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Review

A Review of Axion Lasing in Astrophysics

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Abstract: Axions can be stimulated to decay into photons by ambient photons of the right frequency or by photons from the decay of neighboring axions. If the axion density is high enough, the photon intensity can be amplified, which is a type of lasing or an axion maser. Here, we review the astrophysical situations where axion lasing can appear and possibly be detected.

Keywords: axions; laser; maser; parametric resonance; superradiance; Kerr black holes

1. Introduction

The Peccei–Quinn mechanism [1] is of particular interest because it is still viewed as the most credible solution to the strong CP problem. The spontaneous breaking of Peccei–Quinn symmetry implies the existence of the spin-0 pseudo-Goldstone boson referred to as the axion [2], which then becomes a candidate for dark matter. Effective field theory reduces the coupling between axion and Standard Model particles to

$$\frac{\alpha_s}{8\pi f_a} a G^a_{\mu\nu} \tilde{G}^{a\mu\nu} + \frac{\alpha K}{8\pi f_a} a F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{1}{f_a} J^\mu \partial_\mu a \,, \tag{1}$$

where α and α_s are fine structure coefficients of electromagnetic and strong interactions, respectively; a, $F_{\mu\nu}$ and $G_{\mu\nu}$ are fields of the axion, QED and QCD, respectively; $\tilde{F}_{\mu\nu}$ and $\tilde{G}_{\mu\nu}$ are the corresponding dual fields of QED and QCD, respectively; J^μ is the Noether current of the broken symmetry, K is a model dependent coefficient, and the decay constant f_a is the energy scale that suppresses the coupling of axions to Standard Model particles. Axions with f_a of electroweak symmetry breaking scale $v_{\text{weak}} \sim 250$ GeV were ruled out [3,4] by experiments, which left models, namely, KSVZ [5,6] and DFSZ [7,8], predicting invisible axions with $f_a \gg v_{\text{weak}}$. Also, effective field theory [9] determines the product of axion mass m_a and decay constant f_a via $m_a f_a = m_\pi f_\pi = [75.5\ \text{MeV}]^2$, where m_π and f_π are the mass and decay constant of the pion, respectively.

The axion couples to photons via the interaction $\frac{\alpha K}{8\pi f_a} a F_{\mu\nu} \tilde{F}^{\mu\nu}$, which suggests the axion lifetime is

$$\tau_a = \frac{256\pi^3}{K^2 \alpha^2 m_a} \left(\frac{f_a}{m_a} \right)^2 \tag{2}$$

where model dependent coefficient K could be on the order of 1 to 10 [10]. The lifetime (2) of the axion could exceed the age of the universe if $m_a \lesssim$ a few eV. This coupling also gained astrophysical interest because of axion–photon conversion in external magnetic fields [11] and can become a strategy for detecting axions. Extensive reviews and references to most of the early work on axions in astrophysics and cosmology can be found in [12–16].

Axions can be stimulated to decay into two photons by ambient photons of the right frequency or by photons from the decay of other neighboring axions. Formation of axion

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miniclusters and Bose stars from axion self-interaction or gravity are discussed in [17–19]. If the axion density is high enough, the photon intensity can be amplified, which is a type of lasing or masing. There are several astrophysical situations where axion lasing appears possible. These include various dense dark matter axion cluster [20,21] configurations and axions produced by superradience around primordial black holes (pBHs), also known as the black hole lasers powered by axion superradiant instabilities (BLASTs) [22]. We review and summarize the current possibility of detecting these astrophysical lasers.

2. Axion Masers

2.1. Maser Luminosity

Tkachev [20] examined the possibility that the growing axion density in the center of a gravitational well gives rise to a coherent cosmic maser source through the stimulated $a \to \gamma\gamma$ process during the galaxy formation era. For one model, upon collapse of the irregularities in the axion medium, a substantial luminosity develops: $L \sim m_a^3 \Delta p \, r_{\rm p}^2 \exp(D)$, where $r_{\rm p}$ is a gravity-related distance, Δp is the uncertainty in axion momentum, and D is the integrated amplification factor, as defined below. The luminosity L is comparable with that of the brightest quasars, provided $D \sim 10^2$, which is attainable if $M < 10^8[(10^{12}\text{GeV})/f_a]^4 M_\odot$, where M is the mass of irregularities and $M_\odot$ is the solar mass.

