helium-3 abundances provide relatively weaker constraints compared to helium-4 and deuterium. Nevertheless, existing measurements remain broadly consistent with standard BBN predictions of approximately ${}^3\text{He}/\text{H} \sim 10^{-5}$ [153].

Finally, lithium-7 abundances are typically measured spectroscopically in the atmospheres of old, metal-poor halo stars in the Milky Way. These stars, presumed to retain nearly primordial composition, exhibit lithium abundances significantly lower (by a factor of approximately 2–3) than the standard BBN prediction of $({}^7\text{Li}/\text{H})_p \sim 5 \times 10^{-10}$ [154]. This discrepancy, known as the “cosmological lithium problem,” remains unresolved and is actively studied. However, it mostly likely reflects complexities in stellar astrophysics, such as lithium depletion through mixing processes within stars, rather than a fundamental issue in the standard BBN framework [155].

Overall, the remarkable agreement between the observed primordial abundances of helium-4 and deuterium and their precise BBN predictions provides powerful validation for the standard cosmological model and provides an inferred value of the expansion rate today in agreement with the Λ CDM models predictions.

3.2 Cosmic microwave background

The CMB is, perhaps, the most fundamental and informative observables in modern cosmology. The photons that comprise the CMB were emitted merely $\sim 380,000$ years after the Big Bang and are observable today in the microwave regime. As a nearly perfect isotropic

and homogeneous radiation field, the anisotropies of the CMB encode unparalleled insights into the conditions of the Universe at this early time in cosmic history.

3.2.1 Physical origin of the CMB

Following BBN, the visible Universe primarily consists of fully ionized hydrogen and helium nuclei, free electrons, and a photon-dominated radiation field. Together, these components form a tightly coupled photon-baryon plasma, characterized by frequent interactions through Thomson scattering. Due to frequent Thomson scattering, the early Universe is opaque, as the photons have a mean free path significantly smaller than the horizon size. As the Universe expands and cools, it becomes energetically favorable for electrons to bind to helium and eventually hydrogen atoms in a process known as recombination⁴. This drastically reduces the free electron density and consequently decreases the rate of Thomson scattering, enabling photons to freestream, likely for the first time in cosmic history. These photons subsequently travel largely unimpeded and constitute the radiation we observe today as the CMB.

3.2.2 Recombination

Recombination occurs in two distinct regimes: an initial equilibrium phase followed by a non-equilibrium phase. In equilibrium, recombination is accurately described by the Saha approximation, while the subsequent non-equilibrium regime requires a more detailed treatment involving the Peebles equation. In this section, we consider only hydrogen recombination, as the CMB offers little support for the specific details of neutral helium recombination as photons are likely to continue to scatter after helium forms. However, it is relevant to note that helium recombination occurs earlier, due to its higher electron binding energy. In general, the timing of recombination for any given species depends on the ratio of its binding energy to the background temperature.

⁴The term “recombination” is somewhat misleading, as electrons and protons combine to form neutral atoms for the first time at this epoch; there was no prior combination event. Nevertheless, the historical usage of “recombination” persists in cosmological literature.

In the equilibrium regime, the evolution of the free electron fraction, X_e , is accurately described by the Saha approximation:

$$\frac{X_e^2}{1 - X_e} \approx \frac{1}{n_b} \left(\frac{m_e T}{2\pi} \right)^{3/2} e^{-B_H/T} \quad (69)$$

where X_e is the free electron fraction, n_b is the visible baryon number density, m_e is the electron mass, T is the photon temperature, and B_H is the atomic binding energy.

Once the free electron fraction and temperature drops sufficiently, the recombination rate falls below the background expansion rate. This causes X_e to freeze out and is no longer describable through equilibrium processes. Now, a more comprehensive Boltzmann treatment is required. This necessitates the use of the Peebles equation given by;

$$\frac{dX_e}{da} = \frac{C\alpha^{(2)}}{aH} \left(\left(\frac{m_e T}{2\pi} \right)^{3/2} e^{-B_H/T} (1 - X_e) - n_b X_e^2 \right). \quad (70)$$

where C is the Peebles factor and $\alpha^{(2)}$ is the case B recombination rate given approximately by [156]

$$\alpha^{(2)} \approx \frac{\alpha^2}{m_e^2} \left(\frac{B_H}{T} \right)^{1/2} \ln \left(\frac{B_H}{T} \right). \quad (71)$$

The Peebles factor encodes the probability that a hydrogen nucleus with an electron in the $n = 2$ shell reaches the ground state before it is photoionized. This is critical as the only transitions which result in ‘net’ recombination are those from the first excited state to the ground state of hydrogen. All other transitions generate a photon with sufficient ionizing energy that it immediately ionize a different neutral hydrogen atom. The Peebles factor is parameterized by three key rates given by

$$C = \frac{R_{Ly\alpha} + \Lambda_{2s \rightarrow 1s}}{\beta_B + R_{Ly\alpha} + \Lambda_{2s \rightarrow 1s}}. \quad (72)$$

The escape rate of Lyman- α photons, $R_{\text{Ly}\alpha}$, is

$$R_{\text{Ly}\alpha} = \frac{8\pi H}{\lambda_{\text{Ly}\alpha}^3 n_{\text{H}} x_{1\text{s}}}, \quad (73)$$

where n_{H} is the number density of hydrogen, $x_{1\text{s}}$ is the fraction of hydrogen with an electron in the ground state, and $\lambda_{\text{Ly}\alpha}$ is the Lyman- α wavelength given by $\frac{1}{\lambda_{\text{Ly}\alpha}} = \frac{3}{4}B_{\text{H}}$. The factor of $\frac{3}{4}$ arises from the electron transitioning from the first excited state to the ground state. The dominant contribution to the C factor and the overall amount of net recombination is $R_{\text{Ly}\alpha}$.

The two-photon decay rate, $\Lambda_{2\text{s}-1\text{s}}$, describes transitions where an electron transitions from the metastable 2s state to the 1s ground state, emitting two photons whose combined energy equals the energy difference between these states. It is a fundamental atomic parameter and is well established through quantum electrodynamics calculations and experimental measurements. The commonly adopted theoretical value for hydrogen is approximately [157]:

$$\Lambda_{2\text{s}\rightarrow 1\text{s}} \approx 8.22458 \text{ s}^{-1}. \quad (74)$$

Finally, the case-B photoionization rate, β_{B} , is expressed as

$$\beta_{\text{B}} = \left(\frac{m_{\text{e}} T}{2\pi} \right)^{3/2} e^{-B_{\text{H}}/T} \alpha^{(2)}. \quad (75)$$

The recombination process moves out of equilibrium due primarily to the finite escape probability of Lyman- α photons ($R_{\text{Ly}\alpha}$), rather than the intrinsic atomic physics rates alone ($\Lambda_{2\text{s}\rightarrow 1\text{s}}$ and β_{B}). It is the gradual escape of Lyman- α photons out of resonance due to cosmological redshift that ultimately regulates the timescale and progress of recombination in the non-equilibrium regime.

It is clear from Eqs. (69), (70), and the following discussion that recombination is not an instantaneous event. In fact, the Universe somewhat gradually transitions from being optically thick to thin over about 100,000 years. Therefore, there must be an agreed upon standard in which the surface of last scattering is defined. To compute the redshift of the surface of last scattering, we consider the visibility function. The visibility function represents the likelihood that a photon scatters for the last time before free-streaming away and is given by

$$g(z) = e^{-\tau} \frac{d\tau}{dz} \quad (76)$$

where τ is the optical depth to photons at redshift z and is given by

$$\tau(z) = \int_0^z \frac{\dot{\kappa}(z')}{(1+z')H(z')} dz' \quad (77)$$

where $\dot{\kappa}$ is given by

$$\dot{\kappa} = \frac{n_e \sigma_T}{1+z} \quad (78)$$

where n_e is the free electron number density and σ_T is the Thomson cross section. There are two critical features of the visibility function: the width and peak of the function. The peak represents the time where photons are most likely to scatter for the last time and sets the distance to the surface of last scattering, and the width corresponds to the duration of decoupling [158].

3.2.3 The CMB blackbody

The radiation field of the CMB exhibits a nearly perfect blackbody spectrum, providing strong evidence that the early Universe was in thermal equilibrium and supporting the hot Big Bang model. The remarkable precision of this spectrum, first accurately measured by the COBE satellite (shown in Fig. 1) and subsequently refined later, tightly constrains possible energy injections into the early plasma and late-time photon interactions, both of which would produce measurable deviations from a pure blackbody. Any non-standard energy in-

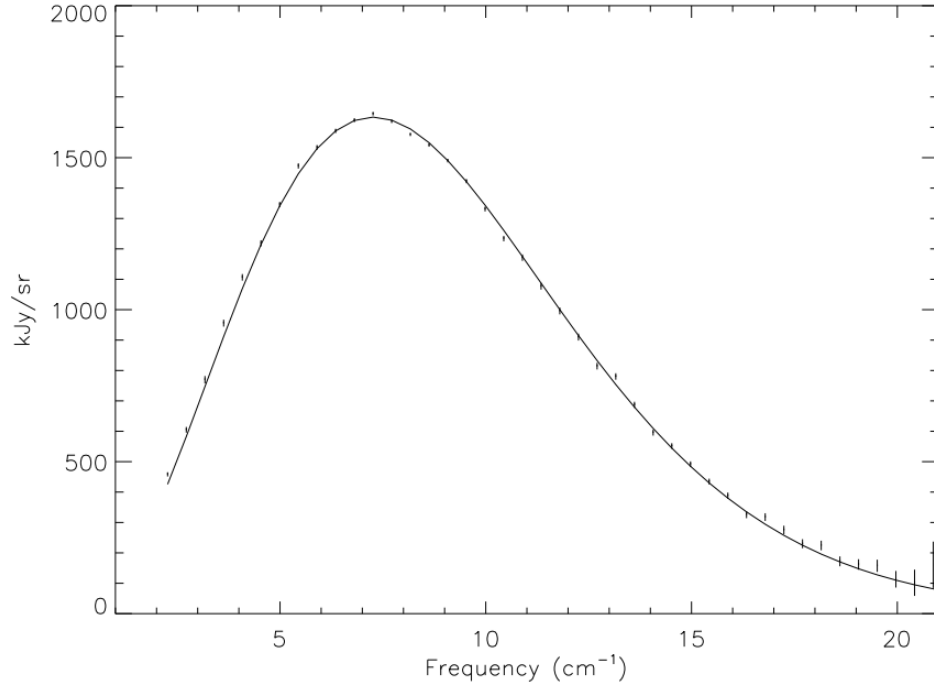


Figure 1: The intensity of a $T = 2.7255$ K blackbody curve compared to original COBE-FIRAS data. Note that even in the initial data release, the data strongly agrees with the predicted Planck blackbody spectrum. Figure from Ref. [12].

jection processes, such as particle decay, annihilation, or exotic physics phenomena occurring after thermalization, would distort the blackbody spectrum. The lack of observable spectral distortions strongly limits interactions of CMB photons with matter or radiation fields since recombination. Additionally, processes such as Compton scattering by hot electrons (the Sunyaev–Zel’dovich effect), absorption by dust, or photon-photon interactions must be extremely limited to preserve the blackbody form observed today. The current temperature of the CMB is measured to be $T_0 = 2.7255 \pm 0.0005$ K, which is one of the most precisely known quantities in all of cosmology [14].

3.2.4 CMB anisotropies

However, small fluctuations (anisotropies) in the temperature of the CMB exist at the level of $\Delta T/T \sim 10^{-5}$. These anisotropies encode critical information about primordial density

perturbations, the geometry of the Universe, and the relative abundances of baryonic and dark matter. The temperature variations of the cosmic microwave background can be formally expressed as fluctuations around the mean temperature T_0 , defined at a particular direction $\hat{n}$ on the sky as:

$$\frac{\Delta T(\hat{n})}{T} = \frac{T(\hat{n}) - T_0}{T_0}. \quad (79)$$

The primary source of temperature anisotropies arise from gravitational potentials in a process called the *Sachs-Wolfe effect*. Consider a gravitational potential Φ with small Gaussian-distributed perturbations, Φ , about a uniform mean potential. Regions with negative perturbations ($\Phi < 0$) correspond to over-densities, whereas positive perturbations ($\Phi > 0$) correspond to under-densities. Photons originating from areas of where there is a positive (negative) perturbation will lose (gain) energy due to gravitational redshift as it leaves the region, resulting in different regions having slightly different temperatures compared to T_0 . Thus, temperature anisotropies generated by the Sachs-Wolfe effect directly encode information about the spatial distribution of matter at the surface of last scattering.

To statistically quantify the observed anisotropies of the CMB in a way that cosmological models can be compared to data, the temperature perturbations are expanded into our familiar spherical harmonics, $Y_{\ell m}(\hat{n})$:

$$\frac{\Delta T(\hat{n})}{T} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\hat{n}), \quad (80)$$

where the expansion coefficients $a_{\ell m}$ fully characterize the temperature field as they quantify the amplitude of anisotropies at each angular scale on the sky and are given by

$$a_{\ell m} = \int d\hat{n} \frac{\Delta T(\hat{n})}{T} Y_{\ell m}^*(\hat{n}). \quad (81)$$

Under the assumption of isotropy and Gaussianity, the statistics of the temperature anisotropies

are completely determined by the angular power spectrum C_ℓ , defined via the ensemble average:

$$\langle a_{\ell m} a_{\ell' m'}^* \rangle = C_\ell \delta_{\ell\ell'} \delta_{mm'}, \quad (82)$$

where the Kronecker deltas, $\delta_{\ell\ell'}$ and $\delta_{mm'}$, enforce statistical isotropy and independence of modes. In practice, the observed angular power spectrum, C_ℓ , is estimated by averaging over all measured $|a_{\ell m}|^2$ for each multipole ℓ :

$$C_\ell^{\text{obs}} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2. \quad (83)$$

Perhaps a more intuitive way to consider the angular power spectrum is to think of it as a two point correlation function given by

$$C(\theta) = \frac{1}{T^2} \langle \Delta T(\hat{n}) \Delta T(\hat{n}') \rangle \quad (84)$$

where θ is the angular separation of $\hat{n}$ and $\hat{n}'$. Then, to convert from $C(\theta)$ to the more familiar angular power spectrum, some spherical harmonics are required to appropriately describe the ‘distortion’ we observe due to the 2D spherical nature of the sky, where the C_ℓ ’s are given by

$$C_\ell = \frac{1}{4\pi} \int d\hat{n} d\hat{n}' P_\ell(\cos \theta) C(\theta) \quad (85)$$

where $P_\ell(\cos \theta)$ are the standard Legendre polynomials. As a final note, it is common to plot the dimensionful quantity $\mathcal{D}_\ell$ in units of $[\mu K]^2$ rather than the dimensionless C_ℓ ’s, given by

$$\mathcal{D}_\ell \equiv \frac{\ell(\ell + 1)}{2\pi} T_0^2 C_\ell \quad (86)$$

where T_0 is the CMB temperature today. This convention is common as there is much more power on large scales compared to small scales, and because $\mathcal{D}_\ell$ is proportional to the power per logarithmic interval in ℓ , the features in the small scale power spectrum are more

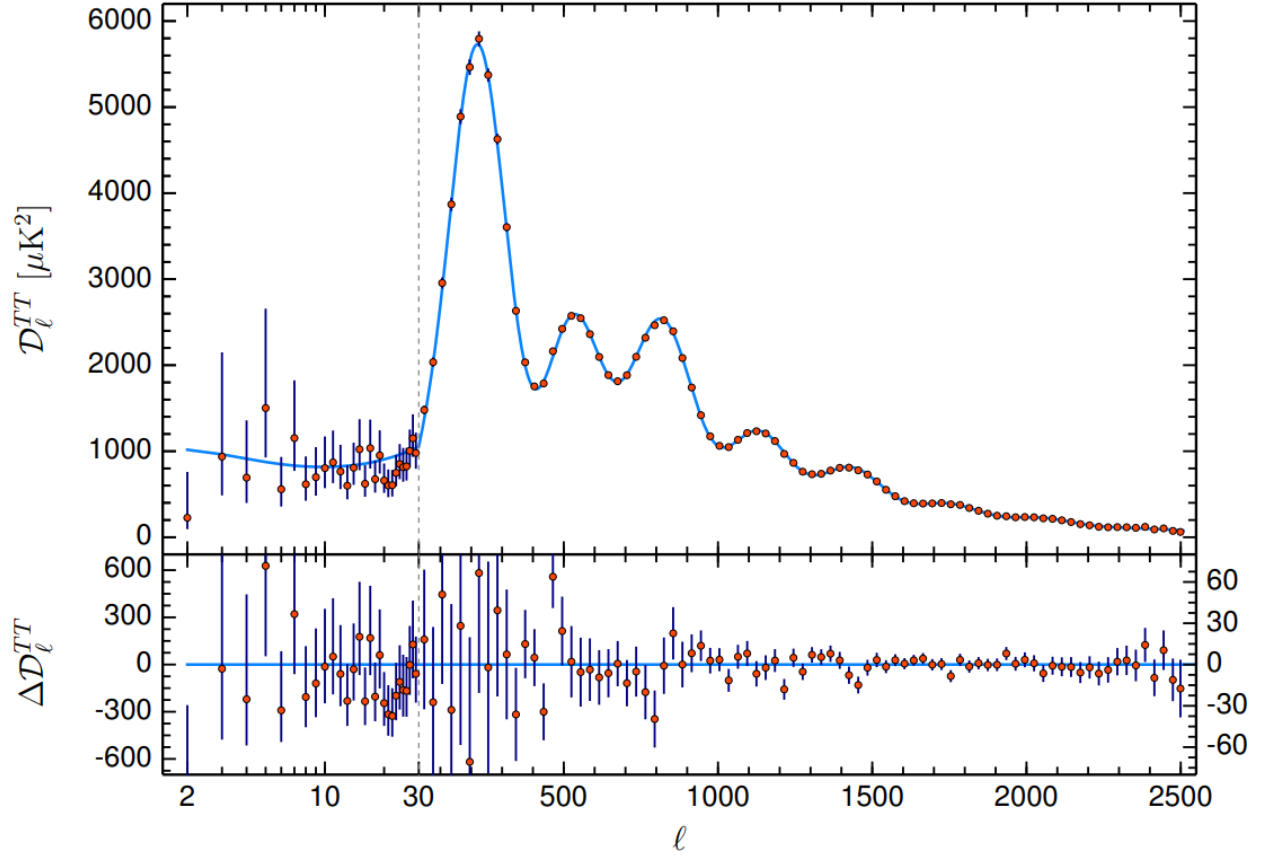


Figure 2: Planck 2018 temperature power spectrum. Red dots show Planck satellite observations with corresponding error bars indicating $\pm 1\sigma$ uncertainties. The blue line shows the best-fit ΛCDM prediction using combined TT, TE, EE, lowE, and lensing likelihoods. The lower panel displays residuals relative to this best-fit ΛCDM model. Figure from Ref. [144].

pronounced and easier to interpret. Effectively, it's just a way to narrow the large dynamic range that is found in the C_ℓ spectra.

3.2.5 The TT spectrum

The angular power spectrum of temperature anisotropies in the CMB, shown in Fig. 2, exhibits distinctive structure across different multipole ranges, each associated with unique physical processes occurring at vastly different angular scales. Broadly, this angular spectrum is divided into three regimes: the Sachs–Wolfe plateau ($\ell < 100$), the acoustic peak regime ($100 \lesssim \ell \lesssim 1000$), and the diffusion damping tail ($\ell > 1000$).

$\ell \lesssim 100$: At large angular scales, temperature anisotropies predominantly arise from gravitational effects rather than acoustic effects. Photons leaving gravitational potential wells at recombination experience gravitational redshifts, producing observed temperature fluctuations. Over-dense regions correspond to deeper gravitational potentials, resulting in cooler observed photon temperatures, while under-dense regions yield the inverse effect. These gravitationally induced fluctuations form a characteristic flat plateau at low multipoles, termed the *Sachs–Wolfe plateau*. On such large scales, causal processes have insufficient time to significantly alter initial density perturbations, thus preserving their primordial structure. The power seen on these scales serves as a probe of the initial conditions of the Universe. However, evolving gravitational potentials encountered after recombination generate further anisotropies, known as the *integrated Sachs–Wolfe (ISW) effect*. The ISW effect emerges whenever the Universe deviates from matter-dominated expansion (equation of state $\omega = 0$). Notably this effect influences observation today, as the Universe has recently entered the epoch of dark energy domination ($\omega \approx -1$), and can be seen as a slight rise in power at very low ℓ modes.

$100 \lesssim \ell \lesssim 1000$: In this intermediate regime, temperature anisotropies reflect gravity-driven acoustic oscillations occurring in the photon–baryon plasma prior to recombination. Unlike the largest scales ($\ell \lesssim 100$), these modes have had sufficient time to evolve causally, forming distinct acoustic patterns. These oscillations behave analogously to harmonic oscillators, where dark matter dominated gravitational potentials provide the driving force, with photons providing pressure (restoring force) and baryons contributing inertia. Because perturbations remain small, each Fourier mode evolves independently as a linear harmonic oscillator, whose characteristic frequency is governed by the fluid’s sound speed. At recombination, photons stream freely, effectively freezing the oscillation patterns into observable anisotropies. This mechanism naturally explains the characteristic peaks and troughs in the C_ℓ^{TT} spectrum. The first peak near $\ell \sim 225$ corresponds to modes at maximal compression

(density maximum) at recombination (1/4 oscillation period), while even-numbered peaks represent maximal rarefactions (density minima). Odd peaks beyond the first represent higher order compression phases. Troughs between peaks do not reach zero amplitude, as they correspond to velocity extrema rather than density extrema, where baryonic matter is falling into or out of the wells at their fastest rate.

$\ell \gtrsim 1000$: At smaller angular scales, temperature anisotropies predominantly reflect photon diffusion processes rather than gravitational and acoustic phenomena seen at lower multipoles. At these scales, photons remain tightly coupled to electrons through Thomson scattering, yet their mean free path becomes comparable to or larger than the size of density perturbations. Photon diffusion effectively smooths out density fluctuations, suppressing power at scales smaller than the photon diffusion scale. This suppression appears as the exponential decline in power known as the damping tail (Fig. 2). Naively, one may think that because there is exponentially declining correlation here this regime is less information dense than those controlled by gravitational or acoustic physics. However, the damping tail tells us a considerable amount about the relationship between photons and baryons at this time, setting constraints on the ratio of the Thomson scattering rate to background expansion rate, as well as the spectral index of primordial fluctuations.

3.2.6 The EE spectrum

The radiation field from the CMB also carries linear polarization at approximately the 10% level, primarily due to Thomson scattering. This polarization forms gradient-like patterns known as E-modes. Polarization arises only when incident radiation exhibits anisotropy; specifically, quadrupole anisotropies in the photon temperature distribution generate the observed polarization. These quadrupole anisotropies naturally arise from acoustic oscillations in the photon–baryon plasma before recombination, creating local temperature gradients that, when scattered by electrons, result in net polarization. As we will see, polarization is

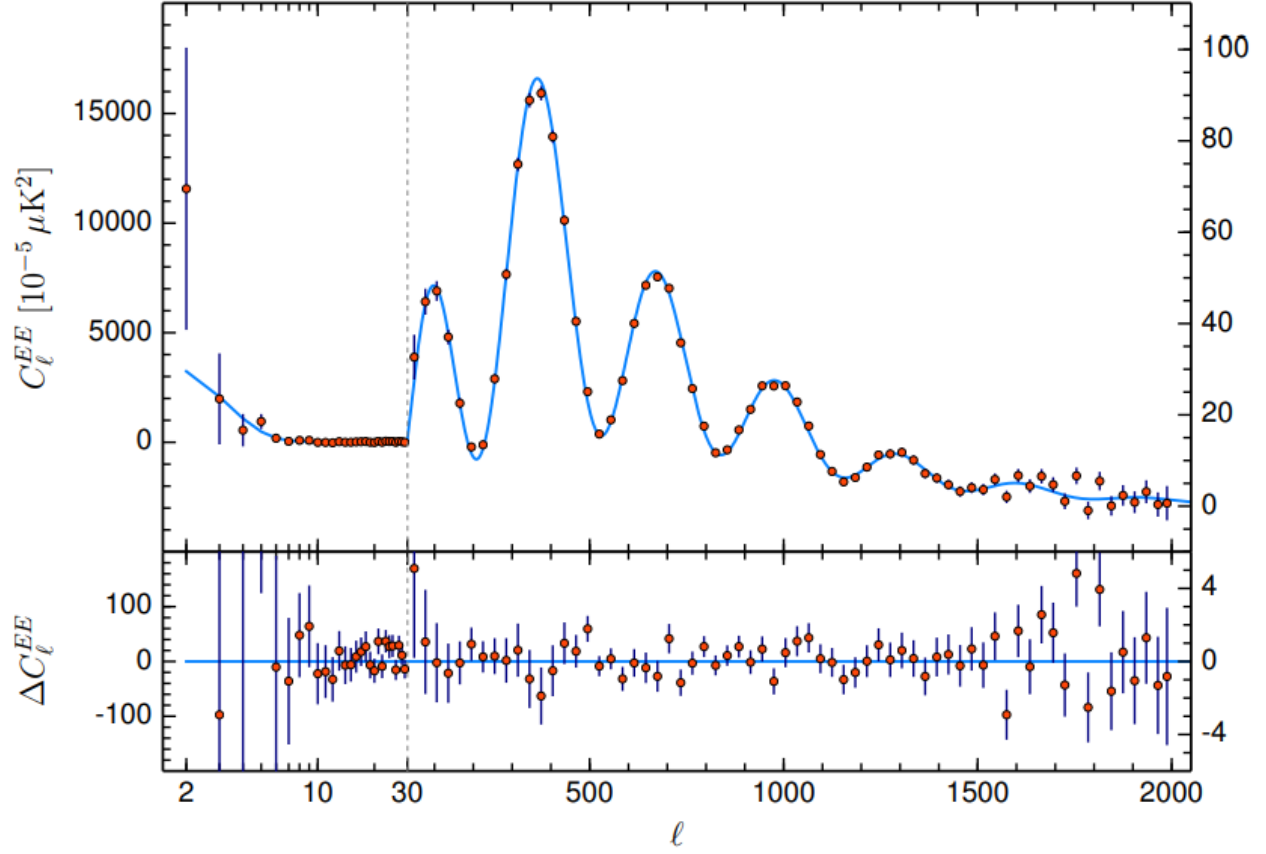


Figure 3: Planck 2018 polarization (EE) power spectrum. Red dots indicate Planck satellite observations with $\pm 1\sigma$ uncertainties. The blue line represents the best-fit Λ CDM prediction based on combined TT, TE, EE, lowE, and lensing likelihoods. The lower panel shows residuals relative to this best-fit model. Figure from Ref. [144].

also generated at later times during reionization, when the first stars and galaxies ionized the intergalactic medium, reintroducing free electrons and producing additional Thomson scattering events.

$\ell \lesssim 10$: At very low multipoles, the C_ℓ^{EE} spectrum shows a broad enhancement from additional Thomson scattering during reionization. The amplitude of this feature directly constrains the optical depth τ_{reio} , providing detailed information on how and when the first stars and galaxies turned on and reionized the intergalactic medium.

$10 \lesssim \ell \lesssim 1000$: Analogous to temperature anisotropies, peaks in the EE spectrum reflect acoustic oscillations before recombination. However, polarization peaks are shifted to

smaller angular scales, appearing approximately half a period out of phase with the temperature peaks. This phase shift occurs because polarization intensity is maximized at velocity extrema rather than at density extrema.

$\ell \gtrsim 1000$: At small angular scales, the EE spectrum experiences suppression from photon diffusion damping, analogous to the TT spectrum. Notably, photon diffusion more strongly affects polarization anisotropies since they have greater dependence on local velocity gradients rather than local density contrasts.

Measurements of the polarization spectrum are critical as it offers not only a complementary data set to go with the TT spectrum data, but also offers an independent constraint on cosmological parameters, like the physical baryon density $\Omega_b h^2$, the physical matter density $\Omega_m h^2$, and the scalar spectral index n_s . This serves as a critical independent check on the Λ CDM model which was first designed to explain just the temperature anisotropies.

3.2.7 The TE spectrum

The TE power spectrum represents the cross-correlation between temperature and E-mode polarization anisotropies. Unlike TT and EE spectra, the TE spectrum can exhibit both positive and negative values, reflecting correlations and anti-correlations between temperature and polarization fluctuations. The structure of the TE spectrum arises directly from acoustic oscillations and their associated velocity and density perturbations, with peaks and troughs closely tied to the oscillation phases. In fact, TE peaks generally appear at intermediate positions between the TT and EE peaks due to their mixed nature, combining contributions from both density extrema (TT-dominated) and velocity extrema (EE-dominated).

3.3 Lyman alpha forest

The term “Lyman- α forest” refers to a series of absorption lines in spectroscopic measurements when observing the spectra of distant quasars. These features are caused by intervening neutral hydrogen clouds along the line of sight and arise due to the Lyman- α electron transition in hydrogen, corresponding to photons with a wavelength of approximately 121.6 nm. What makes the Lyman- α forest useful for cosmology is that the hydrogen clouds that exist on the line of sight to the distant quasar appear at different redshifts. Because of the vast distances to and between the intervening clouds, the Universe expands and redshifts the absorption lines we observe forming the characteristic “forest” of absorption features, as seen in Fig. 4.

After recombination, hydrogen gas is almost entirely neutral until the era of reionization where the first luminous objects (like stars and quasars) began emitting significant amounts of photons into the intergalactic medium, ionizing the hydrogen gas. The Lyman- α forest feature naturally emerges from the line-of-sight intersections between quasar photons and neutral hydrogen clouds. When the quasar produces high energy photons, as quasars tend to do, most of them pass through the neutral hydrogen clouds, but some of the photons are resonantly absorbed by neutral hydrogen atoms. Each absorption feature in the Lyman- α forest corresponds to one or more clouds along the line of sight at a particular redshift. Because each intervening neutral hydrogen cloud exists at a different cosmological redshift, each cloud imprints its own distinct absorption line that is shifted according to the redshift at which the cloud resides.

Observations of Lyman- α forest are a valuable probe of the Universe around redshifts of $z \sim 2 - 6$ as they provide an unique and direct insight into the distribution of neutral hydrogen. By observing the distribution and state of neutral hydrogen through these absorption features, we are able to better understand the nature of cosmic structure evolution and

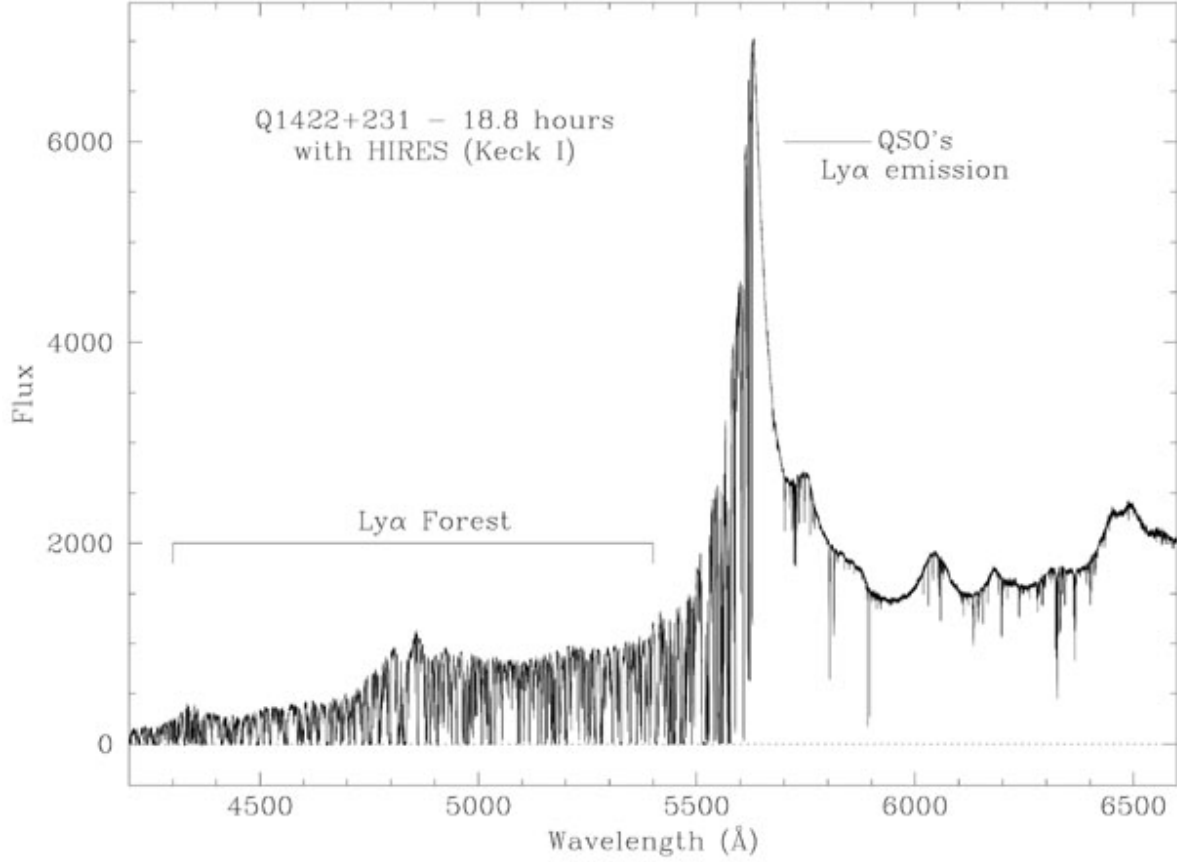


Figure 4: A beautiful example of the Lyman- α forest from a quasar at $z = 3.725$. At these high redshifts, the Lyman- α forest dominates the quasar spectra at wavelengths less than the quasars own Lyman- α line. Each dip seen in the forest corresponds to an absorption feature from intervening clouds of neutral hydrogen. Figure from Ref. [159].

the thermal history of the Universe at this time. Perhaps the most important information the Lyman- α forest provides is regarding the primordial matter power spectrum. As is the typical story in Λ CDM, the clustering of hydrogen clouds closely traces the underlying dark matter distribution, especially at small and intermediate scales. By inferring the physical and number density of the intervening hydrogen clouds through forest observations, we can place constraints on the amplitude and slope of the matter power spectrum at small scales. Additionally, the evolution in redshift of the number density and opacity of the absorption features reveals detailed information on how and when reionization occurred, as well as the temperature and ionization state of the neutral hydrogen clouds. Finally, these observations also provide constraints on the sum of the neutrino masses, the spectral index of scalar perturbations, and provide an independent test ground for more advanced dark matter models.

3.4 Baryon acoustic oscillations

Baryon Acoustic Oscillations (BAO) represents an instance of the acoustic physics driving the dynamics of the prerecombination plasma appearing in observables much later in cosmic history. The BAO signal arises as a direct consequence of the acoustic waves which propagated through the prerecombination plasma by leaving a characteristic length scale imprinted in the large-scale structure of the Universe. The acoustic waves and shocks present in the plasma during and after recombination were effectively “frozen” in place, where their density contrasts remain constant as sound waves no longer had a medium to effectively propagate through. The characteristic scale (commonly referred to as the sound horizon) of these acoustic oscillations are now visible in the distribution of large scale structure and form a powerful standard ruler in cosmology.

Indeed, the BAO feature manifests itself as a statistical excess in the clustering of galaxies precisely at the same scale as the sound horizon. This excess appears as an oscillatory feature in the matter power spectrum at comoving scales corresponding to the scale of the

sound horizon and as a bump in the two-point galaxy correlation function. BAO serves as a powerful probe of the Universe in the redshift range of $z \sim 0.1 - 2.5$ providing a robust standard ruler. The robustness arises from the lack of dependence on astrophysical processes (and thus their uncertainties). The physical size of the sound horizon is well constrained from CMB measurements, allowing BAO to be used as powerful probe of distances and thus expansion rates. Additionally, because the BAO feature is observable across matter-dark energy equality, it serves as a powerful tool to constrain the nature of dark energy and its equation of state ω . Finally, it also provides a critical complementary and independent check on cosmological parameters obtained from the CMB, forcing cosmological models to simultaneously predict the CMB and BAO features simultaneously for a robust and compelling model.

4 Monte Carlo methods

The purpose of computing is insight, not numbers.

- Richard Hamming

A Monte Carlo method is any computational technique that utilizes random sampling to numerically estimate a quantity of interest. Monte Carlo methods are extensively applied across numerous scientific disciplines for numerical computations and simulations, particularly when dealing with complex or high-dimensional probability distributions. In cosmology, these methods allow us to numerically explore complicated probability spaces where analytic solutions are typically inaccessible.

Broadly, Monte Carlo methods fall into two categories. In the first category, independent samples X_i are drawn directly from a known target distribution. The law of large numbers ensures that the sample mean converges to the true mean μ as the number of samples increases:

$$\mu = \frac{1}{n} \sum_{i=1}^n X_i. \quad (87)$$

This is referred to as direct Monte carlo.

The second category comprises Markov Chain Monte Carlo (MCMC) methods, and is the category most relevant for cosmological applications. To properly understand these, we first define what comprises a Markov chain. A Markov chain is a stochastic process consisting of a sequence of random variables X_i , where each step satisfies the *Markov property*. This property states that the conditional probability distribution of each future state depends solely on the current state and not on any prior states, formally written as:

$$P(X_{n+1}|X_n, X_{n-1}, \dots, X_1) = P(X_{n+1}|X_n). \quad (88)$$

A Markov chain is fully specified by:

- a set of states (in cosmology, typically the free parameters of a model),
- an initial probability distribution specifying the starting state, and
- a transition probability governing how the system moves between states.

MCMC methods rely on constructing Markov chains whose stationary distribution matches the target posterior distribution. The stationary distribution of a Markov chain is defined as the distribution toward which the chain converges as the number of iterations approaches infinity. This stationary distribution precisely matches the desired posterior distribution used in Bayesian inference. After an initial “burn-in” period, during which the chain approaches equilibrium, subsequent samples from the chain provide correlated but representative samples from the posterior probability distribution. These samples enable us to robustly characterize the parameter estimates and uncertainties of cosmological models.

It is crucial that the Markov chain satisfies the conditions of *ergodicity* and *detailed balance*. Ergodicity ensures that the chain will eventually explore the entire parameter space, independent of its initial state, while detailed balance guarantees that the chain maintains the posterior as its stationary distribution. Practical implementations of MCMC, such as those used in cosmological parameter inference, explicitly design transition probabilities to fulfill these conditions, which we will see later.

4.1 Bayesian basics

In Bayesian inference and statistics, all uncertainty is described using probability. In general, the statistics which most cosmological studies employ are Bayesian in nature. Put simply, Bayesian inference allows one to find the probability of a event occurring given information on what causes it.

Bayesian statistics begins with a choice of *prior distribution*, which we will call $p(\theta)$, which encodes our knowledge (or in practice, typically our ignorance) about the parameters describing the cosmological model being tested. There also exists a *likelihood function* $p(D | \theta)$ which specifies the probability of observing the data D for a given choice of parameters θ . Note that this step is dependent on the cosmological model of choice. Bayes' theorem combines the prior distribution and the likelihood function to yield the *posterior distribution* $p(\theta | D)$ which represented our updated understanding of the parameters given the data. In this form, Bayes' theorem is given by

$$p(\theta | D) = \frac{p(D | \theta) p(\theta)}{p(D)} \quad (89)$$

where

$$p(D) = \int p(D | \theta) p(\theta) d\theta \quad (90)$$

is the marginal likelihood (or evidence) ensuring the posterior is properly normalized. In this way, we can clearly see how the data and prior influence the inference of parameters θ : if the data are highly informative, say like the Planck 2018 spectra, the likelihood $p(D | \theta)$ will dominate the resulting posterior $p(\theta | D)$ but if the data is sparse or has large uncertainty, the prior $p(\theta)$ will dominate the posterior.

It is generally difficult to compute the marginal likelihood $p(D)$, as θ is typically multivariate and the posterior distribution will typically be of high dimension with no closed form solution. However, for most cases in cosmology, we don't actually need the marginal likelihood, as the unnormalized posterior distribution is sufficient to estimate parameter means and uncertainties. This is because most of what we are interested in calculating using Bayesian inference actually involves the ratio of posteriors. For example, three common results of a MCMC analysis are

1. computing expectation values or posterior means,

2. determining relative posterior odds, and
3. sampling from the posterior to infer variance.

In all three cases, which is most of MCMC analysis in a cosmological context, you end up forming ratios of the posterior distribution at two different sets of θ , such that

$$\frac{p(\theta' | D)}{p(\theta | D)} = \frac{p(D | \theta') p(\theta')}{p(D | \theta) p(\theta)} \quad (91)$$

where, notably, the marginal likelihood $p(D)$ cancels in the ratios.

