Quantum gravity phenomenology from thermodynamics of spacetime

A. Alonso-Serrano
Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut)
Am Mühlenberg 1, 14476 Potsdam, Germany
E-mail: ana.alonso.serrano@aei.mpg.de

M. Liška
Institute of Theoretical Physics, Faculty of Mathematics and Physics, Charles University
V Holešovičkách 2, 180 00 Prague 8, Czech Republic,
Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut)
Am Mühlenberg 1, Potsdam, Germany
E-mail: liska.mk@seznam.cz

In this review, we discuss a derivation of effective low energy quantum gravitational dynamics from thermodynamics. The derivation is based on the formalism developed in semiclassical thermodynamics of spacetime that allows to obtain Einstein equations from the proportionality of entropy to the area. We first introduce the relevant ingredients of semiclassical thermodynamics of spacetime, paying special attention to the various concepts of entropy involved and their relations. We then extend the semiclassical formalism by considering low energy quantum gravity effects which imply a modified entropy formula with an additional term logarithmic in the area. Upon discussing the derivation of effective gravitational dynamics from this modified entropy, we comment on the most important features of our proposal. Moreover, we show its physical implications on a simple cosmological model and show that it suggests the replacement of the Big Bang singularity by a regular bounce.

Keywords: Quantum gravity phenomenology, Thermodynamics of spacetime

1. Introduction

The search for a consistent theory unifying gravity and quantum physics has been an important direction of research in the last decades. While promising candidates have been put forward, none of them presently provides a complete and consistent final theory. In the absence of a fully developed theoretical framework, quantum gravitational phenomenology offers a way to gain information about possible low energy dynamical effects of quantum gravity.[1–5] Phenomenological models concerning strong gravitational fields are mainly available for physics in the vicinity of classical singularities.[6–10] However, these models face limitations coming from the simplicity of the studied geometries and their results cannot be directly applied in more general settings. Motivated by overcoming these issues, the authors have employed thermodynamics of spacetime to study quantum gravitational phenomenology and propose effective equations of motion applicable in generic spacetimes.[11] As we will see, thermodynamic methods allow us to look for model-independent low energy phenomenological dynamics of quantum gravity without choosing any specific background spacetime. One can then particularise the resulting dynamics to any case of interest. Furthermore, the relevant thermodynamic predictions are common for most of the candidate theories of quantum gravity and can even be obtained by model independent considerations.

Thermodynamics of spacetime presents a useful tool for understanding gravitational dynamics. Following the seminal developments of black hole thermodynamics,[12–14] the framework has been extended both to more general spacetimes[15] and beyond general relativity.[16] A key step forward has been the derivation of Einstein equations from thermodynamics of local Rindler horizons.[17]The original approach has since been improved and generalised to work for different constructions of local horizons,[18–22] certain modified theories of gravity[20–26] and even to derive semiclassical gravitational equations of motion.[19,21,22]

Taking a step further, the authors have included quantum gravity effects on the thermodynamics of local causal horizons, obtaining equations governing low energy effective dynamics of quantum gravity.[11] To get sufficiently robust and general results, we concentrated on the leading order quantum correction to Bekenstein entropy, a term logarithmic in the horizon area. This is advantageous, since its presence is predicted by many different approaches to quantum gravity, including loop quantum gravity (LQG),[27,28] string theory[29,30] and AdS/CFT correspondence,[31] non-local effective field theory,[32] entanglement entropy calculations,[33] as well as model independent considerations based on generalised uncertainty principle (GUP) phenomenology[7] and analysis of statistical fluctuations.[34,35] Thus, the effective dynamics we propose is relevant to most of the main approaches to quantum gravity.

While we are chiefly concerned with low energy quantum gravity effects, thermodynamics of spacetime provides interesting insights already on the semiclassical level. Furthermore, a complete understanding of the semiclassical regime is necessary to extend the thermodynamic formalism to the realm of quantum gravity. Therefore, we include a detailed review of the concepts considered in semiclassical thermodynamics of spacetime, especially various notions of entropy. Since derivations of gravitational dynamics rely on equilibrium conditions for local causal horizons which involve Bekenstein, entanglement and Clausius entropy, we further discuss to what extend are these entropies equivalent.

In the present work, we review the main features of our proposal, paying special attention to the various concepts of entropy involved as well as to the unimodular nature of the resulting gravitational dynamics.[36] Section 2 introduces the necessary entropy definitions and discusses their relations both in the semiclassical and low energy quantum gravity settings. In section 3, we first briefly recap derivation of the effective equations. Then we discuss their properties and illustrate their physical implications on a simple cosmological toy model (finding that the Big Bang singularity can be resolved). Lastly, section 4 sums up our results and presents possible directions for future research.

Throughout the paper, we work in four spacetime dimensions and use metric signature $(-,+,+,+)$. Definitions of the curvature-related quantities follow.[37] We use lower case Greek letters to denote abstract spacetime indices and lower case Latin letters for spatial indices with respect to a (local) Cartesian basis. Unless otherwise explicitly stated, we use the SI units.