Assuming that cosmological strings [23] are the seeds of gravitational condensation, and considering the formation of spherically symmetric axionic configuration due to the velocity dispersion, phase space density (occupation number) f_a is isotropic within the "core" region [24]. If the photon phase space density is far less than the axion phase space density ($f_\gamma \ll f_a$) and the gravitational field is weak, then for a wide class of axion distribution f_a, the photon phase space density f_γ evolves according to

$$\frac{df_\gamma}{dt} = \frac{\pi^2 \Gamma_a}{m_a^4 \beta} \epsilon (1 + f_\gamma), \tag{3}$$

where

$$\epsilon = m_a \int f_a(\vec{p}, \vec{r}) \frac{d^3p}{(2\pi)^3} \tag{4}$$

is the axion energy density, and β is the axion escape velocity characterizing the depth of the gravitational well. f_γ increases exponentially with time if $D > 1$ is satisfied, where the amplification coefficient D is introduced as

$$D(r) = \int_0^r \frac{d}{dt} f_\gamma(r') dr' \,. \tag{5}$$

For a self-gravitating system, the amplification becomes [24]

$$D \sim \frac{f_\pi}{r_c f_a} \,, \tag{6}$$

where r_c is the radius of the core region. $D > 1$ gives [24]

$$r_c < 10^7 \beta \frac{10^{12}\ \text{GeV}}{f_a}\ \text{cm} \,. \tag{7}$$

If the characteristic energy scale of the symmetry breaking during phase transitions in the early Universe is $\sqrt{\mu}$, then the linear mass density of cosmic strings is μ, and the network of strings produces oscillating loops of size $r_s \sim ct$ at time t. A spherical shell of radius r_s and mass $2\pi r_s \mu$ may approximate the gravitational effect of the string loop [25]. If axions initially were inside the loop and form the core, then they can constitute galactic halos [25] and maintain the energetics of the core. A rapidly moving string alters axion trajectories, which suggests that if r_c is equal to the loop size r_s, this may violate the stimulated emission condition (7). But the effect of higher multipole moments is small in distant regions [26], and axions are not disturbed by string motion; and, if the core formed by axions can reach a smaller radius, then the enhancement coefficient (6) is still a valid choice. Thus, this region can provide a luminosity of [24]

$$L_0 \sim 10^{-15} M_{\rm pl}^2 \frac{m_s}{M_\odot} \left(\frac{1\ {\rm yr}}{t_c}\right)^{\frac{1}{3}} , \tag{8}$$

where t_c is the recollapse time for the axions and $M_{\rm pl}$ is the Planck mass.

Axion accretion on strings may result in a core with a very high stimulated emission rate or a quasar-like energy release. A pure axionic object produces a monochromatic spectrum with frequency centered at $m_a/2$. To produce the complicated spectrum of quasars, the axionic core in a gravitational well may be surrounded by ordinary (baryonic) matter [20]. If one cannot identify certain lines in spectra with any molecular or atomic level, then the axion mass can be determined [24]. Odd-integer harmonics of the fundamental frequency can also be produced by interactions $3a \to 2\gamma$ at a much lower magnitude [27].

Tkachev [28] proposes that the properties (energy release, duration, event rate) of FRB can be matched with the explosive maser effect of axion miniclusters or the decay of axions in an external magnetic field. Related effects have been found in localized axion–photon states in a strong magnetic field, e.g., there can be a back reaction from the axion field and electromagnetic waves on the external magnetic field. Guendelman [29] takes this back reaction into account on a strongly time-independent external magnetic field and finds a localized axion–photon soliton.

2.2. Parametric Resonance

Levkov et al. [30] have developed a quasistationary formalism of parametric resonance in a finite volume for nonrelativistic axions, incorporating (in)coherence, finite-volume effects, axion velocities, binding energy, gravitational redshift, and back reaction of photons on axions. Axions with large occupation numbers are described by a classical field $a(t,\vec{x}) = \frac{f_a}{\sqrt{2}}[\psi(t,\vec{x})e^{-im_a t} + \text{h.c.}]$ and affected by a potential

$$V = \frac{m_a^2}{2}a^2 - \frac{g_4^2 m_a^2}{4! f_a^2}a^4 + \ldots\ . \tag{9}$$

If λ_a is the wavelength of the axion, then the nonrelativistic approximation reads

$$m_a \lambda_a \gg 1\ ,\ \partial_t \psi \sim \psi/(m_a \lambda_a^2)\ ,\ \partial_{\vec{x}} \sim \psi/\lambda_a\ . \tag{10}$$

A stationary solution of the Schrödinger–Poisson system gives a Bose–Einstein condensate of axions in the ground state of a nonrelativistic gravitational potential Φ, for $\psi = e^{-i\omega_s t}\psi_s(r)$, where $\omega_s < 0$ is the binding energy of axions. Since the phase of ψ_s is independent of spacetime, the axions are coherent [30]. Consider the electromagnetic potential A along an arbitrary z-direction,

$$A_i = C_i^+ e^{im_a(z+t)/2} + C_i^- e^{im_a(z-t)/2} + \text{h.c.} \tag{11}$$

Substituting A_i into Maxwell's equations shows that the electromagnetic field changes fast, $\partial_t C \sim C/\lambda$, which prompts adiabatically the ansatz [30],

$$C_i^\pm = e^{\int^t \mu(t')dt'} c_i^\pm(t,\vec{x})\ , \tag{12}$$

where μ and the quasistationary amplitudes and $c_i^\pm$ evolve on the same time scales $m_a\lambda^2$ as ψ. Static homogeneous axions in an infinite volume give $\partial_z c_i^\pm = 0$ and $\mu_\infty = g m_a|\psi|$, where $g \sim \alpha K$ is the dimensionless axion–photon coupling. Let L be the size of an axion cloud; photons accumulate if $\mu_\infty L \gtrsim 1$.