However, there are cases in cosmology where the marginal likelihood is critical. I won't cover them all in this thesis, but arguably the most prevalent need to calculate $p(D)$ arises during Bayesian model selection and comparison. Bayesian inference provides a natural way to assess the relative probability of two models using Bayes factors. Consider the situation where you have two cosmological models and you want to assess their relative probability, being sure to account for the model complexity differences. The Bayes factor for a given model M is

$$p(D | M) = \int p(D | \theta, M) p(\theta | M) d\theta. \quad (92)$$

Here, $p(D | M)$ represents the probability of observing the data D , integrated over all possible values of the parameters θ in model M . Therefore, the integral of equation Eq. (92) encodes two very important pieces of information. First, it rewards models and parameter sets which fit the observed data, as they will have high likelihood values and thus increase the evidence. Second, models with more free parameters and wide priors tend to have their likelihood “spread” over the increasingly complex parameter space, resulting in lower evidence if the additional parameters are unneeded.

Now, consider two cosmological models, M_1 and M_2 . The ratio of their evidences is called

the Bayes factor, and is given simply by

$$B_{12} = \frac{p(D \mid M_1)}{p(D \mid M_2)}. \quad (93)$$

The Bayes factor quantifies how strongly data prefer one model over the other. The Jefferys' scale is the most common interpretation of the Bayes factor and goes as:

- $1 < B_{12} < 3$: Weak evidence for model M_1
- $3 < B_{12} < 10$: Moderate evidence
- $10 < B_{12} < 100$: Strong evidence
- $B_{12} > 100$: Decisive evidence

4.2 MCMC algorithms

There are many algorithms which support MCMC investigations, and I will cover two here. The first, the Metropolis-Hastings algorithm, is perhaps the most common MCMC algorithm and is very straightforward to apply. The second, the Hamiltonian Monte Carlo approach, is a much more advanced MCMC algorithm which excels in high-dimensional spaces or when the target distribution has strong correlations between parameters.

4.2.1 Metropolis-Hastings

The Metropolis-Hastings (MH) algorithm generates a Markov chain by iteratively proposing steps and accepting or rejecting them in such a way that the chain's stationary distribution *is* the desired posterior $p(\theta \mid D)$. The process can be described in general as

1. Start with an initial set of parameter values, θ_{ini} . Ultimately, this can be a totally arbitrary choice, but the better guess you have for θ_{ini} , the faster your chains will converge.

2. Once a current state θ_i is available, draw a new proposal state θ' from the proposal distribution $q(\theta' | \theta_i)$. The proposal distribution (also sometimes called the jumping distribution) can be any distribution that is samplable. Typically, symmetric distributions are chosen for simplicity, like a Gaussian centered at θ_i .
3. Once a proposal distribution θ' is selected, the acceptance ratio must be computed, given by

$$r = \frac{p(D | \theta')p(\theta')}{p(D | \theta_i)p(\theta_i)} \quad (94)$$

where I have assumed we used a symmetric proposal distribution in step 2.

4. Finally, generate a uniform random number u between 0 and 1. If $u \leq r$, accept the proposed state θ' and set it as the next current state $\theta_{i+1} = \theta'$. Otherwise, if $u \geq r$, reject the proposed state θ' and set the next current state $\theta_{i+1} = \theta_i$. This process may seem somewhat ad-hoc, but it ensures that you always move to a higher probability state (because as long as $r > 1$ you will always satisfy $u \leq r$). Occasionally, you will move to a lower-probability state, but this is somewhat of a feature of the algorithm as it ensures thorough exploration. The more unlikely a proposed state θ' is to explain the data D , the less likely you will move to the proposed state.

By repeating the process of proposal, computation of r , and accepting/rejecting many times, the Markov chain of $\{\theta_1, \theta_2, \theta_3, \dots\}$ is generated. While not presented here, as it would detract from the point of this section, the acceptance presented in step 4 preserves the concept of detailed balance, which ensures that the stationary distribution of the chain *is* the posterior distribution we seek, $p(\theta | D)$. Because the algorithm actually tends to spend more time in regions of high posterior probability (as these are the steps that are more likely to be accepted), the fraction of time that the chain spends in any region of θ space is proportional to the actual posterior $p(\theta | D)$. In other words, the chain actually does sample from the posterior. The major advantages of MH is that it only requires computing the posterior distribution up to a constant, which is exactly what most cosmological studies

need as the evidence (or marginal likelihood) is generally unknown. Note that in Eq. (94), we never need to compute $p(D)$ explicitly.

4.2.2 Hamiltonian

While the Metropolis-Hastings algorithm is notably simple and effective for many cosmological needs, it can struggle in high-dimensional spaces or when the posterior distribution has strong correlations between parameters. To explore higher-dimensional spaces, the Hamiltonian approach is gaining wide acceptance as the successor to MH methods. In Hamiltonian Monte Carlo, gradient information of the posterior distribution is used to guide the proposal process. This algorithm is based on very familiar Hamiltonian dynamics, where each parameter in the set θ is treated as being a position in space with corresponding momentum variables. The algorithm then simulates the motion of the fictitious particles representing our parameters in the posterior distribution, where the posterior defines a potential energy surface.

Imagine having a bumpy dining room table, where there are ridges and valleys on the surface, and then pouring a large amount of small marbles on the table. The marbles will naturally find local minima where they will come to rest, and by counting the density of marbles on your table as a function of position, you can calculate the curvature of your table. This is exactly how Hamiltonian Monte Carlo works; the table is your posterior distribution and the marbles are your parameter sets θ .

More formally, we define the Hamiltonian function

$$H(\theta, \rho) = U(\theta) + K(\rho) \tag{95}$$

where $U(\theta)$ is the ‘potential energy’ of the posterior distribution and $K(\rho)$ is the ‘kinetic energy’ of the fictitious particle representing a point in the parameter space. In most approaches, the potential energy is related to the poster distribution via

$$U(\theta) = -\ln[p(\theta \mid D)] \quad (96)$$

such that low potential energy corresponds to high posterior probability. The kinetic energy is typically chosen such that

$$K(\rho) = \frac{1}{2}\rho^T M^{-1}\rho \quad (97)$$

where ρ is a momentum vector that describes the position and M is a mass matrix (often set to the identity or learned during a tuning phase). Together, $\{\theta, \rho\}$ form a familiar, joint “canonical” density

$$\exp[-H(\theta, \rho)] = \exp[-U(\theta)] \exp[-K(\rho)] \quad (98)$$

where several simplifications have become apparent. First, it is clear that

$$\exp[-U(\theta)] = p(\theta \mid D). \quad (99)$$

Second, we have the freedom to choose the form of $\exp[-K(\rho)]$ and will typically choose it to be Gaussian. With these two simplifications, the marginal distribution for a parameter set θ in this join density *is exactly* the posterior distribution we seek.

In practice, the Hamiltonian Monte Carlo works in much the same way Hamiltonians work in classical physics:

$$\frac{d\theta}{dt} = \nabla_{\rho} H(\theta, \rho) \quad \& \quad \frac{d\rho}{dt} = -\nabla_{\theta} H(\theta, \rho). \quad (100)$$

These dynamics are then simulated in a discretized time coordinate system for a certain

number of steps with given step size, commonly referred to as a “leapfrog integrator”. This generates a proposal in the space $\{\theta', \rho'\}$ by roughly “leaping” from the current state $\{\theta, \rho\}$. Critically, in this step we benefit from the notion of energy conservation. The proposal state will be on the same contour of constant energy as the current state.

Once a proposal state is generated, the acceptance step is essentially the same as in the MH approach. The acceptance probability for the Hamiltonian approach is given by

$$\alpha = \min(1, \exp[H(\theta, \rho) - H(\theta', \rho')]) \quad (101)$$

meaning the move to the new state is accepted with probability 1 if the Hamiltonian H is exactly conserved (i.e. $H(\text{new}) = H(\text{old})$) and otherwise accepted with probability $\exp[-\Delta H]$ for an increase in energy. Of course, if the proposal is rejected, the chain remains at the current state. Critically, because the integrator is time-reversible *and* volume preserving, the chain still satisfies detailed balance such that we are directly sampling the posterior distribution, especially posteriors which are highly uniform or have extended tails.

Before, I briefly mentioned that the Hamiltonian approach benefits from the conservation of energy. By following trajectories that can ‘see’ the gradient of the posterior distribution, the sampler can propose states that are far from the current state in parameter space, but still have a high probability of acceptance. This is in stark contrast to the Metropolis-Hastings approach which steps through the parameter space in diffusive manner, which is characterized by taking small steps to nearby points in parameter space. Therefore, the Hamiltonian approach can traverse the posterior landscape much more efficiently, requiring fewer steps to converge.

4.2.3 MCMC in cosmology

In application, Bayesian parameter inference is a foundational tool in modern cosmology, as it allows for comparing different cosmic models to observational data based on how well they explain the data. The most common way to perform Bayesian style analysis is through MCMC methods, which are heavily used to estimate the posterior distribution of parameters for a given cosmological model. However, performing MCMC analysis is not a trivial task and requires the use of several publicly available and widely used code bases. Essentially, to perform a MCMC analysis you need a Boltzmann solver to predict cosmological observables, and a MCMC engine to sample the cosmological parameter space

The Boltzmann solver of choice for the numerical results shown in this thesis is the Cosmic Linear Anisotropy Solving System (CLASS) [160]. For a given set of cosmological parameters, CLASS computes the theoretical predictions for cosmological observables like the angular power spectra, matter power spectrum, and timing of events like recombination and reionization, to name a few. The MCMC tool of choice for this thesis is MontePython [161]. MontePython performs three critical tasks: it generates proposal states θ' , it calls CLASS to compute cosmological observables for the proposed state θ' , and then it evaluates the likelihood of those predictions against actual data and decides to accept/reject the proposal state. By doing this many times, MontePython uses CLASS to map out the posterior distribution of the free parameters in the model being tested.

However, getting the MCMC running is only half the battle; one must also ensure that the Markov chains have *converged*. In this case, *convergence* means that the the Markov chain has arrived at the stationary distribution (the posterior) and is fairly sampling it. In practice, we never actually know the true posterior, so we must assess convergence based on the behavior of the Markov chains through their evolution. The Gelman-Rubin statistic R is the most common diagnostic to assess convergence and requires that multiple, independent

Markov chains are constructed. Once the collection of chains is thought to be converged, the Gelman-Rubin statistic R compares the variance within each chain to the variance between the chains. If the chains have all converged to the same posterior distribution, the variances of an individual chain should match the variance of the collection of chains and $R \approx 1$. In practice, our chains will never fully converge as this requires an infinite number of steps, but as a general rule of thumb $R - 1 < 0.05$ is a decent criteria to satisfy convergence.

Another concern that is much more difficult to detect and probe, especially with MH algorithms, is ensuring that the chains have fully explored the posterior distribution. If the posterior is multi-modal or has extended tails, chains may seem converged but they may not have explored all the parameter space. This issue can be compensated by running the chains for a very long time, or by starting chains in different parts of the distribution. By starting in different points, one can test if the chains converge to the same final distribution. If they don't, this is strong evidence that the posterior distribution is highly non-trivial and contains multi-modal features. Newer sampling methods like the Hamiltonian method described above can help to alleviate this issue, but it is not a perfect resolution.

Finally, once we are satisfied that our chains are converged and have fully explored the posterior landscape, its time to actually analyze the chains and extract information like the 2D posteriors, best-fit values, and variances for the free parameters. There are a few different tools publicly available to perform these tasks. The two that are used for the numeric investigation in this thesis are *Procoli* [162] and *GetDist*. Procoli is a powerful tool which augments MontePython, enabling the sampler to perform profile likelihood analysis. A profile likelihood analysis essentially determines how the probability of a single parameter explains the data by locking the parameter in question to a single value, and then running the MCMC with the rest of the parameters free. Each time the chains converge, the parameter in question's value is shifted by a discrete amount, and the process is repeated. This provides

information on how the parameter's probabilistic value changes at discrete given values. It also provides information on what likelihoods provide the constraints on the parameter space as we step through the parameter value. Finally, Procoli also has an advanced best-fit value routine to calculate the true best fits for the parameter space. GetDist is a powerful plotting tool for displaying the 1D and 2D posterior distributions. It also has the ability to compute bestfit values and variances, but typically this is left to Procoli.

5 Cosmological questions

We are trying to prove ourselves wrong as quickly as possible,

because only in that way can we find progress.

- Richard Feynman

5.1 The Hubble tension

There exists a tension between the various absolute distance scales used to infer the Hubble constant H_0 within the Λ CDM model [143, 163–167]. Most notably, cosmic microwave background measurements [144, 168, 169] and the Cepheid-calibrated local distance ladder [142, 170–173] infer Hubble constant values that are discrepant at the critical 5σ level [20]. On the other hand, an alternative calibration of the local distance ladder based on the tip of the giant branch (TRGB) [130–138, 174, 175] leads to a Hubble constant value that is somewhat intermediate between that inferred from the CMB and the Cepheid-calibrated ladder, while strong-lensing time delays [176–179] generally find H_0 values that are consistent with the Cepheid-calibrated distance ladder, albeit with larger possible systematic errors [180]. A major question in cosmology now is whether this discrepancy is the result of a yet-to-be-discovered systematics or is caused by a breakdown of our current cosmological paradigm [181–183]. Below, I will briefly present some recent approaches at resolving the tension in these different time regimes that are not covered later in the research chapters.

5.1.1 Late Universe approaches

Late-time dark energy evolution One of the more popular routes to resolving the Hubble tension in the late Universe involves including time dependence on the energy density of dark energy. In Λ CDM, dark energy is described by the cosmological constant Λ with a fixed equation-of-state parameter $\omega = -1$. Recall that the equation of state of a perfect

fluid is given by

$$\omega \equiv \frac{p}{\rho} \tag{102}$$

where p is the pressure of the fluid and ρ is its energy density. Dark energy having an equation of state equation equal to -1 implies that, in a very unusual way, dark energy has a negative pressure. Late-time dark energy models modify this foundational assumption to Λ CDM by allowing ω to evolve with time. For instance, models which send $\omega(t) < -1$ cause the expansion rate of the Universe to accelerate more rapidly at recent epochs. This would increase the locally inferred value of H_0 from the distance ladder, but because the transition occurs late in the Universe, the distance to the CMB (and thus the inferred value of H_0 from the CMB) are largely unchanged.

However, by having $\omega(t) < -1$, several theoretical challenges arise. Perhaps the most prominent is that this situation represents the crossing of the “phantom barrier”. In this regime, dark energy density increases over time leading to a runaway acceleration of the expansion rate of the Universe. However, phantom-like dark energy inherently violates known energy conditions such as the

- **Null energy condition**, where the energy density experienced by a light ray must be non-negative,
- **Weak energy condition**, where the energy density experienced by any observer must be non-negative, and
- **Strong energy condition**, where the average energy density must be greater than or equal to the average pressure, implying that the energy density along any timelike vector must be positive or zero.

While the cosmological constant $\omega = -1$ already violates the strong energy condition, by allowing $\omega(t) < -1$ will also violate the weak and null energy conditions.

Recent results from the Dark Energy Spectroscopic Instrument (DESI) have provided intriguing evidence that could be consistent with time-evolving dark energy [184]. In their recent results, which I note do not directly address the Hubble tension but analyze the general form of late-time dark energy, they consider a form of time evolving dark energy parameterized by

$$\omega(a) = \omega_0 + \omega_a(1 - a) \quad (103)$$

where a is the global scale factor, and ω_0 and ω_a are fitted parameters representing the intercept and slope. For a Λ CDM model, $\omega_0 = -1$ and $\omega_a = 0$. DESI finds moderate evidence deviation from the Λ CDM scenario, where the precise values of ω_0 and ω_a vary depending on what data is considered. These findings suggest a subtle evolution in dark energy behavior, however, there is a challenging consequence of this parameterization. The $\omega_0\omega_a$ parameterization implies that we are observing the *global* maximum of dark energy density, an unlikely scenario often criticized for its anthropic fine-tuning.

Systematics in the distance ladder Although less exciting than the tantalizing prospective of physics beyond the standard model, it may be that the H_0 tension is a result of unknown systematics in the distance ladder. While this has been extensively scrutinized over the last decade, I present a few of the leading sources of systematic uncertainty in the local inference of H_0 .

Recent analyses have significantly improved color corrections and standardization procedures for Type Ia supernovae (SNe Ia), yet uncertainties remain, particularly concerning intrinsic color variations of the supernovae themselves and dust distributions within host galaxies. These factors can systematically bias inferred distances and, consequently, the inference of H_0 . For instance, the standardized brightness of SNe Ia exhibits a strong dependence on the host galaxy’s environment, suggesting that dust properties and intrinsic color variations can play a significant role in these measurements [185]. Additionally, the intrinsic

scatter in SNe Ia brightness is closely linked to variations in dust extinction parameters, further complicating accurate distance estimations [186]. Moreover, analyses focusing on blue SNe Ia, which are presumed to be less affected by dust extinction, have found that the derived H_0 values are consistently lower [187]. This suggests potential systematic effects in dust corrections within standard supernova cosmology, but fails to explain why the TRGB and Cepheid methods arrive at different conclusions, even though they use similar SNe Ia data sets. Therefore, while advancements have been made, addressing the residual uncertainties related to intrinsic color variations and dust distributions may be crucial for resolving the Hubble tension.

Next, there are a pair of uncertainties surrounding Cepheid variable stars which can impact the overall distance ladder inference of H_0 . First are questions surrounding the effect of metallicity on Cepheids, where the abundance of elements heavier than helium in the stellar atmosphere can influence luminosities and thus distance measurements. However, studies have shown that this effect is relatively minor, with variations in metallicity contributing to less than 2% change in distance measurements, and unlikely to resolve the existing tension in H_0 measurements [188]. Second are doubts around the actual photometry of the Cepheids regarding a phenomena known as crowding. When Cepheids (or any stellar objects) are imaged in dense stellar fields, the light of multiple objects can blend together, artificially boosting the measured brightness resulting in an underestimation of distance. Recent high-resolution observations from the James Webb Space Telescope (JWST) have addressed this issue. JWST’s superior resolution reduces the impact of crowding, providing more accurate photometry of Cepheids. These observations have demonstrated that unrecognized crowding does not significantly contribute to the Hubble tension, effectively ruling it out as a major systematic error [189].

Then, there exists similar types of uncertainties for the tip of the red giant branch cal-

ibration of the distance ladder. The determination of the tip of the red giant branch is primarily sensitive to color, which dust extinction in the host galaxy may add unaccounted for effects. Additionally, variations in color and luminosity as a result of different ages and metallicity of the red giant branch stars can further influence the determination of the ‘tip’ of the population for distance purposes. However, these effects have been shown to also be subdominant and unlikely to explain the tension [190]. In general, systematics surrounding the phenomena used to construct the distance ladder are unlikely to resolve the tension due to their relative impact on distance determinations, but their careful characterization is essential for robust and accurate statements about the nature of our local Universe.

5.1.2 Early Universe approaches

Early dark energy Early dark energy (EDE) models introduce a brief but significant injection of dark energy around the time of matter-radiation equality. By injecting dark energy into the early Universe at this specific time, the expansion rate prior to recombination is greater than predicted by Λ CDM [191, 192]. This shortens the duration in which the photon-baryon plasma dominates the Universe, reducing the amount of time which sound waves could propagate through the plasma, thereby reducing the sound horizon scale in the CMB (and also the BAO characteristic scale). Consequently, matching the angular scales of the acoustic peaks in the CMB now requires a higher H_0 bringing the early Universe inference inline with distance ladder inferences.

Despite the simple story and conceptual appeal, EDE models face significant theoretical and observational challenges. Firstly, the exact timing, magnitude, and duration of the injected energy density is highly fine tuned to avoid violating observational constraints arising from the CMB, BAO, and large scale structure observations. This fine-tuning remains a significant challenge, as without a justification from fundamental physics principles the model is highly unnatural. However, some recent models have attempted to address the timing

issue by invoking novel neutrino interactions and coupling mechanisms, providing a dynamic explanation for the apparent fine tuning [193].

In addition to the issues surrounding the specific timing of the energy injection, the EDE must also rapidly dissipate following its brief dominance to avoid strongly conflicting with CMB observations. Mechanisms capable of rapidly diluting this energy density, such as phase transitions or scalar field oscillations, have been proposed, yet these solutions typically require additional, and sometimes exotic, physics beyond standard paradigms which takes away from the simplistic appeal of EDE.

Primordial magnetic fields Primordial magnetic fields (PMFs) represent another early-Universe solution. Strong magnetic fields introduce anisotropic pressure gradients into the photon-baryon plasma, which can have dramatic effects on its phenomenology. Critically, it can directly alter the speed and propagation of acoustic waves in the plasma, as well as provide pressure support against photon diffusion processes [194]. Consequently, similar to EDE scenarios, this affects the sound horizon scale at recombination, potentially increasing H_0 inferred from the observed angular scales in the CMB.

Despite their apparent potential at resolving the tension, as usual, PMFs face strict constraints from multiple other observational avenues. Magnetic fields induce Faraday rotation in plasmas, which would change the polarization plane of the CMB photons and generate distinct non-gaussian signatures in CMB maps. Observations from Planck and ACT have already limited the allowed strength of PMFs at recombination to extremely low levels, typically around the nanogauss scale [195]. Nanogauss strength fields are unfortunately not nearly strong enough to effect the plasma dynamics in a way to resolve the expansion rate tension.

5.2 The S_8 tension

The S_8 tension describes a persistent discrepancy between the amount of matter clustering from early-Universe predictions and late-time observations. The CMB predicts there to be much more ‘clumpy’ structure than what is actually measured in the late Universe using large scale structure surveys and weak gravitational lensing studies. Here, S_8 combines the amplitude of matter fluctuations σ_8 and matter density Ω_m as

$$S_8 \equiv \sigma_8 \sqrt{\frac{\Omega_m}{0.3}} \quad (104)$$

where σ_8 represents the amplitude of matter fluctuations on scales of approximately $8h^{-1}$ Mpc.

Observations of the CMB with Planck, assuming a Λ CDM model of cosmology, yields a precise prediction for $S_8 = 0.832 \pm 0.013$ [144]. In contrast, weak lensing data from the Dark Energy Survey (DES) [196] and the Kilo-Degree survey (KiDS) [197], consistently indicate lower values of $S_8 \sim 0.76$ – 0.78 . This difference, while smaller in magnitude than the well-known Hubble tension, has been steadily persistent and statistically significant at levels typically around $2 - 3\sigma$, hinting that we may be missing something in our understanding of the growth of structure. Additionally, it seems that most approaches to resolve the Hubble tension do so at the cost of the S_8 tension, and vice versa. Combined, these two tensions point to something subtly wrong with our current understanding of the growth of structure and the expansion of the Universe.

One prominent resolution to the S_8 tension involves introducing higher total neutrino masses or additional light relics. These particles effectively dampen structure formation on small scales due to their free-streaming nature, aligning predictions more closely with late-Universe observations. However, this route to resolution faces (as usual) strict observational

constraints. Lab based neutrino oscillation experiments have tightly constrained the sum of neutrino masses, placing upper bounds around $\sum m_\nu = 0.12 - 0.15$ eV. Moreover, significant increases in neutrino mass or additional relativistic species can conflict with precise measurements of other cosmological parameters, such as the matter power spectrum, baryon acoustic oscillations, and lensing patterns in the CMB itself.

5.3 Other tensions

Baryon acoustic oscillations (BAO) Baryon Acoustic Oscillation measurements are crucial for establishing cosmological distance scales and serve as a robust standard ruler due to their origin in well-understood early-Universe physics. However, recent analyses have uncovered subtle but persistent discrepancies between angular-only BAO measurements (which rely solely on angular separations on the sky) and full three-dimensional (3D) analyses that incorporate both angular and radial distance information from spectroscopic redshift surveys [198, 199]. These discrepancies are significant because differing BAO methodologies yield slightly different inferred cosmological parameters, such as the Hubble parameter $H(z)$ and angular diameter distances.

Furthermore, mild but notable tensions have emerged between BAO measurements obtained at different redshifts [200, 201]. The internal consistency of BAO measurements across cosmic epochs is essential since deviations could signify unaccounted redshift-dependent physics or previously unrecognized observational systematics. Resolving these internal BAO discrepancies remains crucial, as they directly impact the reliability of the BAO scale as a standard ruler and thereby influence confidence in our cosmological inferences, particularly those related to dark energy and curvature [202].

Cosmic dipole anisotropy The cosmic dipole anisotropy is traditionally interpreted as arising primarily from our peculiar velocity relative to the cosmic rest frame. However, recent

observations, particularly from surveys of distant quasars and radio galaxies, report dipole anisotropies significantly larger than those predicted by standard models accounting solely for local peculiar motion [203, 204]. This enhanced dipole signal, if confirmed, directly challenges the foundational assumption of large-scale isotropy inherent in the cosmological principle, potentially indicating large-scale inhomogeneities or anisotropic structures far beyond those predicted by Λ CDM. Such large-scale anisotropies could require either a reconsideration of local motions and structure formation on very large scales or even revisions to fundamental cosmological principles such as isotropy and homogeneity [205, 206].

6 Hubble distancing: Focusing on distance measurements in cosmology

The beginning of wisdom is to call things by their proper names.

- Confucius

This chapter is a modified version of the publication, “Hubble distancing: Focusing on distance measurements in cosmology,” by Kylar Greene and Francis-Yan Cyr-Racine published in the Journal of Cosmology and Astroparticle Physics in June of 2022 [207].

6.1 Introduction

The pioneering work of Slipher, Lemaître, Robertson, Leavitt, and Hubble in the late 1920s established that the Universe is expanding [5, 6, 29, 123, 208]. Since then, cosmological distance measurements have been at the forefront of understanding the expansion history of the Universe.

As briefly presented in Ch. 5, Ref. [209] summarizes many possible new-physics solutions to alleviate the Hubble tension, while Ref. [210] ranks the proposed models with respect to a common data set. Generally speaking, new-physics scenarios can be classified into two broad categories, depending on whether they primarily modify the early or late Universe. Early-time solutions aim to decrease the value of the sound horizon r_s [68–72], either by injecting energy into the pre-recombination Universe (see e.g. refs. [145, 161, 192, 193, 211–226]) or by other means [227–230] in such a way that also preserves the CMB measurements.

Late-time solutions aim to increase the value of H_0 locally by modifying the expansion history at $z \lesssim 1$ (see e.g. refs. [191, 231–263]). At face value, these late solutions may appear to resolve the tension because a sudden increase in the Hubble expansion rate at very-late times could yield a larger apparent H_0 value while only causing a small fractional change in the overall distance to the CMB last-scattering surface. However, such solutions are mis-

leading in the notion that the local distance ladder *directly* measures the Hubble expansion rate at $z = 0$ [166]. In practice, the local distance ladder *infers* H_0 from measurements of the peak absolute type Ia supernova (SNe Ia) magnitude M_{sn} and of Hubble flow SNe Ia with redshift $z \gtrsim 0.02$. To be successful, late-time solutions therefore need to accurately fit the measured distances to Hubble-flow supernovae, rather than trying to obtain a given H_0 value derived using Λ CDM assumptions.

Indeed, the singular focus on H_0 rather than on the calibrated distances to Hubble flow objects often obscures what model ingredients are necessary to address the root cause of the discrepancy [165, 264–268]. In cosmological models that are phenomenologically close to Λ CDM at late times (including models that only affect the early Universe), this focus on H_0 is warranted as distances to Hubble-flow objects are inversely proportional to the Hubble constant, and the cosmographic expansion of the luminosity distance is accurate at low redshifts. For these, using a Gaussian prior on H_0 to represent the entire distance ladder is likely sufficient, albeit at the price of discarding information about the actual goodness of fit of model distances to low-redshift objects. On the other hand, for cosmologies that differ significantly from Λ CDM at late times, the relationship between H_0 and distances is more complex as other model parameters could enter the computation of the latter. In this case, simply focusing on H_0 can be extremely misleading as it is not possible to boil down the entirety of the distance ladder to a single number without the loss of important information. Instead, the distances to Hubble-flow objects must be directly accounted for in assessing the success of such cosmological scenarios.

In this Chapter, I show that the Hubble tension is not simply about a single number – namely H_0 – but rather about a large array of distance measurements at low redshifts. In this language, “solving” this discrepancy is really about finding a self-consistent expansion history of the Universe that can describe all cosmological data, including the CMB, large-

scale structure, BBN abundance yields, Lyman- α observations, baryon acoustic oscillations, and the distances to nearby Hubble-flow objects such as SNe Ia. Using the newly developed, custom designed likelihood package *distanceladder*⁵, which provides a fast yet accurate fit to the entirety of the local distance ladder, I show that apparently promising late-time solutions that have been presented in the literature, in fact, provide poor fits to the actual measured distances to low-redshift objects. Even more concerning, I show that some evidence for new physics that has been reported in the literature using a Gaussian H_0 prior appears to be spurious once the actual distance measurements are properly taken into account. Our work strongly indicates that cosmological analyses should focus on fitting cosmological distances rather than argue about the “correct” value of the current expansion rate.

In a significant number of cosmological analyses to date for late-time changes to cosmology (see e.g. refs. [235, 248, 250, 253, 254, 261, 269–272]), all of the richness and complexity of the distance ladder measurements described in the previous section are reduced to a simple Gaussian constraint on H_0 (see Refs. [237, 268] for exceptions to this). However, such Gaussian prior on the $z = 0$ Hubble rate is an obvious over-simplification since it neglects that all of the information on the expansion rate gathered from the distance ladder comes from redshifts higher than zero. In particular, all information on the shape of the SNe Ia magnitude-redshift relation is lost when boiling down the whole distance ladder to a simple Gaussian prior on H_0 [166, 273]. It is thus worth emphasizing that to resolve the Hubble tension; it is in general *not* sufficient for a cosmological model to have a $z = 0$ Hubble rate compatible with the value reported by the distance ladder measurements. Instead, the model must be able to successfully fit the *calibrated distances* to SNe Ia in the Hubble flow. Indeed, while there is a one-to-one relation between distances and the Hubble constant for Λ CDM models with fixed acceleration and jerk parameters q_0 and j_0 [232], this is in general not the case for models with a sudden late-time transition for which the cosmographic expansion

⁵The package is publicly available on github at [distanceladder](https://github.com/mturkmen/distance_ladder).

of the luminosity distance breaks down. Such theories can appear to resolve the tension by having a Hubble constant compatible with that inferred from the distance ladder but do not necessarily provide a good fit to the actual supernova distances.

To remedy this, I introduce the *distanceladder* numerical likelihood package, which replaces the standard Gaussian prior on H_0 (or M_{sn}) used in many analyses with a direct fit to the calibrated distances of the SNe Ia in the Hubble flow. Our goal here is not to replace the detailed analyses performed in, e.g. , refs. [20, 136, 142, 170], but instead provide the user with an end-to-end distance-ladder package such that the impact of different analysis assumptions and choices on cosmological constraints can be fully understood. I briefly describe below the content of the numerical package and its usage and then illustrate in section 6.6 how a model that appears to resolve the Hubble tension can be strongly disfavored once the actual distances are taken into account, using a dynamic late-time dark energy (LDE) model as a case example. For the remainder of this discussion, when a Gaussian prior is used, it utilizes the results of Ref. [20] with $H_0 = 73.2 \pm 1.3$ km/s/Mpc, hereafter referred to as R21.

6.2 Implementation and usage

As its core, the ‘*distanceladder*’ likelihood package incorporates the local distance ladder as described by section 2.4 into the Monte-Carlo-Markov-Chain tool **MontePython** [274] by directly comparing the measured luminosity distances of Hubble flow SNe Ia to the distances predicted by a cosmological model. The *distanceladder* nullifies the need for a Gaussian prior on H_0 and allows all cosmological models, even those with baroque late-time dynamics, to properly account for the distances measured by the local distance ladder.

User inputs – *distanceladder* requires two inputs from the user; a choice of calibrating standard candles and a choice of anchor measurements. The available options of calibrating

standard candles are Cepheids, the TRGB, or a subsample in which the distances to anchor SNe Ia calculated by both methods agree to 1σ referred to as the ‘concordant’ sample. The concordant option allows the user to select the SNe Ia anchors which agree on distance measurements between the TRGB and the Cepheid calibration providing a unique tool to analyze the tension seen between the two methodologies. The choice of anchors is only available for the Cepheid calibration presently. A future update will include the 42 Cepheid calibrators [20] as well as the Pantheon+ sample for Hubble flow SNe Ia [275, 276]⁶. Additionally, future iterations may include an option to use the Carnegie Supernova Project (CSP) Hubble flow SNe Ia data [277] as the Hubble flow supernova data, if there is interest from the community.

Calibrating standard candles – If Cepheids are chosen as the calibrating standard candle of choice, the absolute magnitude of a Cepheid variable with one day period M_{ceph} is first calculated based on the choice of anchors. Currently, the LMC and NGC4258 are supported as anchor choices. The distance to the anchors are provided directly and are not recalculated, using the results of Ref. [106] for the LMC and Ref. [79] for NGC4258. If both anchors are chosen, M_{ceph} is calculated independently for both anchors then combined using a weighted average using data from refs. [278, 279]. Once M_{ceph} is calculated, the weighted average slope of Leavitt’s Law is determined using the Cepheid data directly from the 19 host galaxies presented by Ref. [142] in the Wesenheit near-infrared magnitude band [280]. The correlation value R between Cepheid intrinsic color and luminosity is assumed to be $R = 0.386$ as done by SH_0ES . Using the Copernican principle, the *distanceladder* assumes that all Cepheids in the data sample provided will have identical slopes. The global slope of Leavitt’s Law is then applied to the Cepheid samples of the 19 SNe Ia host galaxies to determine the intercept of the relation, as shown in Fig. 5.

⁶A few weeks after the release of *distanceladder*, the Pantheon+ likelihood was released for MontePython which includes the 42 Cepheid calibrators, the new SNe Ia, and also a careful treatment of how the distances to said observations impact the posterior.

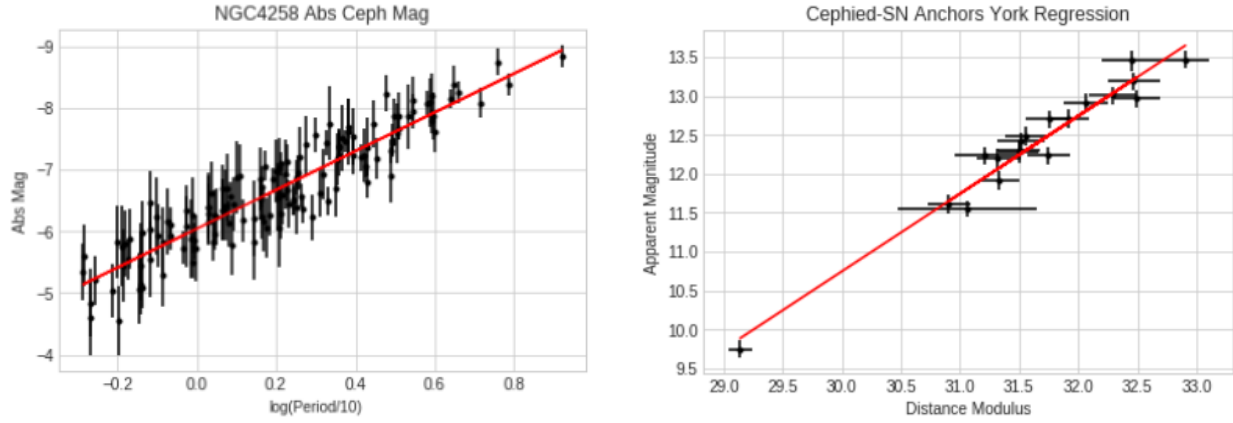


Figure 5: **Left:** This figure shows how Cepheid observations in the anchor host galaxy NGC4258 (with a known distance) are used to determine the absolute magnitude M_{ceph} and the slope of the period-luminosity relationship. From this linear regression, the slope is -3.15 ± 0.1 and the intercept (corresponding to M_{ceph}) is -6.04 ± 0.04 . This information is then used to calibrate the broader Cepheid sample that exist in host galaxies without anchor distance measurements, but have recorded SNe Ia. **Right:** The result of utilizing the calibrated Cepheid sample is shown, where the distances to the 19 SNe Ia host galaxies is used along with apparent magnitude data of the SNe Ia to calculate the absolute magnitude of the supernova. This is done using a York linear regression algorithm which accounts for error in both the x and y directions. From this analysis, I find that the intercept (corresponding to M_{sn}) is -19.26 ± 0.04 , in agreement with Ref. [20].

As discussed previously, there may exist a break in Leavitt’s law in which case this method can be generalized to include a broken power-law feature in the future. The distance modulus given by Eq. (43) determines the distance to the host galaxies by using M_{ceph} and the intercept of Leavitt’s Law for each host galaxy. Finally, a linear regression calculates the intercept of the relation between the distance to the host galaxies and SNe Ia’s apparent magnitude. The intercept corresponds to the absolute magnitude of SNe Ia, M_{sn} .

If the TRGB is chosen as the calibrating standard candle of choice, the distance modulus to the host galaxies is not calculated by the likelihood package but instead taken from table 3 from Ref. [136]. The TRGB method of calculating distances is more involved than the Cepheid method, so it is not explicitly included in this release but may be included in future releases if there is interest. A linear regression using the distance modulus from table 3 of Ref. [136], and the apparent magnitude of the local SNe Ia calculates M_{sn} in the same manner as in the Cepheid method. In all calculations, the Hubble photometry B magnitudes are used as the standard filter of choice.

If the concordant sample is chosen as the calibrating standard candle, the distance modulus again is not calculated but instead taken from table 3 of Ref. [136]. Only distance measurements in which the TRGB and Cepheid calibration agree to less than 1σ are considered. This cut results in a smaller sample size of 6 local SNe Ia rather than the 19 local SNe Ia calibrated by Cepheids or the 18 local SNe calibrated by the TRGB. M_{sn} is then calculated in the same manner as in the prior two methods.

SNe Ia and tension calculation – Regardless of calibration method, Eq. (43) calculates the distance to Hubble flow SNe Ia using M_{sn} and apparent magnitude measurements from the Pantheon data set [281]. Since our analysis does not rely on the cosmographic expansion of the luminosity distance, the *distanceladder* uses the *entire* Pantheon dataset with no

upper bound on redshift to ensure the shape of the redshift-luminosity relationship is fully accounted for. Separately from the *distanceladder* likelihood, the Boltzmann code CLASS [160] then calculates the luminosity distances to these Hubble flow SNe Ia for the cosmological model under consideration. The Hubble (if present in the model) is then quantified as a disagreement of distances, not of H_0 values, by comparing the distance moduli of the Hubble flow SNe Ia calculated from M_{sn} to those inferred from the cosmological model.

Redshift errors – Redshift errors are included whenever possible for the Hubble flow supernovae from Ref. [282]. Not all supernovae in Pantheon have errors that are available presently (this has been changed in the recent Pantheon+ release [275]). The average fractional error of the 831 supernovae with errors available in the overall Pantheon data set is 0.83%. The average fractional error of the 184 supernovae with errors available with $z < 0.15$ (corresponding to the upper redshift cut of R21 and similar studies) in the Pantheon data set is 1.0%. While the fractional error is small, the redshift error does need to be incorporated in future studies. The covariant matrix used by *distanceladder* in minimizing the distance disagreement is generated by adding the observational errors associated with apparent magnitude and redshift as a diagonal matrix to a dense matrix where each element is the error associated with M_{sn} , representing the systematic error.

