1-D and 2-D Seebeck coefficient of a thermal QCD medium with chiral-mode dependent quasiparticle masses

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Introduction

Large fluctuations in the initial energy density in the heavy-ion collisions[1] translate to significant temperature gradients between the central and peripheral regions of the produced fireball, providing the ideal ground to study thermoelectric phenomena. The medium created in such collisions may be exposed to magnetic fields (B) arising from non-central nucleus-nucleus collisions; its strength depending on the time scale of evolution. A weak B leads to the lifting of the degeneracy between left (L-) and right (R-) chiral modes of massless quark flavors. This, in turn, leads to chiral-mode dependent transport coefficients. The thermoelectric response of a hot QCD medium can be quantified by the Seebeck and Nernst coefficients, which can be visualized as a response tensor to the temperature gradient in the medium:

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} S & N|\boldsymbol{B}| \\ N|\boldsymbol{B}| & S \end{pmatrix} \begin{pmatrix} \nabla T|_x \\ \nabla T|_y \end{pmatrix}.$$

The quasifermion mass in weak B is such that the range of B and T values is constrained in such a manner so as to enforce the weak magnetic field ($eB \ll T^2$) condition. Thus, it serves as a consistency check. We also study the impact of the dimensionality of quark dynamics on the Seebeck coefficient, wherein we find that the coefficient magnitude is significantly enhanced in the 2-dimensional setup. As is explained later, a strong background magnetic field ($eB \gg T^2$) makes the fermion dynamics one-dimensional, and can thus be used to probe the effects of dimensional reduction on transport coefficients.

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Quasiquark mass in weak B: Chiral modes

The medium generated masses of quarks is obtained from the zeros of the inverse resummed quark propagator:

$$S^{-1}(K) = S_0^{-1}(K) - \Sigma(K),$$

where, $\Sigma(K)$ is the one-loop quark self-energy in weak B. The bare quark propagator $S_0(K)$ in weak B, up to $\mathcal{O}(q_f B)$, is given by

$$iS_0(K) = \frac{i\,(\not{K})}{K^2 - m_f^2} - \frac{i\gamma_5[(K.b)\not{u} - (K.u)\not{b}]}{(K^2 - m_f^2)^2}(q_f B).$$

with $u^\mu = (1, 0, 0, 0)$ and $b^\mu = (0, 0, 0, 1)$. Diagrammatically, the quark self energy is

$$\Sigma(P) = g^2 C_F T \sum_n \int \frac{d^3k}{(2\pi)^3}\gamma_\mu\left(\frac{\not{K}}{(K^2 - m_f^2)} - \frac{\gamma_5[(K.b)\not{u} - (K.u)\not{b}]}{(K^2 - m_f^2)^2}(|q_f B|)\right)\gamma^\mu \frac{1}{(P-K)^2}$$

In a covariant tensor basis, the above self energy can be expressed as

$$\Sigma(P) = -\mathcal{A}\not{P} - \mathcal{B}\not{u} - \mathcal{C}\gamma_5\not{u} - \mathcal{D}\gamma_5\not{b},$$

The self energy and the full propagator can then be written in terms of projection operators $P_{L/R} = (\mathbb{I} \mp \gamma_5)/2$ as

$$\Sigma(P) = -P_R \not{A}' P_L - P_L \not{B}' P_R$$
$$S(P) = \frac{1}{2}\Big[P_L \frac{\not{L}}{L^2/2} P_R + \frac{1}{2} P_R \frac{\not{R}}{R^2/2} P_L\Big],$$

where, L^2, $R^2 = f(\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D})$. The $p_0 = 0$, $\mathbf{p} \to 0$ limit of the denominator of the effective propagator yields the quasiparticle masses as[2]

$$m_{L^2/R^2} = \frac{L^2/R^2}{2}|_{p_0=0,|\mathbf{p}|\to 0} = m_{th}^2 \pm 4g^2 C_F M^2,$$

thus lifting the degeneracy. Here,

$$M^2 = \frac{|q_f B|}{16\pi^2}\left(\frac{\pi T}{2m_f} - \ln 2 + \frac{7\mu^2\zeta(3)}{8\pi^2 T^2}\right),$$