Assuming that ψ is real, which means the axions are static and coherent, there exist localized solutions with

$$c_i^+|_{z\to+\infty} = c_i^-|_{z\to-\infty} = 0\ ,\ \mathrm{Re}\ \mu \geq 0\ , \tag{13}$$

representing resonance instabilities. Defining $D(z)$ as

$$D(z) = g m_a \int_{-\infty}^{z} dz' \psi(z')\ , \tag{14}$$

parametric resonance along the arbitrary z-direction corresponds to $D(+\infty) \geq \frac{\pi}{2}$. Resonance starts with a small exponent $\mu \ll L^{-1}$ immediately after the condition (14) is met, which means initial growth is tiny. When the electromagnetic amplitude become large, the back reaction will cause the resonant flux to fall immediately, but a long-lived quasistationary electromagnetic field could appear, causing a glowing axion star to be formed [30].

Levkov et al. [30] argue that the Bose stars with $D(+\infty) \ll 1$ are better amplifiers (stimulated decay) than diffuse axions when an external radio wave of frequency $\sim m_a/2$ travels through the axions. For a diffuse axion cloud, photon fluxes could be amplified but exponential growth is not expected. The case of collapsing axion stars is also investigated in [30]: the star initially contracts without an electromagnetic effect, and growth of the luminosity begins once the localized solution appears.

Other notable findings in [30] include: the modes with different angular harmonic number l of a spherical axion star grow at rate $\sim \mu$, resonance may develop when two axion stars come close to each other with negligible relative velocity even if resonance does not occur for individual Bose stars, and as expected relative velocities among axions prohibit resonance, etc.

Another important type of astrophysical axion objects are axion Bose condensates, found in the work of Sikivie et al. [31,32] and further analyzed by Hertzberg and Schiappacasse [33], focusing on axion clump resonance of photons. These objects correspond to unstable (resonant) and stable solutions of the Mathieu equation, which is the equation of motion for homogeneous small amplitude axion fields. The electromagnetic modes have a maximum exponential growth rate of [33]

$$\mu_H^* \approx \frac{\alpha K}{8\pi f_a} m_a a_0\ , \tag{15}$$

in the first resonant region, where a_0 is the amplitude of homogeneous axion oscillation $a(t)$. Note that μ_H^* in [33] is comparable with μ_∞ in [30].

The homogeneous axions may eventually become unstable and collapse/condensate towards an axion clump from gravity and attractive self-interactions. A spherically symmetric axion clump was found [34] to be accurately approximated by

$$\psi = \sqrt{\frac{3N}{\pi^3 R^3}}\ \mathrm{sech}\left(\frac{r}{R}\right) e^{-i\mu t}\ , \tag{16}$$

where R is the radius of the clump, μ serves as a correction to the frequency $m_a/\hbar$, and $N = \int d^3x \psi^* \psi$ is the particle number of axions. A clump of N axions cannot resonate if $N < N_c$, where N_c is a critical value of the particle number. In the case of attractive axion self-interactions, there is a maximum particle number $N_{\max}$ in an axion clump [33],

$$N_{\max} \approx \frac{10.12 f_a}{|g_4|\sqrt{G} m_a^2}\ . \tag{17}$$

If a clump of QCD axions has $N_{\rm max}$ and attractive self-interactions and $g_4^2 = 0.3$ (the preferred value for conventional QCD axions) [9], there would be no resonance due to axion–photon coupling coefficient K being too small in current models (however, resonance of hidden sector photons could still occur [33]). The general criteria for clump resonance is that a pair of photons being produced has to stimulate another pair of photons before escaping the axion clump, which is succinctly expressed as

$$\mu_H^* \approx \frac{\alpha K}{8\pi f_a} m_a a_0 > \frac{c}{2R}\,, \quad \text{where} \quad a_0 = \sqrt{\frac{2}{m_a}}\sqrt{\frac{3N}{\pi^3 R^3}}\,, \tag{18}$$

i.e., the homogenous axion field growth rate must be higher than the photon escape rate [33]. A typical growth time for a spherical clump of QCD axions with $m_a = 10^{-5}$ eV is estimated to be $1/\mu^* \lesssim 2 \times 10^{-4}$ s, which is similar to the duration ($\sim$ms) of pulses calculated in [22].

A non-spherical clump profile was found to be accurately approximated by a modified Gaussian [35], which leads to a maximally allowed number $N_{\rm max}$ of particles in the clump being larger than that of a spherical clump. $N_{\rm max}$ of a non-spherical clump increases rapidly with angular momentum l, and thus, it is easier to achieve resonance of photons. The condition for resonance that $\mu_H^* > \frac{c}{2R}$ carries over from spherical clumps to non-spherical axion configurations. For conventional QCD axions, resonance does not occur in non-spherical clumps unless the angular momentum is very large $l \gtrsim \mathcal{O}(10^3)$ [33].