6.3 Consistency with previous results

As a consistency check, figure 6 demonstrates the ability of the *distanceladder* likelihood to recover the results of both F21 and R21 for a Λ CDM model using the Monte-Carlo Markov Chain tool `MontePython`. The Λ CDM model utilizes Planck high- ℓ TT, TE, EE, low- ℓ EE, and low- ℓ TT data to represent the cosmological measurement of H_0 . The local measurements utilize the *distanceladder* likelihood with the respective calibration choice and a prior on the baryon density to represent the local measurement of H_0 . The TRGB calibration accurately recovers the F21 mean value result but underestimates the error slightly as the

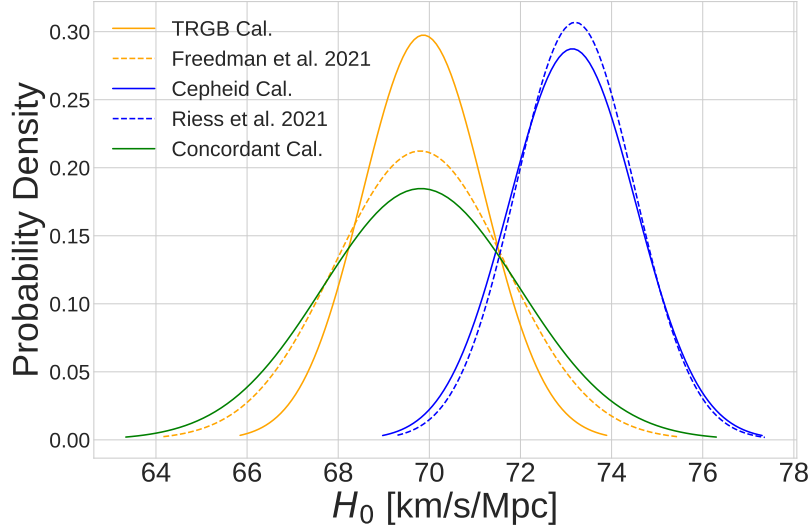


Figure 6: 1D posterior distributions of H_0 using only the *distanceladder* likelihood, comparing the user input choices of using the TRGB, Cepheid, or concordant calibration. The solid lines correspond to *distanceladder* results using the tip of the red giant branch calibration (yellow), Cepheid calibration (blue), or a combination of the results in which both calibration schemes agree on the host galaxy distances (green). The dashed lines correspond to the previous results obtained by Ref. [136] (yellow) and Ref. [173] (blue).

larger Pantheon supernova data set in B magnitude is used, rather than the smaller CSP data set in the B' magnitude, which uses a different photometric calibration scheme and different assumptions of blackbody physics. However, because we can reproduce the mean results of the F21 study nearly precisely using the Pantheon Hubble flow supernovae with B magnitude photometry and the photometric results of Ref. [142] for the anchor supernova, the results support that the CSP and Pantheon supernovae data sets are mostly consistent and do not contribute to the tension nor does the photometry of Ref. [142]. The Cepheid calibration recovers an accurate mean value of H_0 compared to the R21 value, with the slight difference stemming from the *distanceladder* likelihood only having access to the LMC and NGC4258 as anchors while the R21 value uses the LMC, NGC4258, and Milky Way Cepheids as anchors. Interestingly, the concordant sample finds a value of H_0 similar to the F21 result with larger error bars due to less anchor SNe Ia available, which should be investigated in future studies.

	Cepheid Cal.	TRGB Cal.	Concordant Cal.
M_{sn}	-19.226 ± 0.039	-19.325 ± 0.039	-19.267 ± 0.064
H_0	73.14 ± 1.39	69.87 ± 1.34	69.82 ± 2.16

Table 2: The distance ladder parameters derived from *distanceladder* for the three calibration options available. M_{sn} is given in the B apparent, K corrected peak magnitude. H_0 is given in units of km/s/Mpc. The concordant sample finds a value of H_0 similar to the TRGB calibration at the cost of increasing the error nearly two-fold.

Table 2 summarizes the values of M_{sn} and H_0 for our different calibration choices in a Λ CDM model. An unexpected behavior of the results is that when increasing the error in M_{sn} as seen in the Concordant sample case, a smaller value of H_0 is preferred from the MCMC breaking the typical relationship between H_0 and M_{sn} . As a check to determine if it was indeed the error in M_{sn} driving this difference, we reran the concordant calibration scheme artificially lowering the error in M_{sn} by a factor of 2. By forcing the error in M_{sn} to be similar to that seen in the Cepheid and TRGB calibration schemes, we find that the typical relationship between M_{sn} and H_0 is recovered.

6.4 A new metric to assess the tension between data sets

Now that we have a likelihood focusing on cosmological distances rather than H_0 itself, the next question is which metric should be used to compare local distance measurements with predictions from high-redshift observations such as the CMB? The intercept β presented in figure 7 may be what future studies should focus on to resolve the Hubble tension, rather than H_0 itself. In this figure, the y -axis represents the distance moduli to Hubble flow SNe Ia as inferred by the local distance ladder. The x -axis represents the distance moduli to the Hubble flow SNe Ia as inferred in a Λ CDM cosmological model fitted to Planck data, including errors associated with redshift expansion rate [144]. The intercept β is calculated for the three calibration schemes and Λ CDM using a simple linear regression with a fixed slope of unity. Allowing the slope to vary is possible and will be sensitive to changes in

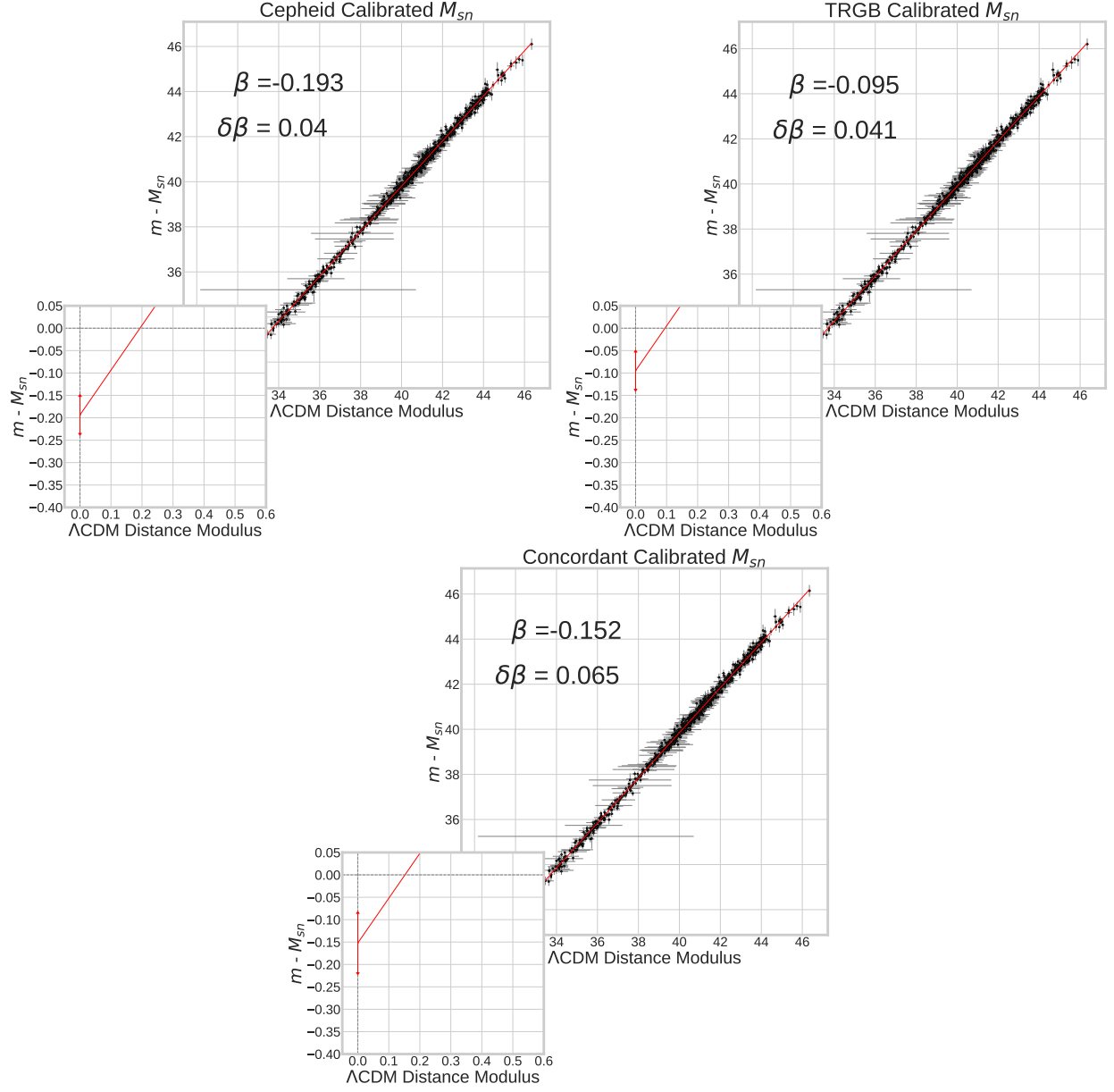


Figure 7: (left top) Graph comparing the calculated distance moduli for Hubble flow supernova. The y -axis describes the distance modulus as calculated by the local distance ladder using the M_{sn} absolute distance scale, calibrated by Cepheid variable stars. The x -axis describes the distance modulus as calculated from Planck CMB measurements assuming a ΛCDM model. (right top) Graph describing an identical situation as in the previous panel but using the TRGB method to calibrate M_{sn} . (bottom) Graph describing an identical situation as in the previous two panels, but using the concordant calibration for M_{sn} .

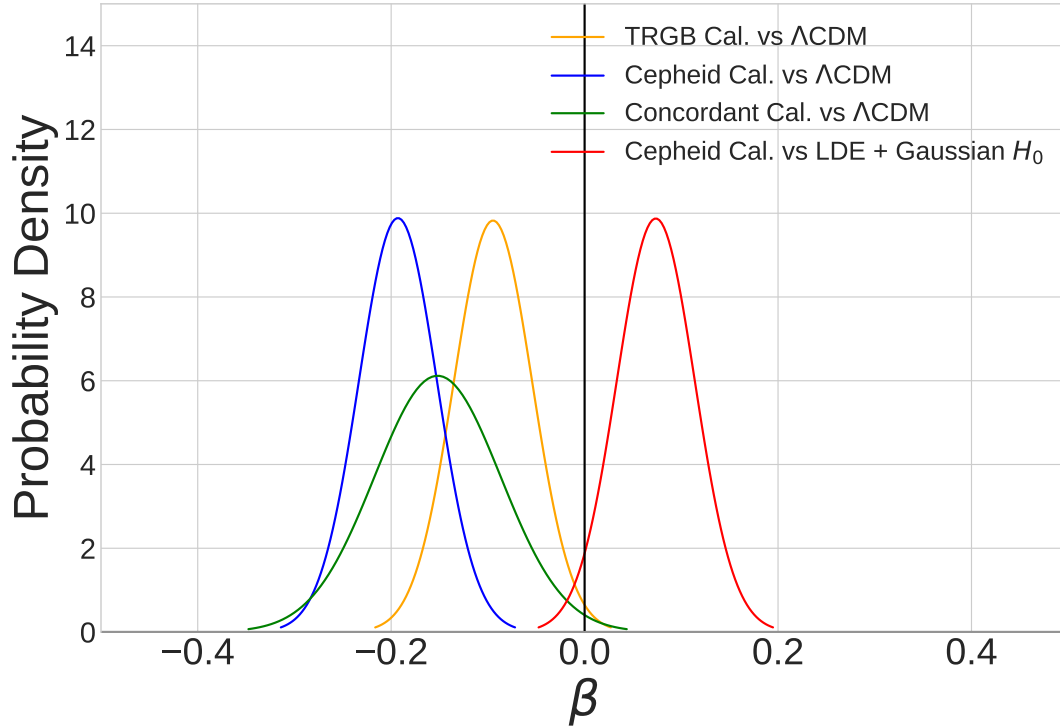


Figure 8: The probability densities of β . The solid black vertical line indicates an agreement between the distance ladder and Planck CMB measurements. The blue, green, and orange lines on the left represent the tension between the chosen calibration scheme and a Λ CDM calibrated to Planck CMB data. The red line on the left represents the tension between a Cepheid calibration scheme and a late-time dark energy model using a Gaussian H_0 to represent the local measurements of the distance ladder. The Gaussians are shown out to the 3σ level.

late-time cosmology, as it will modify the shape of the SN-magnitude redshift relationship, potentially detecting geometric inconsistencies.

If no Hubble tension existed, the distance moduli established by M_{sn} and those inferred from a cosmological fit to the CMB would agree, and the intercept would be statistically consistent with zero. As we can see from Figure 8 however, the β values for our three calibration schemes differ from zero at more than 3σ in a Λ CDM model calibrated to Planck CMB measurements, indicating the presence of the well-known Hubble tension. Additionally, all cases find a slope statistically identical to unity when allowing the slope to vary, indicating

no underlying geometric issues regarding the nature of the background spacetime. The width of the β posteriors is similar as all three case examples utilize the same Pantheon data set. Beyond just being a parameter in linear regression, the intercept β also has a clear physical meaning: $-\beta$ represents the change to the absolute SNe Ia magnitude necessary to bring the low-redshift distances to supernovae in agreement with those inferred from the CMB.

6.5 Equivalence with gaussian prior for early-time dynamics

As an additional consistency check, we consider a simple extension of Λ CDM in which the parameter N_{eff} is allowed to freely vary. Since N_{eff} is a proxy for the abundance of free-streaming radiation (including neutrinos) at early times [283, 284], it has only indirect impacts (i.e. through parameter degeneracies) on the late Universe. As the late-time cosmology in such an extension is phenomenologically similar to Λ CDM for which the standard cosmographic expansion in terms of q_0 and j_0 is valid, we expect that the standard Gaussian H_0 prior and the *distanceladder* likelihood should find nearly equivalent posterior distribution for the Hubble constant when fitted in combination with CMB data. As such, this model constitutes a simple test case to assess the equivalency of our *distanceladder* likelihood with the standard Gaussian prior on H_0 for cosmological models differing from Λ CDM only through their early-Universe evolution (e.g. Ref. [192]).

To perform this test, we run MCMC analyses with the code `MontePython` [274], letting the 6 standard Λ CDM parameters and N_{eff} vary (as well as necessary nuisance parameters) in the fit. As shown in figure 9, we find that our *distanceladder* likelihood reproduces the results obtained from a Gaussian prior on H_0 when performing a joint analysis with CMB data for this one-parameter extension of Λ CDM. In detail, when combined with Planck high- ℓ TTTEE, low- ℓ EE, and low- ℓ TT data [144], our *distanceladder* likelihood finds $N_{\text{eff}} = 3.340 \pm 0.148$ ($H_0 = 69.96 \pm 1.01$ km/s/Mpc), which is to be compared with $N_{\text{eff}} = 3.344 \pm 0.149$ ($H_0 = 69.95 \pm 1.02$ km/s/Mpc) when using the Gaussian prior on the Hubble constant. The

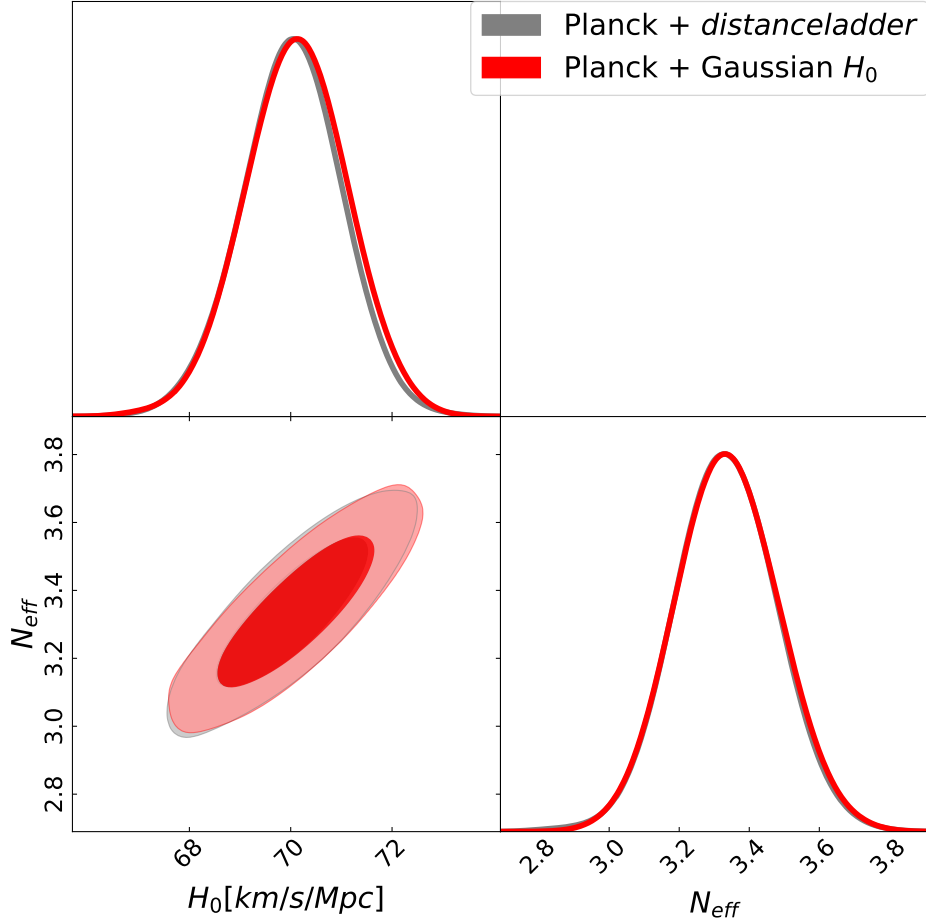


Figure 9: Marginalized posterior distributions for H_0 and N_{eff} using either our *distanceladder* likelihood (DL) or a Gaussian H_0 prior, in combination with Planck temperature and polarization data for both cases [144]. The strong agreement between the *distanceladder* and the Gaussian prior results supports that early-time changes to cosmology only affect distance ladder measurements through H_0 .

agreement between the *distanceladder* and the Gaussian H_0 prior, in this case, demonstrates that the latter is sufficient when analyzing models modifying the cosmological expansion history at early times. However, as we will see in the next section, this is generally not the case for models changing the Hubble expansion history at late times.

6.6 Models impacting late-time cosmology: issues with gaussian H_0 prior

Our *distanceladder* likelihood package allows one to concentrate on the key issue behind the Hubble tension: the distances to SNe Ia in the Hubble flow. Refocusing the discussion

onto distance disagreements immediately begins to narrow the field of possible solutions to the Hubble tension, specifically those which propose changes to late-time cosmology. Such models may seem at the surface to resolve the tension through their accommodation of larger H_0 values compared to Λ CDM. However, they generally do so at the cost of worsening the tension with the actual distance measurements, indicating that such models do not fundamentally address the root cause of the discrepancy and are therefore only of modest interest.

As an example of how a Gaussian prior on H_0 standing in for the entirety of the local distance ladder can be misleading, we consider below a late-time dark energy (LDE) model in which a sudden transition in the abundance of dark energy increases the value of the Hubble rate close to $z = 0$ [236]. Using our *distanceladder* likelihood package, we show below that such LDE models with a high H_0 value poorly fits the measured SNe Ia distances. While we focus here on this simple phenomenological LDE model, we note that other proposed models such as chameleon dark energy [191, 285] and phenomenological emergent dark energy (see e.g. refs. [286, 287]) will also suffer from this problem.

6.6.1 Test case: A sudden dark energy transition at late times

Prior work has demonstrated that introducing a dynamic component of dark energy, particularly at redshifts $z \lesssim 0.02$, successfully increases the value of H_0 locally while also preserving the CMB temperature and polarization spectra [231, 269]. These solutions generally modify the Friedmann equation to allow for a redshift dependent dark energy density component to be included, such as

$$H^2(z) = H_0^2 [\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_{DE}h(z) + \Omega_k(1+z)^2], \quad (105)$$

$$h(z) = \exp \left[3 \int_0^{\ln(1+z)} d\ln(1+z') (1 + w_{DE}(z')) \right], \quad (106)$$

$$w_{\text{DE}} \equiv \frac{p_{\text{DE}}}{\rho_{\text{DE}}}. \quad (107)$$

where Ω_{r} , Ω_{m} , and Ω_{DE} are the ratio of the current energy density in radiation, matter, and dark energy to the critical density of the Universe. Ω_{k} is the underlying curvature density parameter, which we set to 0 here to enforce flatness, p_{DE} is the dark energy pressure, and ρ_{DE} is the dark energy density. A subclass of late-time dark energy models which use the above set of equations also allow the phantom regime $w_{\text{DE}} < -1$. Recall that the limit of $w_{\text{DE}} = -1$ represents the standard cosmological constant.

6.6.2 Model setup

To illustrate how changes to late-time cosmology in the presence of a Gaussian H_0 prior can lead to misleading results, we focus here on the simple phenomenological model presented in Ref. [236] in which the dark energy density acquires a time dependence according to

$$\rho_{\text{DE}}(z) = [1 + f(z)]\tilde{\rho}_{\text{DE}}, \quad (108)$$

where

$$f(z) = \frac{2\delta}{\tilde{\Omega}_{\text{DE}}} \frac{S(z)}{S(0)}, \quad (109)$$

$$S(z) = \frac{1}{2} \left[1 - \tanh \left(\frac{z - z_{\text{t}}}{\Delta z} \right) \right], \quad (110)$$

where $\tilde{\rho}_{\text{DE}}$ and $\tilde{\Omega}_{\text{DE}}$ are the dark energy density and density parameter in a standard Λ CDM model. In this model, the dark energy density suddenly increases at redshift z_{t} as compared to its Λ CDM value by a fractional factor of 2δ , with Δz parameterizing the duration of the transition. As a result, this model boosts the $z = 0$ Hubble rate according to

$$H_0^2 = (1 + 2\delta)\tilde{H}_0^2, \quad (111)$$

where the tilde indicates the original Λ CDM value.

The critical feature of this model is that when $z \gg z_t$, it is indistinguishable from the Λ CDM model. In order to achieve a positive value of δ , the equation of state must be allowed to enter the phantom regime of $w < -1$, which can make a physical implementation of such a phenomenological model difficult. With its sudden late-time transition, this scenario nevertheless achieves the desired result of ‘hiding’ the reduced distance to the CMB last-scattering surface that usually results from increasing H_0 . Indeed, using CMB and BAO data in conjunction with a Gaussian H_0 prior (based on the results of Ref. [288]), Ref. [236] showed that such model could achieve $H_0 = 72.5 \pm 1.85$ km/s/Mpc, hence apparently alleviating the tension.

6.6.3 Taking into account the distance measurements: results

From the analysis quoted above, the LDE model considered here may seem like a promising way to resolve the Hubble tension. However, such models are much less promising once the local distance ladder is properly accounted for [236]. As explained above, LDE-based solutions can increase the value of H_0 while causing a negligible fractional change in distance to the CMB last-scattering surface and thus to θ_* . However, the fractional change in distance to low-redshift SNe Ia from such late-time transition is, in general, not negligible. This can result in a poor fit to the actual distances, with the lowest redshift objects being the most affected. Therefore, LDE models, in general, are ideal examples of being too focused on modifying the value of H_0 itself while entirely missing the actual source of the tension (i.e. the distances).

The red line in figure 8 shows the distribution of intercepts values β (using the Cepheid calibration) for a LDE model fitted to Planck data and a Gaussian prior on H_0 from the R21 measurement. Recall that for no tension to exist, the intercept should be identically 0. In

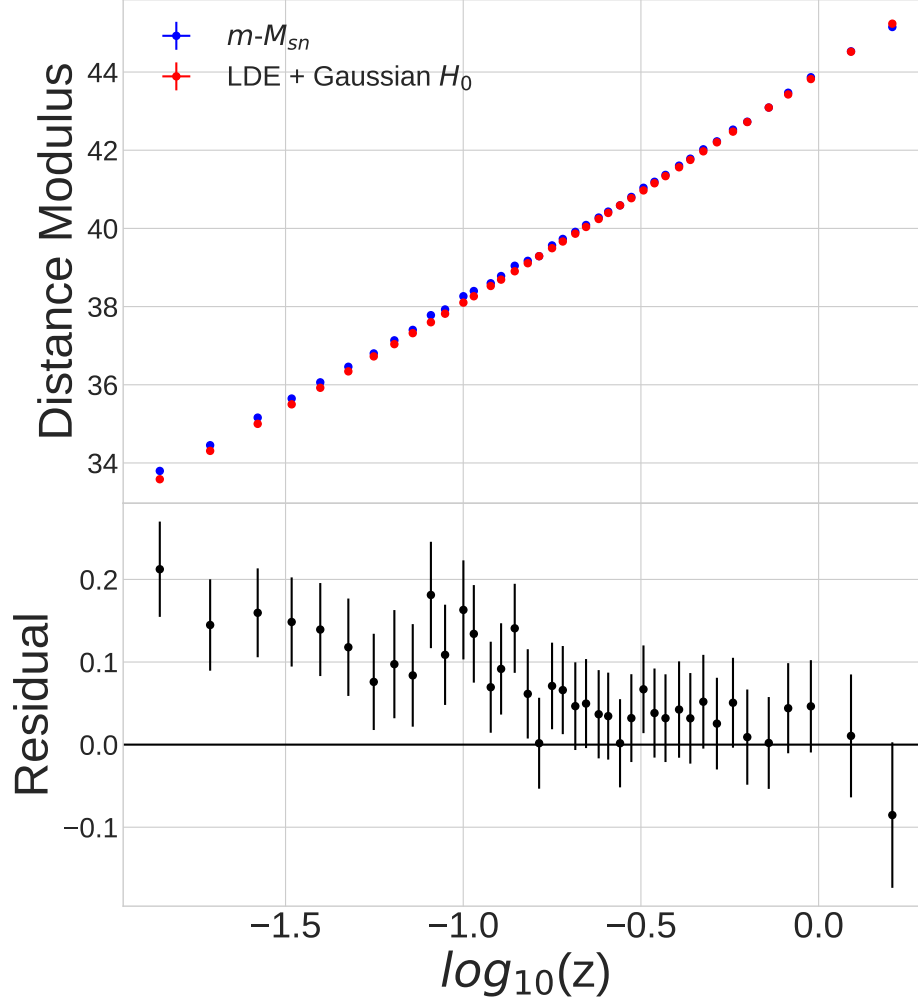


Figure 10: Comparison of the Cepheid-calibrated SNe Ia distance moduli from the Pantheon sample (blue) to the distance moduli predicted by a LDE model fitted with Planck CMB data, BAO data, and a Gaussian prior on H_0 (red). The residual difference between the measurements and the LDE predictions are shown in the bottom panel. The error bars on the blue and red points are smaller than the points themselves and are not easily seen. A systematic disagreement is seen between the Cepheid-calibrated distances and the LDE prediction with a Gaussian H_0 prior, especially at lower redshifts.

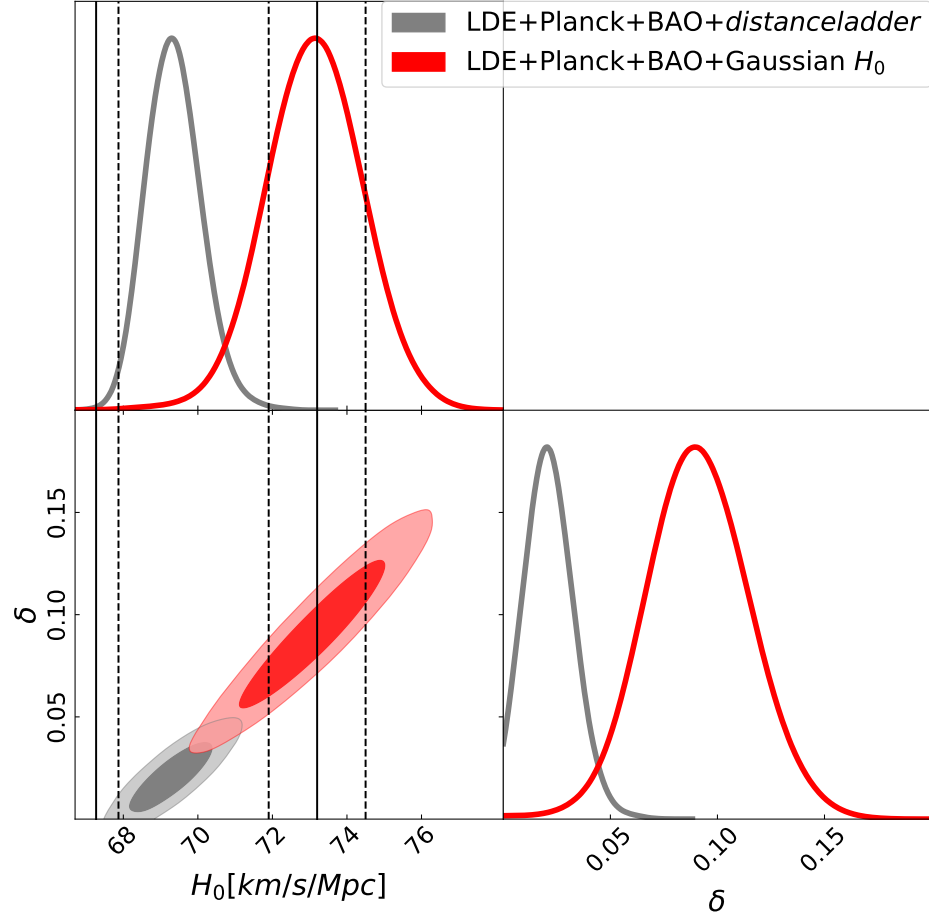


Figure 11: Posterior distributions involving a late-time dark energy model using the traditional Gaussian prior on H_0 indicated by red, and *distanceladder* likelihood package in gray. The solid black lines indicate the mean value from Ref. [144] (left) and Ref. [173] (right) with the dashed lines indicating the 1σ differences.

this case, the intercept value of the LDE model is $\beta = 0.073 \pm 0.04$, which is consistent with zero at less than two standard deviations. While this seems to alleviate the tension between CMB and the Cepheid calibration, this solution is much less promising when looking at the individual distances to nearby supernovae.

This is shown in figure 10 where we observe a clear offset at low redshifts ($z \lesssim 0.1$) between the actual distances to SNe Ia calibrated with Cepheids (blue points), and the corresponding distances predicted in the LDE model (red points) fitted with a Gaussian prior on H_0 . The residual plot in the lower panel of figure 10 describes the difference in distance modulus required to bring the LDE-predicted SNe Ia distances into agreement with their measurements. We can further quantify this low-redshift distance tension by fitting the relationship between the predicted distance moduli μ and the measured $m - M_{\text{sn}}$ values with a linear regression with a free slope. Recall that if this slope differs from unity, it would indicate a breakdown of the standard relationship between absolute luminosity and measured flux in an expanding Universe. Performing such fit, we find a slope of 0.983 ± 0.002 , which differs from unity at 8.5σ indicating a clear issue in this late-time dark energy model. The corresponding intercept is $\beta = 0.729 \pm 0.09$, which differs significantly from zero at several standard deviations. Clearly, a late dark energy transition does nothing to address the root cause of the tension and can even make the global fit to cosmological data worse. In fact, comparing the LDE model fitted with a Gaussian prior on H_0 to the LDE model fitted with the *distanceladder* likelihood results in a $\Delta\chi^2 \sim 21$, demonstrating that the former provides a poorer fit to the distances than the latter. While these results are broadly consistent with those presented in Ref. [236], our work clarifies why these models fail to properly resolve the tension: they are unable to fit the actual distance to low-redshift SNe Ia in the Hubble flow, as shown in figure 10.

Perhaps even more concerning is that using a Gaussian prior, in this case, could lead

LDE	Distance Ladder H_0	Gaussian H_0
$100\theta_s$	1.04200 ± 0.00029	1.04170 ± 0.00029
Ω_b	0.02246 ± 0.00014	0.02247 ± 0.00014
Ω_c	0.11867 ± 0.00094	0.11890 ± 0.00097
τ	0.0570 ± 0.0076	0.0596 ± 0.0073
$\ln(10^{10}A_s)$	3.050 ± 0.015	3.052 ± 0.015
n_s	0.9694 ± 0.0038	0.9689 ± 0.0038
δ	0.022 ± 0.022	0.091 ± 0.048
H_0	69.74 ± 1.28	73.78 ± 2.15

Table 3: Table describing the mean and 68% confidence intervals of a MCMC in which the *distanceladder* likelihood and the standard Gaussian likelihood on H_0 are used. Prior limited implies that the value is unconstrained within the prior. Δz and z_t are prior limited in all cases.

to spurious detection of new physics. As shown in figure 11 (see also table 3), the LDE model analyzed with Planck and BAO data in addition to a Gaussian H_0 prior leads to a significant detection ($> 3.5\sigma$) of a non-vanishing δ value, indicating a strong statistical preference for a late transition in the abundance of dark energy. However, when the actual calibrated distance measurements are taken into account as in our *distanceladder* likelihood, the value of δ becomes entirely consistent with zero, and no significant evidence for a late-time transition in the abundance of dark energy is observed. In this case, the failure of the Gaussian H_0 prior can be traced back to the breakdown of the standard cosmographic expansion of the luminosity distance to SNe Ia in the presence of a sharp transition in the Hubble rate at late times.

6.7 Discussion

The above section demonstrates that a Gaussian prior on H_0 can lead to a statistically significant detection of a dark energy transition at very late times. However, once the actual distances to SNe Ia are properly taken into account, such preference largely disappears. Fig-

ure 10 clearly illustrates why this is happening: the Gaussian prior on H_0 leads to distances to low-redshift SNe Ia that are too small compared with their actual measured values. Ultimately, this problem arises because the LDE models break the assumptions (in particular, the cosmographic expansion of the luminosity distance) that were used to infer H_0 from carefully calibrated distances to SNe Ia in the Hubble flow [142, 173]. This should serve as a cautionary tale when analyzing models that modify the Universe’s expansion history at very late times. In general, we recommend that cosmological models that deviate significantly from Λ CDM at $z \lesssim 1$ should never be analyzed with a Gaussian prior on H_0 . Instead, the calibrated distances to SNe Ia in the Hubble flow should directly be used in statistical analyses of such models. Our *distanceladder* likelihood package make such analyses straightforward.

Beyond the possible breakdown of the cosmographic expansion, another issue that has plagued many cosmological analyses using a Gaussian H_0 prior in conjunction with CMB data is the resulting ambiguity in the underlying absolute SNe Ia calibration. For example, R16 finds a value of $M_{\text{sn}} = -19.214 \pm 0.037$. Imposing the corresponding Gaussian prior on H_0 implicitly assumes this M_{sn} value, which does not necessarily agree with the value of M_{sn} implied by the other likelihoods used. For instance, the Pantheon data set calibrated with Planck data finds a value of $M_{\text{sn}} = -19.420 \pm 0.014$ in Λ CDM, which is statistically inconsistent with the above value as shown in figure 12. However, since most of the attention is focused on H_0 rather than the distances themselves, this underlying inconsistency is often missed in cosmological analyses. Of course, this $\beta \sim -0.2$ mag discrepancy between M_{sn} values is the root cause of the Hubble tension within Λ CDM. This point was recently emphasized in refs. [143, 166, 268, 289], and recent analyses of possible solutions to the tension have used M_{sn} instead of H_0 as their key parameters (see e.g. Ref. [210]). However, as we have seen in figure 8, simply making the values of M_{sn} between the local and inverse distance ladders compatible (i.e. $\beta \sim 0$) is not necessarily sufficient to ensure a successful

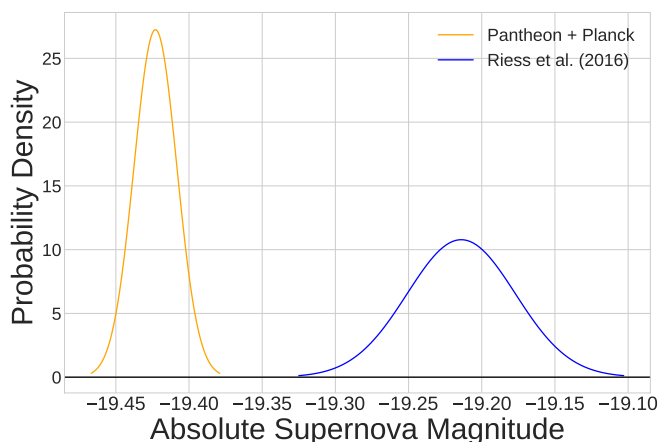


Figure 12: Comparison of the values of M_{sn} gotten by calibrating the Pantheon supernova data set with Planck data in Λ CDM with that obtained from the local distance ladder [142].

cosmological solution: the model must also fit the actual distances to SNe Ia.

6.8 Conclusion

An impressive amount of literature has been written on the Hubble tension, and it remains a real possibility that it could be the result of some missing pieces in our cosmological model. This impressive literature includes cosmological models that modify the cosmological expansion history at $z < 1$. Using a simple Gaussian prior distribution on H_0 to represent the entirety of the local distance ladder, some of these solutions can appear to be promising at relieving the tension. However, in using such a H_0 prior, the user implicitly assumes that the local distance ladder directly measures the Hubble expansion rate at $z = 0$. In reality, regardless of calibration choice, the local distance ladder actually measures distances to SNe Ia in the Hubble flow in the redshift range $z \gtrsim 0.02$. Therefore, to properly assess the effectiveness of cosmological models with modified late-time dynamics at relieving the Hubble tension, a simple Gaussian prior can not be used to represent the entire distance ladder. Instead, a likelihood that reconstructs the distance ladder and the distance measurements to the Hubble flow supernova is required for any model which proposes changes to the late-time Universe.

To aid in this, we have developed a likelihood package aptly named *distanceladder* which focuses on fitting the actual distances to SNe Ia, rather than simply fitting for H_0 . The *distanceladder* likelihood uses the actual luminosity distance given by the proposed cosmological model and is therefore sensitive to any changes in cosmology presented. This publicly available numerical package allows the user to test the different assumptions made in analyzing the cosmological distance ladder. It is therefore much more flexible and insightful than simply specifying a prior on the absolute supernova magnitude. To demonstrate the effectiveness of the likelihood, we have used the example of a late-time transition in the abundance of dark energy to showcase how focusing purely on H_0 can lead to the spurious detection of new physics, and a false sense that the Hubble tension has been relieved. When the distances are correctly accounted for, such late-time dark energy transition are severely constrained in their ability to resolve the Hubble tension. This behaviour is not limited to only this late-time dark energy model but to any model which dramatically changes the late-time expansion history of the Universe as compared to Λ CDM. In general, the success of a cosmological model should be assessed by how well it can fit the actual distances to Hubble-flow objects (including supernovae, strong gravitational lenses, and gravitational waves standard sirens, among others), rather than whether it can reproduce a specific value of H_0 . We encourage the observational cosmology community to make their actual distance measurements broadly available instead of emphasizing the derived Hubble constant values, which contain assumptions that might not be applicable to all cosmological scenarios.

Within the standard Λ CDM model, the Hubble tension between Cepheid-calibrated SNe Ia distances and Planck CMB data is now at the critical 5σ threshold [20]. Addressing this crucial cosmological problem requires the community to focus on the root cause of the tension – the distances to low-redshift Hubble-flow objects – rather than argue about which value of the Hubble constant might be the correct one.

7 Thomson scattering: one rate to rule them all

All we have to decide is what to do with the time that is given us.

- Gandalf

This chapter is a modified version of the publication, “Thomson scattering: one rate to rule them all,” by Kylar Greene and Francis-Yan Cyr-Racine published in the Journal of Cosmology and Astroparticle Physics in October of 2023 [290].

7.1 Introduction

As we have seen thus far, the path to resolving the Hubble tension may lie either in modifications to the local or distant Universe. Both strategies entail invoking physics beyond the Standard Model, typically by introducing new dynamics within the unseen dark sector. Efforts towards a model that locally resolves this discrepancy, whilst maintaining unchanged distances to Type Ia supernovae, are underway, albeit challenging [191, 231–263, 291–295]. Consequently, our focus shifts towards innovations in the distant Universe [163–165, 167, 296]. This endeavor is undoubtedly challenging due to the tight constraints imposed by detailed observations of the CMB and large scale structure (LSS) [144, 168, 169]. Nonetheless, recent explorations into the Universe’s dynamics have uncovered an overlooked symmetry of cosmological observables, known as the Free-Fall, Amplitude, and Thomson (FFAT) scaling [297]. This scaling embodies a universal rescaling of the Universe’s various length and rate scales, permitting significantly higher values of the Hubble parameter while preserving CMB spectra and LSS observations.

Essential to the FFAT scaling is the rescaling of the Thomson scattering rate in the early Universe. The Thomson scattering rate is the dominant process controlling photon diffusion in the pre-recombination plasma and is the second most important rate to understanding the structure of the CMB anisotropies, behind only the background expansion rate. The

scattering rate is often assumed to be a fixed feature of the cosmological model, but its variation is essential to exploring the overarching tension. Indeed, CMB spectra are sensitive to the ratio $\frac{\dot{\kappa}(z)}{H(z)}$, where $\dot{\kappa}(z)$ is the Thomson scattering rate with respect to redshift z . Any variation to $H(z)$ without equally scaling $\dot{\kappa}(z)$ will change the timing of photon decoupling, the thickness of the last scattering surface, and the suppression of power at high ℓ in CMB spectra. The observational constraints of the CMB may imply a relationship between the Hubble tension and the Thomson scattering rate.

