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$\mathcal{N}=2$ Calogero models within superspace

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We provide a superspace description of $\mathcal{N}=2$ supersymmetric Calogero models associated with the classical A_n, B_n, C_n and D_n Lie algebras and their hyperbolic/trigonometric versions. These models are described by n bosonic and $2n(n-1)$ fermionic $\mathcal{N}=2$ superfields, the latter being subject to a nonlinear chirality constraint. This constraint has a universal form valid for all Calogero models and guarantees an existence of more general supercharges (and a superspace Lagrangian), which provide the $\mathcal{N}=2$ supersymmetrization for bosonic potentials with arbitrary repulsive pairwise interactions.

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1. Introduction

The famous Calogero model of n interacting identical particles on a line [1, 2], pertaining to the roots of $A_1 \oplus A_{n-1}$ and given by the classical Hamiltonian

$$H = \frac{1}{2}\sum_{i=1}^{n} p_i^2 + \frac{1}{2}\sum_{i\neq j}\frac{g^2}{(x^i - x^j)^2}\,, \tag{1}$$

plays a significant role in mathematical and theoretical physics. Being the prime example of an integrable and solvable many-body system, it appears in many areas of modern mathematical physics, from high-energy to condensed-matter physics (see e.g. the review [3] and refs. therein).

Subsequently, the Calogero model has often been the subject of "supersymmetrization". In this endeavor, *extended* supersymmetry has turned out to be surprisingly rich. After the straightforward formulation of $\mathcal{N}=2$ supersymmetric Calogero models by Freedman and Mende [4], a barrier was encountered at $\mathcal{N}=4$ [5]. An important step forward then was the explicit construction of the supercharges and the Hamiltonian for the $\mathcal{N}=4$ supersymmetric three-particle Calogero model [6, 7], which introduced a second prepotential F besides the familiar prepotential U. However, it was found that quantum corrections modify the potential in (1), and that F is subject to intricate nonlinear differential equations, the WDVV equations, beyond the three-particle case. These results were then confirmed and elucidated in a superspace description [8]. Finally, extending the system by a single harmonic degree of freedom ($su(2)$ spin variables [9]) it was possible to write down a unique $osp(4|2)$ symmetric four-particle Calogero model [10]. [1] A detailed discussion concerning the supersymmetrization of the Calogero models can be found in the review [11].

A common property of all these models is the limited number of fermionic components accompanying the bosonic coordinates x_i (four fermions for each x_i in the case of $\mathcal{N}$=4). It seems that a guiding principle was missing for the construction of extended supersymmetric Calogero models. Indeed, while for $n \leq 3$ translation and (super-)conformal symmetry almost completely defines the system, the $n \geq 4$ cases admit a lot of freedom which cannot *a priori* be fixed. In the bosonic case, such a guiding principle exists [12]. The Calogero model as well as its different extensions (see, e.g. [13–15]) are closely related with matrix models and can be obtained from them by a reduction procedure (see [16] for first results and [3] for a review).

A different approach to supersymmetric Calogero-like models has been proposed in [9, 17–19]. Starting from a supersymmetrization of the Hermitian matrix model, the resulting matrix fermionic degrees of freedom are packaged in $\mathcal{N}$=4 superfields. This approach, developed in [20, 21] for the rational spin-Calogero models with $\mathcal{N}=2,4$ supersymmetry, was recently extended to $\mathcal{N}=2,4$ supersymmetric hyperbolic Calogero models [22–24]. A similar extended set of fermions appeared in [25], however their bosonic sector contains no interaction.

Inspired by these results we developed a supersymmetrization of the free Hermitian matrix model and constructed an $\mathcal{N}$-extended supersymmetric $su(n)$ spin-Calogero model [26]. By employing a generalized Hamiltonian reduction adopted to the supersymmetric case, we derived a novel rational n-particle Calogero model with an arbitrary even number of supersymmetries. Like

[1] Here and in the above history, the goal is a bosonic potential exactly as in (1). Models with more general interactions can be found for any number of particles.

in the models of [9, 17–19, 25], it features $\mathcal{N}n^2$ rather than $\mathcal{N}n$ fermionic coordinates and increasingly high fermionic powers in the supercharges and the Hamiltonian. While quite satisfactory from a mathematical point of view, the new $\mathcal{N}$-extended supersymmetric Calogero models [26] look very complicated for possible applications. The reason for this is expressions $\sqrt{g+\Pi_{ii}}$, where g is a coupling constant and $\Pi_{ij} \sim (\text{fermions})^2$, present in the supercharges and the Hamiltonian. Since the Π_{ij} are nilpotent, the Taylor expansion of the square root eventually terminates, but for n particles the series will end with a term proportional to $(\Pi_{ii})^{\mathcal{N}(n-1)}$, generating higher-degree monomials in the fermions, both for the supercharges and for the Hamiltonian.

In [27] we found a non-trivial redefinition of the matrix fermions, which brings the supercharges of $\mathcal{N}$-extended supersymmetric Calogero models [26] to the standard form, maximally cubic in the fermions. Then it was demonstrated that the complexity of the initial supercharges is shifted to a non-canonical and nonlinear conjugation property of the redefined fermions. The simple form of the supercharges admits a supersymmetric generalization of a "folding" procedure [28, 29], which relates the $A_{2n-1} \oplus A_1$ Calogero model with the B_n, C_n and D_n ones. Based on this idea, it was provided a supersymmetric extension of the B_n, C_n and D_n rational Calogero models with an arbitrary even number of supersymmetries in [27] and their trigonometric/hyperbolic $\mathcal{N}$=4 extensions in [30].

The Hamiltonian approach turned out to be very fruitful for the extended Calogero models. However, in the construction of the supersymmetric models, it is also important to find a superspace formulation for the Calogero models at least for the lowest number of its supersymmetric extension, namely, for $\mathcal{N}$=2. In our recent paper [31] this description was provided for all types of $\mathcal{N}$=2 extended Calogero models, including both their rational and their trigonometric/hyperbolic versions.[2]

This paper is organized as follows. In Section 2 we start with a review of the Hamiltonian description of the supersymmetric Calogero models. Then in Sections 3 and 4 we give a superfield representation for Calogero models of type $A_1 \oplus A_{n-1}$ and types B_n, C_n, D_n, respectively, We demonstrate that there is a universal nonlinear fermionic chiral supermultiplet which collects all matrix fermions occurring in all super-extended Calogero models. Finally, in Section 5 we present more general supercharges (and a superspace Lagrangian), which correspond to an $\mathcal{N}$=2 supersymmetrization for a bosonic potential $\frac{g^2}{2}\sum_{i<j} f(x_i-x_j)^2$ with an arbitrary function f.