A scenario of clump resonance in astrophysics would be: clumps formed under gravity in the past with particle number $N > N_c$ resonate into photons and lose energy individually, which inevitably results in $N < N_c$. Clumps with $N < N_c$ merge together and become a clump with $N > N_c$, which could still resonate today [33].

3. Lasing Axions as Particles

3.1. Spontaneous Emission

Kephart and Weiler [21] conservatively proposed that the luminosity of an axion cluster is comparable to that of an astrophysical object (star, galaxy, or galaxy cluster) of similar mass, considering solely the mechanism of spontaneous decay of the axion. Let $N_a \sim 10^{66}(M/M_\odot)[(1\text{ eV})/m_a]$ be the total number of axions in a cluster of mass M, where $L_\odot$ is the solar luminosity. The luminosity of photons L from spontaneous emission of the cluster is then

$$L = \frac{m_a N_a}{\tau_a} \sim 4 \times 10^{29}\left(\frac{m_a}{1\text{ eV}}\right)^5 \frac{M}{M_\odot}\frac{\text{erg}}{\text{s}} \sim 10^{-4}\left(\frac{m_a}{1\text{ eV}}\right)^5 \frac{M}{M_\odot} L_\odot\,. \tag{19}$$

If an axion cluster is at a distance of $D = 300h^{-1}(z_c/0.1)$ from the earth, the flux observed at the earth becomes

$$F = \frac{L}{4\pi D^2} = 4.7 \times 10^{-16} h^2 f(z_g) \frac{r}{1\text{ pc}}\left(\frac{0.1}{z_c}\right)^2 \left(\frac{m_a}{1\text{ eV}}\right)^5 \frac{\text{W}}{\text{m}^2}\,, \tag{20}$$

where z_c is the cosmological redshift, h is related to the present Hubble parameter H_0 by $H_0 = 100h$ km/s Mpc, r is the radius of the cluster, and $f(z_g)$ describes the surface gravitational red shift through $f(z_g) \equiv z_g(2+z_g)/(1+z_g)^2 = 2GM/r$. Suppose that a mass fraction x_a of a galactic halo is axionic; the spectral photon radiance of the Milky Way and the spectral photon irradiance of Andromeda are then found to be

$$\frac{dI}{d\Omega\Delta\lambda} \sim 10^4 x_a \left(\frac{m_a}{1\text{ eV}}\right)^5 (\text{cm}^2\cdot\text{s}\cdot\text{sr}\cdot\text{Å})^{-1} \quad \text{and} \qquad \frac{I}{\Delta\lambda} \simeq 20 x_a \left(\frac{m_a}{1\text{ eV}}\right)^5 (\text{cm}^2\cdot\text{s}\cdot\text{Å})^{-1}\,, \tag{21}$$

respectively. Comparing these with the background photons of the relevant spectrum in both galaxies, one can find lower bounds on x_a that allow axions to be detectable.

3.2. Stimulated Emission Rate Equations

For any species of particles with occupation number $f(\vec{p},\vec{r},t)$, the particle number density $n(\vec{r},t)$ and the total particle number N in a volume V are, respectively,

$$n(\vec{r},t) = \int \frac{d^3p}{8\pi^3}\, f(\vec{p},\vec{r},t) \text{ and } N = \int_V d^3r\, n(\vec{r},t). \tag{22}$$

The rate of change of the photon number density n_λ of helicity $\lambda = \pm 1$ due to the decay process $a \to \gamma\gamma$ is given by the Boltzmann equation,

$$\frac{dn_\lambda}{dt} = \int dX^{(3)}_{\text{LIPS}}[f_a(1+f_{1\lambda})(1+f_{2\lambda}) - f_{1\lambda}f_{2\lambda}(1+f_a)]|M(a\to\gamma\gamma)|^2\,, \tag{23}$$

where $f_a(\vec{p})$, $f_{i\lambda} = f_\lambda(\vec{k}_i)$ are occupation numbers of axion and photon, respectively. $M(a \to \gamma\gamma)$ is the decay amplitude of the coupling term $\frac{\alpha K}{8\pi f_a} a F_{\mu\nu}\tilde{F}^{\mu\nu}$, and $dX^{(3)}_{\text{LIPS}}$ is the Lorentz invariant three-body phase space of the axion and two photons. Assuming spherical symmetry $f_a(\vec{p}) = f_a(|\vec{p}|) \equiv f_a(p)$, $f_\lambda(\vec{k}) = f_\lambda(|\vec{k}|) \equiv f_\lambda(k)$ for both the axion and photon occupation numbers, one could carry out the integration over the momentum space of one of the photons,

$$\frac{dn_\lambda}{dt} = \frac{m_a\Gamma_a}{2\pi^2}\int_{m_a} dp^0 \int_{k_{\min}}^{k_{\max}} dk\{f_a(p^0)[1+2f_\lambda(k)] - f_\lambda(k)f_\lambda(p^0-k)\}\,, \tag{24}$$