While our comprehension of the relationship between the Hubble tension and the FFAT scaling is yet nascent in contrast to other established approaches like early dark energy models [192, 215], progress is being made exploring this new model space. Ref. [297] effectively scaled $\dot{\kappa}$ in the FFAT scaling direction, generating CMB spectra that aligns with observational data and allows for higher values of H . Their methodology, however, necessitated a decrease in the primordial helium abundance, a parameter stringently constrained by Big Bang Nucleosynthesis. Inspired by this line of research and the tantalizing opportunity to explore new physics, we propose a different path: varying the electromagnetic fine-structure constant and the electron mass to achieve the desired scaling of $\dot{\kappa}$. The idea of varying fundamental constants is not new, it was first suggested by Dirac in the 1930s, who proposed that the values of physical constants could be related to the age of the Universe [298, 299]. Presently, lab measurements and astrophysical observations restrict possible variations of α and m_e up to a redshift of $z \approx 7$, assuming the constants vary as a function of a simple power law. Fundamental constant variation also affects the recombination history which was previously thought to be well constrained in the Λ Cold-Dark-Matter (Λ CDM) model of cosmology [300–304]. In this work, we highlight the protective role of the FFAT scaling against alterations to the recombination history caused by variations in fundamental constants. This relaxation of observational constraints set by the Λ CDM model on fundamental constant variation widens the scope for model builders to delve into the FFAT scaling di-

rection without disrupting other cosmological observables. We demonstrate the efficacy of the FFAT scaling and fundamental constant variation to reconcile CMB, baryon acoustic oscillations (BAO), and local observations of H_0 , without worsening the fit to any of these observations. This constitutes a significant stride towards comprehending the interplay between the Thomson scattering rate and the Hubble tension. Additionally, I would like to emphasize that this is, in part, a phenomenological model of fundamental constant variation chosen to explore the relationship between the Thomson scattering rate and the Hubble rate.

7.2 The FFAT scaling

The FFAT scaling, first presented in Ref. [297] and used in Ref. [305], provides a powerful tool to understand the origins of cosmological constraints on new physics parameters. The transformation consists of a simple rescaling of rate and length scales by a constant factor λ that play key roles in the dynamical evolution of the Universe. Since CMB and LSS predictions are left invariant under the transformation, it also allows the exploration of theoretical models that predict a high value of H_0 that are automatically compatible with these observations. The set of scaling transformations is given by

$$\begin{aligned}\mathbf{FF}: \sqrt{G\rho_i} &\xrightarrow{\lambda} \lambda\sqrt{G\rho_i}, \\ \mathbf{A}: A_s &\xrightarrow{\lambda} A_s/\lambda^{n_s-1}, \\ \mathbf{T}: \sigma_T n_e &\xrightarrow{\lambda} \lambda\sigma_T n_e,\end{aligned}\tag{112}$$

where $\xrightarrow{\lambda}$ indicates the FFAT scaling, G is Newton's gravitational constant, ρ_i is the energy density of the i^{th} component, A_s is the amplitude of the primordial curvature power spectrum, n_s is the scalar spectral index, σ_T is the Thomson cross-section, and n_e is the free electron number density. We would like to emphasize that λ is not a physical parameter in the model, but rather λ captures the change in H of a derived model compared to the baseline model, like Λ CDM. A restricted version of this transformation (i.e. $\mathbf{FF} + \mathbf{A}$ in the above), valid only on

very large scales, was initially presented in Ref. [301]. By adding the Thomson rate rescaling (**T**), Ref. [297] showed that this transformation leaves CMB and LSS observables invariant over a broad range of scales. This paper focuses on a possible physical implementation of a transformation involving variation of fundamental constants and the **T** scaling. For clarity, we briefly review each element of the FFAT scaling, emphasizing their role in maintaining the invariance of cosmological observables.

7.2.1 Free Fall

The scaling of the gravitational free fall rate (**FF**), $\sqrt{G\rho_i} \xrightarrow{\lambda} \lambda\sqrt{G\rho_i}$, is possible by scaling the energy density $\rho_i \xrightarrow{\lambda} \lambda^2\rho_i$ ^{7,8}. The energy density of the Universe is directly related to the expansion rate, $H(z)$, as described by the Friedmann equation [4]

$$H(z) = \sqrt{\frac{8\pi G\rho_i(z)}{3}}. \quad (113)$$

Therefore, the FFAT scaling enables model builders to increase the value of $H(z)$ such that $H \xrightarrow{\lambda} \lambda H$. A model realizing $\lambda \sim 1.08$ would bring the CMB inference of H_0 into an agreement with the locally observed value and resolves the Hubble tension without putting other observations into further disagreement if the complete FFAT scaling is realized [20, 144]. The **FF** scaling keeps key cosmological quantities unchanged, including; (1) cosmological angles, (2) the redshift of matter-radiation equality, (3) the free-streaming fraction of the radiation density $\frac{\rho_{fs}}{\rho_r}$, and (4) the matter clustering fraction.

- *Cosmological Angles* – The FFAT scaling leaves cosmological angles tied to the baryon-photon sound horizon invariant. This includes the angular size of the acoustic sound horizon as measured directly in CMB observation θ_* , as well as the angular size of the baryon acoustic oscillation (BAO) feature imprinted on the distribution of galaxies at

⁷The FF transformation can also be achieved by scaling the gravitational constant G (as in Ref. [301]), perhaps including the FFAT transformation into a theory of modified gravity.

⁸For species with non-constant equations of state, such as massive neutrinos, it's imperative to scale the pressure in the same way as the density.

lower redshifts. To understand this invariance, consider $\theta_\star$, which is given by

$$\theta_\star = \frac{r_\star}{D(z_\star)}, \quad (114)$$

where $r_\star$ is the comoving sound horizon, $D(z_\star)$ is the comoving angular diameter distance, and a $\star$ references the value at the surface of last scattering. $r_\star$ and $D(z_\star)$ are given by

$$r_\star = \int_{z_\star}^{\infty} \frac{c_s(z)}{H(z)} dz, \quad (115)$$

$$D(z_\star) = \int_0^{z_\star} \frac{1}{H(z)} dz, \quad (116)$$

where c_s is the sound speed of the photon-baryon fluid given by $c_s = 1/\sqrt{3\left(1 + \frac{3\rho_b}{4\rho_\gamma}\right)}$ [70, 306]. Since the FFAT scaling leaves ratios of energy densities invariant, the sound speed c_s is left unchanged. Equations (115) and (116) are then both scaled by a factor of $1/\lambda$ (from the $1/H(z)$ factor), leaving the ratio $\theta_\star = r_\star/D(z_\star)$ invariant.⁹ Similarly, the FFAT scaling leaves the angular size of the baryon acoustic oscillation (BAO) feature imprinted on the distribution of galaxies at lower redshifts invariant.

- *Redshift of Matter-Radiation Equality* – The redshift of matter-radiation equality, z_{eq} , marks the transition from radiation domination to matter domination in the Universe. CMB spectra observations are sensitive to z_{eq} via the early integrated Sachs-Wolfe effect (eISW) where photons interact with a time-dependent gravitational potential and introduce additional CMB anisotropies [307–310]. To keep the CMB temperature spectra invariant under the FFAT scaling, z_{eq} must remain invariant. z_{eq} is given by

$$z_{\text{eq}} = \frac{\rho_m^0}{\rho_r^0} - 1 \quad (117)$$

⁹In principle, the integration bounds of equations (115)-(116) are shifted when generally changing the background cosmology. However, the FFAT scaling ensures that the redshift of decoupling is left invariant compared to the Λ CDM model.

where ρ_{m}^0 and ρ_{r}^0 are the present-day energy densities of matter and radiation. Similarly, the FFAT scaling leaves energy density ratios invariant, so the eISW effect is left unchanged under the FFAT scaling, which in turn leaves the low ℓ C_ℓ 's invariant.

- *Free-Streaming Radiation Ratio* – Free-streaming particles that propagate faster than the sound speed of the pre-recombination plasma contribute to the decay of gravitational potentials which drive acoustic oscillations [311–313]. The free-streaming to radiation fraction is given by

$$f_{\text{fs}} = \frac{\rho_{\text{fs}}}{\rho_{\text{r}}} \quad (118)$$

where ρ_{fs} is the energy density of free-streaming radiation. Including additional free streaming radiation pulls on the wavefront of the slower propagating plasma, smoothing the perturbation [314]. This damping would have noticeable effects on the CMB by introducing a phase and amplitude shift into the peaks of the temperature spectra. Again, the FFAT transformation uniformly scales all energy densities such that ρ_{fs} and ρ_{r} are evenly scaled, and f_{fs} is left invariant.

- *Matter Clustering Fraction* – The growth of matter structure at early times is dictated in part by the fraction of the total matter density that is pressure-supported [315]. Indeed, baryons cannot contribute to density fluctuations until the end of the baryon drag epoch, leaving only the dark matter to establish gravitational potential wells prior to this time. These potentials impact the CMB through the Sachs-Wolfe effect and set up the initial conditions for what would eventually become the large-scale structure of the Universe. Subtle changes to these potential is tightly bounded by the data and is a major source of constraints on physics beyond Λ CDM [305]. Since the **FF** transformation equally scales the pressure-supported and non-supported matter content of the Universe, it leaves these gravitational potentials unchanged. Once coupled to the **A** and **T** scalings (see below), the **FF** transformation thus keeps observables related to matter clustering such as σ_8 , the matter power spectrum, and lensing of the CMB

invariant.

7.2.2 Amplitude

The Boltzmann equations that describe the evolution of photon, baryon, neutrino, and dark matter perturbations depend on the Fourier wavenumber k . This introduces another length scale into the problem (beyond the Hubble and Thomson rates; see next section for the latter) which must be appropriately scaled (i.e. $k \xrightarrow{\lambda} \lambda k$) to leave the equations of motion invariant. However, as shown in references [301, 305], one could bypass this scaling by noting that the solution to the Boltzmann equations $\tilde{\Phi}$ under the scaling of the Hubble and Thomson rates can be written in terms of the original solution Φ (i.e. in the absence of scaling) but with a different wavenumber $k' = k/\lambda$,

$$\tilde{\Phi}(k, a, \lambda) = \Phi(k', a, \lambda = 1). \quad (119)$$

Here, $\Phi, \tilde{\Phi}$ stand for any perturbation variable entering the Boltzmann equations. The above identity is important as it allows us to show that the impact of simultaneously scaling the Hubble and Thomson rates on the CMB and large-scale structure can be entirely absorbed into a modification of the primordial amplitude of scalar fluctuations, as we now show. Taking the CMB temperature power spectra as an example, we can write its rescaled version as

$$C_\ell^{\text{TT}}(\lambda) = \int \frac{dk}{k} P(k) |\tilde{\Delta}_{\text{T}\ell}(k, \lambda)|^2, \quad (120)$$

where $P(k)$ is the primordial spectrum of curvature fluctuations and $\tilde{\Delta}_{\text{T}\ell}(k, \lambda)$ is the photon transfer function, which only depends on $\tilde{\Phi}$ [302, 316–328].¹⁰ This means that the transfer

¹⁰As noted earlier, the specific scaling of the Thomson scattering rate preserves the visibility function relative to the Λ CDM model. This ensures a high degree of similarity between the photon transfer functions for both models.

function obeys the identity $\tilde{\Delta}_{\text{T}\ell}(k, \lambda) = \Delta_{\text{T}\ell}(k/\lambda, \lambda = 1)$. We take $P(k)$ to have the form

$$P(k) = A_s \left(\frac{k}{k_p} \right)^{n_s-1}, \quad (121)$$

where A_s is the power spectrum amplitude at some pivot wavenumber k_p , and n_s is the scalar spectral index [329]. Using the scaling of the photon transfer function, it is straightforward to show that the CMB temperature spectrum is left unchanged by the **FF** transformation if A_s is simultaneously scaled by $A_s \xrightarrow{\lambda} \frac{A_s}{\lambda^{n_s-1}}$ such that

$$\begin{aligned} C_\ell^{\text{TT}}(\lambda) &\rightarrow \int \frac{dk}{k} \frac{A_s}{\lambda^{n_s-1}} \left(\frac{k}{k_p} \right)^{n_s-1} |\Delta_{\text{T}\ell}(k/\lambda, \lambda = 1)|^2 \\ &= \int \frac{dk'}{k'} \frac{A_s}{\lambda^{n_s-1}} \left(\frac{\lambda k'}{k_p} \right)^{n_s-1} |\Delta_{\text{T}\ell}(k', \lambda = 1)|^2 \\ &= C_\ell^{\text{TT}}(\lambda = 1). \end{aligned} \quad (122)$$

A similar argument applies to the polarization and cross temperature-polarization power spectra, as well as to several large-scale structure observables including cosmic shear power spectra.

7.2.3 Thomson

After the Hubble expansion rate, the Thomson scattering rate is the next most important rate affecting the structure of the CMB and LSS as it governs interactions between photons and free electrons before and during the recombination epoch. It is given by

$$\dot{\kappa} = an_e \sigma_T, \quad (123)$$

where σ_T is the Thomson cross section given by

$$\sigma_T = \frac{8\pi}{3} \left(\frac{\alpha}{m_e} \right)^2. \quad (124)$$

$\dot{\kappa}$ controls photon diffusion, and thus the damping of small-scale CMB anisotropies [330] and polarization generation [331, 332]. The overall level of photon diffusion is determined by the ratio

$$\frac{\dot{\kappa}(z)}{H(z)}, \quad (125)$$

which must be kept fixed to preserve the shape of CMB temperature and polarization spectra. To preserve equation (125) under a $H \xrightarrow{\lambda} \lambda H$ scaling, $\dot{\kappa}$ must also be scaled as $\dot{\kappa} \xrightarrow{\lambda} \lambda \dot{\kappa}$. In the familiar language of the CMB sound horizon and Silk damping scales [69], simultaneously scaling both $H(z)$ and $\dot{\kappa}(z)$ automatically leaves both scales invariant. Conversely, any cosmological model that does not preserve the $\dot{\kappa}(z)/H(z)$ ratio lead to a differential change to the CMB sound and damping scales, which typically limits the capability of these models to address the Hubble tension. It is interesting to highlight that, although the scattering rates of photons to baryons differ greatly from those of baryons to photons, attributable to their respective number densities differing by order 10^9 , both species undergo equivalent density scaling. This ensures that the photon to baryon energy density ratio remains consistent with Λ CDM, thus leaving both scattering rates remain invariant. This highlights the key role that Thomson scattering plays in our understanding of cosmology.

7.2.4 Cosmological anchors: breaking the scaling invariance

In our Universe, the FFAT scaling invariance is broken by key observations that essentially anchor the transformation, and thus underpin our entire knowledge of cosmology. Here we list some of these cosmological anchors, ordered roughly from most impactful to least impactful.

- *COBE-FIRAS* – The CMB spectral energy distribution measurements by COBE-FIRAS indicate a nearly perfect black-body with a temperature of $T_0 = 2.726 \pm 0.002$ K today, placing strict constraints on the photon energy density of the Universe [12, 14]. This introduces a significant anchoring effect for the **FF** scaling of the FFAT symmetry,

which requires a uniform scaling of all energy densities, including the photon energy density. Getting around this COBE-FIRAS anchor is the most important model-building challenge necessary to fully exploit the FFAT transformation as a means to address the Hubble tension.

- *Big Bang Nucleosynthesis* – BBN depends on the background temperature and expansion rate as they affect the neutron-to-proton ratio at the onset of nucleosynthesis. Changing these factors affects the creation of light elements as the neutron abundance will be changed. The measured primordial helium and deuterium abundances, $Y_{\text{P}} = 0.2449 \pm 0.0040$ and $D/H = 2.53 \pm 0.15 \times 10^{-5}$ respectively, tightly constrain the Hubble rate during BBN [150, 151, 333–336], and hence anchor the transformation $H \xrightarrow{\lambda} \lambda H$. A way around this BBN constraint is to assume that whatever physics exploits the FFAT scaling to yield a large Hubble constant today was not active at the time of nucleosynthesis. Unless otherwise mentioned, we will work under this assumption in what follows.
- *Recombination* – The ionization history of the Universe near the CMB epoch depends sensitively on the ratio of the hydrogen recombination rate to the Hubble expansion rate. Changing the latter without a corresponding rescaling of the former would thus modify the recombination history, which would in turn change the CMB’s last scattering surface. However, since Thomson scattering and hydrogen recombination are similar electromagnetic processes, they have nearly identical parametric dependence, with both rates proportional to $\sigma_{\text{T}} n_{\text{e}}$ at leading order (i.e. when neglecting the details of hydrogen atomic physics). This means that the FFAT transformation leaves the recombination history of the Universe *approximately* invariant. It does not, however, leave it *exactly* invariant as other subleading atomic rates also enter the detailed evolution of the hydrogen ionization history, most notably the forbidden $2s \rightarrow 1s$ transition rate. These residual changes to the recombination history under the FFAT scaling

were studied in refs. [297, 305], where it was shown that they only become important for relatively large values of $\lambda \gtrsim 1.1$.

7.2.5 Avoiding symmetry-breaking with a mirror world dark sector

We now present a phenomenological model which can exploit the FFAT scaling while avoiding the constraints from the prior section. The strictest constraint is imposed by COBE-FIRAS. The FFAT scaling requires a uniform scaling across all energy densities, including those of photons. However, COBE-FIRAS only detects the *visible* photon energy densities. One possible avenue to circumvent the COBE-FIRAS constraint while satisfying the FFAT scaling is to introduce an energy component within the dark sector that behaves as radiation. The required dark radiation [337, 338] can be included into the dark sector utilizing a “mirror world dark sector” (MWDS) containing a copy of photons, baryons, and neutrinos [339–366]. A MWDS presents a compelling avenue for moving along the FFAT scaling direction, particularly within the realm of dark sector models containing an explicit Z_2 symmetry, as exemplified by twin Higgs models [367, 368]. In this setup, the dark sector mirrors the Standard Model in terms of particle content and gauge interactions, but can have different masses for the corresponding particles. When combined with an Affleck-Dine condensate mechanism which populates both sectors, it inherently circumvents numerous fine-tuning challenges and minimizes the total number of additional parameters required [369]. This includes fine-tuning concerns regarding the timing of the dark sector recombination compared to the visible sector [361]. Indeed the approach of Ref. [369] naturally leads to the ratio of binding energy to background temperature to be similar in both sectors. This preserves the radiation free-streaming fraction evolution, which must be unchanged to stay along the FFAT scaling direction.

Such a model also naturally explains the additional baryon density requirement. We can introduce the required baryon density as atomic dark matter, thus satisfying the baryon scal-

ing requirement and preserving the effective (visible + dark) baryon-photon ratio [370–375]. This form of atomic dark matter constitutes a fractional percentage of the total dark matter budget and consists of two fundamental fermions charged under a $U(1)$ symmetry, analogous to the visible proton and electron [376, 377]. Lastly, the additional neutrinos ensure the ratio of free-streaming to coupled radiation energy density remains invariant.

The next constraint originates from BBN. Present knowledge places limits on fluctuations in $H(z)$ during BBN, considering the formation of light elements is intricately linked to the expansion rate. Consequently, we work under the assumption that the mechanism responsible for reheating the dark sector must activate post-BBN, see e.g. refs. [225, 369, 378–383]. We thus set $Y_P = 0.2449$ in our model to avoid possible symmetry-breaking effects arising from BBN. Similarly, we assume that the physics responsible for changes in fundamental constants is not present during BBN.

The final and weakest constraint comes from recombination. Recall that along the FFAT scaling direction, recombination is only slightly broken. We choose to leave this symmetry breaking intact and in section 7.4, explore its impact numerically. However, implementing the MWDS with FFAT scaling and fundamental constant variation in existing cosmological code poses computational challenges. Consequently, in subsection 7.4.1, we propose an alternative methodology to mimic the MWDS with FFAT scaling into the CLASS code [160, 384–386]. This approach successfully reproduces the leading-order gravitational effects that the MWDS exerts on the observable Universe, while keeping the recombination history and cosmological observables intact. A later publication will present the full MWDS with FFAT scaling implementation, and probe the nature of the more complex dark sector.

7.3 Fundamental constant variation

To implement the FFAT scaling, one must choose how to scale the Thomson scattering rate to achieve the required phenomenology. Recent studies [297, 305] have opted to increase $\dot{\kappa}$ by scaling the helium fraction Y_{P} , while keeping the baryon density constant. The scaling $(1 - Y_{\text{P}}) \xrightarrow{\lambda} \lambda(1 - Y_{\text{P}})$ effectively raises n_{e} and thus $\dot{\kappa}$ by a factor of λ , preserving the invariance of the ratio in Eq. (125). These studies confirm that the FFAT scaling allows for increased values of $H(z)$ while keeping other CMB and LSS observables in agreement. It however requires a value of Y_{P} that is strong tension [297] with direct measurements of the helium abundance [150, 151, 333–336].

In this study, we postulate that the Thomson cross-section in the early Universe diverged *slightly* from its current value, rather than change the predicted helium abundance. This deviation suggests that the fundamental constants appearing in the Thomson cross section, specifically the fine-structure constant α and the electron mass m_{e} , had subtly different values around the time of decoupling compared to their present-day measurements. While fundamental constant variation has been explored previously in different contexts (see e.g. refs. [229, 387–415]), prior works did not consider the invariance of certain cosmological observables protected by the FFAT scaling, thus deriving overly strict constraints on the possible variation of α and m_{e} [300–304]. The following sections elucidate the nature and magnitude of the fundamental constant variations required to realize the desired Thomson rate scaling, while minimally altering the intricacies of hydrogen atomic physics.

7.3.1 Thomson cross section rescaling

When scaling the Thomson cross section, care must be taken to leave the atomic physics of hydrogen nearly invariant as to not change the process of recombination. The most important quantity to leave unchanged is the hydrogen binding energy $B_{\text{H}} \propto m_{\text{e}}\alpha^2$, which controls the onset of hydrogen recombination and, ultimately, the redshift of photon decoupling. In order

to meet the condition $\sigma_T \xrightarrow{\lambda} \lambda \sigma_T$ without disturbing B_H , a specific variation is,

$$m_e \xrightarrow{\lambda} \lambda^{-1/3} m_e \ \& \ \alpha \xrightarrow{\lambda} \lambda^{1/6} \alpha. \quad (126)$$

Addressing the Hubble tension using the FFAT scaling combined with fundamental constant variation necessitates a value of $\lambda \approx 1.088$ (corresponding to $H_0 = 73.3$ km/s/Mpc) [20]. This corresponds to a modest, yet significant, variation of 2.76% in m_e and 1.41% in α from their respectively measured local values. While these changes might appear small, these early-Universe variations are magnitudes greater than those permissible in the low-redshift Universe (see section 7.3.2). Also, conventional understanding of fundamental constant variation during the recombination epoch suggests that the Universe’s recombination history places constraints on possible variations, a topic that will be revisited in the following subsection. The implementation of the FFAT scaling with fundamental constant variations in Eq. (126) safeguards the crucial ratio depicted in Eq. (125), but introduces minor alterations to hydrogen recombination given the different parametric dependence of other atomic rates on α and m_e which we discuss in section 7.3.3. We emphasize that any cosmological model that increases H must account for the change in the ratio given by equation (125) to maintain agreement with observables.

7.3.2 Fundamental constant variation constraints

A variety of methods, including atomic clock comparisons at $z = 0$ [416–419], the Oklo phenomenon (a naturally occurring fission reaction) at $z \approx 0.14$ [420–422], assessments of radioactive elements derived from meteorite dating at $z \approx 0.45$ [423], observations of the Sunyaev-Zeldovich effect at $z \approx 0.5$ [58], and inspection of quasar spectra at $z \approx 7$ [424, 425], offer an extensive set of measurements to constrain the time variation of α and m_e . This is not a complete list of methodologies, and there are several reviews which provide a more complete discussion [426, 427]. The collection of these methods finds that there is nearly no

time variation allowed for the constants in the redshift range $0 \leq z \leq 7$. However, they do not place bounds on what the allowed level of variation is at the time of photon decoupling, $z \sim 1100$.

Prior understanding was that the CMB imposed constraints on the permissible variation of constants, due to the variation's inability to keep the recombination history unchanged from the Λ CDM model. References [146, 300] determined that the distance to the surface of last scattering, the overall level of diffusion damping [330], and the early integrated Sachs-Wolfe effect [428] are all related and will all be altered under fundamental constant variation. The tFFAT scaling preserves these quantities, as discussed in subsection 7.2.1, thereby avoiding the constraints set by the CMB on fundamental constant variation. This scaling also mostly maintains the integrity of hydrogen recombination, as we will discuss in the next section.

Delving further into redshift space allows us to constrain the permitted variation of constants during BBN at $z \sim 10^9$. Alterations in the values of these constants could potentially disturb the predicted abundances of light elements, an outcome not safeguarded against by the FFAT scaling. Perturbations to α and m_e could possibly modify the lifetime of neutrons and the mass differential between protons and neutrons [429]. Such changes would adjust the neutron-to-proton ratio, thereby introducing potential deviations in the predicted abundances of light elements. Accurately calculating these effects requires intricate QCD and QED lattice calculations [430]. Therefore, it is of interest to investigate the impact of fundamental constant variation on BBN abundances to constrain the allowed variation of α and m_e when combined with the broader FFAT symmetry (particularly the expansion rate scaling) to understand this potential symmetry-breaking effect, but this is beyond the scope of this thesis.

To avoid these constraints set on the variation of fundamental constants, we make the following conservative assumptions; the mechanism which varies the constants must become active after BBN, the time-scale in which the constants vary must be greater than the timescale of recombination, and the constants must be at their stable, locally measured values by $z \sim 7$.

7.3.3 Recombination with fundamental constant variation

To track how the FFAT scaling influences the recombination history, we compute the free electron fraction X_e and visibility function, limiting our discussion to hydrogen recombination as the visibility function has little support from the specific details of helium recombination. In our analysis, we consider quantities not protected by the tFFAT scaling and assume a simplified description of recombination for clarity. A more detailed, multi-level approach like that used by the cosmological code HyRec [431, 432] reveals similar levels of symmetry breaking in appendix A.

The Saha approximation given in Eq. (127) is employed to describe systems in equilibrium and estimate the redshift of the onset of recombination:

$$\frac{X_e^2}{1 - X_e} \approx \frac{1}{n_b} \left(\frac{m_e T}{2\pi} \right)^{3/2} e^{-B_H/T} \equiv s \quad (127)$$

where X_e is the free electron fraction, n_b is the visible baryon number density¹¹, and T is the background temperature. However, it is not invariant under the FFAT scaling of m_e , leading to a symmetry-breaking effect of $s \xrightarrow{\lambda} \lambda^{-1/2}s$. While this effect has negligible influence on the overall X_e history, it does shift the equilibrium value of X_e . However, X_e

¹¹In the tFFAT scaling, the visible baryon density is consistent with Λ CDM, being anchored by BBN predictions. It is up to model builders to introduce additional baryon density without putting BBN constraints into tension. A plausible approach is to invoke a more complex dark sector like that described in section 7.2.5 in which a fractional amount of dark matter is baryonic.

falls out of equilibrium at the beginning of recombination and we must transition to a more comprehensive Boltzmann treatment. This transition necessitates the use of the Peebles equation [433], which describes the recombination history by evaluating the rate of change of X_e with respect to the cosmological scale factor a and is given by

$$\frac{dX_e}{da} = \frac{C\alpha^{(2)}}{aH} \left(\left(\frac{m_e T}{2\pi} \right)^{3/2} e^{-B_H/T} (1 - X_e) - n_b X_e^2 \right). \quad (128)$$

Here, $\alpha^{(2)}$ is the case B recombination rate given approximately by [156]

$$\alpha^{(2)} \approx \frac{\alpha^2}{m_e^2} \left(\frac{B_H}{T} \right)^{1/2} \ln \left(\frac{B_H}{T} \right). \quad (129)$$

In Equation (128), two factors are not protected by the FFAT scaling: $\left(\frac{m_e T}{2\pi} \right)^{3/2} e^{-B_H/T}$ and $\frac{C\alpha^{(2)}}{aH}$. The first term mirrors the symmetry-breaking term in the Saha equation, introducing a factor of $\lambda^{-1/2}$. The second term demands a more thorough examination. Specifically, $\alpha^{(2)}$ is not invariant under the tFFAT scaling, where the $\frac{\alpha^2}{m_e^2}$ term introduces a symmetry breaking such that $\alpha^{(2)} \xrightarrow{\lambda} \lambda \alpha^{(2)}$. Fortunately, this cancels out the factor of λ introduced in the denominator of $\frac{C\alpha^{(2)}}{aH}$ by H . However, the Peebles C -factor, representing the probability that a hydrogen nucleus with an electron in the $n = 2$ shell reaches the ground state before it is photoionized, is not left invariant and introduces a symmetry breaking term [433]. Recall that C is parameterized by three key rates and is given by

$$C = \frac{R_{Ly\alpha} + \Lambda_{2s-1s}}{\beta_B + R_{Ly\alpha} + \Lambda_{2s-1s}}. \quad (130)$$

The escape rate of Lyman- α photons, $R_{Ly\alpha}$, is

$$R_{Ly\alpha} = \frac{8\pi H}{\lambda_{Ly\alpha}^3 n_H x_{1s}}, \quad (131)$$

where n_H is the number density of hydrogen, x_{1s} is the fraction of hydrogen with an electron

in the ground state, and $\lambda_{\text{Ly}\alpha}$ is the Lyman- α wavelength given by $\frac{1}{\lambda_{\text{Ly}\alpha}} = \frac{3}{4}B_{\text{H}}$. The factor of $\frac{3}{4}$ arises from the electron transitioning from the first excited state to the ground state. We observe that $R_{\text{Ly}\alpha} \propto \lambda_{\text{Ly}\alpha}^{-3} H n_{\text{H}}^{-1}$ which under the tFFAT scaling becomes

$$R_{\text{Ly}\alpha} \propto \lambda_{\text{Ly}\alpha}^{-3} H n_{\text{H}}^{-1} \xrightarrow{\lambda} \lambda_{\text{Ly}\alpha}^{-3} (\lambda H) n_{\text{H}}^{-1} \propto \lambda R_{\text{Ly}\alpha}. \quad (132)$$

The two-photon decay rate, $\Lambda_{2\text{s}-1\text{s}}$, scales as [157]

$$\Lambda_{2\text{s}-1\text{s}} \propto \alpha^8 m_{\text{e}} \xrightarrow{\lambda} (\lambda^{1/6} \alpha)^8 (\lambda^{-1/3} m_{\text{e}}) \propto \lambda \Lambda_{2\text{s}-1\text{s}}. \quad (133)$$

The case-B photoionization rate, β_{B} , is expressed as

$$\beta_{\text{B}} = \left(\frac{m_{\text{e}} T}{2\pi} \right)^{3/2} e^{-B_{\text{H}}/T} \alpha^{(2)}. \quad (134)$$

Under the tFFAT scaling, β_{B} transforms as:

$$\beta_{\text{B}} \propto m_{\text{e}}^{3/2} \alpha^{(2)} T^{3/2} \xrightarrow{\lambda} (\lambda^{-1/3} m_{\text{e}})^{3/2} (\lambda \alpha^{(2)}) T^{3/2} = \lambda^{1/2} \beta_{\text{B}}. \quad (135)$$

Employing equations (131), (133), and (135), we scale the Peebles C -factor as

$$C \xrightarrow{\lambda} \frac{R_{\text{Ly}\alpha} + \Lambda_{2\text{s}-1\text{s}}}{\lambda^{-1/2} \beta_{\text{B}} + R_{\text{Ly}\alpha} + \Lambda_{2\text{s}-1\text{s}}} \quad (136)$$

This introduces a minor symmetry-breaking term due to the photoionization rate β_{B} scaling as $\lambda^{-1/2}$, slightly increasing the net rate of hydrogen recombination. In the following section, we numerically investigate the magnitude of symmetry breaking introduced by recombination.

7.4 Phenomenological and numerical implementation

7.4.1 Mimicking the MWDS in CLASS

Here we present the mimic FFAT (mFFAT) scaling which modifies the visible baryon and photon densities according to equation (113). The mFFAT scaling replicates the *gravitational* impacts introduced by a MWDS, mimics the recombination history of the true FFAT scaling, and keeps the observables protected by the FFAT scaling in check. For brevity, from here on the ‘true FFAT’ scaling will be referred to as tFFAT. While this model is schematic instead of physical (since it technically violates the COBE-FIRAS bounds), it captures all relevant physics to explore variations in α and m_e as credible pathways to approaching the Hubble tension. It also highlights the crucial role of the Thomson scattering rate in maintaining observational congruence and reaffirms the utility of FFAT scaling as a robust mechanism to safeguard cosmological observables. In Ch. 8, we investigate numerically the full FFAT scaling with a mirror world dark sector.

To mimic the gravitational influence that a MWDS would have on the observable Universe, we remove the COBE-FIRAS constraint from CLASS and replace dark radiation with visible radiation by increasing T_{CMB} such that $\rho_r \propto T_{\text{CMB}}^4 \xrightarrow{\lambda} \lambda^2 \rho_r \propto (\sqrt{\lambda} T_{\text{CMB}})^4$. However, the recombination history is exponentially sensitive to the ratio $\frac{B_{\text{H}}}{T_{\text{CMB}}}$. Thus, we must ensure that the ratio of the two remains invariant such that

$$\frac{B_{\text{H}}}{T_{\text{CMB}}} \xrightarrow{\lambda} \frac{\sqrt{\lambda} B_{\text{H}}}{\sqrt{\lambda} T_{\text{CMB}}}. \quad (137)$$

This maintains the ratio in the exponential of equations (127) and (128), ensuring that the changes to the recombination history remain negligible.

We also remove the observed baryon density constraint [434] and add visible baryons as a facsimile for dark baryons. Scaling the visible baryon density equally scales the electron number density n_e by λ^2 . To preserve the ratio in equation (125), we must choose a different

scaling of σ_T as $\sigma_T \xrightarrow{\lambda} \frac{1}{\lambda} \sigma_T$ to counterbalance the increased n_e . To stay in the FFAT scaling direction, we also add additional dark matter and neutrinos, thus preserving the free streaming-radiation fraction and observables related to matter clustering. The new constraints provided by equation (137) and the Thomson cross section result in a different variation of fundamental constants given by

$$m_e \xrightarrow{\lambda} \lambda^{1/2} m_e \quad \& \quad \alpha \xrightarrow{\lambda} \alpha. \quad (138)$$

This transformation effects recombination in a subtly different way than the FFAT scaling, since the Peebles factor (eq. (130)) is serendipitously left invariant in this case. The escape rate of Lyman- α photons in the mFFAT scaling scales as

$$R_{Ly\alpha} \propto \lambda_{Ly\alpha}^3 H n_H^{-1} \xrightarrow{\lambda} (\lambda^{1/2} \lambda_{Ly\alpha})^3 (\lambda H) (\lambda^2 n_H)^{-1} \propto \lambda^{1/2} R_{Ly\alpha}. \quad (139)$$

The two-photon decay rate Λ_{2s-1s} scales as

$$\Lambda_{2s-1s} \propto \alpha^8 m_e \xrightarrow{\lambda} \alpha^8 (\lambda^{1/2} m_e) \propto \lambda^{1/2} \Lambda_{2s-1s}. \quad (140)$$

The case B photoionization rate scales as

$$\beta_B \propto m_e^{3/2} \alpha^{(2)} T^{3/2} \xrightarrow{\lambda} (\lambda^{1/2} m_e)^{3/2} (\lambda^{-1} \alpha^{(2)}) (\lambda^{1/2} T)^{3/2} = \lambda^{1/2} \beta_B. \quad (141)$$

As a result, the mFFAT scaling leaves the Peebles factor invariant such that

$$C \xrightarrow{\lambda} \frac{\lambda^{1/2} R_{Ly\alpha} + \lambda^{1/2} \Lambda_{2s-1s}}{\lambda^{1/2} \beta_B + \lambda^{1/2} R_{Ly\alpha} + \lambda^{1/2} \Lambda_{2s-1s}} = C. \quad (142)$$

Therefore the mFFAT scaling avoids the introduction of a symmetry-breaking term associated with the photoionization rate, β_B , as seen in the tFFAT case. The mFFAT scaling can thus be considered as an idealized version of the tFFAT scaling, in which the recombination

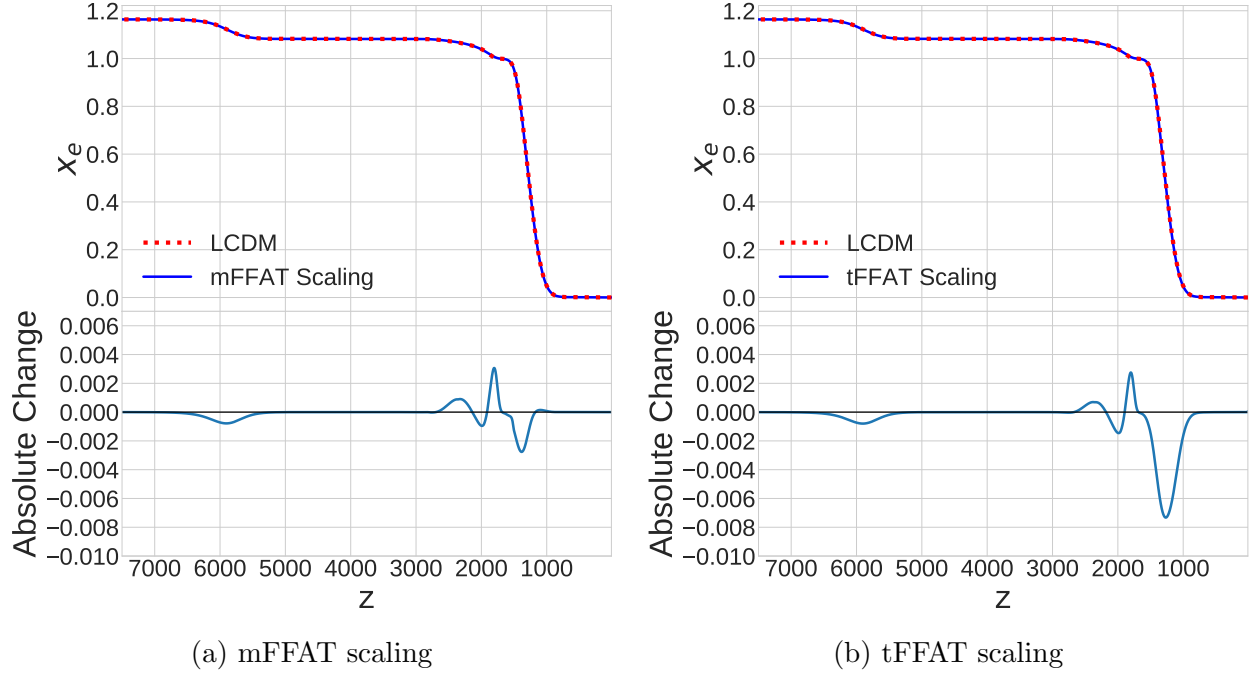


Figure 13: Figures 13a and 13b compare the expected change in the free electron fraction for the mFFAT and tFFAT models (blue) to the Λ CDM model (dotted red), scaled by $\lambda = 1.088$, corresponding to $H_0 = 73.3$ km/s/Mpc. The tFFAT X_e history in figure 13b is similar to the mFFAT scaling until the onset of hydrogen recombination, where the symmetry-breaking effect from the Peebles C-factor becomes relevant.

symmetry-breaking effects have been suppressed.

7.4.2 Testing our implementation: the ionization history

To quantify the impact of the Peebles factor's symmetry-breaking in the tFFAT scaling on the recombination history and visibility function, we use the mFFAT scaling as a baseline for comparison and perform a series of numerical tests (figures 13 and 14) to confirm that the mFFAT and tFFAT scalings accurately approximate the Λ CDM recombination history and visibility function. Figure 13 shows X_e as a function of redshift with $\lambda = 1.088$ corresponding to $H_0 = 73.3$ km/s/Mpc for both the mFFAT and tFFAT scalings. The mFFAT scaling exhibits at most a 0.3% deviation in X_e from Λ CDM due to the small symmetry-breaking term introduced from the Saha approximation, where the increased visible baryon density has shifted the equilibrium value. This subtle alteration is clearly seen around redshifts

$z \sim 6000$ in both models¹², where the equilibrium process HeIII to HeII is very slightly changed and also influences the slight changes found at redshift $z \sim 2000$ during HeII to HeI recombination. While both scalings display a similar deviation from Λ CDM up until $z \sim 1500$, a significant difference arises at the onset of hydrogen recombination.¹³ This is when the recombination symmetry-breaking becomes relevant for the tFFAT model, as described at the end of section 7.3.3, causing at most a 0.8% deviation. If the level of symmetry-breaking from the fundamental constant variation is significant enough ($\lambda \gtrsim 1.1$), it can lead to changes in CMB spectra that deviate from observations. However, in both cases the level of symmetry-breaking is small for values required to resolve the Hubble tension, as evidenced by the change to the visibility function seen in figure 14.