2. Hamiltonian description of $\mathcal{N}$=2 supersymmetric Calogero models

In the series of papers [26, 27, 30], the Hamiltonian approach was used to formulate the supersymmetric extensions of Calogero models. With this approach, it was obtained an ansatz for supercharges, which provides a description of all Calogero models associated with the classical A_n, B_n, C_n and D_n Lie algebras and their trigonometric/hyperbolic extensions. Since the Hamiltonian approach was successfully developed for $\mathcal{N}$-extended supersymmetric Calogero models (in the rational case) as well as for $\mathcal{N}$=2, 4 trigonometric/hyperbolic Calogero models, it was important to obtain also its superfield formulation, for the simplest case with $\mathcal{N}$=2 supersymmetry. Such

[2] A superspace description of the $\mathcal{N}$-extended supersymmetric Euler-Calogero-Moser system has been provided in [32].

a construction has been performed in $\mathcal{N}{=}2$ superspace in our recent paper [31]. But before proceeding to the construction in superspace, let us review the Hamiltonian description of the $\mathcal{N}{=}2$ supersymmetric Calogero models.

2.1 The case of $A_1 \oplus A_{n-1}$ Calogero models

In the Hamiltonian approach the n-particle supersymmetric Calogero model with $\mathcal{N}{=}2$-extended supersymmetry [26, 27, 30] is described by the following degrees of freedom:

- n bosonic coordinates x_i and momenta p_i with $i = 1, \ldots, n$,
- $2\,n$ fermions ψ_i and $\bar{\psi}_i$,
- $2\,n(n{-}1)$ fermions ξ_{ij} and $\bar{\xi}_{ij}$ with $\xi_{ii} = \bar{\xi}_{ii} = 0$.

The non-vanishing Poisson brackets read

$$\left\{x_i, p_j\right\} = \delta_{ij}, \qquad \left\{\psi_i, \bar{\psi}_j\right\} = -\mathrm{i}\,\delta_{ij}, \qquad \left\{\xi_{ij}, \bar{\xi}_{km}\right\} = -\mathrm{i}\left(1{-}\delta_{ij}\right)\left(1{-}\delta_{km}\right)\delta_{im}\delta_{jk}. \tag{2}$$

In the Hamiltonian construction a central role plays the composite objects Π_{ij} and $\widetilde{\Pi}_{ij}$, which are defined by the following expressions

$$\Pi_{ij} \;=\; \left(\psi_i{-}\psi_j\right)\bar{\xi}_{ij} + \left(\bar{\psi}_i{-}\bar{\psi}_j\right)\xi_{ij} + \sum_{k=1}^{n}\left(\xi_{ik}\bar{\xi}_{kj} + \bar{\xi}_{ik}\xi_{kj}\right), \tag{3}$$

$$\widetilde{\Pi}_{ij} \;=\; 2\delta_{ij}\psi_i\bar{\psi}_i + \left(\psi_i{+}\psi_j\right)\bar{\xi}_{ij} - \left(\bar{\psi}_i{+}\bar{\psi}_j\right)\xi_{ij} + \sum_{k=1}^{n}\left(\xi_{ik}\bar{\xi}_{kj} - \bar{\xi}_{ik}\xi_{kj}\right). \tag{4}$$

They form an $s(u(n) \oplus u(n))$ algebra with respect to the introduced Poisson brackets (2),[3]

$$\begin{array}{ll}\left\{\Pi_{ij}, \Pi_{km}\right\} = \mathrm{i}\big(\delta_{im}\Pi_{kj} - \delta_{kj}\Pi_{im}\big), & \left\{\Pi_{ij}, \widetilde{\Pi}_{km}\right\} = \mathrm{i}\big(\delta_{im}\widetilde{\Pi}_{kj} - \delta_{kj}\widetilde{\Pi}_{im}\big), \\ \left\{\widetilde{\Pi}_{ij}, \widetilde{\Pi}_{km}\right\} = \mathrm{i}\big(\delta_{im}\Pi_{kj} - \delta_{kj}\Pi_{im}\big). & \end{array} \tag{5}$$

The $\mathcal{N}{=}2$ supersymmetric Calogero models of A-type [27, 30] are defined by a generic form of their supercharges,

$$\begin{aligned} Q &= \sum_{i=1}^{n} p_i\psi_i - \mathrm{i}\sum_{i\neq j}^{n}\Big[\left(g+\Pi_{jj}\right)f(z_{ij}) + \frac{f'(z_{ij})}{f(z_{ij})}\Pi_{ij}\Big]\xi_{ji}, \\ \overline{Q} &= \sum_{i=1}^{n} p_i\bar{\psi}_i - \mathrm{i}\sum_{i\neq j}^{n}\Big[\left(g+\Pi_{jj}\right)f(z_{ij}) + \frac{f'(z_{ij})}{f(z_{ij})}\Pi_{ij}\Big]\bar{\xi}_{ji}, \end{aligned} \tag{6}$$

with some function f, to be specified in a moment. These supercharges form an $\mathcal{N}{=}2$ super-Poincaré algebra,

$$\left\{Q, \overline{Q}\right\} = -2\mathrm{i}\,\mathcal{H} \qquad \text{and} \qquad \left\{Q, Q\right\} = \left\{\overline{Q}, \overline{Q}\right\} = 0\,, \tag{7}$$

together with the Hamiltonian

$$\mathcal{H} \;=\; \frac{1}{2}\sum_{i=1}^{n} p_i^2 \;+\; \frac{1}{2}\sum_{i\neq j}^{n}\Big[\left(g+\Pi_{jj}\right)f(z_{ij}) + \frac{f'(z_{ij})}{f(z_{ij})}\Pi_{ij}\Big]\Big[\left(g+\Pi_{ii}\right)f(z_{ij}) + \frac{f'(z_{ij})}{f(z_{ij})}\Pi_{ji}\Big] + \frac{\alpha}{2}\sum_{i,j}^{n}\Pi_{ij}\Pi_{ji}. \tag{8}$$

[3] It is to remind that $\sum_i \Pi_{ii} = 0$.

