# $V_{cb}$ FROM INCLUSIVE $b \to c$ DECAYS: AN ALTERNATIVE METHOD [a]

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These proceedings review how reparametrization invariance, a symmetry within the heavy quark expansion (HQE) reflecting Lorentz invariance of the underlying QCD, reduces the number of independent HQE parameters necessary to predict total rate and $q^2$ moments in inclusive semileptonic $B$ decays. An alternative extraction of $V_{cb}$ based on $q^2$-moment measurements at $B$ factories is proposed. This novel method could push the $V_{cb}$ determination up to order $1/m_b^4$ without prior estimates of the higher order terms in the $1/m_b$ expansion and access their size in a model independent way.

## 1 Introduction

The current extraction of $V_{cb}$ from inclusive semileptonic $B$ decays ($B \to X_c \ell \nu$) is based on the possibility to predict the total rate and the spectral moments as a double series in $\Lambda_{\rm QCD}/m_b$ and $\alpha_s$. The moments of the electron energy spectrum and the hadronic invariant mass were measured by BABAR [1,2] and Belle [3,4] and previously by CDF, [5] CLEO [6] and DELPHI. [7]

Using moments of the semileptonic $b$ decay spectra, Gambino *et al.* [8,9,10] performed global fits of $V_{cb}$, the heavy quark masses and the non perturbative parameters of the heavy quark expansion (HQE), obtaining [10]

$$|V_{cb}| = (42.11 \pm 0.74) \times 10^{-3}, \quad (1)$$

whose fractional uncertainty is about 1.8%. They relayed on NNLO perturbative corrections to the partonic rate, as well as NLO corrections to the $1/m_b^2$ terms. They also included at tree-level the HQE parameters up to $1/m_b^5$. Up to $1/m_b^3$ there are only four HQE parameters, starting at $1/m_b^4$ their number grows factorially and increases up to 32 when $1/m_b^5$ corrections are considered. This constitutes a theoretical challenge as the extraction of all HQE parameters in a fully data driven way becomes complicated. Therefore, one has to relay on *a priori* estimates of the expectation value of the parameters at order $1/m_b^4$ and $1/m_b^5$.

The $V_{cb}$ value in (1) was determined estimating local operators of the form $\bar{b}_v iD_{\mu_1} \dots iD_{\mu_N} b_v$ using the lowest-lying state saturation approximation (LLSA), that splits chains of covariant

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derivatives into shorter ones, [11,12]

$$\langle B|\, \bar{b}_v X_1^k X_k^N b_v \,|B\rangle = \frac{1}{2m_B}\sum_n \langle B|\, \bar{b}_v X_1^k b_v \,|H_n\rangle \langle H_n|\, \bar{b}_v X_k^N b_v \,|B\rangle\,, \tag{2}$$

where $H_n$ are hadronic states and $X_i^j = iD_{\mu_i}\dots iD_{\mu_j}$. The LLSA assumes that the sum is saturated by the ground state multiplets $B$ or $B^*$. The fit was performed starting with the LLSA central values (2) and assigning 60% gaussian priors. A sub-percent reduction in $V_{cb}$ was found.

It is thus desirable to confirm the smallness of the higher order terms in the $1/m_b$ expansion in a model-independent approach, also in light of the future experimental precision at the Belle II experiment which might allow a measurement of $V_{cb}$ below 1%. In these proceedings we review the alternative method for the determination of $V_{cb}$ proposed by Fael, Mannel & Vos. [13] The method is based on the measurement of the leptonic invariant mass ($q^2$) moments and the fractional branching ratio as a function of a lower cut on $q^2$. These observables are invariant under reparametrization, a symmetry within the HQE reflecting Lorenz invariance of the underlying QCD, and therefore they depend on a reduced set of HQE parameters, as it was shown for the total rate by Mannel & Vos. [14] The smaller set of parameters necessary in a global fit of these observables (eight instead of 13 up to $1/m_b^4$) opens the possibility to extract $V_{cb}$ in a completely data-driven way, without making use of the LLSA, and thus to independently check and validate the finding of Gambino *et al.* [10] about the size of the higher order terms in the HQE.