The visibility function represents the likelihood that a photon scatters for the last time before free-streaming away and is given by

$$g(z) = e^{-\tau} \frac{d\tau}{dz} \quad (143)$$

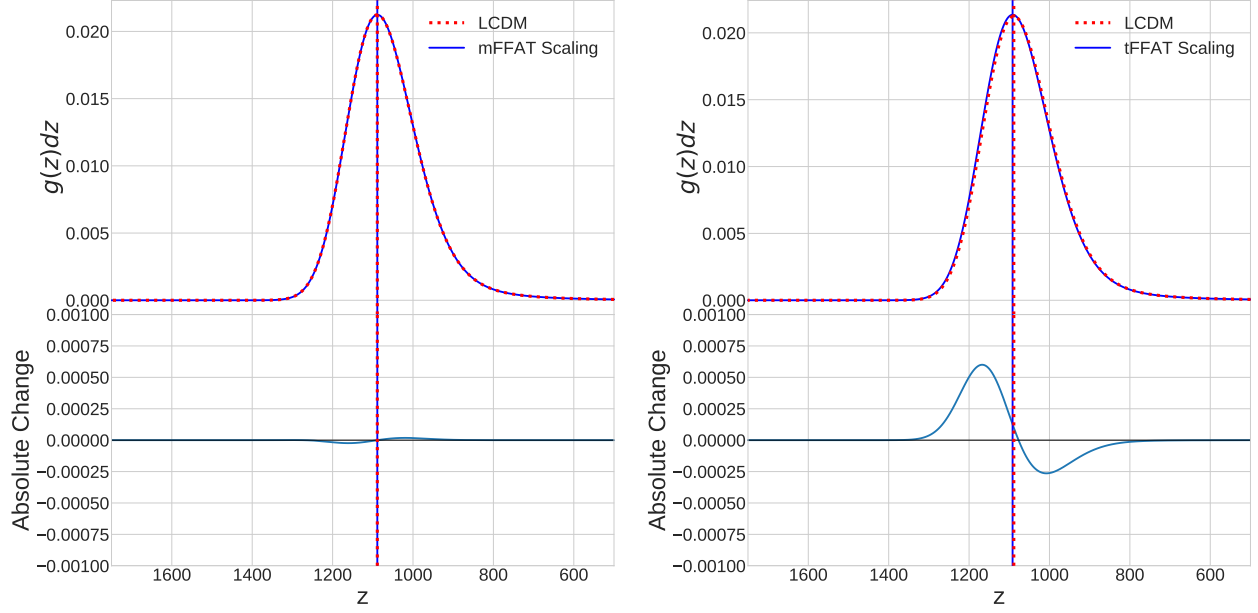
where τ is the optical depth to photons at redshift z and is given by

$$\tau(z) = \int_0^z \frac{\dot{\kappa}(z)}{(1+z)H(z)} dz. \quad (144)$$

There are two critical features of the visibility function: the width and peak of the function. The peak sets the distance to the surface of last scattering, and the width corresponds to the duration of decoupling [158]. The mFFAT scaling remains nearly invariant for both the maximum and width of the visibility function, whereas the tFFAT scaling negligibly shifts the peak to a slightly earlier redshift. Though the visibility function for the tFFAT scaling

¹²Coincidentally, the tFFAT model has the same level of symmetry-breaking for the equilibrium value, but it is due to the change in m_e and not the change in the visible baryon density.

¹³See appendix A for more details regarding the slightly different symmetry breaking introduced by the full four level model HyRec uses to compute X_e .



(a) Mimic model with fundamental constant variation given by eq. (138) (b) MWDS model with fundamental constant variation given by eq. (126)

Figure 14: $g(z)dz$ is the visibility function with respect to redshift. The vertical lines indicate the location of the maximum value of the function. This figure shows that both the mFFAT and tFFAT scalings keep the visibility function's maximum and width nearly invariant compared to the Λ CDM model, even when $H_0 = 73.3$ km/s/Mpc.

is not precisely invariant due to its integration of $\dot{\kappa}$ (and thus X_e) over all redshifts, the changes in the visibility function are similar to other models that increase H_0 while keeping CMB spectra in agreement with observation [212].

Figures 13 and 14 also demonstrate that the free electron history does not severely constrain the variation of fundamental constants, as previously thought in Ref. [300, 303, 304], when combined with the FFAT scaling. This allows for the exploration of different cosmological models with permissible constant variations that maintain a mostly invariant recombination history. Since the mFFAT and tFFAT scalings have similar effects on recombination and maintain the similar invariance of cosmological observables, we now focus on the mFFAT scaling for computational simplicity for the remainder of this paper. In a future publication, we will explore the tFFAT scaling in the same manner as below for the mFFAT scaling to

explore the parametric freedom of the dark sector.

7.4.3 Explicitly exploring the FFAT symmetry direction

As an initial test, we confirm numerically that the mFFAT scaling produces invariant CMB spectra while accommodating high values of H_0 . To do this, we fix all cosmological parameters to the typical Λ CDM values from Planck [144] and introduce the variable λ from equations (112). This forces Monte Python [274], the Monte-Carlo-Markov-chain sampler we use to compare the mFFAT model to data, to explicitly explore the FFAT symmetry direction. As expected, figure 15 shows that this ‘fixed’ mFFAT scaling invariantly scales H_0 , flattening it’s one-dimensional posterior. Only H_0 , n_s , and τ_{reio} are displayed, while traditional parameters such as ω_b , ω_{cdm} , and A_s are scaled from their Λ CDM values according to equations (112). The flattening of the posterior demonstrates the effectiveness of the mFFAT scaling in invariantly increasing H_0 , while maintaining consistency with observed CMB spectra.

7.5 Results

7.5.1 Naturally exploring the FFAT scaling

In our second test of the idealized mFFAT scaling, we allow all cosmological parameters to vary and remove the explicit enforcement of the FFAT scaling. As illustrated in Figure 16, the FFAT scaling direction, indicated by the dashed red lines, naturally emerges from the MCMC exploration. By allowing all cosmological parameters to vary, we find that the MCMC sampler naturally broadens the posterior on H_0 , enhancing our understanding of the relationship between the Thomson scattering rate and the Hubble tension. We find the correct scaling between N_{eff} , ω_{cdm} , and ω_b , in accordance with equation (113). We also define the variation of m_e by $\delta_e = \frac{\tilde{m}_e}{m_e} - 1$ where $\tilde{m}_e$ is the varied electron mass. For a list of priors used to generate figures 15 and 16, please see table 4.

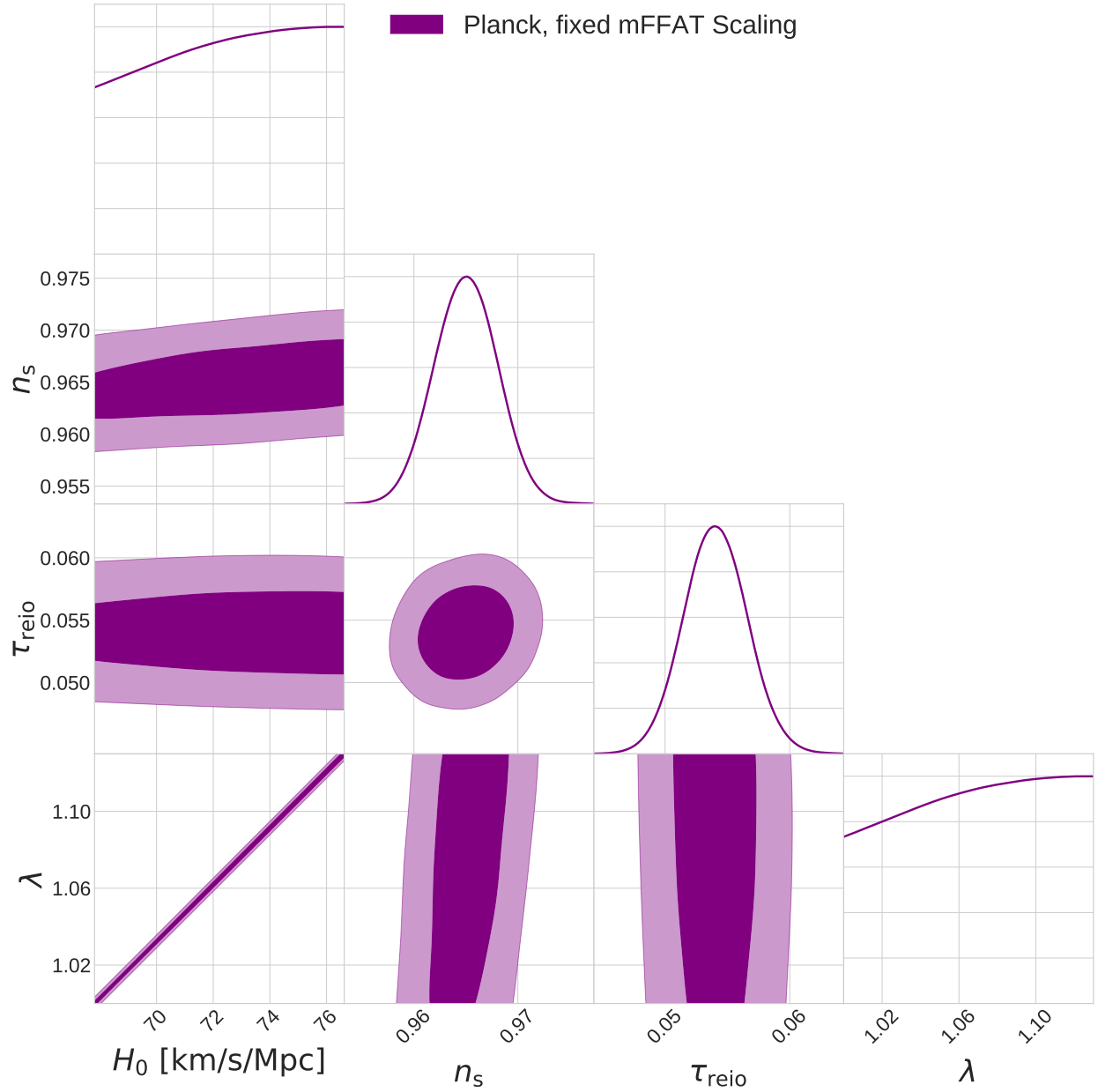


Figure 15: This triangle plot shows that the mFFAT scaling allows the CMB to be made consistent with a broad range of H_0 values. Planck data used include TT, TE, EE, and lensing data [144]. In this case, we fix the energy densities ρ_i , A_s , α , m_e , and H to their Λ CDM values, and allow them to vary exclusively through the FFAT scaling parameter λ , directing MontePython to explore the symmetry direction. We have included the scaling parameter λ to demonstrate the tight degeneracy between it and H_0 . The upper bound for H_0 is determined by a limitation in the default HyRec implementation, where the interpolation tables are not expansive enough to accommodate the necessary photon energy densities.

	Figure 15	Figure 16
ω_b	0.02238*	[0.02, 0.0322]
ω_{cdm}	0.12011*	[0.1, 0.173]
h	0.6781*	[0.6, 0.75]
A_s	2.101e-9*	[None, None]
n_s	[None, None]	[None, None]
τ_{reio}	[0.004, None]	[0.004, None]
T_{CMB}	2.7255*	[2.7, 2.87]
$\delta_e + 1$	1.0*	[1.0, 1.095]
λ	[1.0, 1.13]	-

Table 4: This table enumerates the prior bounds used to generate the contour plots of figures 15 and 16. A * indicates the reference values ($\lambda = 1$) of the parameters that are then scaled according to eq. (112) as λ is varied in the MCMC chains.

Note that as δ_e approaches 5%, corresponding to a value of $\lambda \sim 1.1$ ($H_0 = 74.4$ km/s/Mpc), the ionization balance shifts, and the symmetry breaking effects introduced by recombination cause a deviation from the FFAT symmetry direction. This is reflected in the one-dimensional posterior for δ_e , which finds very high values of δ_e to be increasingly less likely. ΔN_{eff} shown in figure 16 is a derived parameter given by

$$\Delta N_{\text{eff}} = \left(\frac{\tilde{T}^4}{T_{\text{COBE}}^4} - 1 \right) \left(3.044 + \frac{1}{\frac{7}{8} \left(\frac{4}{11} \right)^{4/3}} \right) \quad (145)$$

where $\tilde{T}$ is the CMB temperature that is allowed to vary by the MCMC representing the additional radiation density, and T_{COBE} is the COBE-FIRAS temperature measurement of the CMB equal to 2.7255 K [14]. The prior volume is limited for ΔN_{eff} from a limitation of default HyRec interpolation tables for the CMB temperature.

When comparing the best-fit parameters from figure 16 to the best-fit Λ CDM values, it becomes evident that the mFFAT scaling prefers a higher value of H_0 and is able to fit Planck and BAO data as effectively as the Λ CDM model, with a corresponding $\Delta\chi^2 = -2.3$. This improvement of χ^2 is expected as we have included two additional parameters into the

model. However, the prior volume effect listed above may influence the shape of posterior, and a larger volume may find a preference for even higher values of H_0 . Taken together, these findings indicate that fundamental constant variation with the FFAT scaling offers a promising approach for investigating the Hubble tension while emphasizing the effect that the Thomson scattering rate has on the underlying nature of the tension.

The CMB temperature-temperature (TT) and polarization-polarization (EE) spectra obtained with the mFFAT scaling best fit values from figure 16 are compared to those of a Λ CDM model in Fig. 17. The CMB spectra exhibit a high degree of invariance, concurrently accommodating both cosmological and local observations of the expansion rate, aligning with the predictions made in Section 7.2.2. This example serves as a successful demonstration of how the FFAT scaling can be utilized to explore a wider range of cosmological models, without sacrificing agreement with observational data.

7.5.2 Relationship with other work

The interplay between variations in fundamental constants and the Hubble tension has been explored in previous works, though in different scenarios [210, 229, 230, 387–415, 435–438]. For example, reference [229] investigated the effect of varying the electron mass to allow for an earlier recombination time. They outlined four criteria for a “successful” cosmological solution to the Hubble tension. These criteria include: maintaining the CMB power spectra similar to Λ CDM, reducing sound horizon size by $\sim 10\%$, ensuring the angular size of the sound horizon aligns with Λ CDM, and not worsening the fit to BAO, SNeIa, and other low-redshift distance measurements. Notably, these four criteria are also naturally satisfied along the FFAT scaling direction. However, a deviation from the FFAT scaling occurs in their model when increasing the physical matter density, which alters the redshift of matter-dark energy equality and limits their model in increasing H_0 . Furthermore, the variation of

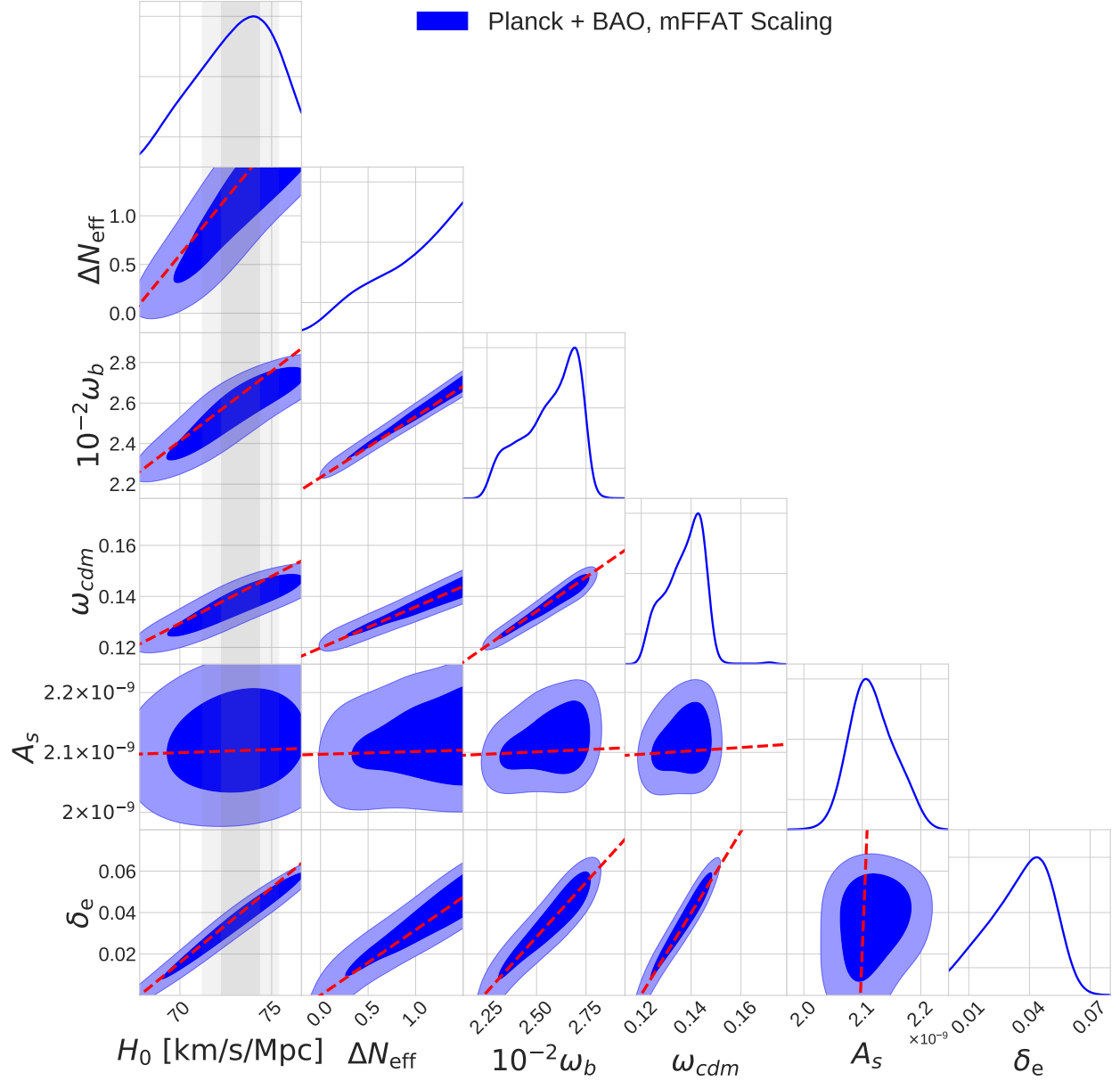
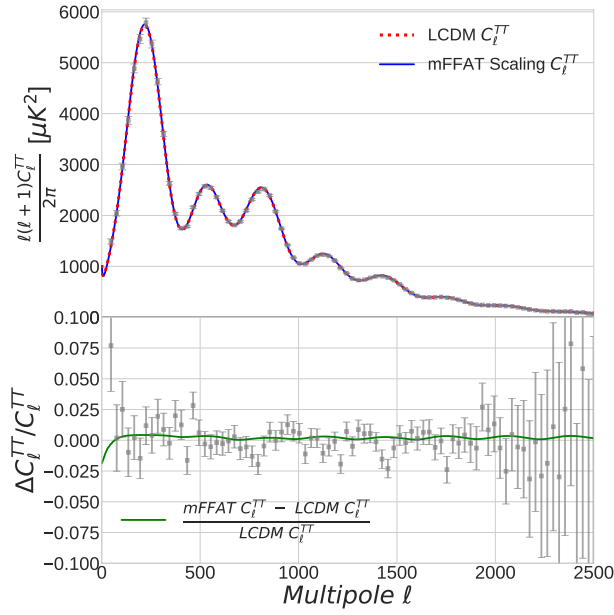
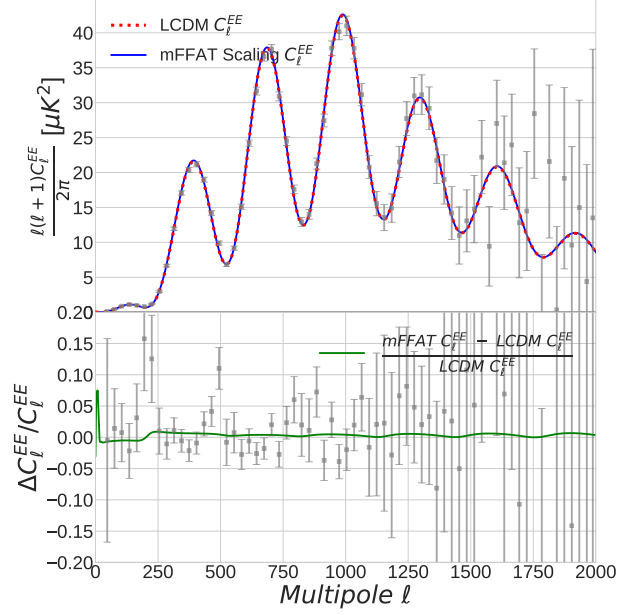


Figure 16: This triangle plot demonstrates the natural realization of the FFAT scaling. The red dashed lines indicate the direction of the FFAT symmetry. Note that the contours agree with the red lines within 1σ . We see a 1σ range for the posterior on H_0 that extends well beyond the typical Λ CDM model. A high value of H_0 is preferred with no data representing the local measurement. The grey band represents the local measurement of $H_0 = 73.3 \pm 1.04$ km/s/Mpc from Ref. [20].



(a) CMB Temperature Spectra



(b) CMB Polarization Spectra

Figure 17: These plots compare the TT and EE CMB spectra for the Λ CDM model and the mFFAT scaling utilizing the best fit values from figure 16 ($H_0 = 73.97$ km/s/Mpc). The grey points represent a subsample of measured values from Planck, accompanied by their respective error bars. The mFFAT scaling demonstrates remarkable agreement with Planck observations in both the TT and EE spectra, while preserving locally measured H_0 values.

m_e proposed in reference [229] is inverse to what the FFAT scaling suggests, aiming for a larger electron mass. These distinctions could illuminate why models on the FFAT scaling direction demonstrate an increase in H_0 compared to the model presented by reference [229], where only m_e is varied.

In a different approach, reference [438] utilizes Fisher-bias information to identify data-driven extensions to Λ CDM involving variation of fundamental constants. Their findings align with the notion that a modified recombination history could ease the Hubble tension when solely considering CMB data. They find that when, for example, the electron mass is allowed to vary it is accompanied by a decrease in the physical baryon and cold dark matter densities. However, much like reference [229], their methodology encounters similar limitations: the change in the physical matter density changes the redshift of matter-dark energy equality, veering the models away from the FFAT scaling direction. Indeed, because the FFAT scaling direction ensures that ratio of the observed matter density to the critical density is left consistent compared to Λ CDM (see sec. 7.2.1), it is able to accommodate values of Ω_m which are less than 2σ discrepant with recent supernova surveys like Pantheon+ [439], as shown in figure 18, avoiding the constraints prior models encountered.

7.6 Conclusion

In this chapter, we have furthered the understanding of the Hubble tension through the employment of the Free-Fall, Amplitude, and Thomson (FFAT) scaling in conjunction with a variation of fundamental constants given by equation (126). This alternative approach substantiates the findings of previous studies [297, 305], demonstrating that the FFAT scaling successfully can bring the local and distant measurements of H_0 into concordance with each other. Here, we briefly outline the key results of our study:

- The FFAT scaling with fundamental constant variation successfully achieves high val-

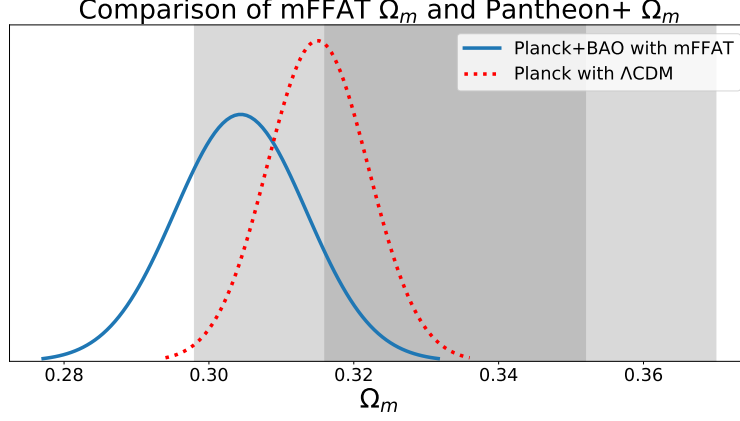


Figure 18: Posteriors of Ω_m shown up to 3σ for the mFFAT model with Planck+BAO data and the baseline Λ CDM model with Planck data. In grey is the Pantheon+ derived value of Ω_m representing the 1σ (dark grey) and 2σ (light grey) regimes.

ues of H_0 , corroborating the results of refs. [297, 305] through an alternative approach and emphasizes the critical role the Thomson scattering rate takes in the context of the Hubble tension.

- By safeguarding the ratio of the Thomson scattering rate to the background expansion rate, the FFAT scaling combined with the variation of fundamental constants significantly mitigates the restrictions on fundamental constant variation during the epoch of recombination. This protection of the recombination history allows for greater flexibility and exploration in cosmological models which vary fundamental constants.
- An MCMC exploration of the FFAT scaling broadens the posterior for H_0 , reconciling the local and distant measurements of H_0 utilizing only LSS and CMB data. Including a prior on the local measurement of H_0 would result in a significant improvement to the fit compared to Λ CDM.

The methodology proposed in this research lays a robust groundwork for subsequent investigations into the Hubble tension and physics beyond the standard models of particle physics and cosmology. Our study highlights three promising avenues for future exploration. Firstly, the incorporation of the complete MWDS into CLASS will allow for exploration of the dark

sector parameter space beyond simple energy densities. Secondly, a thorough examination of the relationship between the FFAT scaling and BBN may broaden the tight constraints set on the early Universe by primordial element abundances. Lastly, the development of particle physics models which encapsulates the MWDS and fundamental constant variation can potentially elucidate the enigmatic nature of the dark sector.

8 A ratio-preserving approach to cosmological concordance

We do not see things as they are, we see them as we are.

- Anaïs Nin

This chapter is a modified version of the publication, “A ratio-preserving approach to cosmological concordance,” by Kylar Greene and Francis-Yan Cyr-Racine published in Physical Review D in August of 2024 [440].

8.1 Introduction

Building on the work presented in Ch. 7, we investigate beyond the Λ CDM paradigm to produce concordant cosmologies, drawing on several observational clues as our guide. The initial hint originates from Λ CDM directly. The model tightly constrains several critical ratios in the early Universe: the baryon-photon ratio, the matter-radiation ratio, the fluid-freestreaming radiation ratio, and the balance between pressure-supported and pressureless matter. Λ CDM’s effectiveness at predicting CMB features hinges on its precise predictions of these ratios while maintaining the minimal number of degrees of freedom possible.

The second clue emerges from the significant similarities between the baryon density and the estimated dark matter density, suggesting a potential link in the mechanisms which populate the two sectors. Given the visible sector’s dynamic and complex nature, it is reasonable that if there is a connection between the population of both sectors, the dark sector might be more intricate than just a single particle species. However, this notion goes against one of Λ CDM’s foundational assumptions. It assumes that the majority of the Universe’s matter consists of a single type of cold, non-interacting, and collisionless dark matter, termed cold dark matter (CDM), detectable solely through its gravitational effects. Despite its elusive nature and absence from the Standard Model, it is evident that a significant portion of the dark sector must conform to CDM’s phenomenological criteria. However, the question arises:

Must the *entire* dark sector adhere to the CDM paradigm?

Consequently, guided by these clues, our investigation suggests the necessity for a subdominant addition to the dark sector that: maintains the ratios predicted by Λ CDM and has more dynamic degrees of freedom than CDM. Separately, we require that the introduction of new physics becomes cosmologically relevant after Big Bang Nucleosynthesis to not interfere with light elemental abundances such as in Refs. [441, 442] where the dark sector thermalizes with the visible sector after BBN.

These clues converge toward a singular solution: a mirror sector inspired by $\mathbb{Z}_2$ symmetries [338–365, 367, 368, 378, 443–455]. Introducing mirror sectors as atomic dark matter (ADM), realizable through broken $\mathbb{Z}_2$ symmetries, offers remarkable potential for resolving tensions in both particle physics and cosmology [309, 312, 313, 337, 366, 370–372, 374, 376, 377, 382, 383, 456–495]. Here, a mirror sector can succinctly satisfy both outlined phenomenological criteria. Firstly, they provide a natural link between the two sectors, elucidating the similarity in energy densities through connections in baryogenesis [369, 373, 496–499]. Secondly, they inherently justify how the mirror sector contributes the required radiation and pressure supported matter preserving Λ CDM’s predictive accuracy for the specified ratios [290, 297].

We utilize a symmetry of cosmological observables to guide our model building process, the Free-Fall, Amplitude, and Thomson (FFAT) scaling as covered in the previous Ch. 7 [297, 305]. The FFAT symmetry prescribes a way to increase the Hubble expansion rate of the Universe by proportionally adjusting the length and rate scales of the Universe. Specifically, the following rescaling of the energy density ρ_i of species i , the Thomson scattering rate $\dot{\kappa}$, and the amplitude of scalar fluctuations A_s by a real parameter $\lambda > 1$ leaves dimensionless

cosmological observables invariant

$$\{\rho_i \xrightarrow{\lambda} \lambda^2 \rho_i, \dot{\kappa} \xrightarrow{\lambda} \lambda \dot{\kappa}, A_s \xrightarrow{\lambda} A_s / \lambda^{n_s-1}\}, \quad (146)$$

while leading to a larger Hubble expansion rate $H \xrightarrow{\lambda} \lambda H$. In the above n_s is the scalar spectral index. Integrating the FFAT scaling into a cosmological scenario requires addressing two, nontrivial model building questions:

- What is the form of the additional energy density needed to satisfy $\rho_i \rightarrow \lambda^2 \rho_i$?
- How do we modify the visible Thomson scattering rate to satisfy $\dot{\kappa} \rightarrow \lambda \dot{\kappa}$?

Notably, ADM naturally satisfies the first model building criteria of the FFAT symmetry by allowing the inclusion of proportional additions of radiation and pressure supported matter while preserving the critical ratios predicted by Λ CDM. However, as a generic feature of models seeking to alleviate the Hubble tension, the increase in H_0 will begin to cause issues with CMB observables as the ratio of the Thomson scattering rate to background expansion rate (the characteristic timescale of recombination) moves away from the Λ CDM predicted value.

To protect cosmic observables from alterations to the characteristic time scale of recombination, Thomson scattering rate must be proportionally scaled compared to the expansion rate as part of its second model building criteria. Scaling the Thomson scattering rate, which depends on electron number density and the Thomson cross section, introduces theoretical challenges. In this study, we utilize the strategy first suggested by Ref. [290], modifying the parameter values for the electromagnetic fine-structure constant and electron mass at the percent level. This adjustment achieves the necessary scaling of the Thomson scattering rate while maintaining the binding energy of light elements at the Standard Model value. As we will see, this effectively renders the CMB insensitive to these changes at first order. However, varying fundamental constants is notoriously difficult to realize in a physical Lagrangian, for

example see Refs. [500, 501]. We point out that this method is not the only way to scale the Thomson scattering rate. It represents a straightforward phenomenological model that captures interactions beyond the Standard Model potentially affecting the Thomson scattering rate. Potential interactions between the dark and visible sectors, currently beyond experimental detection limits, could be incorporated within the mirror sector framework. Such integration may provide a compelling ‘two-for-one’ realization of the necessary phenomenology, effectively leveraging the unique capabilities of the new, subdominant ADM component [502, 503]. Moreover, *any* effect which reduces overall amplitude of photon diffusion will satisfy the phenomenological demand of the required Thomson scaling. A phenomenological model of fundamental constant variation is simply the most straight forward route to testing the overall applicability of the FFAT scaling. The relationship between fundamental constant variation and cosmology is a well discussed topic, for more models and recent discussion please see Refs. [229, 404, 436–438, 504–515].

8.2 The synchronous mirror recombination model

To understand how a mirror sector containing a subdominant ADM component may allow for concordant cosmology, we explain here how its parameters can be chosen to exploit the FFAT symmetry direction, ensuring alignment with CMB and LSS data. This requires satisfying the phenomenological criteria outlined above: $\rho_i \rightarrow \lambda^2 \rho_i$ and $\dot{\kappa} \rightarrow \lambda \dot{\kappa}$. Subsection 8.2.1 details how we fulfill the energy density requirement to increase H , and subsection 8.2.3 describes the scaling of the Thomson scattering rate to ensure consistent recombination history.

8.2.1 Atomic dark matter

Mirror-like dark sectors can vary widely, ranging from those featuring multiple, exact replicas of the Standard Model to others with only partial duplications [338–365, 367, 368, 378, 443–454]. Importantly, these frameworks often allow for masses and coupling strengths of mirror

particles to be comparable in magnitude to the visible sector. Given the variety and scope of models exploring ADM and $\mathbb{Z}_2$ symmetries, our approach in this study is broad, aiming to delineate dark sector phenomenology that could be compatible with various mirror ADM implementations [309, 312, 313, 337, 366, 370–372, 374, 376, 377, 456–495].

We assume the mirror sector primarily consists of two massive stable particles, each bearing a charge under a dark $U(1)$ symmetry reminiscent of the visible electromagnetic sector. For the sake of simplicity in modeling, these particles are assumed to be fundamental fermions, though we recognize that composite particles could serve equally well in a more specific realization. These fermions are distinguished by their masses: one mirroring the mass of the visible electron and the other mirroring the mass of the visible proton. We refer to the lighter of these as the dark electron and the heavier as the dark proton, with their interactions mediated by a massless dark photon introducing a dark radiation component to the Universe. These additions to the dark sector interact with the visible sector exclusively through gravitational forces and represent a subdominant portion of the overall dark sector energy density. For the scope of this work, we do not define the specific characteristics of the remaining dark matter, aside from its adherence to CDM behavior. However, we note an interesting possibility and connection to recent research: the CDM could for example be a stable ‘dark neutron’ left over from a dark nucleosynthesis era in the early Universe which may naturally agree with our ADM approach [450].

This schematic model presents three significant phenomenological implications. Firstly, it enables the formation of bound states akin to visible baryons. Secondly, the capacity to form bound states means the mirror sector will undergo a recombination process analogous to that experienced by the visible sector. Thirdly, ADM models allow the critical ratios established by Λ CDM to be preserved:

- The precisely measured baryon-photon ratio is preserved through the proportional

inclusion of ADM and dark radiation.

- The matter-radiation ratio is maintained through the proportional inclusion of CDM, ADM, neutrinos, and dark radiation.
- Dark radiation lends pressure support to ADM, and the integration of extra cold dark matter ensures a balance between pressure-supported and pressureless matter.
- At appropriate temperatures, radiation interacting with ADM behaves as a fluid, maintaining this state up to ‘mirror recombination’, akin to the recombination process in the visible sector. Following this, the dark radiation shifts to freestreaming, thereby maintaining the fluid-freestreaming radiation ratio when the recombination events align.

Models of atomic dark matter, especially those derived from partial $\mathbb{Z}_2$ symmetries, naturally define a parameter space encompassing mirror recombination which occur near visible recombination. The proportional inclusion of the mirror sector densities is the same statement as the second clue outlined above. It is natural that the ratios of dark sector energies matches that of the visible sector if they are populated by the same criteria. Together, this approach circumvents numerous fine-tuning challenges.

8.2.2 Thermal history of mirror sector

To limit the number of free parameters of the theory during our numerical investigation, we fix the dark proton mass $m_{p,d}$, dark electron mass $m_{e,d}$, and dark fine structure constant α_d to be functions of the mirror sector temperature ξ_d such that mirror recombination occurs at the same time as the visible sector, ensuring the fluid-freestreaming radiation ratio is preserved. The fraction of total dark matter which is atomic, f_{ADM} , is left as a free parameter. We call this scenario the ‘synchronous mirror recombination’ case. Later, in section 8.4.3 we will consider a model where the mirror sector is allowed to recombine independently of the

visible sector in the ‘asynchronous mirror recombination’ case.

To ensure synchronous mirror recombination and enforce the fourth condition regarding the fluid-freestreaming radiation ratio, we make three assumptions regarding the mirror sector: (1) $\dot{\kappa}$ is similar in both sectors, (2) the sound speeds in both the sectors are similar to first order, and (3) the ratio of binding energy to temperature is congruent between the two sectors. The first constraint ensures dark photons appropriately couple to dark electrons. The second constraint guarantees that pressure waves in the mirror plasma propagate at speeds matching those in the visible sector; otherwise, differing speeds of dark pressure waves would gravitationally affect visible sector pressure waves, altering their profile slightly. The third constraint determines the conditions under which recombination in the mirror sector becomes energetically favorable.

The first assumption is that the Thomson scattering rates for both sectors is the same:

$$\lambda n_e a \sigma_T = n_{e,d} a \sigma_{T,d}. \quad (147)$$

where $\lambda = \tilde{H}_0/H_{0,\Lambda\text{CDM}}$ with $\tilde{H}_0$ being the increased value and the subscript d denotes dark sector values. This formulation introduces a new variable; the dark electron number density. Similar to the visible sector, we account for this by assuming global neutrality under the dark $U(1)$ symmetry, leading to the relation

$$n_{e,d} = n_{p,d} \approx \frac{\rho_{\text{ADM}}}{m_{p,d}} \quad (148)$$

where $n_{p,d}$ and ρ_{ADM} represent the dark proton number density and ADM energy density, respectively. Therefore, the mass of the dark proton emerges as a key determinant of the dark electron number density. Equation (148) holds true specifically when $m_{e,d} \ll m_{p,d}$, a condition inherently met in mirror-like models.

From the degeneracy direction pointed out by the FFAT symmetry, the visible baryon density is related to the ADM density such that

$$\rho_{\text{ADM}} = (\lambda^2 - 1) \rho_{\text{b}}. \quad (149)$$

Therefore, equation (147) becomes

$$\lambda \frac{\rho_{\text{b}}}{m_{\text{p}}} \sigma_{\text{T}} = \frac{(\lambda^2 - 1) \rho_{\text{b}}}{m_{\text{p,d}}} \sigma_{\text{T,d}}. \quad (150)$$

For clarity, we define the ratio of a quantity χ and its dark sector component χ_{d} as $r_{\chi} = \frac{\chi_{\text{d}}}{\chi}$. With this notation, the constraint from (147) becomes

$$\frac{r_{\text{mp}} r_{\text{me}}^2}{r_{\alpha}^2} = \frac{\xi_{\text{d}}^4}{\sqrt{\xi_{\text{d}}^4 + 1}} \quad (151)$$

where α is the visible fine-structure constant and we have made the substitution

$$\xi_{\text{d}} \equiv \frac{T_{\gamma,\text{d}}}{T_{\gamma}} = (\lambda^2 - 1)^{1/4} \quad (152)$$

with T_{γ} being the visible sector temperature and $T_{\gamma,\text{d}}$ being the mirror sector temperature.

We now seek to write r_{p} in terms of ξ_{d} and can do so while enforcing the second assumption. The sound speed of the primordial plasma is given by

$$c_{\text{s}}^2 = \frac{T_{\text{b}}}{\bar{\mu}} \left(1 - \frac{1}{3} \frac{d \ln T_{\text{b}}}{d \ln a} \right) \quad (153)$$

where T_{b} is the temperature of baryons and $\bar{\mu}$ is the reduced molecular weight. At first order,

we can ensure $c_s = c_{s,d}$ by enforcing

$$\frac{T_b}{\bar{\mu}} = \frac{T_{\text{ADM}}}{\bar{\mu}_d} \quad (154)$$

where T_{ADM} is the temperature of the atomic dark matter. Prior to recombination, the baryons and ADM are tightly coupled to their respective radiation baths such that $T_b \approx T_\gamma$ and $T_{\text{ADM}} \approx T_{\gamma,d}$. Additionally, $\bar{\mu} \approx m_p$ as long as $m_p \gg m_e$. Therefore, (154) implies

$$\begin{aligned} \frac{T_{\text{ADM}}}{T_b} &\approx \frac{T_{\gamma,d}}{T_\gamma} = \frac{m_{p,d}}{m_p} \\ &\Rightarrow \xi_d = r_{\text{mp}}. \end{aligned} \quad (155)$$

The constraint on the dark electron mass and dark fine structure constant from equation (151) becomes

$$\frac{r_{\text{me}}^2}{r_\alpha^2} = \frac{\xi_d^3}{\sqrt{\xi_d^4 + 1}}. \quad (156)$$

We now turn our attention to the last and perhaps most important constraint: keeping the ratio of the binding energy B_e to temperature in both sectors equal such that

$$\frac{B_e}{T_\gamma} = \frac{B_{e,d}}{T_{\gamma,d}} \quad (157)$$

which simplifies to the constraint

$$\xi_d = r_e r_\alpha^2 \quad (158)$$

where we have used $B_{e(d)} = \frac{1}{2}m_{e(d)}\alpha_{(d)}^2$. Therefore, the system of equations we must solve for r_e and r_α is

$$r_{\text{me}} r_\alpha^2 = \xi_d \quad \& \quad \frac{r_{\text{me}}^2}{r_\alpha^2} = \frac{\xi_d^3}{\sqrt{\xi_d^4 + 1}}. \quad (159)$$

We find the mirror sector parameters to enforce synchronous mirror recombination to be

solely functions of temperature such that

$$\begin{aligned}
r_{\text{me}} &= \left(\frac{\xi_{\text{d}}^4}{\sqrt{\xi_{\text{d}}^4 + 1}} \right)^{1/3} \\
r_{\text{mp}} &= \xi_{\text{d}} \\
r_{\alpha} &= \left(\frac{\sqrt{\xi_{\text{d}}^4 + 1}}{\xi_{\text{d}}} \right)^{1/6}.
\end{aligned} \tag{160}$$

This approach ensures that the forth condition of preserving the fluid-freestreaming radiation ratio is exactly preserved. However, we note that the mirror sector’s subdominant nature offers flexibility in these relationships; exact matches are not required. We explore this flexibility when we permit the mirror sector to recombine independently from the visible sector in section 8.4.3. We note that this parameterization begins to break down for very cold mirror sectors where $\xi_{\text{d}} \rightarrow 0$, as not only do the masses and couplings diverge, but the introduced ADM will be cold and behave as CDM which we describe in section 8.4.2.