For further convenience, a new variable is introduced

$$z_{ij} = x_i - x_j, \tag{9}$$

and the constant parameter α and the function f are given as follows,

$$\begin{array}{rll} \text{rational Calogero model} & \alpha = 0, & f(z_{ij}) = \dfrac{1}{z_{ij}} = \dfrac{1}{x_i - x_j}, \\ \text{hyperbolic Calogero-Moser model} & \alpha = -1, & f(z_{ij}) = \dfrac{1}{\sinh(z_{ij})} = \dfrac{1}{\sinh(x_i - x_j)}, \\ \text{trigonometric Calogero-Moser model} & \alpha = 1, & f(z_{ij}) = \dfrac{1}{\sin(z_{ij})} = \dfrac{1}{\sin(x_i - x_j)}. \end{array} \tag{10}$$

The bosonic part of Hamiltonian (8) is given by

$$\mathcal{H}_{bos} = \frac{1}{2}\sum_{i=1}^{n} p_i^2 + \frac{g^2}{2}\sum_{i\neq j}^{n} f^2(z_{ij}). \tag{11}$$

2.2 The case of B_n, C_n and D_n Calogero models

The structure of supercharges for the B, C and D-type Calodgero models have a more complicated generic form [27, 30], because they include both composite objects Π_{ij} and $\widetilde{\Pi}_{ij}$

$$\begin{aligned} Q &= \sum_{i=1}^{n} p_i\psi_i - \mathrm{i}\sum_{i\neq j}^{n}\Big[\big(g+\Pi_{jj}\big) f(z_{ij}) + \frac{f'(z_{ij})}{f(z_{ij})}\Pi_{ij}\Big]\xi_{ji} + \mathrm{i}\sum_{i\neq j}^{n}\Big[\big(g+\Pi_{jj}\big) f(y_{ij}) - \frac{f'(y_{ij})}{f(y_{ij})}\widetilde{\Pi}_{ij}\Big]\xi_{ji} \\ &\quad + \mathrm{i}\sum_{i}^{n}\Big[\big(g'+\Pi_{ii}\big) f(y_{ii}) - \frac{f'(y_{ii})}{f(y_{ii})}\widetilde{\Pi}_{ii}\Big]\psi_i, \\ \overline{Q} &= \sum_{i=1}^{n} p_i\bar{\psi}_i - \mathrm{i}\sum_{i\neq j}^{n}\Big[\big(g+\Pi_{jj}\big) f(z_{ij}) + \frac{f'(z_{ij})}{f(z_{ij})}\Pi_{ij}\Big]\bar{\xi}_{ji} - \mathrm{i}\sum_{i\neq j}^{n}\Big[\big(g+\Pi_{jj}\big) f(y_{ij}) - \frac{f'(y_{ij})}{f(y_{ij})}\widetilde{\Pi}_{ij}\Big]\bar{\xi}_{ji} \\ &\quad - \mathrm{i}\sum_{i}^{n}\Big[\big(g'+\Pi_{ii}\big) f(y_{ii}) - \frac{f'(y_{ii})}{f(y_{ii})}\widetilde{\Pi}_{ii}\Big]\bar{\psi}_i. \end{aligned} \tag{12}$$

In addition, the following variable is introduced here to simplify the notation

$$y_{ij} = x_i + x_j, \tag{13}$$

while the function f is the same as in (10). The supercharges (12) form the same $\mathcal{N}\!=2$ super-Poincaré algebra (7) together with the Hamiltonian

$$\begin{aligned} \mathcal{H} &= \frac{1}{2}\sum_{i=1}^{n} p_i^2 + \frac{1}{2}\sum_{i\neq j}^{n}\Big[\big(g+\Pi_{jj}\big) f(z_{ij}) + \frac{f'(z_{ij})}{f(z_{ij})}\Pi_{ij}\Big]\Big[\big(g+\Pi_{ii}\big) f(z_{ij}) + \frac{f'(z_{ij})}{f(z_{ij})}\Pi_{ji}\Big] \\ &\quad + \frac{1}{2}\sum_{i\neq j}^{n}\Big[\big(g+\Pi_{jj}\big) f(y_{ij}) - \frac{f'(y_{ij})}{f(y_{ij})}\widetilde{\Pi}_{ij}\Big]\Big[\big(g+\Pi_{ii}\big) f(y_{ij}) - \frac{f'(y_{ij})}{f(y_{ij})}\widetilde{\Pi}_{ji}\Big] \\ &\quad + \frac{1}{2}\sum_{i}^{n}\Big[\big(g'+\Pi_{ii}\big) f(y_{ii}) - \frac{f'(y_{ii})}{f(y_{ii})}\widetilde{\Pi}_{ii}\Big]\Big[\big(g'+\Pi_{ii}\big) f(y_{ii}) - \frac{f'(y_{ii})}{f(y_{ii})}\widetilde{\Pi}_{ii}\Big]. \end{aligned} \tag{14}$$

Its bosonic sector reads

$$\mathcal{H}_{bos} = \frac{1}{2}\sum_{i=1}^{n} p_i^2 + \frac{g^2}{2}\sum_{i\neq j}^{n}\left(f^2(z_{ij}) + f^2(y_{ij})\right) + \frac{g'^2}{2}\sum_{i}^{n} f^2(y_{ii}). \tag{15}$$

Note that due to the presence of only two coupling constants, g and g', in supercharges (12) and Hamiltonian (14), it is possible to describe B, C and D-type models in the rational case and C and D (but not B)-type models in the hyperbolic/trigonometric case.

3. $\mathcal{N}$=2 superspace $A_1 \oplus A_{n-1}$ Calogero models

In a superspace description of $\mathcal{N}$=2 supersymmetric Calogero models, the physical components x_i, ψ_i, $\bar\psi_i$, ξ_{ij} and $\bar\xi_{ij}$ are collected in some $\mathcal{N} = 2$ superfields, which, possibly, must satisfy irreducible constraints.

As follows from the structure of the supercharges Q and $\overline{Q}$ (6), the coordinates x_i transform into the fermions ψ_i and $\bar\psi_i$ under $\mathcal{N}$=2 supersymmetry:

$$\delta x_i \equiv \left\{x_i, \mathrm{i}\,\bar\epsilon Q + \mathrm{i}\,\epsilon\overline{Q}\right\} = \mathrm{i}\,\bar\epsilon\psi_i + \mathrm{i}\,\epsilon\bar\psi_i\ . \tag{16}$$

Thus, one is let to n bosonic $\mathcal{N}$=2 superfields $\boldsymbol{x}_i$ with the components [4]

$$x_i = \boldsymbol{x}_i|, \quad \psi_i = -\mathrm{i}D\boldsymbol{x}_i|, \quad \bar\psi_i = -\mathrm{i}\overline{D}\boldsymbol{x}_i|, \quad A_i = \tfrac{1}{2}\left[\overline{D}, D\right]\boldsymbol{x}_i|. \tag{17}$$