## 2 Reparametrization invariance in HQE

The semileptonic decays of a $b$ quark to final states with a charm are due to the weak Hamiltonian

$$\mathcal{H}_W = \frac{4G_F}{\sqrt{2}} V_{cb}\,(\bar{c}\gamma^\mu P_L b)\left(\bar{\ell}\gamma^\mu P_L \nu\right) + \text{h.c.} = \frac{4G_F}{\sqrt{2}} V_{cb} J_q^\mu J_{\ell\mu} + \text{h.c.}, \tag{3}$$

where $P_L$ is the left-handed projector, $J_q^\mu$ and $J_\ell^\mu$ are the hadronic and the leptonic currents, respectively. The rate for the inclusive decay $B(p_B) \to X_c(p_X)\ell(p_\ell)\nu(p_\nu)$ is determined by the hadronic tensor

$$W^{\mu\nu} = (2\pi)^4 \sum_X \delta^4(p_B - q - p_X)\,\langle B|\, J_q^{\dagger\mu} \,|X\rangle \langle X|\, J_q^\nu \,|B\rangle\,, \tag{4}$$

where $q = p_e + p_\nu$. The differential rate can be written as $d\Gamma \propto L_{\mu\nu}(p_e, p_\nu) W^{\mu\nu}(q^2, q\cdot v)$, where $v = p_B/m_B$ is the $B$ meson velocity and $L_{\mu\nu}$ is the lepton tensor. The non-perturbative hadronic tensor $W^{\mu\nu}$ is related via the optical theorem to the imaginary part of the forward scattering amplitude

$$W^{\mu\nu} = 2\,\text{Im}\,\langle B|\, R(S) \,|B\rangle = 2\,\text{Im}\,\langle B|\, i\int d^4x\, e^{-im_b S\cdot x}\, T\left\{\bar{b}_v(x)\gamma^\mu P_L c(x)\,\bar{c}(0)\gamma_\mu P_L b_v(0)\right\} |B\rangle\,, \tag{5}$$

where $S = v - q/m_b$ and $b_v(x) = \exp(im_b\, v\cdot x) b(x)$ is the re-phased $b$-quark field. Then, we perform an operator product expansion (OPE) for the time-ordered product:

$$R(S) = \sum_{n=0}^{\infty} \frac{C^{(n)}_{\mu_1\cdots\mu_n}(S)}{m_b^{n+3}} \otimes \bar{b}_v(iD^{\mu_1}\dots iD^{\mu_n})b_v\,, \tag{6}$$

where the symbol $\otimes$ is a shorthand notation for contraction of spinor indices. Taking the forward matrix element $\langle \bar{b}_v \dots b_v\rangle \equiv \langle B(p)|\bar{b}_v \dots b_v|B(p)\rangle$ , we obtain the hadronic tensor for the inclusive transition $B \to X_c\ell\bar{\nu}$ :

$$W(p,q) = 2\,\text{Im}\sum_{n=0}^{\infty} C^{(n)}_{\mu_1\cdots\mu_n}(S) \otimes \langle \bar{b}_v(iD^{\mu_1}\dots iD^{\mu_n})b_v\rangle\,, \tag{7}$$

where we suppressed the Lorentz indices for simplicity.

The hadronic tensor (4), as well as its OPE in (6), do not depend on $v$ as long as all orders in the OPE are taken into account. This means that both are invariant under the reparametrization (RP) transformation $\delta_{\rm RP}$ that shifts $v_\mu \longrightarrow v_\mu + \delta v_\mu$. One can show that the invariance under reparametrization (RPI), which dictates $\delta_{\rm RP} R(S) = 0$, connects subsequent orders in the $1/m_b$ series of Eq. (6). This generates relations between the coefficients $C$ at order $n$ and $n+1$: [14]

$$\delta_{\rm RP} C^{(n)}_{\mu_1\dots\mu_n}(S) = m_b\, \delta v^\alpha \left[ C^{(n+1)}_{\alpha\mu_1\dots\mu_n}(S) + C^{(n+1)}_{\mu_1\alpha\dots\mu_n}(S) + \dots + C^{(n+1)}_{\mu_1\dots\mu_n\alpha}(S) \right]. \tag{8}$$

The hadronic matrix elements $\langle \bar b_v (iD_{\mu_1} \cdots iD_{\mu_n}) b_v \rangle$ can be expressed in terms of scalar matrix elements, such as the kinetic energy parameter $\mu_\pi^2$ and the chromomagnetic parameter $\mu_G^2$ at $n = 2$. However the number of independent parameters grows factorially in the $1/m_b$ expansion (at tree level there are nine and 18 at order $1/m_b^4$ and $1/m_b^5$, respectively [11,15]) and therefore their extraction from data becomes challenging already at order $1/m_b^4$. Because of RPI the total rate depends only on a reduced set of HQE parameters, which are given by fixed linear combination of the matrix elements defined for the general case.[14] Up to order $1/m_b^4$ there are only eight independent parameters at tree level (for their explicit definitions see Mannel *et al.* [13,14]).