8.2.3 Phenomenological changes to $\dot{\kappa}$ using FCV

The dimensionless ratio of the Thomson scattering rate, $\dot{\kappa}$, to the Hubble rate, $H(z)$, is crucial for setting the photon diffusion scale during recombination. This period, vital for shaping the CMB spectra at small angular scales, demands a balance between the expansion and Thomson scattering rates for observational consistency. The high ℓ regions of the CMB spectra, where $\dot{\kappa}$ smooths perturbations resulting in power dampening, are particularly sensitive to this ratio. Thus, the Hubble tension can be reframed as a ‘photon diffusion tension’ in this context [290].

The Thomson cross section dictates the mean free path of photons at this time, and variations in α and m_e can enhance or hinder the level of diffusion. Our method employs a targeted FCV approach, focusing on maintaining constant binding energy and $\dot{\kappa}/H(z)$, the

physical quantities which drives observational constraints. In essence, CMB observations are more attuned to changes in the binding energy and this ratio, rather than specific alterations in α or m_e . Notably, CMB *spectral* observations may be sensitive to specific variations in α or m_e with possible future telescopes but remain out of reach for current observations [12, 516–519]. We remind the reader that this phenomenological approach encapsulates any modification to the Thomson scattering rate potentially achievable through novel interactions between the dark and visible sectors, and is not solely limited to models investigating FCV.

Our proposed scaling for $\dot{\kappa}$ hinges on two constraints. Firstly, the binding energy of light elements $B_e = \frac{1}{2}m_e\alpha^2$ must remain unchanged compared to the Standard Model to ensure visible recombination timing aligns with Λ CDM. This preserves observables related to the distance to the CMB. Secondly, the Thomson cross section $\sigma_T = \frac{8\pi}{3}\alpha^2/m_e^2$ requires a specific scaling ($\sigma_T \rightarrow \lambda\sigma_T$) to maintain the $\dot{\kappa}/H$ ratio, thereby preserving CMB temperature spectra observational consistency. Resolving these constraints yields precise scaling predictions for the fundamental parameters: $\alpha \rightarrow \lambda^{1/6}\alpha$ & $m_e \rightarrow \lambda^{-1/3}m_e$ [290, 305]. These scalings align with the degeneracy direction indicated by the FFAT symmetry, while preserving recombination-related cosmological observables at first order. For more discussion regarding the observational constraints imposed on FCV, see App. B, and for results depicting the relationship between FCV and the binding energy see Sec. 8.2.3.

8.3 Methodology

We integrate the ADM model into the Cosmology Linear Anisotropy Solving System (**CLASS**) to probe the mirror sector parameter space, and allow the prior volume to include the FFAT symmetry direction [384]. To incorporate the FCV necessary to keep $\dot{\kappa}/H$ invariant, we utilize **CLASS**’s default functionality for varying α and m_e and call upon **HyREC** to compute recombination for the mirror and visible sectors [431]. Combining **CLASS** with the Monte-

Carlo-Markov-Chain (MCMC) package `MontePython` [274, 520], we compute the one and two-dimensional posteriors for the free parameters for the synchronous mirror recombination model. To plot our derived 1 and 2 dimensional posteriors, we utilize the python package `GetDist` [521].

Although this approach introduces a considerable expansion in the model’s degrees of freedom, they are well justified by physical principles and guided by underlying symmetries. For clarity, we list the free parameters incorporated into the model, along with their physical justification. First, we consider the six standard Λ CDM parameters $[h, \omega_b, \omega_{\text{cdm}}, A_s, n_s, \tau_{\text{reio}}]$. To maintain a constant ratio of fluid-freestreaming radiation, we introduce a variable neutrino energy density $[N_{\text{ur}}]$ while assuming that neutrinos are massless. Additionally, we incorporate variations in the fundamental parameters $[\alpha, m_e]$ such that the Thomson scattering rate can vary while keeping the binding energy constant. We allow $[f_{\text{ADM}}, \xi_d]$ to vary, enabling the mirror sector to preserve the matter-radiation, baryon-photon, and pressure supported to pressureless matter ratios. The other mirror sector parameters $(m_{\text{p,d}}, m_{\text{p,e}}, \alpha_d)$ are fixed functions of the temperature according to Eq. (160), ensuring a synchronous mirror recombination with the visible sector.

We consider the following observational data sets

- CMB - Planck 2018 high ℓ TT, TE, and EE data, Planck 2018 low ℓ TT and EE data, and Planck Lensing data [144]
- Baryon Acoustic Oscillations (BAO) - 6dF Galaxy Survey [522], SDSS DR7 [523], and BOSS DR12 [524]
- Uncalibrated Pantheon+ Supernova [439]
- Pantheon+ Supernova calibrated by the local SH_0 ES measurement [20]

in three different combinations: (A) CMB and BAO, (B) CMB, BAO, and uncalibrated Pantheon+, and (C) CMB, BAO, and Pantheon+ calibrated by SH_0ES , referred to hereafter as likelihood groups A, B, and C. In all cases, we set the helium abundance to $Y_p = 0.2454$ to ensure alignment with BBN predictions [525]. As mentioned previously, models where the dark sector thermalizes with the visible sector after BBN, thereby avoiding N_{eff} constraints [442, 461], present intriguing connections and are certainly possible, but are beyond the scope of this work.

The MCMC analysis is allowed to run until the chains satisfy a Gelman-Rubin convergence criterion of $R - 1 < 0.05$ in all free parameters. To identify the global minimum of the fits (a significant computational challenge given the extensive parameter space spanned by likelihood groups A and B as we will see) we employ the newly developed **Procoli** package which comes equipped with an integrated global best-fit minimizer [162]. **Procoli** is also utilized for conducting profile likelihood scans, enabling us to understand the relationship between the fit to data and increasing values of H_0 when exploring the FFAT symmetry. Additionally, we define the visible sector variables

$$\begin{aligned}\delta_\alpha &= \frac{\tilde{\alpha} - \alpha_0}{\alpha_0} \\ \delta_e &= \frac{\tilde{m}_e - m_{e,0}}{m_{e,0}}\end{aligned}\tag{161}$$

where the tilde indicates the varied value and 0 indicates the Standard Model value.

8.4 Results and discussion

We tabulate the results for the scans over the synchronous mirror recombination model in table 5 and show the posteriors in figure 19 for likelihood groups A, B, and C. Additionally, we perform the same analysis for the Λ CDM model using the same likelihoods to generate comparative data. In general, we find that all parameters naturally follow the FFAT sym-

metry direction, depicted by red dashed lines, and keep the critical ratios discussed above invariant compared to Λ CDM. The binding energy is left invariant compared to the Standard Model while the Thomson cross section is scaled to match the change in H as expected.

Table 5 shows that without Pantheon+ supernova data and SH_0 ES calibration, likelihood group A best-fit values favour Λ CDM model values. This matches our clues discussed in previously: without a need to introduce an additional signal into CMB data to increase H_0 , Λ CDM out performs other models as it accurately predicts the ratios dictating CMB spectra with minimal parameters. Yet, posteriors are significantly expanded, allowing for H_0 values well over 80 km/s/Mpc within the 68% confidence level - a notable departure from traditional Λ CDM CMB and LSS analysis. Figure 19 demonstrates an expanded H_0 posterior which opens the parameter space to explore cosmologically concordant models. Indeed, as we will see in our discussion of the profile likelihood for H_0 , CMB and LSS observations have little constraining power in the FFAT symmetry direction. The minimizer finds best-fit values consistent with Λ CDM underscoring Λ CDM's efficacy at matching observations, while also highlighting the need for cosmological models to adhere to its foundational ratios (photon-baryon number, matter-radiation ratio, pressure supported to pressureless matter ratio, fluid-freestreaming radiation ratio) for stronger statistical agreement. Adding five parameters to the model marginally improves the fit to CMB and LSS data alone with a $\Delta\chi^2 = -1.0799$.

Free Parameters		CMB & BAO		CMB, BAO, Pantheon+	CMB, BAO, Pantheon+, SH_0ES
ΛCDM	h	$0.7462 (0.6854) \pm 0.0549$	$0.7273 (0.7370) \pm 0.0549$	$0.7380 (0.7380) \pm 0.0102$	$0.7380 (0.7380) \pm 0.0102$
	ω_b	$0.0223 (0.0224) \pm 0.0002$	$0.0223 (0.0224) \pm 0.0002$	$0.0223 (0.0226) \pm 0.0002$	$0.0223 (0.0226) \pm 0.0002$
	ω_{cdm}	$0.1495 (0.1221) \pm 0.0244$	$0.1439 (0.1475) \pm 0.0256$	$0.1472 (0.1473) \pm 0.0065$	$0.1472 (0.1473) \pm 0.0065$
	n_s	$0.9585 (0.9580) \pm 0.0095$	$0.9574 (0.9568) \pm 0.0089$	$0.9585 (0.9589) \pm 0.0096$	$0.9585 (0.9589) \pm 0.0096$
	$A_s 10^9$	$2.0794 (2.0740) \pm 0.0411$	$2.0728 (2.0736) \pm 0.0368$	$2.0792 (2.0802) \pm 0.0417$	$2.0792 (2.0802) \pm 0.0417$
	τ_{reio}	$0.0518 (0.0550) \pm 0.0077$	$0.0520 (0.0512) \pm 0.0072$	$0.0521 (0.0516) \pm 0.0075$	$0.0521 (0.0516) \pm 0.0075$
	ξ_{dark}	$0.6242 (0.4989) \pm 0.1954$	$0.5615 (0.6977) \pm 0.2660$	$0.6849 (0.6893) \pm 0.0707$	$0.6849 (0.6893) \pm 0.0707$
ADM	f_{adm}	$0.0317 (0.0072) \pm 0.0212$	$0.0359 (0.0340) \pm 0.0212$	$0.0322 (0.0346) \pm 0.0103$	$0.0322 (0.0346) \pm 0.0103$
FCV	δ_α	$0.0142 (0.0021) \pm 0.0120$	$0.0122 (0.0152) \pm 0.0118$	$0.0141 (0.0149) \pm 0.0040$	$0.0141 (0.0149) \pm 0.0040$
	δ_{me}	$-0.0264 (-0.0014) \pm 0.0264$	$-0.0256 (-0.0309) \pm 0.0260$	$-0.0293 (-0.0296) \pm 0.0132$	$-0.0293 (-0.0296) \pm 0.0132$
Other	N_{ur}	$3.3860 (2.8063) \pm 0.5624$	$3.2670 (3.3393) \pm 0.5370$	$3.3713 (3.3793) \pm 0.3481$	$3.3713 (3.3793) \pm 0.3481$
Derived Parameters					
Dark	r_α	$1.1291 (1.1286) \pm 0.1305$	$1.1705 (1.0808) \pm 0.2130$	$1.0816 (1.0822) \pm 0.0183$	$1.0816 (1.0822) \pm 0.0183$
	r_{me}	$0.5291 (0.3917) \pm 0.2201$	$0.4996 (0.5973) \pm 0.2524$	$0.5906 (0.5886) \pm 0.0774$	$0.5906 (0.5886) \pm 0.0774$
	r_{mp}	$0.6242 (0.4989) \pm 0.0195$	$0.5615 (0.6977) \pm 0.2660$	$0.6849 (0.6893) \pm 0.0707$	$0.6849 (0.6893) \pm 0.0707$
Visible	δ_{Be}	$0.0004 (0.0028) \pm 0.0086$	$-0.0023 (-0.0012) \pm 0.0085$	$-0.0020 (-0.0005) \pm 0.0087$	$-0.0020 (-0.0005) \pm 0.0087$
	δ_{σ_T}	$0.0901 (0.0070) \pm 0.0831$	$0.0903 (0.0974) \pm 0.0839$	$0.0929 (0.0938) \pm 0.0355$	$0.0929 (0.0938) \pm 0.0355$
	Ω_m	$0.3070 (0.3075) \pm 0.0089$	$0.3124 (0.3123) \pm 0.0077$	$0.3111 (0.3114) \pm 0.0082$	$0.3111 (0.3114) \pm 0.0082$
	S_8	$0.8178 (0.8167) \pm 0.0127$	$0.8113 (0.8146) \pm 0.0125$	$0.8164 (0.8159) \pm 0.0128$	$0.8164 (0.8159) \pm 0.0128$
	$\Delta\chi^2$	-1.08	-1.26	-35.72	-35.72

Table 5: This table presents the mean (best-fit) values for the synchronous mirror recombination model across three different likelihood groups, with all errors representing the 68% CL. A double horizontal line divides the table into two main sections. The upper section lists free parameters, categorized into ΛCDM , atomic dark matter (ADM), and fundamental constant variation (FCV). The lower section details derived parameters from both the dark and visible sectors. At the bottom, the table features $\Delta\chi^2$ values, comparing the synchronous mirror recombination model's χ^2 to a baseline ΛCDM model using identical likelihoods.

When incorporating the uncalibrated Pantheon+ supernovae data into the analysis, the best-fit preferences for the model shift dramatically, naturally leading to a best-fit $H_0 = 73.70$ km/s/Mpc. The inclusion of uncalibrated supernova data makes the model sensitive to the shape of the redshift-luminosity relationship and probes the eras of matter domination and matter-dark energy equality, but in this case is unaware of the SH_0ES calibration. Notably, the best-fits agrees nicely with direct local measurements of the expansion rate without notion of their existence. For both likelihood groups A and B, the error bars are broad because the sampler is traversing the FFAT symmetry direction without a calibration for H_0 . Although the best-fits now align with local measurements of H_0 , the enhancement to the overall fit remains minimal, with a $\Delta\chi^2 = -1.2602$. The two-dimensional posteriors for likelihood groups A and B in figure 19 align well with our predictions from the FFAT symmetry, indicated by the red dashed lines.

The inclusion of the SH_0ES ’ local H_0 measurement into likelihood group C calibrates the FFAT symmetry, significantly reducing parameter uncertainties and anchoring the results. With SH_0ES calibration, we find $H_0 = 73.80 \pm 1.02$ km/s/Mpc, closely matching the local measurement of Ref. [20] with an overall $\Delta\chi^2 = -35.72$ compared to Λ CDM. Furthermore, we detect variations in α and m_e , yet the binding energy remains consistent with the Standard Model, affirming the precise timing of visible recombination and the insensitivity of observables to specific variations in fundamental parameters at first order. Additionally, the FCV represents the phenomenological need to scale the Thomson scattering rate accordingly when exploring models with higher H_0 , regardless of the mechanism which amplifies it. The incorporation of dark radiation and ADM upholds the ratios of matter to radiation, pressure-supported to pressureless matter, and fluid to free-streaming radiation, in alignment with Λ CDM predictions. The required ADM fraction is $f_{\text{adm}} = 0.0322 \pm 0.0103$, and the mirror sector is found to be slightly cooler than the visible sector with $\xi_{\text{dark}} = 0.6849 \pm 0.07$. The variation in neutrino energy density, N_{ur} , yields best-fit values about 10% higher than

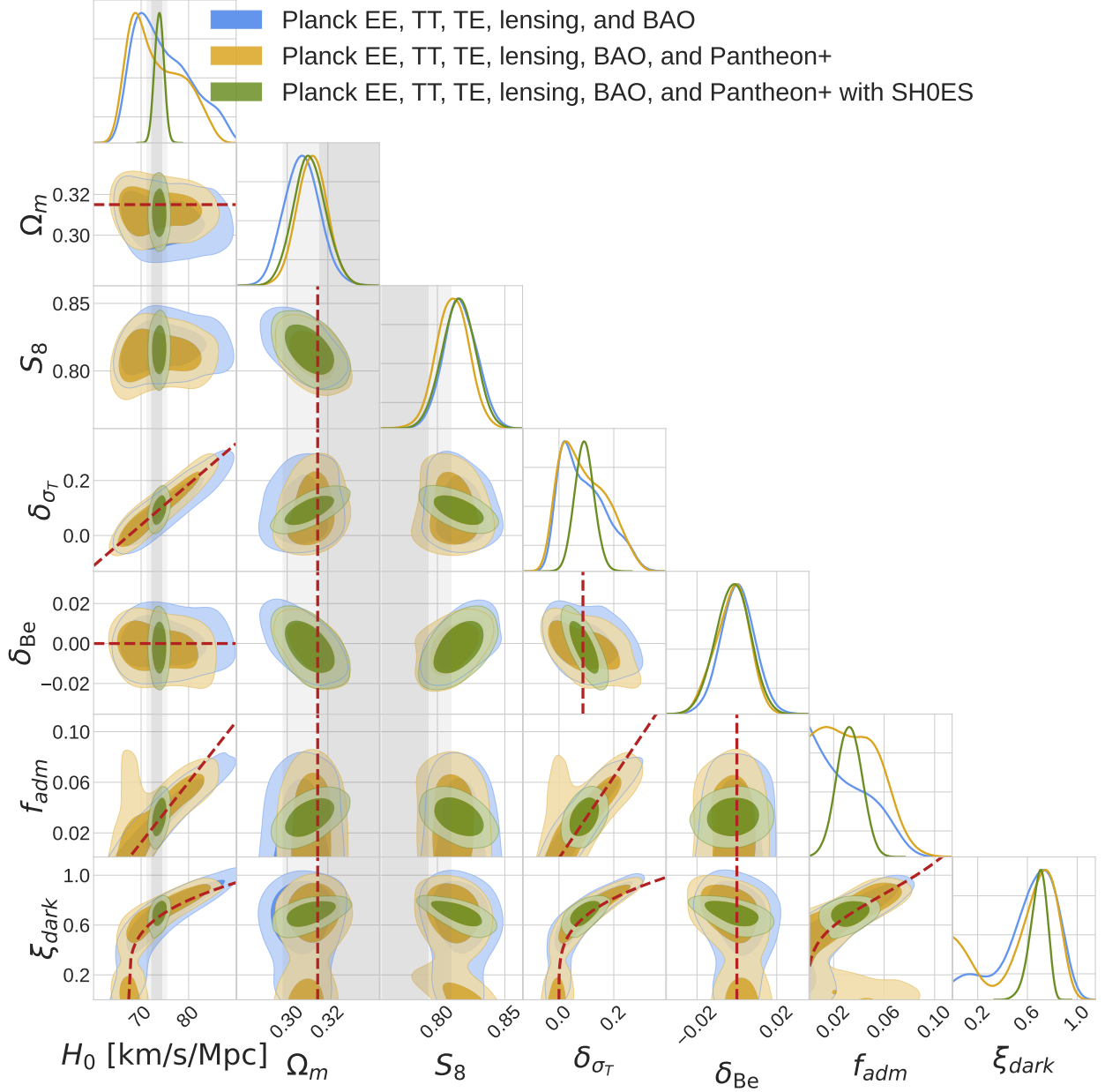
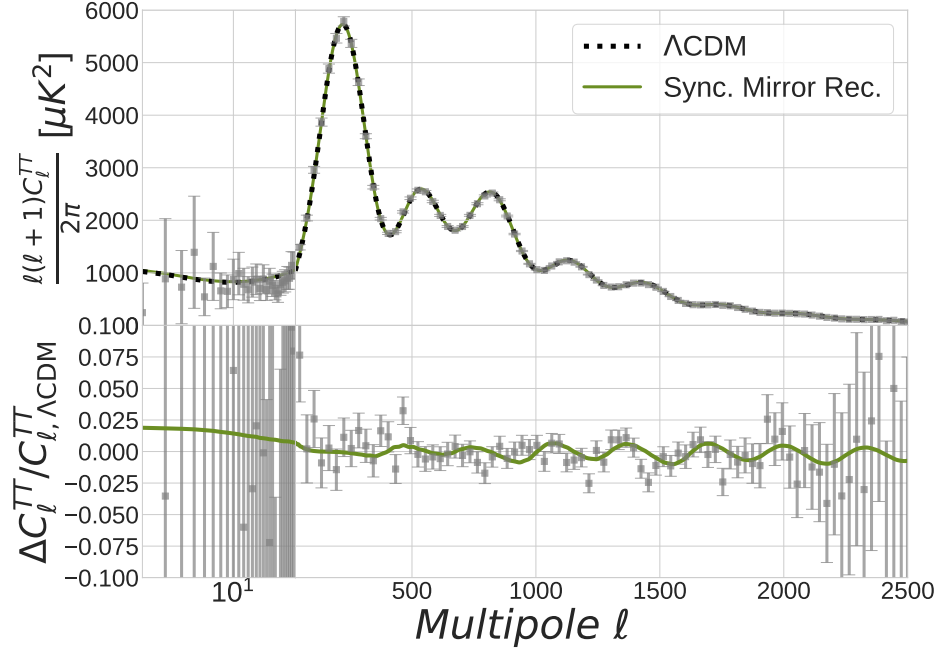
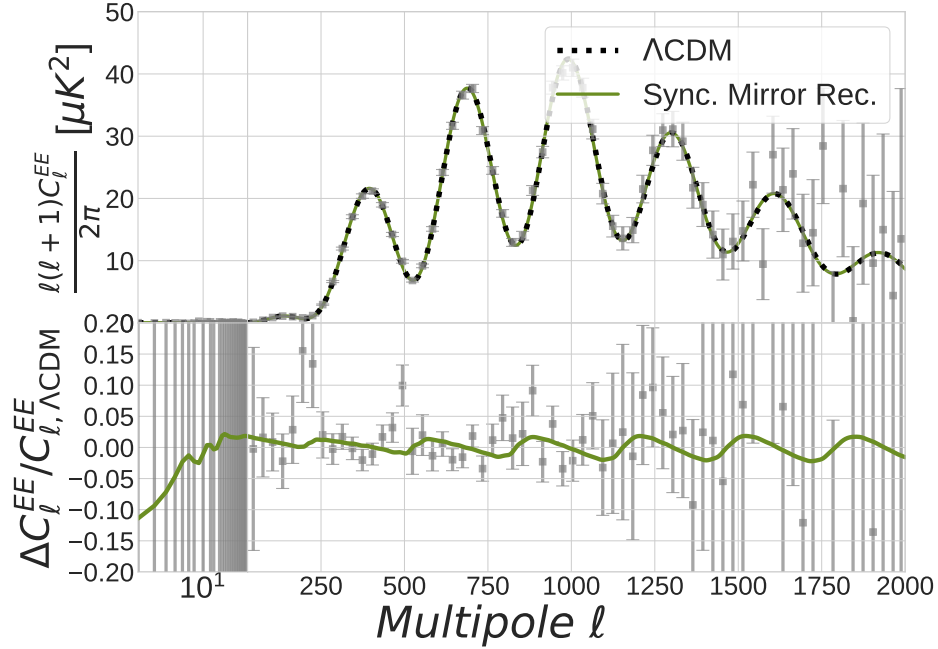


Figure 19: This triangle plot showcases the MCMC analysis results for the synchronous recombination mirror model under three different likelihood groups: A (blue), B (yellow), and C (green). Dark red dashed lines represent the degeneracy direction of the FFAT symmetry. Note that the FFAT symmetry does not make a prediction for S_8 . In the H_0 column, dark and light grey bands mark the 68% and 95% CL from the SH_0ES ' locally measured value of $H_0 = 73.3 \pm 1.04$ [20], while similar bands in the Ω_m and S_8 columns reflect the constraints from Pantheon+'s measured value of $\Omega_m = 0.334 \pm 0.018$ [439] and DES-Y3 result of $S_8 = 0.776 \pm 0.017$ [196]. The MCMC sampler naturally follows the FFAT symmetry direction as it ensures agreement with CMB and LSS observational data. The degeneracy is broken only once the local SH_0ES calibration of H_0 is included, effectively selecting a slice of the posterior.



(a) CMB TT Correlation Spectra



(b) CMB EE Correlation Spectra

Figure 20: These two figures compare the TT and EE spectra from the ΛCDM model best-fit values obtained from Ref. [144] to the synchronous mirror recombination model best-fit values obtained when considering likelihood group C. In both figures, $H_0 = 73.8$ km/s/Mpc. The top panels of the figures demonstrate the spectra's power correlation. The bottom panels demonstrate the fractional difference between the ΛCDM and synchronous model. The grey points with associated error bars are binned Planck 2018 observations of the TT and EE spectra. The plots are logarithmic on the x axis for $2 \leq \ell < 30$ for the low ℓ data, and then linear for $\ell \geq 30$ for the high ℓ data.

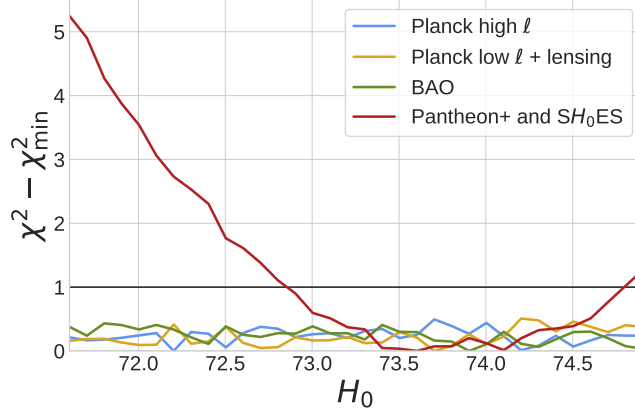


Figure 21: This plot demonstrates the results from a profile likelihood scan over H_0 utilizing the `Procoli` package for the synchronous mirror recombination model when considering likelihood group C. The results are normalized to each datasets minimum χ^2 value, allowing comparison between all data sets. We point out the $\Delta\chi^2 = 1$ line which represents the 68% CL.

the Standard Model predictions and nicely matches the predicted scaling of $N_{\text{ur}} \rightarrow \lambda^2 N_{\text{ur}}$. However, across all likelihood groups, the results consistently align with $N_{\text{ur}} = 3.044$ within the 68% confidence level (CL).

Figure 20 demonstrates the CMB temperature-temperature (TT) and polarization - polarization (EE) spectra for the best-fit values obtained for the mirror model when considering likelihood group C. We find that even with significantly enhanced values of $H_0 = 73.80$ km/s/Mpc compared to Λ CDM, we do not sacrifice agreement to CMB spectra observations both at low ℓ and high ℓ multipoles as seen in Table 6 and Fig. 20, although there is a slight trade for agreement for high ℓ from the low ℓ data.

8.4.1 Profile likelihood with `Procoli`

To delve deeper into the connection between preserving ratios and cosmological observables, we perform a profile likelihood scan over H_0 across all three likelihood groups utilizing the newly developed `Procoli` package [162]. Figure 21 displays the outcomes of one of these

	Λ CDM	Synchronous Recombination	Asynchronous Recombination
Planck High ℓ	2347.79	2343.97	2342.18
Planck Low ℓ TT	22.63	24.16	23.89
Planck Low ℓ EE	396.95	396.09	396.05
Planck Lensing	8.85	8.62	8.70
BOSS DR12	3.57	4.28	4.41
BOA low z	2.32	1.23	1.198
Pantheon+ SH_0 ES	1319.44	1287.49	1287.45
$\Delta\chi^2$	0	-35.72	-37.65

Table 6: This table provides a comparison of the fit to individual datasets within likelihood group C across three models: the baseline Λ CDM, the synchronous mirror recombination model, and the asynchronous mirror recombination model. The $\Delta\chi^2$ value quantifies the change in χ^2 relative to the baseline Λ CDM model.

scans, for likelihood group C, demonstrating that CMB and LSS data are insensitive to changes in H_0 while exploring along the FFAT symmetry direction. The degeneracy is only resolved when a calibrating measurement of H_0 is incorporated. In table 6, we present a comparison of the χ^2 values across each dataset within likelihood group C for three distinct cases: Λ CDM, the synchronous mirror recombination model, and the asynchronous mirror recombination model (see Sec.8.4.3). We observe that both mirror models result in a reduced χ^2 for the Planck high- ℓ measurements and for the Pantheon+ data, once calibrated by the SH_0 ES measurement. The model’s effectiveness at high ℓ stems from maintaining the $\dot{\kappa}/H$ ratio. This preservation allows for higher values of H , driven by the increased ADM and dark radiation density, ensuring alignment with the Pantheon+ with SH_0 ES dataset. The fit to the low ℓ TT spectra and the BOSS DR12 BAO dataset shows a slight deterioration. However, measurements from low redshift BAO largely compensate for this change.

8.4.2 Cold mirror sector volume effects

The one and two dimensional posteriors for likelihood groups A and B are influenced by a potentially large volume effect in the prior space in which f_{adm} becomes unconstrained when the mirror sector is very cold. This result was first pointed out by Ref. [526] and is most

clearly seen when considering likelihood group B.

In scenarios where the mirror sector temperature is cold ($\xi_d \lesssim 0.25$), f_{adm} becomes unconstrained as the ADM begins to simply resemble the background cold dark matter. This similarity introduces a significant volume to the parameter space, echoing Λ CDM characteristics and resulting in ‘mitten’ shaped two-dimensional contours with ‘cold thumbs’. Figure 22 demonstrates that the synchronous mirror recombination model follows the expected FFAT symmetry direction for f_{adm} in the ‘mitten’ when considering likelihood group B. However, as $\xi_d \rightarrow 0$, $f_{\text{adm}} \rightarrow 1$ and the volume effect is introduced creating ‘cold thumbs’ of the mitten. This volume effect is removed when calibrating the Pantheon+ supernovae with SH_0 ES, as the SH_0 ES anchor strongly disfavours cold mirror sectors. Ultimately, this increased volume influences the shape of the one dimensional posteriors when considering likelihoods groups A and B, but do not influence the best-fit parameters. For instance, in figure 22, we point out the best-fit values and note they do not align with the peaks of the distribution.

8.4.3 Additional mirror sector freedom

As a final test of $\mathbb{Z}_2$ inspired ADM models, we now consider a scenario where the mirror sector’s recombination is asynchronous with the visible sector in an asynchronous mirror recombination model. By varying the mirror binding energy to mirror sector temperature ratio through changes in the dark fine structure constant, we determine the recombination epoch for the mirror sector: ratios greater (less) than unity indicate earlier (later) mirror sector recombination compared to the visible sector.

Our findings reveal a statistical preference in $\Delta\chi^2$ for an earlier mirror sector recombination, finishing just before the onset of visible hydrogen recombination. Recall that in this model, the mirror sector interacts with the visible sector only gravitationally. Prior to mirror sector recombination, dark photons act as a fluid. Once the majority of atomic dark

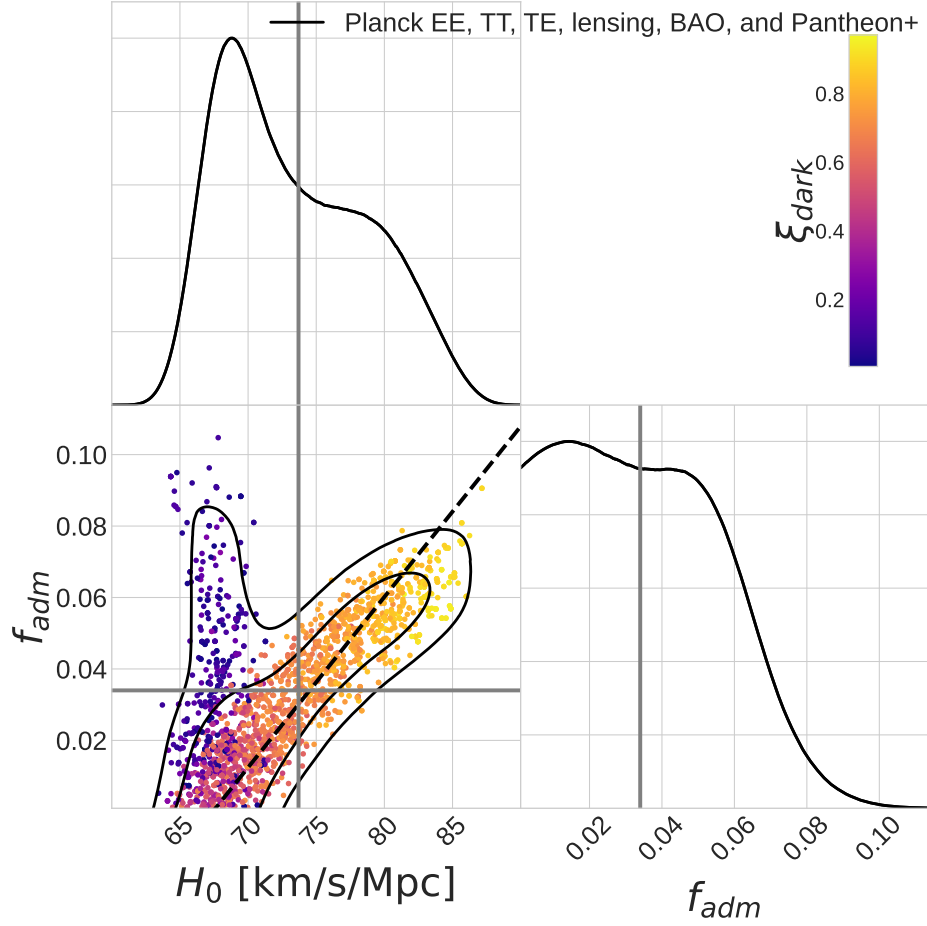


Figure 22: This triangle plot showcases a volume effect shaping the posteriors for likelihood groups A and B in the synchronous mirror recombination model. The one-dimensional posteriors, prominently influenced by this volume effect, display peaks that are shaped accordingly. A black dashed line in the plot indicates the FFAT symmetry direction. The color scale indicates the temperature of the mirror sector, with purple colors being very cold mirror sectors. The grey lines indicate best fit values, and because of the volume effect introduced by cold dark sectors, do not necessarily align with the peak of the one dimensional distribution.

matter recombines, the dark photons start to freestream. Therefore, CMB data seems to favor an injection of freestreaming radiation before hydrogen recombination but after helium recombination which would have the effect of gravitationally pulling on sound horizon wavefronts, and efficiently transporting energy out of the system [311, 314]. The asynchronous mirror recombination model with likelihood group C finds a $\Delta\chi^2 = -37.65$ compared to the Λ CDM model, and H_0 values which agree with local measurements. Figure 24 illustrates the correlation between mirror sector parameters and cosmological tensions for the asymmetric recombination model, with parameter values given in table 7. A bimodal feature related to the timing of mirror recombination is found, where it is disfavoured to recombine during visible HeII to HeI recombination. As a general feature of this model and the synchronous mirror model, the ADM experiences pressure support from the dark radiation until the end of its drag epoch, slowing the infall of dark matter into initial seeds of structure. This allows the one and two dimensional posteriors in figure 24 to penetrate into higher values of Ω_m and lower values of S_8 . We note that this preference for later recombination in the dark sector is phenomenologically similar to recent studies showing that allowing the free electron fraction during visible recombination to vary freely suggests a non-standard recombination history, with part of the visible sector recombining later than predicted by the standard model [527, 528].

While outside the scope of this study, it may be of great interest to investigate how subdominant components of the mirror sector adhering to similar ADM phenomenology influence the growth of structure on scales relevant for the S_8 tension. Local observations of satellite galaxies and searches for thin dark matter disks constrain the energy dissipation rate caused by ADM additions to the dark sector and its impact on the formation and shape of these small nearby objects, providing valuable guidance for future dark sector model development [529–533].

	Parameter	Asymmetric Recombination
Λ CDM	h	$0.7375 (0.7381) \pm 0.0100$
	ω_b	$0.0224 (0.0022) \pm 0.0003$
	ω_{cdm}	$0.1461 (0.1474) \pm 0.0067$
	n_s	$0.9624 (0.9591) \pm 0.0106$
	$A_s 10^9$	$2.0889 (2.0718) \pm 0.0460$
	τ_{reio}	$0.0519 (0.0515) \pm 0.0074$
ADM	ξ_{dark}	$0.6715 (0.7132) \pm 0.0817$
	f_{adm}	$0.0362 (0.0347) \pm 0.0143$
	r_α	$1.1804 (1.2127) \pm 0.3811$
FCV	δ_α	$0.0122 (0.0142) \pm 0.0044$
	δ_{me}	$-0.0254 (-0.0312) \pm 0.0140$
Other	N_{ur}	$3.3757 (3.2859) \pm 0.3628$
Dark	r_{me}	$0.5721 (0.6132) \pm 0.0950$
	r_{mp}	$0.6715 (0.7132) \pm 0.0817$
	$r_{\text{Be}}/\xi_{\text{dark}}$	$1.3077 (1.2645) \pm 0.7095$
Visible	δ_{Be}	$-0.0015 (-0.0035) \pm 0.0084$
	δ_{σ_T}	$0.0796 (0.0959) \pm 0.0385$
	Ω_{m}	$0.3098 (0.3117) \pm 0.0827$
	S_8	$0.8161 (0.8139) \pm 0.0131$
	z_{rec}	$1090.1 (1088.1) \pm 9.3337$
$\Delta\chi^2$		-37.65

Table 7: This table presents the mean (best-fit) values for the asynchronous mirror recombination model when considering likelihood group C, with all errors representing 68% CL. The format is the same as in table 5.

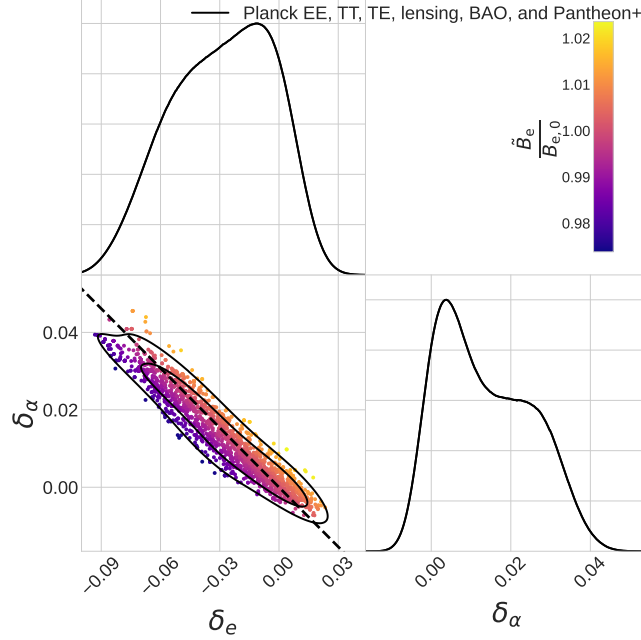


Figure 23: This triangle plot demonstrates the relationship between variations in α and m_e in the synchronous mirror recombination model. A black dashed line marks the Standard Model binding energy. The samples are plotted as points with colors corresponding to the ratio of the varied visible binding energy compared to the Standard Model value.

Finally, an interesting coincidence has appeared in the analysis of the synchronous and asynchronous mirror recombination models when considering likelihood groups B and C. The best-fit temperature for the mirror sector in the asymmetric recombination model is $\xi_{\text{dark}} = 0.7132$ or 1.944 K. This temperature is strikingly similar to the 1.95 K predicted for the neutrino background. Note that this is not simply adding an additional neutrino species and naming it something else, as this radiation behaves as a fluid until mirror recombination. This observation could imply that, at some point in cosmic history, the mirror sector radiation might have coupled to the neutrino or visible sector, allowing the two sectors to achieve thermal equilibrium with one another and explaining this temperature coincidence [534, 535]. A similar phenomenology to this was realized in [441].