The rest of fermionic components $\xi_{ij}, \bar\xi_{ij}$ can be considered as the first components of $2n(n-1)$ new fermionic superfields $\boldsymbol{\xi}_{ij}$ and $\bar{\boldsymbol{\xi}}_{ij}$ with vanishing diagonal parts, i.e.

$$\boldsymbol{\xi}_{ii} = \bar{\boldsymbol{\xi}}_{ii} = 0 \quad \forall\, i. \tag{18}$$

However, since the general $\mathcal{N}$=2 superfields, the $\boldsymbol{\xi}_{ij}$ and $\bar{\boldsymbol{\xi}}_{ij}$ contain a lot of components and, therefore, correspond to reducible superfields, they have to be constrained somehow. The appropriate constraints can be derived from the explicit form of the supercharges Q and $\overline{Q}$ (6), which leads to the following supersymmetry transformations of the leading components ξ_{ij} and $\bar\xi_{ij}$ of these superfields,

$$\begin{aligned}
\delta_Q\xi_{ij} &\sim \mathrm{i}\bar\epsilon\Big[-\frac{f'(z_{ij})}{f(z_{ij})}\left(\psi_i - \psi_j\right)\xi_{ij} + \xi_{ij}\Big(\sum_{k\neq i}^{n} f(z_{ik})\xi_{ik} - \sum_{k\neq j}^{n} f(z_{jk})\xi_{jk}\Big) - \sum_{k\neq i,j}^{n}\Big(\frac{f'(z_{ik})}{f(z_{ik})} + \frac{f'(z_{kj})}{f(z_{kj})}\Big)\xi_{ik}\xi_{kj}\Big],\\
\delta_{\overline{Q}}\bar\xi_{ij} &\sim \mathrm{i}\epsilon\Big[-\frac{f'(z_{ij})}{f(z_{ij})}\left(\bar\psi_i - \bar\psi_j\right)\bar\xi_{ij} + \bar\xi_{ij}\Big(\sum_{k\neq i}^{n} f(z_{ik})\bar\xi_{ik} - \sum_{k\neq j}^{n} f(z_{jk})\bar\xi_{jk}\Big) - \sum_{k\neq i,j}^{n}\Big(\frac{f'(z_{ik})}{f(z_{ik})} + \frac{f'(z_{kj})}{f(z_{kj})}\Big)\bar\xi_{ik}\bar\xi_{kj}\Big].
\end{aligned} \tag{19}$$

[4] We use the $\mathcal{N}$=2 spinor covariant derivatives D and $\overline{D}$ obeying $\left\{D,\overline{D}\right\} = 2\mathrm{i}\partial_t$ and $\{D, D\} = \left\{\overline{D},\overline{D}\right\} = 0$. We denote by $\mathcal{A}|$ the $\theta = \bar\theta = 0$ limit of a superspace expression $\mathcal{A}$.

To realize this transformation property we are forced to impose a nonlinear chirality condition on the superfields $\boldsymbol{\xi}_{ij}$ and $\bar{\boldsymbol{\xi}}_{ij}$,

$$
\begin{aligned}
D\boldsymbol{\xi}_{ij} &= \mathrm{i}\Big[-\frac{f'(z_{ij})}{f(z_{ij})}\left(\boldsymbol{\psi}_i-\boldsymbol{\psi}_j\right)\boldsymbol{\xi}_{ij}+\boldsymbol{\xi}_{ij}\Big(\sum_{k\neq i}^{n} f(z_{ik})\boldsymbol{\xi}_{ik}-\sum_{k\neq j}^{n} f(z_{jk})\boldsymbol{\xi}_{jk}\Big)-\sum_{k\neq i,j}^{n}\Big(\frac{f'(z_{ik})}{f(z_{ik})}+\frac{f'(z_{kj})}{f(z_{kj})}\Big)\boldsymbol{\xi}_{ik}\boldsymbol{\xi}_{kj}\Big],\\
\overline{D}\bar{\boldsymbol{\xi}}_{ij} &= \mathrm{i}\Big[-\frac{f'(z_{ij})}{f(z_{ij})}\left(\bar{\boldsymbol{\psi}}_i-\bar{\boldsymbol{\psi}}_j\right)\bar{\boldsymbol{\xi}}_{ij}+\bar{\boldsymbol{\xi}}_{ij}\Big(\sum_{k\neq i}^{n} f(z_{ik})\bar{\boldsymbol{\xi}}_{ik}-\sum_{k\neq j}^{n} f(z_{jk})\bar{\boldsymbol{\xi}}_{jk}\Big)-\sum_{k\neq i,j}^{n}\Big(\frac{f'(z_{ik})}{f(z_{ik})}+\frac{f'(z_{kj})}{f(z_{kj})}\Big)\bar{\boldsymbol{\xi}}_{ik}\bar{\boldsymbol{\xi}}_{kj}\Big].
\end{aligned}
\tag{20}
$$

These constraints leave in the superfields $\boldsymbol{\xi}_{ij}$ and $\bar{\boldsymbol{\xi}}_{ij}$ the following components

$$
\xi_{ij}=\boldsymbol{\xi}_{ij}|,\quad B_{ij}=\overline{D}\boldsymbol{\xi}_{ij}|,\qquad \bar{\xi}_{ij}=\bar{\boldsymbol{\xi}}_{ij}|,\quad \overline{B}_{ij}=D\bar{\boldsymbol{\xi}}_{ij}|\,. \tag{21}
$$

It is also important to have the correct brackets (2) for $(\psi_i,\bar{\psi}_i)$ and $(\xi_{ij},\bar{\xi}_{ij})$ after passing to the Hamiltonian formalism. This is achieved when the kinetic terms for these fermionic components have the following form in the Lagrangian

$$
\mathcal{L}^{\psi}_{kin}=\frac{\mathrm{i}}{2}\sum_{i=1}^{n}\left(\dot{\psi}_i\bar{\psi}_i-\psi_i\dot{\bar{\psi}}_i\right)\qquad\text{and}\qquad \mathcal{L}^{\xi}_{kin}=\frac{\mathrm{i}}{2}\sum_{i,j}^{n}\left(\dot{\xi}_{ij}\bar{\xi}_{ji}-\xi_{ij}\dot{\bar{\xi}}_{ji}\right). \tag{22}
$$

In $\mathcal{N}=2$ superspace, this amounts to the free action ($g=0$)

$$
S_0\;=\;\int dt\,d^2\theta\,\Big[-\frac{1}{2}\sum_{i=1}^{n} D\boldsymbol{x}_i\,\overline{D}\boldsymbol{x}_i+\frac{1}{2}\sum_{i,j}^{n}\boldsymbol{\xi}_{ij}\bar{\boldsymbol{\xi}}_{ji}\Big]\qquad\text{with}\quad d^2\theta\equiv D\overline{D}. \tag{23}
$$