$$m_{th}^2 = \frac{1}{8}g^2 C_F\left(T^2 + \frac{\mu^2}{\pi^2}\right).$$

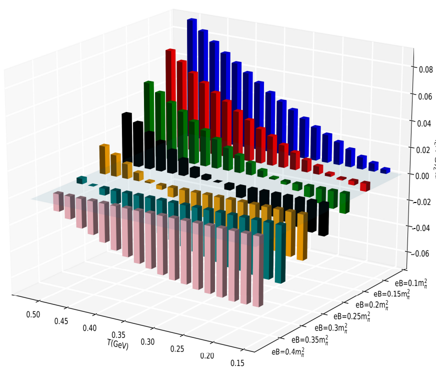

FIG. 1: Medium generated mass (squared) of R mode u quark as a function of T and B.

eB	$\lvert eB/T^2\rvert_{\max}$
0.2 m_π^2	0.050
0.25 m_π^2	0.038
0.30 m_π^2	0.033
0.35 m_π^2	0.027

TABLE I: Maximum value of eB/T^2 that leads to a positive m_R^2, for a given eB.

We see from Fig.(1) that the mass (squared) of the R mode fermion, comes out to be negative for a certain range of combinations of T and B values, and is thus unphysical. As B is increased, the temperature upto which m_R^2 is negative, also increases. This suggests that the perturbative framework used by us to study the chirality dependence of the thermoelectric response is valid only at regions sufficiently far (∼200 MeV) from the crossover region in the QCD phase diagram for $|eB| > 0.2m_\pi^2$. Another way to look at it is that the condition $eB \ll T^2$ is strictly enforced, as can be seen from Table I.

1-D and 2-D Seebeck coefficients

In general, the induced current in the system is expressed as

$$\boldsymbol{J} = \sum_i q_i g_i \int \frac{d^3p}{(2\pi)^3\,\omega_i}\boldsymbol{p}\left[\delta f_i^q(x,p) - \delta f_i^{\bar{q}}(x,p)\right],$$

where, δf is evaluated from the relativistic Boltzmann transport equation within the relaxation time approximation.

$$p^\mu \frac{\partial f_i(x,p)}{\partial x^\mu} + q_i F^{\rho\sigma} p_\sigma \frac{\partial f_i(x,p)}{\partial p^\rho} = -\frac{\delta f_i}{\tau_i}$$

Once $\boldsymbol{J}$ is set equal to zero, we obtain the Seebeck coefficient from the relation $\boldsymbol{E} = S\boldsymbol{\nabla}T$.

In the limit of strong B, the transverse motion of quarks is restricted and the quark dynamic becomes purely longitudinal (1-D), which is seen in the modification of the phase space factor $\int\frac{d^3p}{(2\pi)^3} \to \frac{q_i B}{2\pi}\int\frac{dp_z}{2\pi}$. In the presence of a weak field, however, both the longitudinal and transverse (with respect to $\boldsymbol{B}$) momenta of quarks are non-zero.

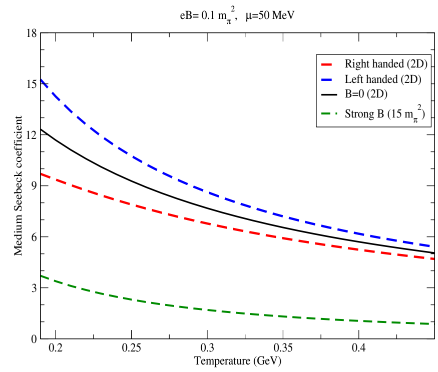

FIG. 2: Variation of individual and total Nernst coefficients with T for $\mu = 50$ MeV, $eB = m_\pi^2$.

As expected, there are 2 modes for the Seebeck coefficient in weak B corresponding to the two quasiparticle modes of different masses[3], whereas only a single one for strong B. The magnitude of the coefficient decreases with T for all values of magnetic fields, and is much enhanced in the presence of weak B (2-D setup).

References

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[2] Pushpa and B. K. Patra, Phys. Rev. D **105**, 116009 (2022).
[3] D. Dey and B. K. Patra, arXiv:2204.06195 (2022).