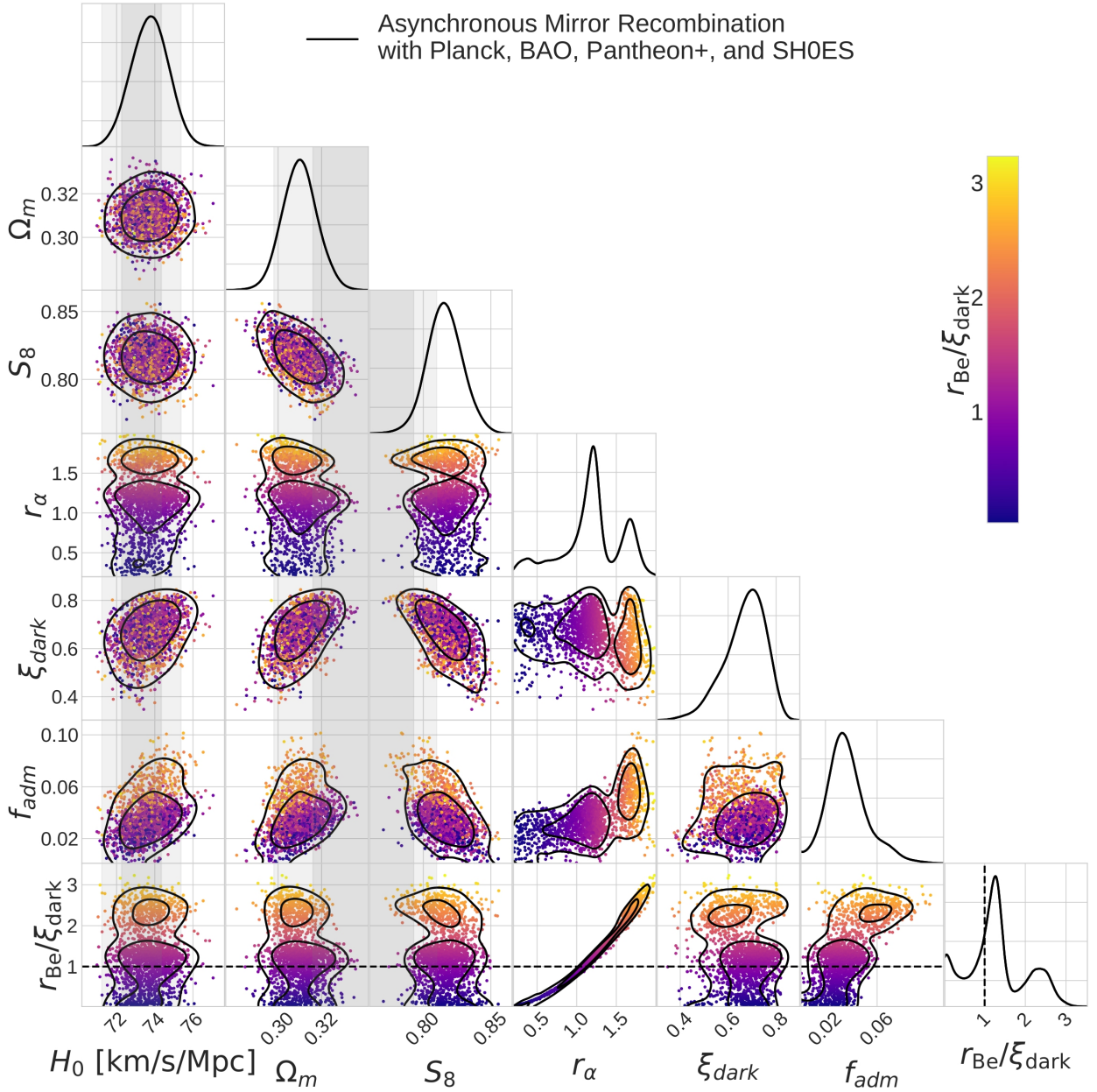


Figure 24: This triangle plot showcases the asynchronous mirror recombination model. It displays three free mirror sector parameters alongside three key observables (H_0 , S_8 , and Ω_m) linked to cosmological tensions. 68% (dark grey) and 95% (light grey) CL limits for H_0 , S_8 , and Ω_m appear in the relevant plots. The color coding reflects the ratio of the mirror sector binding energy to its temperature relative to visible sector values, a determinant of the mirror sector's recombination epoch. Ratios above one suggest earlier recombination compared to the visible sector, while values below one indicate a later recombination. The black dashed line indicates the values which align with the visible sector binding energy to temperature ratio.

8.4.4 Constant binding energy

A particular scaling of α and m_e has been determined to maintain constant binding energy while enabling increased Thomson scattering rates [290]. Without prior knowledge of this specific strategy, the MCMC sampler naturally navigates this path, revealing the insensitivity of cosmic observables to individual changes in either α or m_e , focusing instead on maintaining the invariance of the binding energy within the visible sector. This degeneracy direction is depicted in Figure 23, where the plot’s two-dimensional posterior closely follows lines of constant binding energy, and guarantees that recombination in the visible sector occurs at a time consistent with Λ CDM predictions, thereby preserving distance measurements. Simultaneously, the Thomson scattering rate not left invariant in the direction of constant binding energy, allowing the sampler to explore regimes with constant $\frac{\dot{\kappa}}{H}$ ratios. We show likelihood group B for its capacity to explore the FFAT symmetry’s degeneracy direction without the SH_0 ES calibration which anchors the result.

8.5 Conclusion

In this study, we leverage insights into the dark sector’s connection to the visible sector, and the predictions of ratios by Λ CDM. We introduce a streamlined, universal model of ADM, inspired by recent works investigating mirror sectors. In this model, a subdominant component of the dark sector ($\sim 3\%$) mirrors the visible sector’s $U(1)$ symmetry for electromagnetic interactions. Notably, with parameters analogous to those in the visible sector, the ADM is expected to experience a recombination event within the mirror sector at epochs parallel to those observed in the visible sector. To assess the influence of adding a dynamic, subdominant ADM component to the mirror sector, we employ the FFAT symmetry to guide our modeling approach. This strategy safeguards cosmological observables, guaranteeing alignment with CMB and LSS data. ADM inherently fulfills one of the FFAT symmetry’s phenomenological requirements by contributing proportional amounts of matter, radiation, and cold dark matter densities to the Universe. The second criterion of the FFAT symmetry

involves maintaining the ratio of the Thomson scattering rate to the background expansion rate. To achieve this, we suggest a variation of fundamental constants in a way that scales the Thomson scattering rate appropriately, while keeping the binding energy of hydrogen and helium constant. We discover that the CMB observational agreement remains unaffected by particular variations in α and m_e , provided the binding energy remains unchanged. While integrating fundamental constant variation into a realistic Lagrangian presents challenges, the underlying phenomenology remains essential. A model capable of adjusting the Thomson scattering rate similarly, without relying on FCV, would offer a compelling route to explore the concordant cosmological models. While this study demonstrates the potential of mirror sector models to address certain cosmological tensions, it is important to note that the consistency of these models with BBN is not fully settled. Refs. [441, 442] provide a plausible route to address BBN consistency but requires the introduction of new couplings or mediators between the SM and dark sectors. We hope that this work motivates more detailed studies examining the interplay of BBN and the implications for the viability of mirror sector models.

To investigate this phenomenology and its relationship to cosmic tensions, we conduct a MCMC analysis utilizing a modified version of CLASS which incorporates ADM. We explore two distinct cases of mirror sector recombination: i) the synchronous mirror recombination model depicting the mirror sector recombining concurrently with the visible sector and ii) the asynchronous mirror recombination model allowing the mirror sector to undergo recombination independently from the visible sector. Our findings indicate that under both recombination scenarios, the CMB and LSS observables show almost no sensitivity in response to changes in H_0 . Incorporating data from the local measurement of H_0 , we determine a cosmologically inferred value of $H_0 = 73.8 \pm 1.0$ km/s/Mpc for both models. This results in improved fits, especially for the high ℓ Planck data and Pantheon+ dataset, corresponding to a $\Delta\chi^2 = -35.72$ for the synchronous model and $\Delta\chi^2 = -37.65$ for the asynchronous model. The asynchronous mirror recombination model reveals a preference for the mirror

sector to undergo recombination slightly earlier than the visible sector. This timing places mirror sector recombination between the conclusion of helium recombination and the onset of hydrogen recombination.

This dual approach to investigating the Hubble tension has achieved fruitful results, summarized as follows:

- The inclusion of a subdominant (3%) atomic dark matter component in the mirror sector with a temperature of $\xi_d \sim 0.7$, inspired by $\mathbb{Z}_2$ symmetries, alongside a mechanism that maintains a consistent $\dot{\kappa}/H$, eliminates the tension in H_0 .
- The inclusion of extra mirror sector species, with mass and coupling strengths mirroring those in the visible sector, results in mirror sector recombination occurring at times similar to that of the visible sector. Given the freedom to recombine independently, the mirror sector exhibits a preference for undergoing recombination just before the visible sector hydrogen recombines.
- By scaling the Thomson scattering rate in accordance with the background expansion rate, we achieve cosmological concordance. Our approach employs a phenomenological variation of fundamental constants, demonstrating that targeted adjustments to α and m_e facilitate the necessary scaling while maintaining compatibility with CMB observations

Our work has highlighted several intriguing potential links between mirror models and cosmological tensions while offering avenues for further investigation into possible dark sector interactions. Significant efforts are required from both particle physics and cosmology disciplines to further verify the efficacy of this or similar models at addressing cosmological tensions. However, the phenomenology is clear: 3% – 4% of the dark sector behaving as ADM while ensuring the ratio of $\dot{\kappa}/H$ is left invariant compared to Λ CDM produces concordant cosmology.

9 Observable CMB B-Modes from cosmological phase transitions

The important thing is not to stop questioning.

Curiosity has its own reason for existing.

- Albert Einstein

This chapter is a modified version of the letter, “Observable CMB B-Modes from cosmological phase transitions,” by Kylar Greene, Aurora Ireland, Gordan Krnjaic, and Yuhsin Tsai appearing on ArXiv as [2410.23348] in October 2024 [536].

9.1 Introduction

Cosmological inflation is a compelling framework for dynamically solving the horizon and flatness problems while also generating the density perturbations observed in our universe today [537, 538]. Inflationary models also generically predict nearly scale-invariant tensor perturbations, which induce a characteristic B -mode polarization signal in the cosmic microwave background (CMB) [331, 539–549]. It is widely accepted that observing B -modes above known astrophysical foregrounds would constitute “smoking gun” evidence for inflation [21–24].

In this section, I present a counterexample of post-inflationary B -modes that rival the signal strength of testable inflationary predictions. Indeed, *any* source of large-scale, coherent tensor perturbations produced before reionization can contribute to B -mode signals.¹⁴ Our representative scenario is a late-time, strongly first-order phase transition, which produces gravitational waves (GWs) through bubble collisions.¹⁵ The key difference with respect to inflationary signals is that the tensor power spectrum from sub-horizon sources is not (nearly) scale invariant, but rather white noise on super-horizon scales. As a result, the B -mode

¹⁴See also Ref. [550] for B -mode signals from resonant particle production near reionization.

¹⁵First-order cosmological phase transitions also source GWs through sound waves and turbulence. We consider a supercooled transition for which these contributions are subdominant.

signal has a distinct spectral shape, with more power on smaller scales as compared with the inflationary prediction. Accurately measuring B -modes on different angular scales can therefore distinguish between these sources.

The GW signal from a cosmological phase transition has been extensively studied in pioneering earlier works [551–584], including analytical estimates of the GW spectrum [585–587]. Recently, it has also been shown that the white noise scalar perturbations from bubble nucleation can affect CMB temperature anisotropy measurements [588]. To our knowledge, however, the CMB B -mode signal from a first-order phase transition has not been calculated before.

9.2 B-mode polarization

Temperature anisotropies in the CMB arise from various sources, including scalar, vector, and tensor perturbations. When photons scatter with free electrons, quadrupole anisotropies in the temperature distribution polarize the scattered photons. The majority of CMB polarization is generated during recombination, since afterwards the number density of free electrons drops sharply and Thomson scattering ceases to be efficient. Here, we make the simplifying assumption that all polarization is generated in this last scattering event. This approximation will be relaxed later, where we compute the angular B -mode spectrum exactly using the Boltzmann solver CLASS [589, 590]. The purpose of these formulas is simply to provide intuition as well as a semi-analytic check of our results.

Consider initially unpolarized photons which arrive along direction $\hat{n}'$ to the point $\vec{x}$, where they last scatter at conformal time τ . Letting $\hat{n}$ be the direction of observation and τ_0 be the conformal time today, we can write $\vec{x} = (\tau_0 - \tau)\hat{n}$. A tensor perturbation $h_{ij}(\tau, \vec{k})$

contributes to the CMB temperature anisotropy $\Theta \equiv \Delta T/T$ according to the formula [591]

$$\begin{aligned} \Theta(\tau, \vec{k}; \hat{n}, \hat{n}') &= \frac{1}{2} \int_0^\tau d\tau_1 V(\tau, \tau_1) e^{i\vec{k} \cdot \hat{n}(\tau_0 - \tau)} \\ &\times \int_{\tau_1}^\tau d\tau_2 e^{-i\vec{k} \cdot \hat{n}'(\tau - \tau_2)} \sum_{\lambda=+, \times} n'_i \epsilon_\lambda^{ij} n'_j \partial_{\tau_2} h_{ij}^\lambda(\tau_2, \vec{k}) \end{aligned} \quad (162)$$

where ϵ_λ^{ij} is the polarization tensor for GWs, the sum runs over graviton polarizations, and $V(\tau_1, \tau_2)$ is the visibility function, which satisfies

$$V(\tau_1, \tau_2) = e^{-\kappa(\tau_1, \tau_2)} \frac{d\kappa(\tau_2)}{d\tau_2}, \quad (163)$$

and is defined in terms of the optical depth

$$\kappa(\tau_1, \tau_2) = \int_{\tau_2}^{\tau_1} d\tau a(\tau) \sigma_T n_e(\tau) \quad (164)$$

where n_e is the electron number density and σ_T is the Thomson cross section. This temperature anisotropy, induced by the tensor perturbation $h_{ij}(\tau, \vec{k})$, enters the CMB polarization temperature through Θ as [539, 591]

$$P_{ab}(\vec{k}; \hat{n}) = \frac{3}{4\pi} \int d^2 n' \varepsilon_{ab} \int_0^{\tau_0} d\tau V(\tau_0, \tau) \Theta(\tau, \vec{k}; \hat{n} \cdot \hat{n}'), \quad (165)$$

and we have defined the tensor

$$\varepsilon_{ab} = \frac{1 - (\hat{n} \cdot \hat{n}')^2}{2} g_{ab} - \hat{n}' \mathbf{e}_a \cdot \hat{n}' \mathbf{e}_b, \quad (166)$$

where $\mathbf{e}_a$ is the set of basis vectors on the celestial sphere and g_{ab} is the background metric. Notice that because $\hat{n}'$ enters in a bilinear combination, polarization requires a quadrupole component to the anisotropy.

It is convenient to decompose the polarization tensor into E - and B -modes, which can each be expanded in spherical harmonics on the celestial sphere. Doing so, one can identify the coefficient

$$a_{\ell m}^B(\vec{k}) = - \int d^2n [Y_{\ell m}^{(B)}]_{ab}^*(n) P^{ab}(\vec{k}; \hat{n}), \quad (167)$$

where $Y_{\ell m}^{(B)}$ are the B -mode tensor harmonics on the sphere.¹⁶ The angular spectrum of B -mode polarization is defined in terms of these coefficients as

$$C_\ell^{BB} = \frac{1}{2\ell+1} \sum_{m=\pm 2} \int \frac{d^3k}{(2\pi)^3} \left\langle a_{\ell m}^B(\vec{k}) a_{\ell m}^B(\vec{k})^* \right\rangle, \quad (169)$$

where the angle brackets denote ensemble average. Working in a frame with the azimuthal axis oriented along $\vec{k}$ to perform the integrals, one can show that

$$C_\ell^{BB} = 36\pi \int_0^\infty \frac{dk}{k} \mathcal{P}_h(k) \mathcal{F}_\ell(k)^2, \quad (170)$$

where $\mathcal{P}_h(k)$ is the dimensionless tensor power spectrum evaluated at the initial time and

$$\begin{aligned} \mathcal{F}_\ell(k) &= \int_0^{\tau_0} d\tau V(\tau_0, \tau) \mathcal{S}_\ell(k, \tau_0, \tau) \int_0^\tau d\tau_1 V(\tau, \tau_1) \\ &\quad \times \int_{\tau_1}^\tau d\tau_2 \frac{j_2[k(\tau - \tau_2)]}{k^2(\tau - \tau_2)^2} \left(\frac{\partial \mathcal{T}(\tau_2, k)}{\partial \tau_2} \right), \end{aligned} \quad (171)$$

where we have defined

$$\mathcal{S}_\ell(k, \tau_0, \tau) \equiv \frac{\ell+2}{2\ell+1} j_{\ell-1}[k(\tau_0 - \tau)] - \frac{\ell-1}{2\ell+1} j_{\ell+1}[k(\tau_0 - \tau)]. \quad (172)$$

Note that in deriving this result, we have decomposed the tensor perturbation into an initial

¹⁶The B -mode tensor harmonics are related to the ordinary spherical harmonics as

$$(Y_{\ell m}^{(B)})_{ab}(n) = \sqrt{\frac{(\ell-2)!}{2(\ell+2)!}} (\epsilon_b^c \nabla_a \nabla_c + \epsilon_a^c \nabla_c \nabla_b) Y_{\ell m}(n). \quad (168)$$

perturbation amplitude $h_{ij}^{\text{ini}}(k)$ and the transfer function $\mathcal{T}(\tau, \vec{k})$, where

$$h_{ij}(\tau, \vec{k}) = h_{ij}^{\text{ini}}(\vec{k}) \mathcal{T}(\tau, \vec{k}). \quad (173)$$

This decomposition is useful because it separates the effect of statistical correlations between the initial amplitudes from the deterministic effect of their subsequent evolution, as captured by the transfer function. The statistical properties of the initial perturbations are encoded in the (dimensionful) power spectrum $P_h(k)$,

$$\left\langle h_{ij}^{\text{ini}}(\vec{k}) h_{i'j'}^{\text{ini}}(\vec{k}')^* \right\rangle = \frac{\delta_{ii'} \delta_{jj'}}{2} P_h(k) (2\pi)^3 \delta(\vec{k} - \vec{k}'), \quad (174)$$

and we also introduce the dimensionless power spectrum

$$\mathcal{P}_h(k) = \frac{k^3}{2\pi^2} P_h(k), \quad (175)$$

which also appears in Eq. (170).

In the presence of a nonzero source $\Pi_{ij}(\tau, \vec{k})$, a Fourier mode of the metric perturbation $h_{ij}(\tau, \vec{k})$ evolves according to the wave equation

$$h_{ij}'' + 2\mathcal{H}h_{ij}' + k^2 h_{ij} = 8\pi G a^2 \Pi_{ij}, \quad (176)$$

where $\mathcal{H} = a'/a$ is the conformal Hubble rate, primes denote derivatives with respect to conformal time, and the source term is the Fourier transformed anisotropic stress. When the source is inactive, the transfer function satisfies

$$\mathcal{T}'' + 2\mathcal{H}\mathcal{T}' + k^2 \mathcal{T} = 0. \quad (177)$$

and following Ref. [592], we ignore the late-time contribution from dark energy, considering

only effects from the transition from radiation to matter domination. In this 2-component universe, the Friedmann equations can be solved analytically, and one can derive the following solutions for the transfer function in the radiation-dominated and matter-dominated regimes

$$\mathcal{T}_{\text{RD}}(\tau, k) = A_k j_0(k\tau) - B_k y_0(k\tau) \quad (178)$$

$$\mathcal{T}_{\text{MD}}(\tau, k) = \frac{3}{k\tau} [C_k j_1(k\tau) - D_k y_1(k\tau)]. \quad (179)$$

From the initial conditions and the matching conditions at the characteristic timescale $\tilde{\tau} = 4\sqrt{\Omega_r}/H_0\Omega_m$, the constants in Eq. (178) are $A_k = 1, B_k = 0$, and those in Eq. (179) satisfy

$$\begin{aligned} C_k &= \frac{1}{2} - \frac{\cos(2k\tilde{\tau})}{6} + \frac{\sin(2k\tilde{\tau})}{3k\tilde{\tau}}, \\ D_k &= -\frac{1}{3k\tilde{\tau}} + \frac{k\tilde{\tau}}{3} + \frac{\cos(2k\tilde{\tau})}{3k\tilde{\tau}} + \frac{\sin(2k\tilde{\tau})}{6}. \end{aligned} \quad (180)$$

Given a form for the initial tensor power spectrum, Eq. (170) in conjunction with Eqs. (171), (178), (179), and (180) can be used to compute the B -mode signal. These expressions contain many nested integrals with oscillatory integrands, however, which can lead to numerical instabilities. Approximating the visibility function as a Gaussian of width $\Delta\tau_r \simeq 0.04\tau_r$ about its maximum [591], the conformal time integrals can be simplified considerably to yield

$$C_\ell^{BB} \simeq \frac{36\pi}{25} \left(\frac{\Delta\tau_r}{\tau_r} \right)^2 \int_0^\infty \frac{dk}{k} \mathcal{P}_h(k) j_2(k\tau_r)^2 \mathcal{S}_\ell(k, \tau_0, 0)^2, \quad (181)$$

where we have used the fact that modes entering during matter domination ($\tau > \tau_{\text{eq}}$) obey

$$\mathcal{T}(\tau, k) \rightarrow \frac{3j_1(k\tau)}{k\tau}. \quad (182)$$

Note that the approximation in Eq. (181) is inadequate to properly capture the behavior of

C_ℓ^{BB} for large multipoles $\ell \gtrsim 100$.

9.3 Tensor power spectrum from bubble collisions

We consider first-order cosmological phase transitions in the post-inflationary universe as an example of a source of large-scale, coherent tensor perturbations. Such phase transitions proceed through bubble nucleation, which sources tensor perturbations in three distinct stages.

During the bubble collision stage, bubbles of true vacuum collide and merge, which breaks the spherical symmetry of the system, allowing the gradient energy of the scalar field to source anisotropies [593, 594]. This phase completes quickly; nevertheless, it can be the dominant contribution for strong vacuum transitions. After the bubbles have merged, shells of fluid kinetic energy continue to propagate through the plasma, colliding and sourcing GWs during the acoustic stage [562, 595, 596]. These sound wave collisions can also produce vorticity, turbulence, and shocks in the fluid, which in turn source GWs during the final turbulent stage [597–600].

The relative contribution from each of these stages depends largely on the phase transition strength, as parameterized by α , as well as the bubble wall velocity v_w . For concreteness, we will consider a strongly-supercooled phase transition with runaway bubble wall $v_w \rightarrow 1$, for which the dominant contribution comes from the bubble collision stage [601]. We thus conservatively consider *only* the contribution from this stage.

In deriving the initial tensor power spectrum of this source, we follow a semi-analytic approach based on Refs. [585, 586]. During the phase transition, we solve Eq. (176) with source term $\Pi_{ij}(\tau, \vec{k})$, defined as the Fourier transform of the anisotropic stress — the trans-

verse, traceless part of the energy-momentum tensor

$$\Pi_{ij}(\tau, \vec{k}) = \left(\pi_{ik} \pi_{jl} - \frac{1}{2} \pi_{ij} \pi_{kl} \right) T^{kl}(\tau, \vec{k}), \quad (183)$$

with $\pi_{ij} = \delta_{ij} - \hat{k}_i \hat{k}_j$. We take the source to be a statistically homogeneous, isotropic random variable with unequal-time correlator

$$\left\langle \Pi_{ij}(\tau_1, \vec{k}) \Pi_{ij}^*(\tau_2, \vec{k}') \right\rangle = (2\pi)^3 \delta(\vec{k} - \vec{k}') \Pi(\tau_1, \tau_2, k), \quad (184)$$

and we also assume the phase transition completes within a fraction of a Hubble time $\beta^{-1} < H_*^{-1}$, where β is the inverse duration of the phase transition. This justifies neglecting expansion during the phase transition, allowing us to drop the Hubble friction term in Eq. (176). Finally letting $x \equiv k\tau$, the wave equation during the phase transition $x_i \leq x \leq x_f$ simplifies to

$$h_{ij}'' + h_{ij} \simeq \frac{8\pi G a_*^2}{k^2} \Pi_{ij}, \quad (185)$$

where now primes denote derivatives with respect to x and $a_* = a(\tau_*)$ is evaluated at the “time” of the phase transition.¹⁷ The solution, which may be found using the method of Green’s functions, is

$$h_{ij}(x \leq x_f) = \frac{8\pi G a_*^2}{k^2} \int_{x_i}^x dy \sin(x - y) \Pi_{ij}(y). \quad (186)$$

After the phase transition completes, the source term is no longer active so the right-hand side of Eq. (176) vanishes, but we can no longer neglect expansion. Assuming radiation domination $\mathcal{H} = 1/\tau$, the wave equation for $x \geq x_f$ is then

$$h_{ij}'' + \frac{2}{x} h_{ij}' + h_{ij} = 0, \quad (187)$$

¹⁷Since the phase transition completes in under a Hubble time, we can neglect expansion while the source is active and approximate all insertions of the scale factor as constant $a_i \approx a_f \equiv a_*$.

which has the general solution

$$h_{ij}(x \geq x_f) = A_{ij} \frac{\sin(x - x_f)}{x} + B_{ij} \frac{\cos(x - x_f)}{x}. \quad (188)$$

Matching Eqs. (186) and (188) at x_f allows us to identify the coefficients as

$$A_{ij} = \frac{B_{ij}}{x_f} + \frac{8\pi G a_*^2}{k^2} x_f \int_{x_i}^{x_f} dy \cos(x_f - y) \Pi_{ij}(y) \quad (189)$$

$$B_{ij} = \frac{8\pi G a_*^2}{k^2} x_f \int_{x_i}^{x_f} dy \sin(x_f - y) \Pi_{ij}(y), \quad (190)$$

then using these solutions for h_{ij} , we can now define the initial amplitude h_{ij}^{ini} from Eq. (173) and compute the tensor power spectrum using Eq. (174). Note however that because of subtleties involving super-horizon modes, it will be necessary to differentiate between the super-horizon $x_f < 1$ and sub-horizon $x_f > 1$ regimes.

Sub-horizon regime: For sub-horizon modes, we can simply evaluate the initial spectrum at the end of the phase transition, defining $h_{ij}^{\text{ini}} \equiv h_{ij}(x_f) = B_{ij}/x_f$. Converting back to k and τ variables, this explicitly yields

$$h_{ij}^{\text{ini}}(\vec{k}) = \frac{8\pi G a_*^2}{k} \int_{\tau_i}^{\tau_f} d\tau \sin[k(\tau_f - \tau)] \Pi_{ij}(\tau, \vec{k}). \quad (191)$$

Inserting this solution into Eq. (174) and summing over polarizations, the initial power spectrum becomes¹⁸

$$P_h \simeq \frac{32\pi^2 G^2 a_*^4}{k^2} \int_{\tau_i}^{\tau_f} d\tau_1 \int_{\tau_i}^{\tau_f} d\tau_2 \cos[k(\tau_1 - \tau_2)] \Pi(\tau_1, \tau_2, k), \quad (192)$$

where the unequal-time correlator $\Pi(\tau_1, \tau_2, k)$ is defined in Eq. (184). The double integral here has the same form as Eq. (23) of Ref. [586], which identified the result with the dimensionless

¹⁸We have dropped the term $\propto \cos[2(\tau_f - \tau_1 - \tau_2)]$, whose integral vanishes in the infinite time limit adopted by Ref. [586].

quantity $\Delta(k/\beta)$,

$$\begin{aligned} \int_{t_i}^{t_f} dt_1 \int_{t_i}^{t_f} dt_2 \cos[k(t_1 - t_2)] \Pi(t_1, t_2, k) \\ \equiv \frac{3\kappa^2 H_*^4}{16G^2 \beta^2 k^3} \left(\frac{\alpha}{1 + \alpha} \right)^2 \Delta\left(\frac{k}{\beta}\right), \end{aligned} \quad (193)$$

where κ is the efficiency factor characterizing how much vacuum energy goes into bulk kinetic energy. Note that Δ depends only¹⁹ on the ratio k/β , since the other thermal phase transition parameters have been factored out.

The major contribution of Ref. [586] is the derivation of a semi-analytical expression for Δ working in the thin wall and envelope approximations. In Appendix D we reproduce the full analytical form for Δ ; here we simply note here that the result can be approximated with the following empirical fitting formula, which reproduces the full spectrum to within 8% error [586]

$$\Delta(\tilde{k}) \simeq \Delta_p [c_l \tilde{k}^{-3} + (1 - c_l - c_h) \tilde{k}^{-1} + c_h \tilde{k}]^{-1}, \quad (194)$$

where $\tilde{k} \equiv k/k_p$, $k_p \simeq 1.24(a_*\beta)$ is the peak frequency of the phase transition, $\Delta_p \simeq 0.043$ is the peak amplitude, $c_l \simeq 0.064$, and $c_h \simeq 0.48$.

In order to express our power spectrum in terms of Δ such that we may use these results, note that Ref. [586] carries out their source calculation integrating over the dimensionless quantity βt (where they have set $\beta = 1$ in all intermediate steps). Since we work in conformal time, the corresponding dimensionless quantity here is $a_*\beta\tau$. Thus, we are justified in using their functional form for Δ with the replacement $\beta \rightarrow a_*\beta$, so that the dimensionless tensor

¹⁹In principle, Δ also depends on the wall velocity v_w . We set $v_w \simeq 1$, as is the case for our vacuum-dominated transitions of interest.

power spectrum in the sub-horizon regime becomes

$$\mathcal{P}_h(k) = 3\kappa^2 \left(\frac{\alpha}{1+\alpha} \right)^2 \left(\frac{a_* H_*}{k} \right)^2 \left(\frac{H_*}{\beta} \right)^2 \Delta \left(\frac{k}{a_* \beta} \right). \quad (195)$$

Approximating Eq. (194) in the sub-horizon regime yields

$$\Delta(k \gg k_p) \approx \frac{\Delta_p}{c_h} \left(\frac{k_p}{k} \right) = 0.11 \left(\frac{a_* \beta}{k} \right), \quad (196)$$

so substituting this form into Eq. (195), the sub-horizon spectrum becomes

$$\mathcal{P}_h(k) \approx 0.33 \kappa^2 \left(\frac{\alpha}{1+\alpha} \right)^2 \left(\frac{H_*}{\beta} \right) (k\tau_*)^{-3}, \quad (197)$$

where in the last step we have used $1/(a_* H_*) = \tau_*$.

Super-horizon regime: Activating a super-horizon mode is analogous to exciting an over-damped oscillator [602]; it takes a small but finite amount of time ϵ for the mode to grow to its maximum amplitude, which then remains constant until horizon re-entry at $x = 1$. Thus, for super-horizon modes $x_f < 1$, there are subtleties which prevent us from simply defining h_{ij}^{ini} as either Eq. (186) or Eq. (188) evaluated at $x = x_f$:

- We cannot use Eq. (186) with $x = x_f$ because this expression is only valid before super-horizon modes have reached their peak, frozen-in values.
- We also cannot naively evaluate Eq. (188) at $x = x_f$ because at this time the mode has not had enough time to reach its maximum value. Indeed, the A_{ij} term in Eq. (188) vanishes in this limit, but as we will see below, this term actually governs the super-horizon amplitude.

To properly identify the amplitude, we first note that for phase transitions that complete quickly compared to Hubble, $\beta/H > 1$ and $x_f - x_i = x_f(H/\beta)$, so Eqs. (189) and (190)

satisfy

$$A_{ij} \propto \frac{x_f^2}{\beta/H} \quad , \quad B_{ij} \propto \frac{x_f^3}{(\beta/H)^2} \quad . \quad (198)$$

Since $x_f \ll 1$ in the super-horizon limit, we generically have $A_{ij} \gg B_{ij}$.²⁰ Furthermore, just after the super-horizon mode has acquired its maximum value, the terms in Eq. (188) become

$$A_{ij} \frac{\sin(x - x_f)}{x} \rightarrow A_{ij} \quad , \quad B_{ij} \frac{\cos(x - x_f)}{x} \rightarrow \frac{B_{ij}}{x} \quad , \quad (199)$$

so only the first term of Eq. (188) survives in this limit and $h_{ij}^{\text{ini}} = A_{ij}$. Thus, using $x = k\tau$ and approximating the initial amplitude as

$$h_{ij}^{\text{ini}}(\vec{k}) \simeq 8\pi G a_*^2 \tau_* \int_{\tau_i}^{\tau_f} d\tau \cos[k(\tau_f - \tau)] \Pi_{ij}(\tau, \vec{k}) \quad , \quad (200)$$

we can now safely apply the transfer function using Eq. (173) to track the subsequent evolution of super-horizon modes.

To obtain the super-horizon power spectrum, we combine Eqs. (200) and (174) to obtain

$$P_h \simeq 32\pi^2 G^2 a_*^4 \tau_*^2 \int_{\tau_i}^{\tau_f} d\tau_1 \int_{\tau_i}^{\tau_f} d\tau_2 \cos[k(\tau_1 - \tau_2)] \Pi(\tau_1, \tau_2, k) \quad , \quad (201)$$

which depends on the same double integral from Eq. (196). Thus, using the definition of Δ from Eq. (193), we can write the dimensionless power spectrum as

$$\mathcal{P}_h(k) = 3\kappa^2 \left(\frac{\alpha}{1 + \alpha} \right)^2 \left(\frac{H_*}{\beta} \right)^2 \Delta \left(\frac{k}{a_* \beta} \right) \quad , \quad (202)$$

where we have used the fact that $\mathcal{H}_* = 1/\tau_*$ in radiation domination. Deep in the super-

²⁰Note that for $\beta/H_* \gg 1$, x_i and x_f are comparable, so the second term in Eq. (189) dominates over B_{ij}/x_f . Thus, we omit this term in the scaling expression for A_{ij} in Eq. (198).

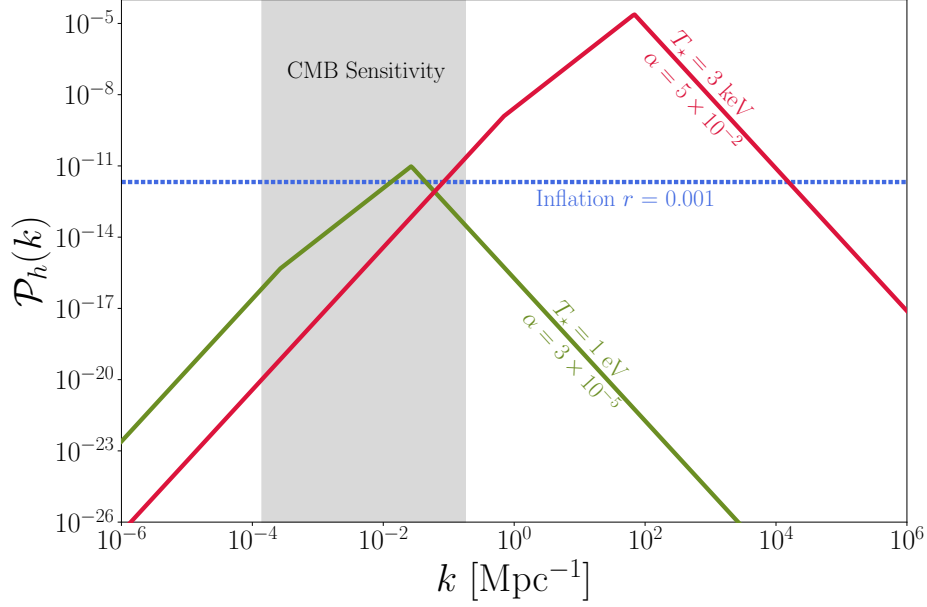


Figure 25: Tensor power spectra corresponding to two sample cosmological phase transitions. We plot $\mathcal{P}_h(k)$ as given by Eq. (211) for two benchmarks with $\kappa = 1$, $\beta/H_* = 2$, $\epsilon_b = 10^{-2}$, and $\{T_*, \alpha\} = \{1 \text{ eV}, 3 \times 10^{-5}\}$ (solid green) and $\{3 \text{ keV}, 5 \times 10^{-2}\}$ (solid red). Note that these parameters are compatible with other limits on cosmological phase transitions, although such limits are model-dependent and vary depending on the post-transition equation of state – see Appendix E for a discussion. We also compare with the inflationary prediction for tensor-to-scalar ratio $r = 10^{-3}$ (dashed blue), which is within the projected sensitivity of CMB-S4 [604]. The gray shaded region corresponds to multipoles $1 < \ell < 2500$ and denotes the linear regime probed by primary CMB anisotropy measurements.

horizon regime $x \ll 1$, the fitting formula of Eq. (194) can be approximated as $\Delta \simeq 0.352(k/a_*\beta)^3$ to yield

$$\mathcal{P}_h(k) \simeq 1.1 \kappa^2 \left(\frac{\alpha}{1 + \alpha} \right)^2 \left(\frac{H_*}{\beta} \right)^5 (k\tau_*)^3, \quad (203)$$

which exhibits the characteristic k^3 scaling for causality-limited modes with wavelengths (and periods) much larger than the spatial (and temporal) correlations of the source [560, 602, 603]. For such modes, the power spectrum is simply white noise and gets sharply suppressed since $(k\tau_*) \ll 1$ outside the horizon.

Full spectrum: Intuitively, power spectrum peaks at the maximum bubble size, which

is somewhat smaller than the horizon. More concretely, the condition for a mode to be “bubble-sized” is $k\tau_* \sim \beta/H_*$, so we expect $k_p \sim (\beta/H_*)\tau_*^{-1}$. A more rigorous calculation in Ref. [586] finds

$$k_p \simeq 1.24 \left(\frac{\beta}{H_*} \right) \tau_*^{-1}. \quad (204)$$

The expression in Eq. (197) governs the power spectrum in the sub-horizon regime for $k \gtrsim k_p$ and Eq. (203) governs the deep super-horizon regime, where $k \ll k_p$. However, the intermediate regime exhibits greater theoretical uncertainty, so here we parameterize our ignorance by defining a “breaking scale”

$$k_b \equiv \epsilon_b k_p, \quad \epsilon_b \ll 1, \quad (205)$$

where the k^3 super-horizon scaling breaks down.

Outside this region of uncertainty, the tensor power spectrum is

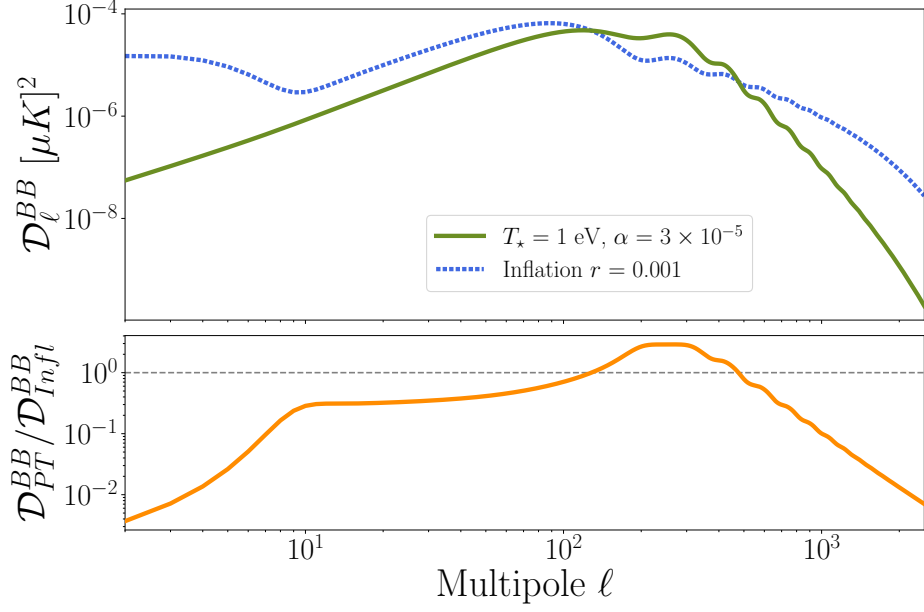
$$\mathcal{P}_h(k) = \begin{cases} 1.1 \kappa^2 \left(\frac{\alpha}{1+\alpha} \right)^2 \left(\frac{H_*}{\beta} \right)^5 (k\tau_*)^3 & k \leq k_b, \\ 0.33 \kappa^2 \left(\frac{\alpha}{1+\alpha} \right)^2 \left(\frac{H_*}{\beta} \right) (k\tau_*)^{-3} & k \geq k_p \end{cases}. \quad (206)$$

and for $k_b \leq k \leq k_p$ there is an intermediate region corresponding to modes which are sub-horizon, but super-bubble. For these modes, we model the spectrum as a power law

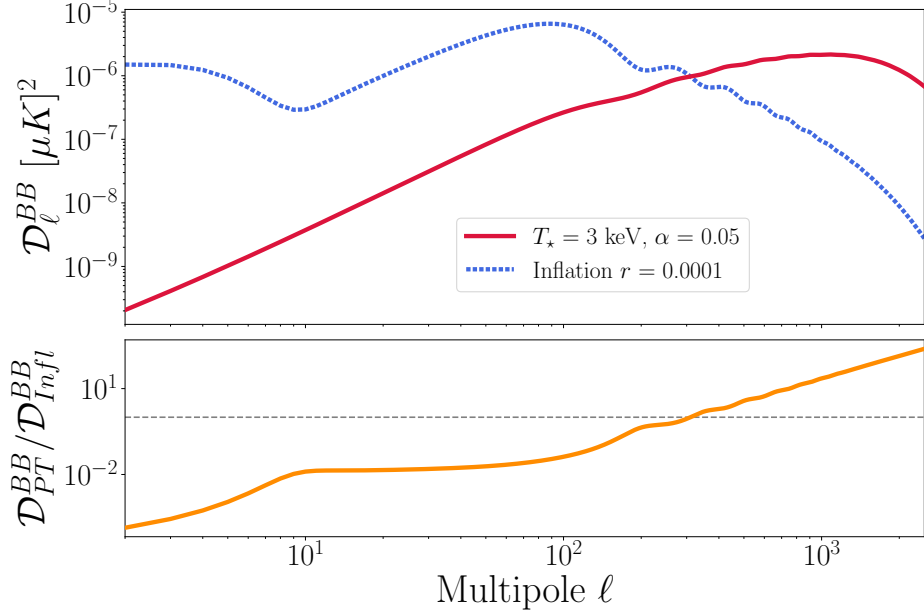
$$\mathcal{P}_h^{\text{int}}(k) = A k^m, \quad (207)$$

whose slope and amplitude satisfy

$$m = \frac{\log(\mathcal{P}_h^p/\mathcal{P}_h^b)}{\log(k_p/k_b)}, \quad A = \frac{\mathcal{P}_h^b}{k_b^m}, \quad (208)$$



(a) CMB TT Correlation Spectra



(b) CMB EE Correlation Spectra

Figure 26: **Top:** B -mode polarization spectra for the green phase transition benchmark of Fig. 25 (solid green) and a minimal inflationary model with $r = 0.001$ (dashed blue). Note that lensing has been removed. **Bottom:** Same as the left panel, but for the red benchmark of Fig. 25. We compare this prediction with that of an inflationary scenario with $r = 10^{-4}$, chosen to match the projected sensitivity of CMB-S4 [604].

in order to match the boundary values

$$\mathcal{P}_h^p \equiv \mathcal{P}_h(k_p) = 0.17\kappa^2 \left(\frac{\alpha}{1+\alpha} \right)^2 \left(\frac{H_*}{\beta} \right)^4, \quad (209)$$

$$\mathcal{P}_h^b \equiv \mathcal{P}_h(k_b) = 2.1\kappa^2 \left(\frac{\alpha}{1+\alpha} \right)^2 \left(\frac{H_*}{\beta} \right)^2 \epsilon_b^3, \quad (210)$$

where we have used the definitions of k_p and k_b in terms of τ_* and ϵ_b . Thus, the full tensor power spectrum is

$$\mathcal{P}_h(k) = \begin{cases} 1.1 \kappa^2 \left(\frac{\alpha}{1+\alpha} \right)^2 \left(\frac{H_*}{\beta} \right)^5 (k\tau_*)^3 & k \leq k_b, \\ \mathcal{P}_h^{\text{int}}(k) & k_b \leq k \leq k_p, \\ 0.33 \kappa^2 \left(\frac{\alpha}{1+\alpha} \right)^2 \left(\frac{H_*}{\beta} \right) (k\tau_*)^{-3} & k \geq k_p, \end{cases} \quad (211)$$

which we plot in Fig. 25 for two sample benchmarks.