The next task is to construct the interaction terms in superspace. Again, some hints come from the transformation properties of the fermions ξ_{ij} and $\bar{\xi}_{ij}$ under $\overline{Q}$ and Q supersymmetry, respectively,

$$
\delta_{\overline{Q}}\xi_{ij}\sim \mathrm{i}\epsilon\, g\, f(z_{ij})+\ldots,\qquad \delta_{Q}\bar{\xi}_{ij}\sim \mathrm{i}\bar{\epsilon}\, g\, f(z_{ij})+\ldots\,. \tag{24}
$$

In order to realize such transformations in superspace, there is the unique possibility to add to the action S_0 (23) a term of the form

$$
S_{int}\;=\;\mathrm{i}\,\frac{g}{2}\int dt\,d\bar{\theta}\,\sum_{i\neq j}^{n} f(z_{ij})\boldsymbol{\xi}_{ij}+\mathrm{i}\,\frac{g}{2}\int dt\,d\theta\,\sum_{i\neq j}^{n} f(z_{ij})\,\bar{\boldsymbol{\xi}}_{ij}. \tag{25}
$$

The integrands in (25) must be chiral and antichiral, respectively, then the action will be invariant. The nonlinear chirality constraint (20) provide this

$$
D\Big(\sum_{i\neq j}^{n} f(z_{ij})\,\boldsymbol{\xi}_{ij}\Big)=0\qquad\text{and}\qquad \overline{D}\Big(\sum_{i\neq j}^{n} f(z_{ij})\,\bar{\boldsymbol{\xi}}_{ij}\Big)=0. \tag{26}
$$

Thus, combining all these facts together, we conclude that the superfield action reads

$$
S\;=\;\int dt\,d^2\theta\Big[-\frac{1}{2}\sum_{i=1}^{n} D\boldsymbol{x}_i\,\overline{D}\boldsymbol{x}_i+\frac{1}{2}\sum_{i,j}^{n}\boldsymbol{\xi}_{ij}\bar{\boldsymbol{\xi}}_{ji}\Big]+\mathrm{i}\,\frac{g}{2}\int dt\,d\bar{\theta}\,\sum_{i\neq j}^{n} f(z_{ij})\boldsymbol{\xi}_{ij}+\mathrm{i}\,\frac{g}{2}\int dt d\theta\sum_{i\neq j}^{n} f(z_{ij})\,\bar{\boldsymbol{\xi}}_{ij}\,, \tag{27}
$$

where the superfields $\boldsymbol{\xi}_{ij}$ and $\bar{\boldsymbol{\xi}}_{ij}$ are subject to the nonlinear chirality constraint (20).

Let us go back to the constraints (20). It turns out that, after passing to new fermionic superfields

$$\boldsymbol{\lambda}_{ij} \equiv f(z_{ij})\,\boldsymbol{\xi}_{ij} \qquad \text{and} \qquad \bar{\boldsymbol{\lambda}}_{ij} \equiv f(z_{ij})\,\bar{\boldsymbol{\xi}}_{ij}\,, \tag{28}$$

the nonlinear constraints (20) are simplified to

$$\begin{aligned} D\boldsymbol{\lambda}_{ij} &= \mathrm{i}\Big[\,\boldsymbol{\lambda}_{ij}\sum_{k\neq i}^{n}\boldsymbol{\lambda}_{ik} - \boldsymbol{\lambda}_{ij}\sum_{k\neq j}^{n}\boldsymbol{\lambda}_{jk} + (1-\delta_{ij})\sum_{k\neq i,j}^{n}\boldsymbol{\lambda}_{ik}\boldsymbol{\lambda}_{kj}\,\Big]\,, \\ \overline{D}\bar{\boldsymbol{\lambda}}_{ij} &= \mathrm{i}\Big[\,\bar{\boldsymbol{\lambda}}_{ij}\sum_{k\neq i}^{n}\bar{\boldsymbol{\lambda}}_{ik} - \bar{\boldsymbol{\lambda}}_{ij}\sum_{k\neq j}^{n}\bar{\boldsymbol{\lambda}}_{jk} + (1-\delta_{ij})\sum_{k\neq i,j}^{n}\bar{\boldsymbol{\lambda}}_{ik}\bar{\boldsymbol{\lambda}}_{kj}\,\Big]\,. \end{aligned} \tag{29}$$

In this form, the constraints have lost any f-dependence, which however will reappear in the action,

$$S = \int dt\, d^2\theta\,\Big[-\frac{1}{2}\sum_{i=1}^{n} D\boldsymbol{x}_i\,\overline{D}\boldsymbol{x}_i + \frac{1}{2}\sum_{i,j}^{n}\frac{\boldsymbol{\lambda}_{ij}\bar{\boldsymbol{\lambda}}_{ji}}{f(z_{ij})\,f(z_{ji})}\Big] + \mathrm{i}\,\frac{g}{2}\int dt\, d\bar{\theta}\sum_{i\neq j}^{n}\boldsymbol{\lambda}_{ij} + \mathrm{i}\,\frac{g}{2}\int dt\, d\theta\sum_{i\neq j}^{n}\bar{\boldsymbol{\lambda}}_{ij}\,. \tag{30}$$

It is also obvious that the component Lagrangian, Hamiltonian and Poisson brackets will be more complicated in terms of the composite superfields $\boldsymbol{\lambda}_{ij}$ and $\bar{\boldsymbol{\lambda}}_{ij}$.

Despite the extremely simple form of the superfield action (27), its component version looks quite complicated due to the constraint (20). Let us consider in more detail how the calculations are carried out in this case. After integration over θ in (27) the off-shell Lagrangian reads

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\rm pot}\,, \qquad \text{where} \tag{31}$$

$$\begin{aligned} \mathcal{L}_0 &= \frac{1}{2}\sum_{i}^{n}\Big(\dot{x}_i\dot{x}_i + A_iA_i\Big) + \frac{\mathrm{i}}{2}\sum_{i}^{n}\Big(\dot{\psi}_i\bar{\psi}_i - \psi_i\dot{\bar{\psi}}_i\Big) + \frac{\mathrm{i}}{2}\sum_{i,j}^{n}\Big(\dot{\xi}_{ij}\bar{\xi}_{ji} - \xi_{ij}\dot{\bar{\xi}}_{ji}\Big) \\ &\quad + \frac{1}{2}\sum_{i,j}^{n}\Big(\,\xi_{ij}D\big(\overline{D}\bar{\xi}_{ji}\big) - \overline{D}\big(D\xi_{ij}\big)\bar{\xi}_{ji} - D\xi_{ij}\overline{D}\bar{\xi}_{ji} + B_{ij}\overline{B}_{ji}\,\Big), \\ \mathcal{L}_{\rm pot} &= -\frac{g}{2}\sum_{i,j}^{n} f'(z_{ij})\Big(\,(\psi_i-\psi_j)\,\bar{\xi}_{ij} + (\bar{\psi}_i-\bar{\psi}_j)\,\xi_{ij}\Big) + \mathrm{i}\,\frac{g}{2}\sum_{i,j}^{n} f(z_{ij})\big(B_{ij}+\overline{B}_{ij}\big)\,. \end{aligned} \tag{32}$$

