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RF SUPERCONDUCTING CAVITY AND ZERO-TEMPERATURE PHYSICAL PHENOMENA

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Abstract

What happens when the temperature reaches absolute zero? Physical phenomena at the zero-temperature limit are studied in accelerator physics. The background temperature of the universe goes down as long as expansion goes on. The BCS resistance of a superconducting cavity is shown as a function of temperature at different frequencies. The surface resistance of the Nb superconducting cavity is reduced to residual resistance and flux-trapped resistance at 0 K. Blackbody radiation is stopped by heat radiation at 0 K. Thermal expansion and thermal diffusion become zero at 0 K. Black holes evaporate at 0 K.

INTRODUCTION

Superconducting cavities are operated at low temperatures with various RF frequencies. The superconducting cavities were studied with field emission effect [1, 2] and magnetic heating effect [3]. The size effect of thermal radiation and generalized thermal radiation were investigated [4-6]. Physical phenomena depend on temperature. In this research, we show zero-temperature phenomena. Photon pressures from Planck temperature and background temperature, BCS resistance from Nb superconducting cavities, energy density from thermal radiation, thermal expansion, thermal diffusion, and black hole evaporation are investigated.

ZERO-TEMPERATURE PHENOMENA

As far as we understand the beginning of the universe, the temperature of the initial universe, the Planck temperature, or the highest temperature, can be expressed as

$$T_P = \sqrt{\frac{\hbar c^5}{G k_B^2}} \tag{1}$$

where $\hbar$ is the reduced Planck constant, G is the gravitational constant, and c is the speed of light. The Planck temperature is $T_P = 1.417\times10^{32}$ K. The Planck length is $l_P = \sqrt{\frac{\hbar G}{c^3}} = 1.616\times10^{-35}\,m$ and the Planck time is $t_P = \sqrt{\frac{\hbar G}{c^5}} = 5.39\times10^{-44}\,s$.

Photon pressure for the Planck temperature and the Planck length in the beginning of the universe can be expressed as

$$P_P = \frac{8\pi^5 k_B^4}{15h^3c^3}\left(\frac{T_P^4}{V_P}\right) \tag{2}$$

The photon pressure of Eq. (2) is $P_P = 7.2\times10^{11}$ Pa.

Photon pressure for the background temperature and expanded universe volume at the current time can be expressed as

$$P_{uni} = \frac{8\pi^5 k_B^4}{15h^3c^3}\left(\frac{T_{bac}^4}{V_{uni}}\right) \tag{3}$$

The photon pressure of Eq. (3) is $P_{uni} = 1\times10^{-92}$ Pa. The superconducting cavity has a very high-quality factor, and it is operated at 2 K, which is lower than the Nb critical temperature. The generalized surface resistance of the superconducting cavity can be expressed as

$$R_{\rm Sur} = R_{\rm BCS} + R_{\rm Res} + R_{Flux} + R_{FE} \tag{4}$$

where $R_{\rm BCS}$ is the BCS resistance, the $R_{\rm Res}$ is the residual resistance, R_{Flux} is the resistance coming from the magnetic flux trap, and R_{FE} is the field emission resistance coming from the field emission effect. When the temperature is less than half of niobium's critical temperature, the BCS surface resistance is approximated to

$$R_{\rm BCS} = \frac{Cf^2}{T}\exp\left(-\frac{\Delta}{k_B T}\right) \tag{5}$$

where T is the temperature in Kelvin, C is the constant depending on the superconducting material, Δ is the band gap of the superconducting material, which represents half the energy required to break a Cooper pair, and f is the resonance frequency of the superconducting cavity. Figure 1 shows the BCS resistance as a function of temperature for different cavity frequencies.

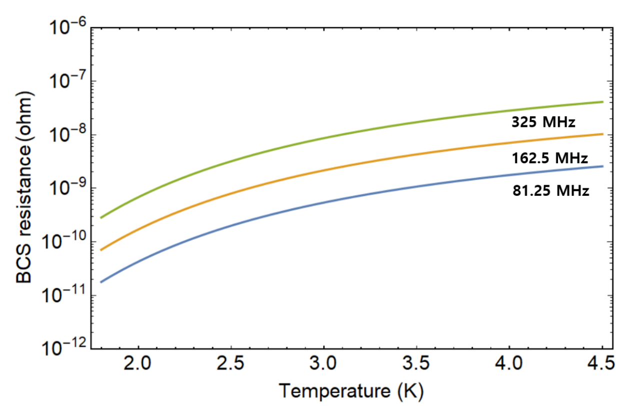

Figure 1: BCS resistance of Nb superconductor for different RF frequencies.