where

$$k_{\max/\min} = \frac{m_a^2}{2(p^0 \mp p)} = \frac{p^0 \pm \sqrt{(p^0)^2 - m_a^2}}{2} \tag{25}$$

is the maximum/minimum photon momentum from the decay of an axion of momentum $\vec{p}$. The rate of change of the axion density is

$$\frac{dn_a}{dt} = -\frac{1}{2}\sum_{\lambda=\pm}\frac{dn_\lambda}{dt}\,, \tag{26}$$

which, together with Equations (22) and (24), become a set of integro-differential equations for the evolution of the axion–photon system [36]. For the approximately static clusters to which the formalism has been applied, it is appropriate to treat the axions as particles [37].

3.3. A Simple Axion Cluster Model

A simple model [36] of lasing axion clusters assumes the following forms

$$f_a(p,r,t) = f_a\Theta(p_{\max} - p)\,\Theta(R-r)\,,\, f_\lambda(k,r,t) = f_\lambda\Theta(R-r)\Theta(k_+ - k)\Theta(k - k_-)\,, \tag{27}$$

for axion and photon occupation numbers, respectively, where f_a and f_λ depend on time only, R is the radius of the cluster, $p_{\max} = m_a\beta$ is the maximum axion momentum, β is the maximum (escape) velocity of an axion, and $k_\pm = \frac{m_a\gamma}{2}(1\pm\beta)$ is the maximum/minimum photon momentum from the decay of an axion with the maximum momentum $m_a\beta$. Integration of Equation (24) over p and k space leads to

$$\frac{dn_\lambda}{dt} = \Gamma_a\left[n_a\left(1 + \frac{16\pi^2 n_\lambda}{\beta m_a^3}\right) - \frac{32\pi^2 n_\lambda^2}{3m_a^3}\left(\beta + \frac{3}{2}\right)\right] - \frac{3cn_\lambda}{2R}\,, \tag{28}$$

where the last term accounts for the surface loss of the photons. The rate of change of the axion number density is half of that of the photon number density without surface losses,

$$\frac{dn_a}{dt} = -\frac{1}{2}\Gamma_a \sum_{\lambda=\pm} \left[n_a \left(1 + \frac{16\pi^2 n_\lambda}{\beta m_a^3} \right) - \frac{32\pi^2 n_\lambda^2}{3m_a^3}\beta \right] , \tag{29}$$

where we subtract the rate of production of axions, with velocity exceeding β from back reaction $\gamma\gamma \to a$. Since these fast axions escape from the cluster, they are labeled as "sterile", with a production rate

$$\frac{dn_s}{dt} = \frac{1}{2}\Gamma_a \sum_{\lambda=\pm} \frac{16\pi^2}{m_a^3} n_\lambda^2 . \tag{30}$$

If one assumes a common initial condition $n_+(0) = n_-(0)$, then helicity densities $n_\pm$ evolve identically, and $n_+(t) = n_-(t) = \frac{1}{2}n_\gamma(t)$ for all time. Substituting n_γ for $n_\pm$ results in equations irrespective of helicity,

$$\begin{aligned} \frac{dn_\gamma}{dt} &= 2\frac{1}{\tau_a} n_a + \frac{16\pi^2}{\beta m_a^3 \tau_a} n_a n_\gamma - \frac{16\pi^2}{3m_a^3 \tau_a}\left(\beta + \frac{3}{2}\right) n_\gamma^2 - \frac{3c}{2R} n_\gamma , \\ \frac{dn_a}{dt} &= -\left(\frac{1}{\tau_a} n_a + \frac{8\pi^2}{\beta m_a^3 \tau_a} n_a n_\gamma - \frac{8\pi^2 \beta}{3m_a^3 \tau_a} n_\gamma^2 \right) , \quad \frac{dn_s}{dt} = \frac{4\pi^2}{m_a^3 \tau_a} n_\gamma^2 . \end{aligned} \tag{31}$$

Expressing time and volume in the units of spontaneous axion lifetime τ_a and Compton volume $\frac{16\pi^2}{m_a^3}$, the set of evolution equations become dimensionless [36,37],

$$\dot{\bar{n}}_\gamma = 2\bar{n}_a + \frac{1}{\beta}\bar{n}_a\bar{n}_\gamma - \left(\frac{\beta}{3} + \frac{1}{2}\right)\bar{n}_\gamma^2 - \frac{3c\tau_a}{2R}\bar{n}_\gamma,\ \dot{\bar{n}}_a = -\bar{n}_a - \frac{1}{2\beta}\bar{n}_a\bar{n}_\gamma + \frac{\beta}{6}\bar{n}_\gamma^2,\ \dot{\bar{n}}_s = \frac{1}{4}\bar{n}_\gamma^2 , \tag{32}$$

where $\bar{n}_a$, $\bar{n}_\gamma$, and $\bar{n}_s$ are dimensionless number densities, and the dot derivative is with respect to dimensionless time t/τ_a. The characteristic time for lasing is

$$\bar{\tau}_{\rm lase} \equiv \tau_{\rm lase}/\tau_a = \left(\frac{\bar{n}_a(0)}{\beta} - \frac{3c}{2R} \right)^{-1} . \tag{33}$$