To eliminate the auxiliary fields A_i and B_{ij} one firstly has to evaluate the terms in the second line of (32) by using the constraint (20). This is a straightforward but rather tedious calculation. After employing the equations of motion we finally obtain the desired result,

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\sum_{i=1}^{n}\dot{x}_i\dot{x}_i + \frac{\mathrm{i}}{2}\sum_{i=1}^{n}\Big(\dot{\psi}_i\bar{\psi}_i - \psi_i\dot{\bar{\psi}}_i\Big) + \frac{\mathrm{i}}{2}\sum_{i,j}^{n}\Big(\dot{\xi}_{ij}\bar{\xi}_{ji} - \xi_{ij}\dot{\bar{\xi}}_{ji}\Big) \\ &\quad - \frac{1}{2}\sum_{i\neq j}^{n}\Big[\big(g+\Pi_{jj}\big)f(z_{ij}) + \frac{f'(z_{ij})}{f(z_{ij})}\Pi_{ij}\Big]\Big[\big(g+\Pi_{ii}\big)f(z_{ij}) + \frac{f'(z_{ij})}{f(z_{ij})}\Pi_{ji}\Big] - \frac{\alpha}{2}\sum_{i,j}^{n}\Pi_{ij}\Pi_{ji}. \end{aligned} \tag{33}$$

Thus, the superfield action (27) with the superfields $\boldsymbol{\xi}_{ij}$ and $\bar{\boldsymbol{\xi}}_{ij}$ subject to the nonlinear chirality constraint (20) indeed describes all $\mathcal{N}=2$ supersymmetric $A_1 \oplus A_{n-1}$ Calogero models.

4. $\mathcal{N}=2$ superspace B_n, C_n and D_n Calogero models

As follows from (12), the supercharges of the $\mathcal{N}=2$ supersymmetric B, C and D-type Calogero models have a more complicated structure than those which are given for Calogero model of A-type in (6). Therefore, it is expected that the nonlinear chirality constraint for the superfields $\boldsymbol{\xi}_{ij}$ and $\bar{\boldsymbol{\xi}}_{ij}$ are more intricate as well. Indeed, the explicit structure of the supercharges (12) uniquely fixes this constraint to be

$$\begin{aligned} D\boldsymbol{\xi}_{ij} &= \mathrm{i}\Big[-\frac{f'(\boldsymbol{z}_{ij})}{f(\boldsymbol{z}_{ij})}(\boldsymbol{\psi}_i-\boldsymbol{\psi}_j)\,\boldsymbol{\xi}_{ij} - \frac{f'(\boldsymbol{y}_{ij})}{f(\boldsymbol{y}_{ij})}(\boldsymbol{\psi}_i+\boldsymbol{\psi}_j)\,\boldsymbol{\xi}_{ij} \\ &\quad + \Big\{\Big(\frac{f'(\boldsymbol{y}_{ii})}{f(\boldsymbol{y}_{ii})} - f(\boldsymbol{y}_{ii})\Big)\boldsymbol{\psi}_i + \Big(\frac{f'(\boldsymbol{y}_{jj})}{f(\boldsymbol{y}_{jj})} + f(\boldsymbol{y}_{jj})\Big)\boldsymbol{\psi}_j\Big\}\boldsymbol{\xi}_{ij} \\ &\quad + \boldsymbol{\xi}_{ij}\Big(\sum_{k\neq i}^{n}(f(\boldsymbol{z}_{ik})+f(\boldsymbol{y}_{ik}))\boldsymbol{\xi}_{ik} - \sum_{k\neq j}^{n}(f(\boldsymbol{z}_{jk})+f(\boldsymbol{y}_{jk}))\boldsymbol{\xi}_{jk}\Big) \\ &\quad - \sum_{k\neq i,j}^{n}\Big(\frac{f'(\boldsymbol{z}_{ik})}{f(\boldsymbol{z}_{ik})} + \frac{f'(\boldsymbol{z}_{kj})}{f(\boldsymbol{z}_{kj})} - \frac{f'(\boldsymbol{y}_{ik})}{f(\boldsymbol{y}_{ik})} + \frac{f'(\boldsymbol{y}_{kj})}{f(\boldsymbol{y}_{kj})}\Big)\boldsymbol{\xi}_{ik}\boldsymbol{\xi}_{kj}\Big], \\ \overline{D}\bar{\boldsymbol{\xi}}_{ij} &= \mathrm{i}\Big[-\frac{f'(\boldsymbol{z}_{ij})}{f(\boldsymbol{z}_{ij})}(\bar{\boldsymbol{\psi}}_i-\bar{\boldsymbol{\psi}}_j)\,\bar{\boldsymbol{\xi}}_{ij} - \frac{f'(\boldsymbol{y}_{ij})}{f(\boldsymbol{y}_{ij})}(\bar{\boldsymbol{\psi}}_i+\bar{\boldsymbol{\psi}}_j)\,\bar{\boldsymbol{\xi}}_{ij} \\ &\quad + \Big\{\Big(\frac{f'(\boldsymbol{y}_{ii})}{f(\boldsymbol{y}_{ii})} + f(\boldsymbol{y}_{ii})\Big)\bar{\boldsymbol{\psi}}_i + \Big(\frac{f'(\boldsymbol{y}_{jj})}{f(\boldsymbol{y}_{jj})} - f(\boldsymbol{y}_{jj})\Big)\bar{\boldsymbol{\psi}}_j\Big\}\bar{\boldsymbol{\xi}}_{ij} \\ &\quad + \bar{\boldsymbol{\xi}}_{ij}\Big(\sum_{k\neq i}^{n}(f(\boldsymbol{z}_{ik})-f(\boldsymbol{y}_{ik}))\bar{\boldsymbol{\xi}}_{ik} - \sum_{k\neq j}^{n}(f(\boldsymbol{z}_{jk})-f(\boldsymbol{y}_{jk}))\bar{\boldsymbol{\xi}}_{jk}\Big) \\ &\quad - \sum_{k\neq i,j}^{n}\Big(\frac{f'(\boldsymbol{z}_{ik})}{f(\boldsymbol{z}_{ik})} + \frac{f'(\boldsymbol{z}_{kj})}{f(\boldsymbol{z}_{kj})} - \frac{f'(\boldsymbol{y}_{ik})}{f(\boldsymbol{y}_{ik})} + \frac{f'(\boldsymbol{y}_{kj})}{f(\boldsymbol{y}_{kj})}\Big)\bar{\boldsymbol{\xi}}_{ik}\bar{\boldsymbol{\xi}}_{kj}\Big]. \end{aligned} \tag{34}$$