Blackbody radiation depends on temperature and dimension. For one-dimensional blackbody radiation, the energy density is [5]

$$u_{1B}(T) = \left(\frac{2\pi^2}{3}\right)\left[\frac{(k_B T)^2}{(hc)}\right] \tag{6}$$

The energy density for two-dimensional blackbody radiation becomes [6]

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$$u_{2B}(T) = 8\pi\zeta[3]\left[\frac{(k_BT)^3}{(hc)^2}\right] \tag{7}$$

where ζ is the Riemann zeta function.
The energy density for three-dimensional blackbody radiation is

$$u_{3B}(T) = \left(\frac{8\pi^5}{15}\right)\left[\frac{(k_BT)^4}{(hc)^3}\right] \tag{8}$$

Figure 2 shows the energy density of thermal radiation as a function of temperature for different dimensions. Energy density decreases as the temperature decreases. Energy density increases as the space dimension increases.

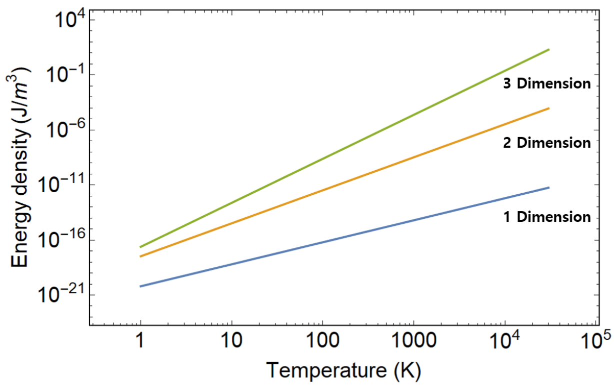

Figure 2: The energy density of thermal radiation is shown as a function of temperature for different dimensions.

Asymmetric potential makes thermal contraction as the temperature decreases. Figure 3 shows the potential energy as a function of atomic separation. The increased atom energy leads to increased average atomic spacing.

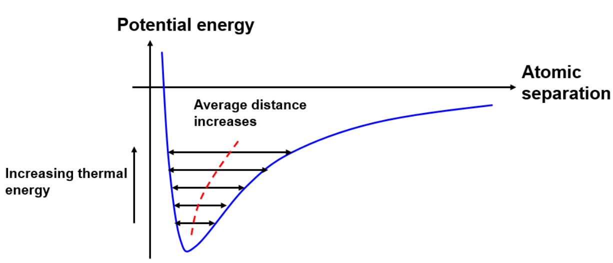

Figure 3: Potential energy vs. atomic separation.

Volume thermal expansion is

$$\alpha_V = \frac{1}{V}\frac{dV}{dT} \tag{9}$$

where V is the volume and α_V is the coefficient of volume thermal expansion. For isotropic solids, the relations of thermal expansion coefficients are $\alpha_A = 2\alpha_L$ and $\alpha_V = 3\alpha_L$, in which α_A is the coefficient of surface thermal expansion and α_L is the coefficient of linear thermal expansion. The linear thermal expansion coefficient decreases as the temperature decreases. As the temperature decreases from 300 K to 2 K, about 95% of the thermal contraction occurs between 300 K and 77 K, and about 5% of the thermal contraction occurs between 77 K and 2 K. The stainless steel of 1 m length is contracted by 3 mm, or 0.3 % from 300 K to 2 K. The Nb of 1 m length is contracted by 1.5 mm, or 0.15 % from 300 K to 2 K.

Fick's first law for steady state diffusion in one dimension is

$$J = -D\frac{dC}{dx} \tag{10}$$

where J is the current density, D is the diffusion coefficient, and C is the concentration.

The diffusion equation for a non-steady state in one dimension can be expressed as

$$\frac{dC}{dt} = D\frac{d^2C}{dx^2} \tag{11}$$

Generalized concentration can be expressed as

$$C(r,t) = \frac{C_o}{(4\pi tD)^{n/2}}\exp\left(-\frac{r^2}{4Dt}\right) \tag{12}$$

where $r^2 = \sum_{i=1}^{n} x_i^2$ and n is the space dimension. Diffusivity depends on the activation energy and temperature as

$$D = D_o e^{-E_a/k_BT} \tag{13}$$

where E_a is the activation energy. Thermal diffusion decreases as its temperature decreases.

Let us show the drift diffusion. The Stokes' force of the water drop and solid body is

$$F = -6\pi\eta Rv \tag{14}$$

where η is the viscosity, R is the radius of the drop, and v is the velocity.
The drift diffusion coefficient for the Stokes' force of the water drop is

$$D = \frac{k_BT}{6\pi\eta R} \tag{15}$$

Mean displacement distance, depending on dimension, diffusion coefficient, and correlation time, can be expressed as

$$<\Delta r^2> = 2nDt \tag{16}$$

where n is the space dimension, D is the diffusion coefficient, r is the distance, and t is the correlation time. The diffusion coefficient becomes zero at absolute zero temperatures. So, the thermal diffusion becomes zero at zero temperature. However, there is still zero-point motion.