3.4. Parameters and Conditions

The axion cluster has an escape velocity of $\sqrt{2GM/R}$, which is set to be the β parameter in the axion lasing model. Axions with higher speed would not be confined by the cluster body of mass M,

$$\beta = \sqrt{\frac{2GM}{R}} = 1.8 \times 10^{-3} \sqrt{\frac{\rho_a}{\rm g/cm^3} \frac{R}{R_\odot}} , \tag{34}$$

where $\rho_a = m_a^4 \bar{n}_a/(16\pi^2)$ is the mass density of the axion cluster in units of $\rm g/cm^3$, and $R_\odot$ is the solar radius.

An estimate of the necessary values of axion cluster parameters for lasing can be found by requiring the photon mean free path to be less than the diameter of the cluster, which gives

$$\left(\frac{m_a}{\rm eV} \right)^2 \frac{\rho_a}{\rm g/cm^3} \geq \frac{6}{K^4} . \tag{35}$$

Another estimation of the values of these parameters for small t neglects the back reaction term in Equation (31) and demands the coefficients of n_γ on the right-hand side of the equation be positive,

$$\frac{16\pi^2 n_a}{\tau_a m_a^3 \beta} - \frac{3c}{2R} > 0\,. \tag{36}$$

These two lasing conditions reproduce each other up to a factor of 3 and provide a lower bound for the density of the lasing axion cluster. Keeping $\beta \lesssim 1/2$ avoids the consideration of black hole effects and provides an upper bound for the density of a lasing cluster at a fixed radius. Considering both bounds places the cluster density in the range [36,37],

$$\frac{6}{K^4}\left(\frac{\text{eV}}{m_a}\right)^2 \leq \frac{\rho_a}{\text{g/cm}^3} \leq 7.7\times 10^4\left(\frac{R_\odot}{R}\right)^2. \tag{37}$$

The maximum radius for lasing to occur is obtained by saturating both bounds,

$$R \lesssim 100K^2\frac{m_a}{\text{eV}}R_\odot\,, \tag{38}$$

which also leads to the maximum mass of the cluster

$$M \leq 6\times 10^6 K^2\frac{m_a}{\text{eV}}M_\odot\,, \tag{39}$$

There are no lower bounds for R and M.

The eventual lasing is inevitable, as it needs only a few photons to trigger lasing, regardless of the initial photon density $\bar{n}_\gamma(0)$ of the cluster, which regulates the timing of the laser profile. Based on the ratio $R/(c\tau_a)$ between the photon diffusion time R/c and the axion life time τ_a, one can write the initial photon density from spontaneous emission of an axion cluster as

$$\bar{n}_\gamma(0) = 320\left(\frac{m_a}{\text{eV}}\right)K^2\beta^2\left(\frac{R_\odot}{R}\right). \tag{40}$$

Photons cannot propagate if the frequencies are below the plasma frequency (due to high electron density in the early universe), which might inhibit axion decay. It has been shown [36] that axion decay proceeds without inhibition as long as the red shift $z \lesssim 10^9(m_a/\text{eV})^{2/3}$.

3.5. Discussion

The flat distribution functions in the simple axion cluster model are idealizations. Deviations from a flat distribution will give regions of higher density, which will be more active for lasing. Gravitational collapse could trigger formation of a high axion density in the vicinity of an object such as a black hole, where gravitational corrections to the rate equations would have to be included. There are a wide range of collisionless, self-gravitating, spherically symmetric systems where the density and velocity distribution functions are known in closed analytic form [38]. To obtain general rate equations for realistic distributions by integration is straightforward.

If compact objects with a mass larger than $10^7 M_\odot$ in the cores of nearby galaxies were axion clusters, they would not be expected to lase since their mass and size fall outside the allowed parameters for lasing. However, spontaneous axion decay may still generate luminous emissions [21]. Furthermore, if these massive cluster were made of smaller dense clusters, lasing might still be possible for individual small clusters.

After the Universe became matter dominated, the energy released by detonation of a typical hadronic axion [39] cluster allowed for the evacuation of matter and formation of a void by multiphoton ionization and the subsequent plasma absorption [36]. The total energy available in cosmic axions is sufficient to power the formation of all the voids in the Universe.