Nevertheless, the complicated form of this constraint disappears after passing to the composite superfields in the same way as in the previous case

$$\boldsymbol{\lambda}_{ij} = \big(f(\boldsymbol{z}_{ij})+f(\boldsymbol{y}_{ij})\big)\boldsymbol{\xi}_{ij} \qquad \text{and} \qquad \bar{\boldsymbol{\lambda}}_{ij} = \big(f(\boldsymbol{z}_{ij})-f(\boldsymbol{y}_{ij})\big)\bar{\boldsymbol{\xi}}_{ij},. \tag{35}$$

In terms of these superfields the constraint acquires its familiar form (29),

$$\begin{aligned} D\boldsymbol{\lambda}_{ij} &= \mathrm{i}\Big[\boldsymbol{\lambda}_{ij}\sum_{k\neq i}^{n}\boldsymbol{\lambda}_{ik} - \boldsymbol{\lambda}_{ij}\sum_{k\neq j}^{n}\boldsymbol{\lambda}_{jk} + (1-\delta_{ij})\sum_{k\neq i,j}^{n}\boldsymbol{\lambda}_{ik}\boldsymbol{\lambda}_{kj}\Big], \\ \overline{D}\bar{\boldsymbol{\lambda}}_{ij} &= \mathrm{i}\Big[\bar{\boldsymbol{\lambda}}_{ij}\sum_{k\neq i}^{n}\bar{\boldsymbol{\lambda}}_{ik} - \bar{\boldsymbol{\lambda}}_{ij}\sum_{k\neq j}^{n}\bar{\boldsymbol{\lambda}}_{jk} + (1-\delta_{ij})\sum_{k\neq i,j}^{n}\bar{\boldsymbol{\lambda}}_{ik}\bar{\boldsymbol{\lambda}}_{kj}\Big]. \end{aligned}$$

Taking into account the arguments used in the analysis of the structure of superfiled action (27), one can conclude that the superfield action for $\mathcal{N}=2$ supersymmetric B, C and D-type Calogero models is given by

$$
\begin{aligned}
S \;=\; & \int dt\, d^2\theta \left[-\frac{1}{2}\sum_{i=1}^{n} \mathcal{D}\boldsymbol{x}_i\, \overline{\mathcal{D}}\boldsymbol{x}_i + \frac{1}{2}\sum_{i,j}^{n} \boldsymbol{\xi}_{ij}\bar{\boldsymbol{\xi}}_{ji} - \frac{1}{2}\, g'\, h(\boldsymbol{y}_{ii}) \right] \\
& + \mathrm{i}\,\frac{g}{2}\int dt\, d\bar\theta \sum_{i\neq j}^{n} \big(f(z_{ij}) + f(\boldsymbol{y}_{ij})\big)\boldsymbol{\xi}_{ij} + \mathrm{i}\,\frac{g}{2}\int dt\, d\theta \sum_{i\neq j}^{n} \big(f(z_{ij}) + f(\boldsymbol{y}_{ij})\big)\, \bar{\boldsymbol{\xi}}_{ij}\,,
\end{aligned}
\tag{36}
$$

where

$$
h'(\boldsymbol{y}_{ii}) = f(\boldsymbol{y}_{ii}). \tag{37}
$$

Compared to the action of the $A_1 \oplus A_{n-1}$ Calogero models (30), only the term $\frac{1}{2}\, g' \int dt\, d^2\theta\; h(\boldsymbol{y}_{ii})$ carrying the new coupling constant g' appears in the action (36). All other terms just mimic those in (30).

It is a matter of straightforward but tedious calculations to check that, after excluding the auxiliary fields by their equations of motion, the final on-shell Lagrangian acquires the expected form

$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{1}{2}\sum_{i=1}^{n} \dot x_i \dot x_i + \frac{\mathrm{i}}{2}\sum_{i=1}^{n}\left(\dot\psi_i\bar\psi_i - \psi_i\dot{\bar\psi}_i\right) + \frac{\mathrm{i}}{2}\sum_{i,j}^{n}\left(\dot\xi_{ij}\bar\xi_{ji} - \xi_{ij}\dot{\bar\xi}_{ji}\right) \\
& - \frac{1}{2}\sum_{i\neq j}^{n}\left[(g+\Pi_{jj})f(z_{ij}) + \frac{f'(z_{ij})}{f(z_{ij})}\,\Pi_{ij}\right]\left[(g+\Pi_{ii})f(z_{ij}) + \frac{f'(z_{ij})}{f(z_{ij})}\,\Pi_{ji}\right] \\
& - \frac{1}{2}\sum_{i\neq j}^{n}\left[(g+\Pi_{jj})f(y_{ij}) - \frac{f'(y_{ij})}{f(y_{ij})}\,\widetilde\Pi_{ij}\right]\left[(g+\Pi_{ii})f(y_{ij}) - \frac{f'(y_{ij})}{f(y_{ij})}\,\widetilde\Pi_{ji}\right] \\
& - \frac{1}{2}\sum_{i}^{n}\left[(g'+\Pi_{ii})f(y_{ii}) - \frac{f'(y_{ii})}{f(y_{ii})}\,\widetilde\Pi_{ii}\right]\left[(g'+\Pi_{ii})f(y_{ii}) - \frac{f'(y_{ii})}{f(y_{ii})}\,\widetilde\Pi_{ii}\right].
\end{aligned}
\tag{38}
$$

Thus, the superfield action (36) with the superfields $\boldsymbol{\xi}_{ij}$ and $\bar{\boldsymbol{\xi}}_{ij}$ subject to the nonlinear chirality constraint (34) indeed describes the $\mathcal{N}=2$ supersymmetric B, C and D-type Calogero models.[5]