## 3 Observables invariant under reparametrization

The $e^+e^-$ colliders measure moments of decay spectra rather than differential rates. We can defined them in a generic way as the phase-space integration of the differential rate multiplied by an appropriate weight function $w$ to some power $n$:

$$\langle M^k[w] \rangle = \int d\Phi\, w^k(v, p_e, p_\nu)\, W^{\mu\nu} L_{\mu\nu}. \tag{9}$$

Spectral moments of the charged lepton energy $E_\ell$ and the hadronic invariant mass $M_X^2$ are obtained by setting $w(v, p_e, p_\nu) = v \cdot p_e$ and $w(v, p_e, p_\nu) = (m_B v - q)^2$, while the moments of the leptonic invariant mass $q^2$ correspond to the weight function $w(v, p_e, p_\nu) = q^2$. In analogy to $R(S)$, we assume that the moment $M$ has an OPE of the form:

$$M^k[w] = \sum_{n=0}^{\infty} \frac{a^{(n)}_{\mu_1\dots\mu_n}}{m_b^{n+3}} \otimes \bar b_v (iD^{\mu_1} \dots iD^{\mu_n}) b_v. \tag{10}$$

Performing a RP transformation in (10), we obtain a similar tower of relations between the coefficients at order $n$ and at order $n+1$, as for the total rate. The key observation is that for RPI weight functions $\delta_{\rm RP} w(v, p_e, p_\nu) = 0$. In this case the relations among the coefficients $a^{(n)}$ are the same relations as for the total rate. Therefore observables that are invariant under reparametrization can be expressed in terms of the reduced set of HQE parameters. For the semileptonic decays the moments of the leptonic invariant mass ($q^2$) have this property, since the corresponding weight function is independent on the velocity $v$. On the contrary electron energy moments and moments of the hadronic invariant mass are not RPI and so they depend on the full set of operators.

## 4 Extracting $V_{cb}$ from $q^2$ moments

Given that $q^2$ moments depend on the reduced set of HQE parameters, we propose a novel strategy for the determination of $V_{cb}$ from inclusive semileptonic $B$ decays. The method is identical to the approach of Gambino *et al.* however it is based only on the measurement of $q^2$ moments,

$$\langle (q^2)^n \rangle_{q^2_{\rm cut}} \equiv \int_{q^2_{\rm cut}}^{(m_b - m_c)^2} dq^2\, (q^2)^n \frac{d\Gamma}{dq^2} \Bigg/ \int_{q^2_{\rm cut}}^{(m_b - m_c)^2} dq^2 \frac{d\Gamma}{dq^2}, \tag{11}$$

and also on the fractional branching ratio $R^* = \Gamma_{q^2>q^2_{\rm cut}}/\Gamma_{\rm tot}$. Their lengthy expressions are attached to the arXiv version of our original paper,[13] where we computed them and verified explicitly that indeed they depend on the reduced set of operators since the redundant ones cancel out in the final results. A cut on $E_e$ in (11) would have broken explicitly the RPI of the $q^2$ moments, with the consequence of reintroducing the complete set of parameters. For this reason, we introduced in the definition (11) a lower cut on $q^2$ instead of $E_e$ because in this way we preserve the RPI property of these observables.

The $V_{cb}$ fits performed by Gambino *et al.*[16,8,9] make use of the electron energy and the hadronic mass moments, including a cut on the electron energy. In fact, the moments up to $n = 4$ and their computable cut-dependence allow for a fully data-driven analysis up to $1/m_b^3$, which means that $V_{cb}$, the quark masses as well as the HQE parameters $\mu_\pi^2, \mu_G^2, \rho_D^3$ and $\rho_{LS}^3$ can be fitted from data. Accessing higher orders in the $1/m_b$ expansion requires to model the HQE parameters of order $1/m_b^4$ and $1/m_b^5$ with the LLSA.[11,12]

We therefore propose to determine $V_{cb}$ from the $q^2$ moments with a possible additional dependence on a lower $q^2$ cut. Since we need eight HQE parameters up to $1/m_b^4$ instead of 13, precise inputs from the $q^2$ spectrum measured at Belle/Belle II would allow us to perform a fully data driven analysis, i.e. an extraction of $V_{cb}$ up to $1/m_b^4$ entirely data based.

## 5 Conclusions

We proposed to measure at $B$ factories the moments of the $q^2$ semileptonic-$B$-decay spectrum with a lower cut $q^2_{\rm cut}$. Since these observables depend on a smaller set of HQE parameters as they are invariant under reparametrization, we suggested the possibility for a novel fit of $V_{cb}$, the heavy quark masses and the non-perturbative parameters up to $1/m_b^4$ in a fully data-driven way based entirely on these kind of observables. This analysis would represent a crucial independent check of the results of Gambino *et al.* [10] about the smallness of the higher order terms in the HQE. It is thus an indispensable ingredient in order to push the $V_{cb}$ uncertainty below the 1% level, given the upcoming precise data of Belle II, and to correctly access the theoretical uncertainty in the $V_{cb}$ extraction.

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