3.6. Application—Superradiant Clouds

In Kerr spacetime, the Klein–Gordon equation admits hydrogenic-like solutions localized in the vicinity of a BH, characterized by integer quantum numbers (n, l, m). Let $M_{\rm BH}$ and $J_{\rm BH}$ be the mass and angular momentum of the black hole, respectively. In the nonrelativistic regime, the spectrum of the quasibound state is [40]

$$\hbar\omega_n \simeq m_a c^2 \left(1 - \frac{\alpha_\mu^2}{2n^2}\right), \text{ where } \alpha_\mu \equiv \frac{G m_a M_{\rm BH}}{\hbar c}. \quad (41)$$

The scalar field extracts the BH's rotational energy, and the axion cloud around the BH grows if $\omega_R < m\Omega_{\rm BH}$, where ω_R is the real part of the complex frequency $\omega = \omega_R + i\omega_I$, and $\Omega_{\rm BH}$ is the horizon's angular velocity. When $\alpha_\mu \ll 1$, the occupation number of the fastest growing mode "$2p$"$(n = 2, l = m = 1)$ grows at a rate

$$\Gamma_s \simeq 4 \times 10^{-4} \tilde{a} \left(\frac{\mu}{10^{-5}\text{eV}}\right) \left(\frac{\alpha_\mu}{0.03}\right)^8 \text{s}^{-1}, \quad (42)$$

where $\tilde{a} = cJ_{\rm BH}/(GM_{\rm BH}^2)$ is the BH's dimensionless spin parameter $(0 < \tilde{a} < 1)$. In the nonrelativistic regime, the axion cloud is localized far away from the horizon; hence, the formalism developed in [36] is still applicable. Although the geometry of the "$2p$" state is more intricate than the flat, spherically symmetric model, the estimates made by Rosa and Kephart [22] by exploiting Equation (31) are sufficiently good in terms of the order of magnitude. They calculated the peak luminosity, total energy, and duration of the black hole lasers powered by axion superradiant instabilities (BLASTs). They are

$$\begin{aligned}
L_B &\simeq \frac{2 \times 10^{42}}{K^2} \tilde{a} \left(\frac{10^{-5}\text{ eV}}{\mu}\right)^2 \left(\frac{\alpha_\mu}{0.03}\right)^7 \left(\frac{\xi}{100}\right) \text{erg/s}, \\
E_B &\simeq \frac{3 \times 10^{39}}{K^2} \sqrt{\tilde{a}} \left(\frac{10^{-5}\text{ eV}}{\mu}\right)^3 \left(\frac{\alpha_\mu}{0.03}\right)^{\frac{5}{2}} \left(\frac{\xi}{100}\right)^{\frac{1}{2}} \text{erg}, \\
\tau_B &\simeq \frac{1}{\sqrt{\tilde{a}}} \left(\frac{10^{-5}\text{ eV}}{\mu}\right) \left(\frac{\alpha_\mu}{0.03}\right)^{-9/2} \left(\frac{\xi}{100}\right)^{-1/2} \text{ms}, \quad (43) \\
\text{where } \xi &= \log\left(\frac{\Gamma_s}{\Gamma_a}\right) \simeq 107 - 4\log\left(\frac{\mu}{10^{-5}\text{ eV}}\right) + 8\log\left(\frac{\alpha_\mu}{0.03}\right) + \log\left(\frac{\tilde{a}}{K^2}\right).
\end{aligned}$$

Note that the duration τ_B is similar to the growth time for a spherical clump of QCD axions, with $m_a = 10^{-5}$ eV estimated in [33]. The brightest bursts could blow away any interstellar plasma surrounding the BH if L_B exceed the BH's Eddington luminosity. However, the brightest bursts generate a e^-e^+ plasma by Schwinger pair production, which is dense enough to prohibit photon propagation and block axion decay. Lasing may stop after a single laser pulse and restart once the plasma becomes subcritical again (via e^-e^+ annihilations), causing repeating bursts. Local plasma density or temperature may also temporarily block lasing. By angular momentum conservation, one of the photons from each decay satisfies the superradiance condition $\omega_R < m\Omega_{\rm BH}$, while the other is in a nonsuperradiant state, which may decrease the laser luminosity and modify its polarization through the spin-helicity effect [41–43].

The parameters (mass and spin) of an axion cloud have to be smaller than those of its associated BH. This restricts the masses of axion and BH for $\alpha_\mu \lesssim 0.05$ to be $\mu \gtrsim 10^{-8}$ eV, $M_{\rm BH} \lesssim 10^{-2} M_\odot$. The notion that lasing can only occur in the non-relativistic regime is confirmed by $\alpha_\mu \lesssim 0.03K$, which is obtained by requiring that quartic axion self-interactions are not significant. BLASTs can only occur for spinning pBHs [44] from the merger of two nonspinning pBHs.

The predicted peak luminosities and durations of the brightest BLASTs exhibit features similar to the fast radio bursts (FRBs) [45–47] observed in recent years. A particular example is the repeating FRB 121102 [48–51]. Geometrical and curved spacetime effects could prevent the observation of bursts along our line of sight and possibly explain why some FRBs do not repeat. The brightest BLASTs are expected to yield up to $\sim 10^5$ FRBs per day across the whole sky. Lighter pBHs may become continuous laser sources (since Schwinger pair production is less likely to occur in these cases) with lower luminosities, but they should be less numerous than repeating BLASTs due to their shorter lifetime [22]. A similar comparison between the photon signature from superradiant ion clouds and the isotropic gamma-ray background (IGRB) is presented in [52]. For a review of superradiance, see [40]. For a more recent development on this topic from a field theoretic approach, see [53].