The thermodynamics of black holes were studied [7, 8]. When the temperature around the black hole is lower than that of the black hole, the black hole should radiate. There are so many black holes in the universe. But most of them do not radiate because their temperature is lower than the background temperature of the universe, 2.72 K. The radius of the black hole is

$$r_{BH} = \frac{2GM}{c^2} \tag{17}$$

The back hole temperature is

$$T_H = \frac{hc^3}{16\pi^2 k_B GM} \tag{18}$$

Blackbody radiation comes from the black hole with

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$$\frac{dE}{dt} = A\sigma T_H^4 \quad (19)$$

where A is the surface area and $\sigma = \frac{2\pi^5}{15}\frac{k_B^4}{c^2 h^3}$.

The black hole mass decreases with blackbody radiation as

$$\frac{dM}{dt} = \frac{4\pi R^2 \sigma T_H^4}{c^2} \quad (20)$$

From Eq. (20), the mass of the black hole decreases with

$$\frac{dM}{dt} = \frac{1}{15360\pi}\left(\frac{\hbar c^4}{G^2 M^2}\right) \quad (21)$$

From Eq. (21), the decay time of the black hole is

$$t = 5120\pi\left(\frac{G^2 M^3}{\hbar c^4}\right) \quad (22)$$

The decay time is proportional to the third power of the mass. The masses for the Sun, Earth, Mars, and Moon are 1.989×10^{30}, 5.972×10^{24}, 6.39×10^{23}, and 7.348×10^{22} kg, respectively. The black hole temperatures for the masses of the Sun, Earth, Mars, and Moon are 6.17×10^{-8}, 0.02, 0.19, and 1.67 K, respectively. Figure 4 shows the black hole evaporation time as a function of mass. The temperature of the black hole is inversely proportional to its mass. The temperature of the light mass is high, so the lifetime of the black hole is short. The light mass of a black hole, whose lifetime is very short, can be produced in very high-energy accelerator. All black holes in the universe radiate at zero temperatures.

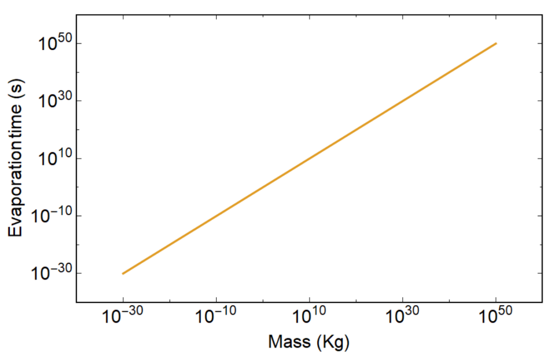

Figure 4: Black hole evaporation time vs. mass.

CONCLUSION

We have shown the physical phenomena of the absolute zero temperature limit in accelerator physics. The photon pressures for Planck temperature and background temperature are calculated. The surface resistance of the superconducting cavity is reduced to the residual resistance and the flux-trapped resistance at 0 K because the BCS resistance goes down to zero as the temperature goes to 0 K. The energy density of blackbody radiation is shown as a function of temperature for different dimensions. Thermal expansion and diffusion are shown in terms of temperature. The evaporation time of black holes is calculated as a function of mass.

REFERENCES

[1] H. Sakai *et al.*, "Field emission studies in vertical test and during cryomodule operation using precise x-ray mapping system", *Phys. Rev. Accel. Beams*, vol. 22, p. 022002, 2019. doi:10.1103/PhysRevAccelBeams.22.022002

[2] S. Jeon *et al.*, "Field emission and x-ray effect on RAON HWR superconducting cavity performance", *Curr. Appl. Phys.*, vol. 38, pp. 67-75, 2022. doi:10.1016/j.cap.2022.03.014

[3] H. Kim, S. Jeon, Y. Jung, J. Kim, "Magnetic Heating Effect for Quarter-Wave Resonator (QWR) Superconducting Cavities", Preprints.org 2023, 2023040905. doi.org/10.20944/preprints202304.0905.v1.

[4] S. J. Yu, S. J. Youn, H. Kim, "Size effect of thermal radiation", *Physica B,* vol. 405, pp. 638-641, 2010. doi.org/10.1016/j.physb.2009.09.079

[5] H. Kim, S. J. Youn, S. J. Yu, "Finite Size Effect of One-dimensional Thermal Radiation", *J. Korean Phys. Soc.*., vol. 56, pp. 554-557, 2010.doi.org/10.3938/jkps.56.554

[6] H. Kim *et al.*, "Generalized thermal radiation from arbitrary fractional dimension", *Infrared Phys. Technol.*, vol. 67, pp. 600-603, 2014. doi.org/10.1016/j.infrared.2014.10.003

[7] J.M. Bardeen, B. Carter, S. W. Hawking, "The four laws of black hole mechanics", *Commun. Math. Phys.* vol. 31, pp. 161-170, 1973.

[8] M. C. LoPresto, "Some Simple Black Hole Thermodynamics", *The Physics Teacher,* vol. 41, 2003.