5. New $\mathcal{N}=2$ supersymmetric n-particle models

The starting point was the explicit form of the $\mathcal{N}=2$ supercharges for the Calogero models (6) and (12) constructed in [27, 30]. In all considered cases the function f to be not arbitrary but to be chosen from the list (10). Indeed, for generic function f the supercharges (6) do not form the closed algebra (7). As a consequence, the nonlinear chirality condition (20) is not self-consistent for an arbitrary function f (i.e. D acting on the r.h.s. of (20) does not vanish). On the other hand, as can be straightforwardly proved, the universal chirality constraint (29) is self-consistent. This

[5]Except again for B-type models in the trigonometric/hyperbolic case.

implies a weaker chirality condition on ξ_{ij} and $\bar{\xi}_{ij}$:[6]

$$
\begin{aligned}
D\xi_{ij} &= \mathrm{i}\Big[-\frac{f'(z_{ij})}{f(z_{ij})}\left(\psi_i-\psi_j\right)\xi_{ij}+\xi_{ij}\Big(\sum_{k\neq i}^{n} f(z_{ik})\xi_{ik}-\sum_{k\neq j}^{n} f(z_{jk})\xi_{jk}\Big)+\sum_{k\neq i,j}^{n}\frac{f(z_{ik})\,f(z_{kj})}{f(z_{ij})}\,\xi_{ik}\xi_{kj}\Big],\\
\overline{D}\bar{\xi}_{ij} &= \mathrm{i}\Big[-\frac{f'(z_{ij})}{f(z_{ij})}\left(\bar{\psi}_i-\bar{\psi}_j\right)\bar{\xi}_{ij}+\bar{\xi}_{ij}\Big(\sum_{k\neq i}^{n} f(z_{ik})\bar{\xi}_{ik}-\sum_{k\neq j}^{n} f(z_{jk})\bar{\xi}_{jk}\Big)+\sum_{k\neq i,j}^{n}\frac{f(z_{ik})\,f(z_{kj})}{f(z_{ij})}\,\bar{\xi}_{ik}\bar{\xi}_{kj}\Big].
\end{aligned}
\tag{39}
$$

One may check that this nonlinear chirality constraint is perfectly self-consistent for an *arbitrary* function f.

Now, let us again start from the superspace action (23), but where the fermionic superfields ξ_{ij} and $\bar{\xi}_{ij}$ must obey the constraint (39). Passing to the components and eliminating the auxiliary components one arrives at

$$
\begin{aligned}
Q &= \sum_{i=1}^{n} p_i\psi_i-\mathrm{i}\sum_{i\neq j}^{n}\Big[\left(g+\Pi_{jj}\right)f(z_{ij})+\frac{f'(z_{ij})}{f(z_{ij})}\left(\psi_i-\psi_j\right)\bar{\xi}_{ij}+\sum_{k\neq i,j}^{n}\frac{f(z_{ik})\,f(z_{ij})}{f(z_{kj})}\xi_{ik}\bar{\xi}_{kj}\Big]\xi_{ji},\\
\overline{Q} &= \sum_{i=1}^{n} p_i\bar{\psi}_i-\mathrm{i}\sum_{i\neq j}^{n}\Big[\left(g+\Pi_{jj}\right)f(z_{ij})+\frac{f'(z_{ij})}{f(z_{ij})}\left(\bar{\psi}_i-\bar{\psi}_j\right)\xi_{ij}+\sum_{k\neq i,j}^{n}\frac{f(z_{ik})\,f(z_{ij})}{f(z_{kj})}\bar{\xi}_{ik}\xi_{kj}\Big]\bar{\xi}_{ji}.
\end{aligned}
\tag{40}
$$

For the functions listed in (10) these supercharges coincide with the ones in (6). However, the supercharges (40) generate the $\mathcal{N}=2$ super-Poincaré algebra (7) for an *arbitrary* function f. It is not too hard to find the bosonic part of the new Hamiltonian,

$$
\mathcal{H}_{\rm bos}=\frac{1}{2}\sum_{i}^{n}p_i^2+\frac{g^2}{2}\sum_{i\neq j}^{n}f^2(z_{ij})\,. \tag{41}
$$

Thus, the supercharges (40) provide an $\mathcal{N}=2$ supersymmetrization of a wide class of multiparticle systems with a bosonic Hamiltonian of the type (41). A detailed analysis of such models will be given elsewhere.

6. Conclusions

In this paper we reviewed the Hamiltonian analysis of the $\mathcal{N}=2$ supersymmetric Calogero models, rational as well as trigonometric/hyperbolic, associated with the classical A_n, B_n, C_n and D_n Lie algebras and then provided a superspace description for all of them. We found a minimal superfield content, in which a set of $2n^2$ fermions can be placed for the $\mathcal{N}=2$ supersymmetric n-particle model. As $2n$ fermions accompany the n bosonic coordinates in general bosonic $\mathcal{N}=2$ superfields, the remaining $2n(n-1)$ fermions must be put into additional fermionic $\mathcal{N}=2$ superfields, which have to be constrained such as to describe those fermions alone. It was shown that the Hamiltonian approach proved to be useful for obtaining the appropriate conditions on superfields. After passing to new composite fermionic superfields, the constraints acquired a form of the nonlinear chirality conditions (29), which solve the task. Although these composite fermionic $\mathcal{N}=2$

[6] For the sake of simplicity the A-type case is considered here, in which the $(\lambda_{ij},\bar{\lambda}_{ij})$ and $(\xi_{ij},\bar{\xi}_{ij})$ are related as in (28).

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superfields make the constraint look simple and universal, the Lagrangian written through them takes a more complicated form. In terms of the fundamental fermions it is the other way around. Finally, we demonstrated that, due to the self-consistency of the nonlinear constraints, there are more general supercharges (and the superspace Lagrangian) which provides an $\mathcal{N}=2$ supersymmetrization of bosonic n-particle systems with an arbitrary repulsive two-body interaction.

It may seem that the approach presented here is unnecessarily complicated, because all $\mathcal{N}=2$ supersymmetric Calogero models can be more or less straightforwardly formulated in the standard way by the use of the minimal set of $2n$ fermions [4]. This, however, is no longer the case with $\mathcal{N}>2$ supersymmetric Calogero models, where our treatment with additional fermions becomes essential. Hence, we consider the results presented here as a preparation for further study of the Calogero systems with more supersymmetry in a superspace approach. Let us finally note that a generalization of the $\mathcal{N}=2$ supercharges for Calogero models to the $\mathcal{N}=4$ case goes in an almost trivial way [30]. Therefore, one can expect that the universality of the nonlinear constraints in the $\mathcal{N}=2$ case can be implemented in a similar way for $\mathcal{N}=4$ Calogero models given in superfields and be useful in a superspace construction of $\mathcal{N}\geq 4$ Calogero models.

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