9.4 Numerical results

In Figs. 26 and 27, we show the angular B -mode power spectrum

$$\mathcal{D}_\ell^{BB} \equiv \frac{\ell(\ell+1)}{2\pi} T_0^2 C_\ell^{BB}, \quad (212)$$

for various phase transition and inflationary scenarios. For illustrative purposes, we exclude the predicted lensing signal from all predictions to highlight the intrinsic shape differences between these sources. Note that the lensing signal is a foreground effect that impacts both models in a similar way [605–616].

The curves in these figures are computed using the Boltzmann solver **CLASS** [590]. In particular for the phase transition signal, we utilize the **external-pk** module with the custom primordial tensor power spectrum from Eq. (211). We then compute the transfer functions **CLASS** using the Einstein-Boltzmann equations and the polarization source functions. These

outputs, combined with the primordial tensor power spectrum, are then used to evaluate the line-of-sight and k -space integrals to obtain the C_ℓ^{BB} spectrum.

The bottom panels of Fig. 26 show the ratio of the phase transition and inflationary B -mode predictions, highlighting their distinct spectral shapes. Notably, the phase transition spectra peaks at much higher multipoles $\ell \sim 1000$ compared to inflationary predictions, which peak around $\ell \sim 100$ (corresponding to recombination).

The scaling behavior for the curves in this figure can be understood from the definition of C_ℓ^{BB} in Eq. (170) and the shape of the power spectrum for each scenario. The domain of support for the function $\mathcal{F}_\ell(k)$ in Eq. (170) roughly corresponds to the gray “CMB sensitivity” region in Fig. 25. This function peaks around $k \sim 0.01 \text{ Mpc}^{-1}$, decreases slowly on larger scales, and decreases very quickly on smaller scales.

The inflationary power spectrum is nearly scale invariant, $\mathcal{P}_h(k) \sim k^0$, so the dominant contribution to the integral in Eq. (170) comes from modes in the vicinity of the peak in $\mathcal{F}_\ell(k)$. In particular by using the approximation $\ell \sim k\tau_0$, one can show that the peak at $k \sim 0.01 \text{ Mpc}^{-1}$ roughly corresponds to a maximal B -mode signal at $\ell \sim 100$, consistent with Fig. 26.

By contrast, the phase transition signal exhibits maximal power on smaller scales $k > 0.01 \text{ Mpc}^{-1}$ and becomes sharply suppressed on large scales since $\mathcal{P}_h(k) \propto k^3$ in the infrared tail. Thus, the dominant contribution to the integral in Eq. (170) comes from these larger k values, despite the competing suppression in $\mathcal{F}_\ell(k)$ at large values of k .

For the $T_* = 1 \text{ eV}$ benchmark, the peak power in $\mathcal{P}_h(k)$ is only at slightly smaller scales $k_p \sim 0.03 \text{ Mpc}^{-1}$, so the B -mode spectrum in the left panel of Fig. 26 peaks at only slightly

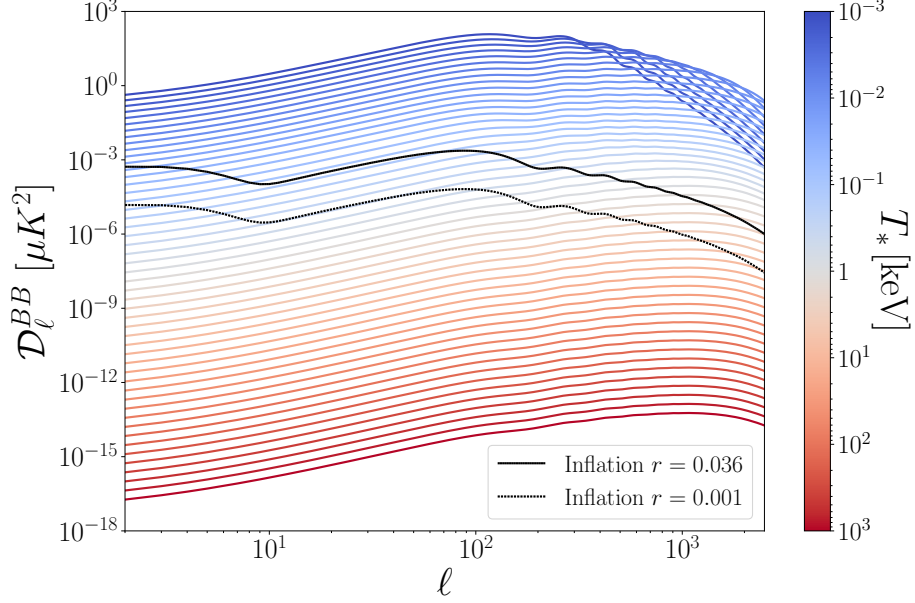


Figure 27: Color map of B -mode power spectra predictions for various T_* assuming $\kappa = 1$, $\beta/H_* = 2$, and $\alpha = 0.05$. The solid black curve is the inflationary prediction for $r = 0.036$, which saturates current BICEP limits [617]. The dotted black curve is the inflationary prediction for $r = 10^{-4}$, corresponding to CMB-S4 sensitivity projections [604].

higher multipoles ($\ell \sim 120$) relative to the inflationary prediction. Note also that for this benchmark, the integral in Eq. (170) receives $\mathcal{P}_h(k)$ contributions from all three regions in Eq. (211). These distinct regions give rise to noticeable features in the angular power spectrum $\mathcal{D}_\ell^{BB}$ for the phase transition predictions; such features do not appear for the inflationary curves. This phenomenon can also be seen in the high- ℓ behavior of the low- T_* contours near the top of Fig. 27.

For the $T_* = 3 \text{ keV}$ benchmark, the tensor power spectrum peaks at significantly smaller scales around $k_p \sim 100 \text{ Mpc}^{-1}$, so $\mathcal{D}_\ell^{BB}$ peaks at significantly higher multipoles $\ell \sim 1000$. From Fig. 25, we see that now $\mathcal{D}_\ell^{BB}$ only receives contributions from the super-horizon tail. Because $\mathcal{P}_h(k) \propto k^3$ in this regime, the net effect on the B -mode spectrum is a suppression of power at small ℓ and an enhancement at high ℓ relative to the (nearly) scale invariant $\mathcal{P}_h \propto k^0$ inflationary prediction, as shown in the right panel of Fig. 26.

9.5 A complementary GW signal

The tensor perturbations sourcing CMB B -mode polarization also result in a stochastic GW background that offers a complementary signal of our scenario. The relative energy density in GWs per logarithmic frequency interval is quantified by the spectral density parameter

$$\Omega_{\text{GW}} = \frac{1}{\rho_{\text{tot}}} \frac{d\rho_{\text{GW}}}{d \ln k}. \quad (213)$$

At the time of the phase transition τ_* , the energy density in GWs is

$$\rho_{\text{GW}}^* = \frac{1}{8\pi G a_*^2} \langle h'_{ij}(\tau_*, \vec{x}) h'_{ij}(\tau_*, \vec{x}) \rangle, \quad (214)$$

where the real-space correlation function can be expressed as

$$\langle h'_{ij}(\tau, \vec{x}) h'_{ij}(\tau, \vec{x}) \rangle = \int d \ln k \mathcal{P}_{h'}(\tau, k), \quad (215)$$

where $\mathcal{P}_{h'}(k)$ is the dimensionless power spectrum of the derivative of the tensor perturbation with respect to conformal time τ . In calculating $\mathcal{P}_{h'}(k)$, there is no longer the subtlety with super-horizon modes encountered before, since the initial condition for the derivative of the field amplitude is non-vanishing. Thus we may simply compute h'_{ij} from Eq. (186), evaluate at τ_f to define $(h'_{ij})^{\text{ini}} \equiv h'_{ij}|_{\tau_f}$, compute the power spectrum, and substitute into the expressions above to find the following spectral density at the time of the phase transition

$$\Omega_{\text{GW}}^* = \kappa^2 \left(\frac{\alpha}{1 + \alpha} \right)^2 \left(\frac{H_*}{\beta} \right)^2 \Delta \left(\frac{2\pi f_*}{\beta} \right), \quad (216)$$

where as a last step we have introduced the physical frequency f , related to the comoving wavenumber as $k = 2\pi a f$.

To extract the present-day GW signal, we redshift the frequency as $f_0 = f_*(a_*/a_0)$ and

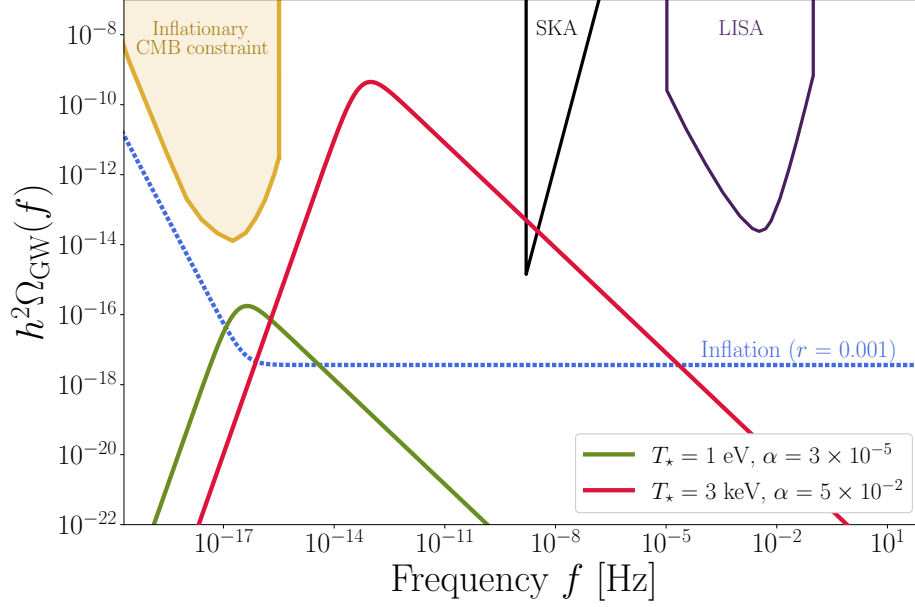


Figure 28: Representative GW spectra from phase transitions (solid red/green) and inflation (dotted blue). Parameter values match the benchmarks of Fig. 25. We show also the CMB constraint on inflation from Ref. [618] (yellow shaded) and sensitivity projections for LISA (purple) [619] and SKA (black) [620].

the energy density as $\rho_{\text{GW}}^0 = \rho_{\text{GW}}^* (a_*/a_0)^4$, where entropy conservation gives

$$\frac{a_*}{a_0} = \left[\frac{g_{*,s}(T_0)}{g_{*,s}(T_*)} \right]^{1/3} \frac{T_0}{T_*}, \quad (217)$$

so the spectral density today is

$$\Omega_{\text{GW}} h^2 \simeq 4 \times 10^{-5} \kappa^2 \alpha^2 \left(\frac{H_*}{\beta} \right)^2 \Delta \left(\frac{2\pi f_0}{a_* \beta} \right), \quad (218)$$

where we have normalized $a_0 = 1$ and taken $g_*(T_*) = 3.36$ and $g_{*,s}(T_*) = 3.91$, since we focus on late-time phase transitions.

In Fig. 28, we show sample Ω_{GW} predictions for the phase transition and inflationary scenarios, presented alongside the CMB constraints from Ref. [618]. Note that the precise location of the CMB limit from B -modes is model dependent, as translating sensitivity

from $\mathcal{D}_\ell^{BB}$ to Ω_{GW} depends on the power spectrum of the source. Notice also that the phase transition and inflationary lines cross around $f \sim 10^{-17}$ Hz, corresponding to $k \sim 0.01 \text{ Mpc}^{-1}$. This is exactly as we would expect, given that these benchmarks lead to similar maximal amplitudes in the B -mode polarization spectrum.

9.6 Discussion

In this chapter, we have found that the gravitational waves produced during a late-time, first-order cosmological phase transition can generate CMB B -mode polarization on observable scales. Although phase transitions occur due to causal processes on sub-horizon scales, they nonetheless exhibit a white noise power spectrum on large scales. Power is suppressed on the scales relevant for the CMB, but it is not necessarily negligible. In particular, for sufficiently strong, late-time transitions, the amplitude can be comparable to inflationary predictions which can be probed with CMB-S4 [604]. Thus, a discovery of B -modes beyond the known lensing effect is no longer a definitive “smoking gun” for inflation, though it is still clear evidence of new physics.

Fortunately, the spectral shape of the phase transition signal is distinct from the inflationary predictions. While the polarization signal from inflation peaks around $\ell \sim 100$, the phase transition predicts peak power at smaller scales (high- ℓ), as well as a relative low- ℓ suppression. Thus, the origin of a potential signal can ultimately be distinguished with sufficiently precise B -mode measurements across multiple angular scales. Furthermore, the cosmological phase transitions we have considered may also predict stochastic GW backgrounds within SKA sensitivity [620], CMB spectral distortions observable with PIXIE [621] and SPECTER [622], and ΔN_{eff} within the reach of CMB-S4 targets [604].

Note that our treatment of GWs from phase transitions is conservative as we have only included contributions from bubble collisions, neglecting the contributions from sound waves

and turbulence. Furthermore, many of the limits on cosmological phase transitions (e.g. ΔN_{eff}) are model dependent, as they assume a specific equation of state for the hidden sector after the transition ends. Fully accounting for these effects and their variations could allow for much louder GW signals within viable parameter space. We leave a more detailed treatment for future work.

While this result heightens the challenge of understanding inflation from a B -mode discovery at a single angular scale, it also opens the door to understanding a wider range of new physics in the event of such a discovery. Indeed, we have shown that existing and future experiments sensitive to B -modes can also be sensitive to power from a new range of non-inflationary scenarios. Similar super-horizon white noise power spectra may also arise from other sources, including cosmic strings or domain walls, whose contributions we leave for future work.

10 Conclusion

Seize the time... Live now! Make now always the most precious time.

Now will never come again.

- Jean-Luc Picard

In this thesis, I have presented original research that critically examines and challenges aspects of the prevailing Λ Cold Dark Matter (Λ CDM) cosmological model. While Λ CDM remains remarkably successful at explaining a wide array of cosmological observations, significant and persistent tensions—particularly regarding the expansion rate of the Universe (H_0)—suggest that it may be incomplete or indicative of new physics.

Firstly, by carefully analyzing cosmological distances, I have shown that the Hubble tension fundamentally reflects discrepancies between local and global distance scales. Furthermore, I demonstrated how the Thomson scattering rate during recombination imposes robust constraints on models attempting to increase early Universe expansion rates without new physics. This constraint provides valuable insights into possible resolutions of cosmological tensions.

Additionally, my exploration of a novel mirror-world dark sector model, incorporating variations in fundamental constants, has provided strong support for a profound observational degeneracy. This degeneracy permits scaling of cosmological observables, achieving consistency with a wide range of expansion rates. Such a discovery has implications not only for the Hubble tension but also for how we interpret cosmological data within the Λ CDM framework.

Finally, I have challenged the long-standing paradigm that primordial B-mode polarization in the Cosmic Microwave Background (CMB) is definitive evidence for inflation. By

demonstrating that detectable tensor perturbations—and therefore observable B-mode signals—can originate from first-order cosmological phase transitions, my work opens exciting avenues for future research beyond standard inflationary scenarios.

Collectively, this thesis underscores the critical importance of continuously questioning established paradigms in cosmology. While Λ CDM remains a powerful model, the tensions and degeneracies explored in this research highlight opportunities for deeper theoretical and observational exploration. Now, before you put this thesis down, I would like to end on some concluding remarks regarding my own approach to science and its place in the broader physics landscape.

10.1 The need to question established paradigms

The history of science is fundamentally marked by major paradigm shifts. By challenging the established scientific narrative, and at times pushing against the prevailing *zeitgeist* of what is considered acceptable or worthwhile research, we maintain continuous progress and deepen our understanding of the Universe. Indeed, many of the most groundbreaking discoveries were initially met with skepticism or even resistance from the broader scientific community.

For example, Galileo Galilei’s defense of heliocentrism in the 17th century directly contradicted the widely accepted geocentric model, fundamentally altering humanity’s view of its place in the cosmos. Similarly, the introduction of quantum mechanics by Max Planck and later developed by Niels Bohr, Werner Heisenberg, and Erwin Schrödinger challenged the deterministic framework established by classical physics. The famous Michelson-Morely experiment attempted to detect the drag of the Earth through the cosmological aether, and definitively dismissed the need for a propagation medium for light. Vera Rubin’s pioneering observations of galaxy rotation curves questioned prevailing gravitational theories, providing some of the most intuitive evidence for dark matter and radically reshaping cosmological

thought.

These are just a few straight forward examples of times questioning the narrative was very successful. Of course, there are countless other research endeavors challenging established paradigms that do not live to see the light of mainstream physics, but I believe that today, more than ever, actively questioning established paradigms is paramount for several critical reasons. First, by interrogating the standard narrative, such as critically evaluating the foundational assumptions of the Λ CDM model, we prevent stagnation in scientific discourse and encourage creativity, both of which are essential catalysts for genuine progress.

Secondly, as competition for research funding intensifies amidst an unprecedented influx of researchers into science [623] during a time where scientific funding is being threatened, there is a significant risk that only projects deemed safely within the current paradigm will receive support. Such a funding environment inherently discourages innovative or unconventional approaches, thus hindering revolutionary scientific breakthroughs.

True scientific progress often emerges precisely from ideas initially regarded as speculative or controversial, as history repeatedly demonstrates. As responsible and ethical scientists, we have an obligation to evaluate all scientifically rigorous ideas, not merely those likely to gain easy acceptance or immediate funding approval. It is this philosophy and methodological approach that drives the original research presented in the subsequent chapters of this thesis and underscores its potential impact. Regardless of whether the current cosmological tensions, such as the Hubble tension, are resolved in the near future, the insights gained through persistently questioning established paradigms always result in something learned, and will continue to profoundly shape our understanding of the Universe.

11 Appendices

Appendix A. Exact scaling transformations for the generalized Peeble’s C-factor . . .	194
Appendix B. Constraints on FCV	196
Appendix C. Comments on BBN and FCV	199
Appendix D. Single and double bubble spectra	201
Appendix E. Scalar perturbation bounds on cosmological PTs	203

Appendix A Exact scaling transformations for the generalized Peeble's C-factor

The following discussion is heavily based on the original HyRec publications. For more details and a clear discussion of the recombination process, please see refs. [431, 432] and the references therein. The generalized Peeble's C-factor [431, 432] used by HyRec-2 to compute the free electron fraction can be given by

$$C_{2\ell} = \frac{\mathcal{R}_{2\ell,1s} + \mathcal{R}_{2\ell,2\ell'} \frac{\mathcal{R}_{2\ell',1s}}{\Gamma_{2\ell'}}}{\mathcal{B}_{2\ell} + \mathcal{R}_{2\ell,2\ell'} \frac{\mathcal{B}_{2\ell'}}{\Gamma_{2\ell'}} + \mathcal{R}_{2\ell,1s} + \mathcal{R}_{2\ell,2\ell'} \frac{\mathcal{R}_{2\ell',1s}}{\Gamma_{2\ell'}}} \quad (219)$$

where $\ell = p$ if $\ell' = s$, $\ell = s$ if $\ell' = p$, and $\Gamma_{2\ell}$ is given by

$$\Gamma_{2\ell} = \mathcal{B}_{2\ell} + \mathcal{R}_{2\ell,2\ell'} + \mathcal{R}_{2\ell,1s}. \quad (220)$$

$\mathcal{B}_{2\ell}$ is the effective photoionization rate of the 2ℓ level, $\mathcal{R}_{i,j}$ is the transition rate from the i^{th} level to the j^{th} level, and $\Gamma_{2\ell}$ is the effective inverse lifetime of the 2ℓ level. This generalized form represents the probability that an atom in the 2ℓ state reaches the ground state by transitioning to the $n = 2$ state first or by direct transitions. Before we analyze the generalized C-factors, we must make the following assumptions that are used by HyREC; first, the rate of transitions from the 2s to 1s state are given by $\Lambda_{2s,1s}$. Second, the rate of transitions from the 2p to 1s state are given by $R_{Ly\alpha}$. Therefore, we can write the C_{2s} and C_{2p} as

$$C_{2s} = \frac{\Lambda_{2s,1s} + \mathcal{R}_{2s \rightarrow 2p} \frac{R_{Ly\alpha}}{\Gamma_{2p}}}{\Gamma_{2s} - \mathcal{R}_{2s \rightarrow 2p} \frac{\mathcal{R}_{2s \rightarrow 2p}}{\Gamma_{2p}}} \quad (221)$$

$$C_{2p} = \frac{R_{Ly\alpha} + \mathcal{R}_{2s \rightarrow 2p} \frac{\Lambda_{2s,1s}}{\Gamma_{2s}}}{\Gamma_{2p} - \mathcal{R}_{2s \rightarrow 2p} \frac{\mathcal{R}_{2s \rightarrow 2p}}{\Gamma_{2s}}} \quad (222)$$

The transition rate from the 2s to 2p state is proportional to the fundamental constants such that

$$\mathcal{R}_{2s \rightarrow 2p} \propto \alpha^5 m_e \quad (223)$$

We see that for both the mFFAT and tFFAT variations, $\mathcal{R}_{2s \rightarrow 2p}$ scales as $\lambda^{1/2}$.

The FFAT scaling retains the symmetry breaking presented above, primarily due to $\Lambda_{2s,1s}$ and $R_{Ly\alpha}$ scaling as λ rather than $\lambda^{1/2}$. This causes the effective inverse lifetimes of the 2ℓ states, $\Gamma_{2\ell}$ to not be invariant under the transformation such that

$$\Gamma_{2\ell} \xrightarrow{\lambda} \lambda^{1/2}(\mathcal{B}_{2\ell} + \mathcal{R}_{2\ell,2\ell'} + (\lambda^{1/2})\mathcal{R}_{2\ell,1s}), \quad (224)$$

thus, reducing the effective lifetime of the 2ℓ level, corresponding to an increase in the rate of overall recombination. This agrees with our conclusions in section 7.3.3.

Appendix B Constraints on FCV

Adjusting fundamental constants introduces challenges that span both theoretical and observational domains. Although we have pinpointed a variation in the CMB during recombination that evades detection, geological records, atomic spectra, quasar observations, and potentially BBN impose much tighter constraints. To ensure the model is not in direct disagreement with current observables, we briefly review the main methods for constraining FCV. For an extensive review, see refs. [426, 435].

Geological constraints, assessing periods comparable to Earth’s age (or $z < 0.5$), are based on measuring relative abundances of various heavy element isotopes. The most stringent of these arises from the fascinating Oklo natural reactor phenomenon, where natural nuclear fission occurred approximately 1.7 billion years ago [624]. Analysis of this event, considering the neutron mass and decay resonance energy, has significantly constrained the variation of α to $|\Delta\alpha/\alpha| < 1.1 \times 10^{-8}$ over this period [625].

Atomic clocks provide a highly precise method for measuring FCV today ($z = 0$). These experiments seek to detect variation in frequencies of long term, stable oscillators and probe the nature of α , the electron to proton mass ratio μ , and the proton gyromagnetic ratio g_p . By measuring frequencies related to the gross, fine-structure, and hyper-fine structure atomic transitions they measure combinations of R_∞ , $\alpha^2 R_\infty$, and $\mu\alpha^2 R_\infty$, respectively, where R_∞ is the Rydberg constant. The scaling of α and m_e proposed in this study specifically leaves the Rydberg constant and $\mu\alpha^2$ invariant, which could leave some atomic clock measurements insensitive to this specific variation. However, the fine structure measurements (αR_∞) effectively break the degeneracy. Current atomic clock data restricts the variability of α to about $|\dot{\alpha}/\alpha| < 10^{-19}$ annually [626].

Quasar (QSO) absorption spectra provide the earliest directly observable constraints on FCV. As some of the Universe’s most luminous sources, QSOs allow observations up to redshifts of $z \sim 7$, reaching back to less than 1 Gyr after the Big Bang. Astronomers analyze absorption spectra from cold gas clouds along QSOs’ line of sight up to $z \sim 3$ (about 2 Gyr old), setting constraints on early Universe FCV. This method, however, introduces additional complexity due to the Universe’s expansion redshifting the spectra. FCV constraints using QSOs rely on identifying chromatic effects in the spectra, impacting all wavelengths and may be difficult to disentangle from redshift. Current constraints from QSO absorption spectra separately limit α variation to $|\Delta\alpha/\alpha| < 10^{-6}$ at $z \sim 2.4$ [627] and $|\Delta\mu/\mu| < 10^{-5}$ at $z \sim 0.9$ [628].

Big Bang nucleosynthesis offers potential constraints on FCV mere minutes after the end of inflation at redshifts of $z \sim 10^8$. Standard BBN models, which successfully predict light elemental abundances, potentially anchor FCV constraints to this early epoch. BBN’s sensitivity extends to various factors, including the expansion rate, neutron-proton number density ratio and mass difference, neutron lifetime, and the Coulomb barrier strength. This complexity opens a challenging avenue, as BBN’s outcomes are not clear when considering variations of α and m_e . However, a comprehensive analysis of FCV’s impact on BBN demands not only advanced nuclear reaction rate calculations but also detailed lattice QCD and QED computations [399, 429, 629, 630].

We summarize the constraints on FCV as follows:

- Atomic clocks confirm the stability of α and m_e at present ($z = 0$) and that linear variations are ruled out.
- Geological and quasar (QSO) observations suggest no significant variation in these parameters individually over the past 10 billion years ($z \sim 3$).

- The exact impact of the FFAT symmetry combined with FCV on Big Bang Nucleosynthesis outcomes remains uncertain at $z \sim 10^8$. See App. C for more discussion.

However, it's interesting to highlight that most discussions herein typically focus on varying a single parameter. Simultaneously varying both α and m_e might weaken some constraints, particularly if they primarily depend on the binding energy, which remains unchanged under this specific FCV. Under the most conservative assumption that simultaneous variations in α and m_e do not impact constraining power, a particle physics model aiming to realize the required Thomson scattering rate increase must vary these parameters while satisfying several criteria: it must preserve BBN predictions, avoid rapid variation compared to the timescale set by recombination, and align with Standard Model values before the earliest QSO observations, maintaining stability thereafter.

Appendix C Comments on BBN and FCV

Big Bang Nucleosynthesis is significantly influenced by two critical quantities that depend on the fine structure constant and the electron mass: the neutron to proton mass difference (Q_{np}) and the neutron lifetime (τ_{n}). Ref. [631] explores the impact of varying α and m_e linearly, establishing two key relationships:

$$\frac{\Delta Q_{\text{np}}}{Q_{\text{np}}} = -0.59 \frac{\Delta \alpha}{\alpha}, \quad (225)$$

$$\frac{\Delta \tau_{\text{n}}}{\tau_{\text{n}}} = 3.86 \frac{\Delta \alpha}{\alpha} + 1.52 \frac{\Delta m_e}{m_e}. \quad (226)$$

In this analysis, we omit variations in the vacuum expectation value and quark masses as examined in [631]. Applying the best-fit values identified in Section 8.4, we ascertain that following the FFAT symmetry direction results in a decrease in Q_{np} by approximately 1% and an increase in τ_{n} by around 1%.

BBN unfolds in three crucial stages, with the neutron to proton mass difference (Q_{np}) and the neutron lifetime (τ_{n}) playing pivotal roles:

1. Initially, the temperature remains high enough for the weak interaction rate to maintain protons and neutrons in equilibrium.
2. Once the weak interaction rate dips below the Hubble rate, inverse reactions cease, and the neutron to proton ratio becomes fixed at an amount sensitive to Q_{np} and the background temperature. However, neutrons are unstable and will continue to decay according to τ_{n} until the onset of light element synthesis.
3. Finally, the temperature falls to a level where primordial nucleosynthesis begins, facilitated by the formation of deuterons.

This simplified overview demonstrates that BBN is influenced by Q_{np} and τ_{n} , as well as

the background expansion rate. Without comprehensive calculations, pinpointing the exact impact of these factors on the predicted helium-4 yields is challenging and very model dependent. However, it seems that an increase in Q_{np} and a decrease in τ_{n} would increase the helium-4 yields. Ultimately, BBN could serve as the decisive assessment for any proposed resolution to the Hubble tension, as elemental abundances offer stringent constraints on the expansion rate during that epoch. For recent discussion regarding the connection between BBN and FCV see Refs. [429, 632–634].

Appendix D Single and double bubble spectra

Here we reproduce the explicit form for the function Δ from Eq. (193) derived originally in Ref. [586] working in the thin-wall and envelope approximations. As in this work, for simplicity we presume a luminal wall velocity $v_w = 1$, though the generalization to $v_w < 1$ is straightforward. We also set $\beta = 1$ for convenience and restore it later as needed using dimensional analysis. We parametrize this function according to $\Delta = \Delta^{(1)} + \Delta^{(2)}$, where the $\Delta^{(1),(2)}$ are the “single-bubble” and “double-bubble” contributions, respectively.

The “single-bubble” term arises when the bubble wall segments passing through two distinct points originate from the same nucleation event, where

$$\Delta^{(1)} = \frac{k^3}{12\pi} \int_0^\infty dy \int_y^\infty \frac{dr}{r^3} \frac{e^{-r/2} \cos(ky)}{\mathcal{I}(y, r)} \mathcal{G}(y, r, k), \quad (227)$$

where the integrand depends on the functions

$$\mathcal{I}(y, r) = e^{y/r} + e^{-y/r} + \frac{y^2 - (r^2 + 4r)}{4r} e^{-r/2}, \quad (228)$$

$$\mathcal{G}(y, r, k) = j_0(kr)F_0 + \frac{j_1(kr)}{kr}F_1 + \frac{j_2(kr)}{(kr)^2}F_2, \quad (229)$$

and we have defined

$$F_0(y, r) = 2(r^2 - y^2)^2(r^2 + 6r + 12) \quad (230)$$

$$F_1(y, r) = 2(r^2 - y^2)[-r^2(r^3 + 4r^2 + 12r + 24) + y^2(r^3 + 12r^2 + 60r + 120)] \quad (231)$$

$$F_2(y, r) = \frac{1}{2}[r^4(r^4 + 4r^3 + 20r^2 + 72r + 144) - 2y^2r^2(r^4 + 12r^3 + 84r^2 + 360r + 720) + y^4(r^4 + 20r^3 + 180r^2 + 840r + 1680)]. \quad (232)$$

The “double-bubble” contribution also arises from bubble wall segments passing through two spatial points, but in this case the two wall segments originate from distinct nucleation events and

$$\Delta^{(2)} = \frac{k^3}{96\pi} \int_0^\infty dy \int_y^\infty \frac{dr \cos(ky) j_2(kr) g(y, r) g(-y, r)}{r^4 \mathcal{I}(y, r)^2 (kr)^2}, \quad (233)$$

where we have defined

$$g(y, r) = (r^2 - y^2)[(r^3 + 2r^2) + y(r^2 + 6r + 12)]e^{-r/2}. \quad (234)$$

Appendix E Scalar perturbation bounds on cosmological PTs

First-order phase transitions generate both tensor and scalar mode perturbations, resulting in curvature and isocurvature fluctuations constrained by observations such as the CMB, Lyman- α , and CMB spectral distortion data.

Several studies have examined scalar perturbation constraints from phase transitions, focusing on super-horizon perturbations [588, 635, 636] or sub-horizon perturbations in thermal phase transitions [637]. Here, we review the findings of Ref. [588] for a hidden sector phase transition that converts latent heat into dark radiation and re-derive the constraints for a scenario where the phase transition energy is converted into kinetic energy which redshifts as $\rho \propto a^{-6}$, as in models of kination [638]. The latter case significantly weakens the ΔN_{eff} and scalar perturbation constraints, allowing for a B -mode signal potentially detectable by CMB-S4.

For a phase transition taking place at comoving time τ_* , the super-horizon perturbation with comoving wavenumber k contains $N \sim (kd_b)^3$ bubbles within the volume of the wavelength region, where $d_b = (8\pi)^{1/3}v_w/\beta$ is the average separation of bubbles. The phase transition time t_c of each spatial point in the wavelength region has a standard deviation that goes as $1/\sqrt{N}$. Based on this, Ref. [588] shows that the super-horizon power spectrum of the phase transition time $\mathcal{P}_{\delta t} \equiv H_*^2 \langle \delta t_c(\vec{x}) \delta t_c(\vec{y}) \rangle$ has a parametric form

$$\mathcal{P}_{\delta t_c} = 8\pi c v_w^3 (k\tau_*)^3 \left(\frac{H_*}{\beta} \right)^5. \quad (235)$$

A detailed calculation of the power spectrum performed in [588], which considers the bubble expansion dynamics, agrees with this parametric form and gives $c \approx 2.8$.

The curvature perturbation can be calculated in the spatially flat gauge by comparing

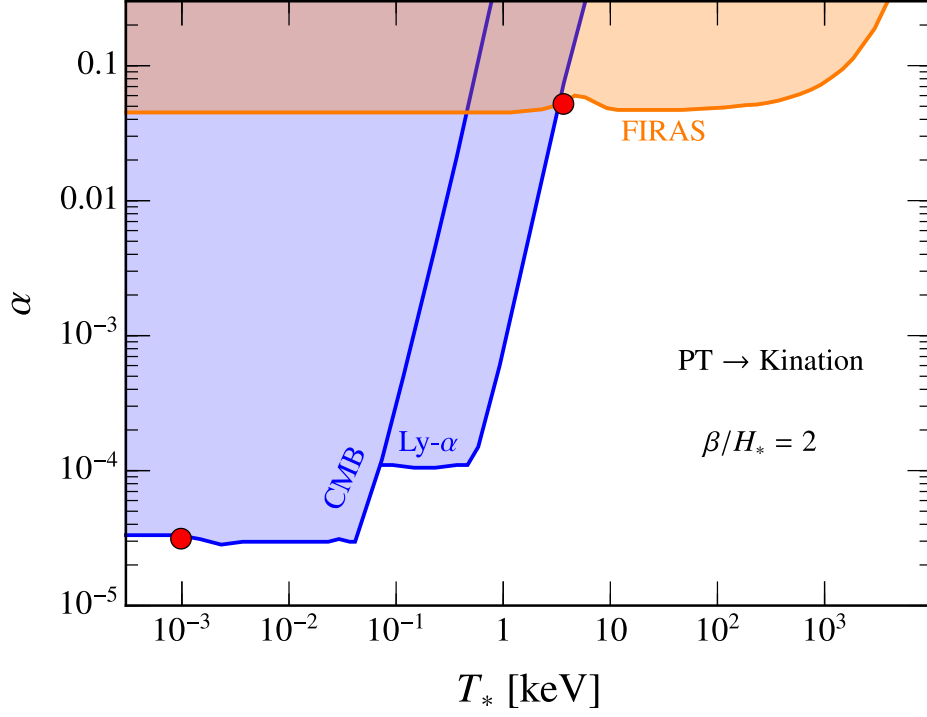


Figure 29: Experimental limits (2σ) on α for various phase transition temperatures, assuming all energy is released into kination with the equation of state $w = 1$. Here we show bounds from Lyman- α [639] (blue shaded), CMB scalar perturbations [640] (blue shaded), CMB spectral distortions COBE-FIRAS, [13] (orange shaded). The red circles denote the benchmark points of Fig. 26. The upper-right point is $(T_*, \alpha) = (3 \text{ keV}, 5 \times 10^{-2})$, and the lower-left point is $(1 \text{ eV}, 3 \times 10^{-5})$. We assume $\kappa = 1$, $v_w = 1$, and $\beta/H_* = 2$ for both cases. See [588] for more detailed derivation of the bounds.

energy densities at different spatial points at a common time when the scale factors are equal. Assuming an instantaneous transfer of the vacuum energy into dark fluid energy density ρ_{dark} , the curvature perturbation given by the dark fluid is given by:

$$\zeta = -\frac{H_{\text{PT}}\delta\rho_{\text{dark}}}{(\dot{\rho}_{\text{dark}} + \dot{\rho}_{\text{SM}})} = \frac{n\alpha H_*\delta t_c}{n\alpha + 4(1-\alpha)(a_{\text{en}}/a_*)^{n-4}}, \quad (236)$$

where we have assumed the latent heat from the phase transition converts into energy density that redshifts as $\rho_{\text{dark}}(a \geq a_*) = \rho_{\text{dark,ini}}(a/a_*)^{-n}$, where a_{en} is the scale factor when the perturbation mode enters the horizon. We can write $(a_*/a_{\text{en}}) = k\tau_*$ during the radiation-dominated era. For $n = 4$, corresponding to standard dark radiation, the dimensionless power spectrum from dark radiation reproduces the expression in [588] as $P_\zeta^{\text{DR}} = \alpha^2 \mathcal{P}_{\delta t_c}$.

In this work, we consider a phase transition example where the latent heat is converted into kination, which redshifts as $n = 6$. Since kination redshifts much faster than dark radiation, it is less affected by the ΔN_{eff} bound. Moreover, its contribution to the total curvature perturbation is reduced due to the smaller energy ratio to the SM radiation just before the horizon entry.

In the kination scenario, the total scalar perturbations right before horizon entry becomes

$$\mathcal{P}_\zeta(k) = \frac{9}{4}\alpha^2(k\tau_*)^4\mathcal{P}_{\delta t_c}(k) + \mathcal{P}_{\text{ad}}(k). \quad (237)$$

Here we have used the fact that $\alpha \ll 1$ to simplify Eq. (236) when setting the exclusion bounds. Comparing the power spectrum to the CMB [640], Lyman- α [639], and CMB spectral distortion constraints [13] on primordial curvature perturbations, one can obtain the exclusion bounds as the shaded regions in Fig. 29 for dark phase transition with $\beta/H_* = 2$. Our two benchmark models are indicated by red dots.

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