3.7. Non-Spherical Cluster and Static Spacetime Modification

The evolution Equations (24) and (31) are based on spherical symmetry $f_a(\vec{p}) = f_a(|\vec{p}|)$, $f_\lambda(\vec{k}) = f_\lambda(|\vec{k}|)$. One considers non-spherically symmetric occupation numbers by allowing dependence on the directions of momenta $\vec{p}$ and $\vec{k}$. Let us write occupation numbers $f_a(\vec{p}) = f_a(p, \Omega_p)$ and $f_\lambda(\vec{k}) = f_a(p, \Omega_k)$ as spherical harmonic expansions,

$$f_a(\vec{p}) = \sum_{lm} a_{lm}(p,t) Y_{lm}(\Omega_p) \ , \quad f_\lambda(\vec{k}) = \sum_{lm} b_{lm}(k,t) Y_{lm}(\Omega_k) \ , \tag{44}$$

where Ω_p and Ω_k are the angular dependences from the directions of momenta $\vec{p}$ and $\vec{k}$, respectively. The evolution equations for individual components b_{lm} of photon occupation number are given by Equation (8) in [54], which is analogous to Equation (24), without imposing spherical symmetry in momentum space.

Many of the equations cited here and elsewhere in this subsection are from [54–57] and so are not reproduced here. We simply provide a guide to them and refer the reader to those works for the details.

The corresponding rate equations for individual components of particle number densities are also obtained as Equations (13)–(15) in [54]. Similarly, one relaxes spherical symmetry in coordinate/spatial space of a simple axion cluster model (27) by expanding occupation numbers $f_a(p, r, \Omega, t)$ and $f_\lambda(k, r, \Omega, t)$ in spherical harmonics [55],

$$\begin{aligned} f_a(p,r,\Omega,t) &= \sum_{lm} f_{alm}(t) Y_{lm}(\Omega) \Theta(p_{\text{max}} - p)\, \Theta(R - r) \ , \\ f_\lambda(k,r,\Omega,t) &= \sum_{lm} f_{\lambda lm}(t) Y_{lm}(\Omega) \Theta(R - r) \Theta(k_+ - k) \Theta(k - k_-) \ , \end{aligned} \tag{45}$$

where Ω is the conventional spatial solid angle. The corresponding rate equations for individual components of particle number densities are then obtained as Equations (19)–(21) in [55], which are later applied to a study [56] of the superradiant growth of non-spherical axions as an update of the earlier analysis [22].

An axion cloud bounded by a host object in static spacetimes is studied, and the rate equations of axion and photon number densities are given as Equations (8.1)–(8.4) in [57]. Compared to the simple axion cluster model in Minkowski spacetime [36], an example [57] shows that the peak number density of photon is about 10% larger and slightly delayed (see Figure 4 in [57]) for a QCD axion ($m_a = 10^{-5}$ eV) cluster of mass $M = 6 \times 10^{21}$ kg a few meters away from the center of a Schwarzschild spacetime produced by a black hole of mass $M_{\text{BH}} = 8 \times 10^{23}$ kg. Climbing out of the Schwarzschild potential well, the associated tidal effects (gravitational redshift) initially slow the decay process, but this wanes quickly and gives a sharper peak signal strength at a delayed time.

4. Comments

We have reviewed two approaches of investigations into the phenomenon of axion lasing, i.e., the field theory methods in Section 2 and the particle perspective in Section 3.

While the field theoretic approach gives the growth rate of photons (e.g., [33]), the particle perspective shows a detailed evolution of the pulses (e.g., [22]). But the results of the two methods are in general agreement in their conclusions.

There are many potential sources of astrophysical axion lasing. Depending on the axion mass and self coupling, these include clumps of axions and axion miniclusters and are limited by the objects' angular momentum. In addition, lasing superradiant clouds of axions around primordial Kerr black holes provides an interesting possibility of discovery when identified with fast radio bursts. If verified, this would constitute the discovery of two components of dark matter, primordial black holes and axions.

More generally, axions and axion-like particles are a compelling class of particles to investigate. They provide numerous and varied possibilities for physics, astrophysics, and cosmological phenomena. They can arise in extensions of the Standard Model, grand unified theories, and string theory. Their discovery would provide considerable insight into aspects beyond standard model physics.

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Abbreviations

The following abbreviations are used in this manuscript:

KSVZ	Kim, Shifman, Vainshtein, Zakharov
DFSZ	Dine, Fischler, Srednicki, Zhitnitsky
BH	black hole
BLASTs	black hole lasers powered by axion superradiant instabilities
pBHs	primordial black holes
FRB	fast radio burst
IGRB	isotropic gamma-ray background

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