2. Entropy in thermodynamics of spacetime

Thermodynamic derivations of gravitational dynamics are based on the observation that gravitational dynamic are encoded in the horizon area equilibrium condition for maximal entropy, $\delta S = 0$, if it holds for all local causal horizons. This condition sums together variations of entropy of the horizon, often interpreted in terms of entanglement (von Neumann) entropy due to quantum correlations across the horizon,[17,19,26] and of entropy of the matter. Since the latter is usually described in terms of thermodynamic Clausius entropy,[17,18] it is not obvious that one can combine both entropies to define a meaningful equilibrium condition (it has actually been suggested that this fails in the case of local Rindler wedges[38]). This combination requires that fluxes of Clausius and von Neumann entropy of matter across the horizon are equal with sufficient precission. In the following, we first introduce our implementation of local causal horizons: geodesic local causal diamonds (GLCD). Then we present the definitions of all the relevant entropies associated with GLCD's and review our previous argument for the interchangeability of Clausius and von Neumann entropy.[36]

2.1. *Geodesic local causal diamonds*

We begin by briefly introducing the construction and most important properties of GLCD's. More detailed description of GLCD's can be found, e.g. in.[39–42]

In an arbitrary spacetime point P choose any unit timelike vector $n(P)$. In every direction orthogonal to n send out of P geodesics of parameter length l. These form a spatial geodesic 3-ball, Σ_0, and the region causally determined by Σ_0 constitutes a GLCD (see figure 1). The boundary, $\mathcal{B}$, of Σ_0 is approximately a 2-sphere. Its area reads[19]

$$\mathcal{A} = 4\pi l^2 - \frac{4\pi}{9} l^4 G_{00}\left(P\right) + O\left(l^5\right), \tag{1}$$

where $G_{00} = G_{\mu\nu} n^\mu n^\nu$. The GLCD possesses an approximate (up to $O\left(l^3\right)$ curvature dependent terms) conformal Killing vector[19]

$$\zeta = C\left(\left(l^2 - t^2 - r^2\right)\frac{\partial}{\partial t} - 2rt\frac{\partial}{\partial r}\right), \tag{2}$$

where C denotes an arbitrary normalisation constant. It has been argued that one can assign Hawking temperature to the GLCD's conformal horizon, $T_H = \hbar\kappa/2\pi k_B c$, where $\kappa = C/2l$ is the surface gravity corresponding to ζ.[19,42]

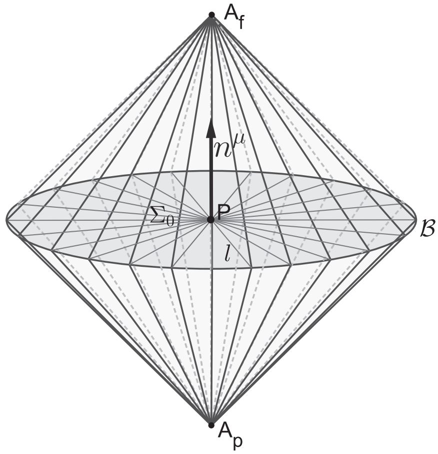

Fig. 1. A sketch of a GLCD with the origin in point P (the angular coordinate θ is suppressed). Σ_0 is a spatial geodesic ball of radius l (several of the geodesics forming it are depicted as grey lines), its boundary an approximate 2-sphere $\mathcal{B}$. Unit timelike vector n^μ is a normal of Σ_0. The tilted lines from the past apex A_p ($t = -l/c$) to the future apex A_f ($t = l/c$) represent the null geodesic generators of the GLCD boundary. The diamond's base Σ_0 is the spatial cross-section of both the future domain of dependence of A_p and the past domain of dependence of A_f at $t = 0$.

2.2. *Bekenstein entropy*

To derive the Einstein equations from thermodynamics of GLCD's (or, conversely, to interpret the first law of GLCD dynamics implied by Einstein equations in thermodynamic terms[42]), one must assume that entropy associated with its horizon obeys the Bekenstein formula[19,22]

$$S_{BH} = \frac{k_B \mathcal{A}}{4 l_P^2}, \tag{3}$$

where $\mathcal{A}$ denotes the horizon area. However, it is far from clear to what types of horizons can one assign Bekenstein entropy. Since its interpretation in terms of quantum entanglement allows a natural extension of the Bekenstein formula to any causal horizon, it is often assumed in the context of thermodynamics of spacetime (although it was originally proposed in a different context[43,44]). To understand the entanglement interpretation of Bekenstein entropy, consider two causally separated spacetime regions. An observer in one region cannot measure vacuum fluctuations in the other one. Since the fluctuations are correlated between regions, some information is inaccessible to the observer and non-zero entanglement entropy appears. Its value is proportional to the area of causal horizon, $S_e = \eta \mathcal{A}$.[33,43,44] The proportionality constant η is infinite unless one introduces a cutoff. Then η becomes finite and depends on the cutoff length, the number and type of quantum fields considered in the theory and even on the position in spacetime.[26,33] To recover the Bekenstein entropy, a universal value $\eta = k_B/4l_P^2$ is necessary (see, e.g.[33,45] for discussions of feasibility and shortcomings of this assumption). Let us stress that the entanglement interpretation of Bekenstein entropy is in no way necessary to derive gravitational dynamics from thermodynamics. Any microscopic interpretation of black hole entropy which also applies to observer-dependent causal horizons would work just as well. Nevertheless, here we focus on the entanglement entropy as it is currently the most developed proposal with this property.

Beyond this semiclassical picture, many different models indicate that the leading order quantum gravity correction to Bekenstein entropy is a non-local term logarithmic in horizon area[7,9,27–35,46–49]

$$S = \frac{k_B \mathcal{A}}{4 l_P^2} + k_B \mathcal{C} \ln\left(\frac{\mathcal{A}}{\mathcal{A}_0}\right), \tag{4}$$

where $\mathcal{C}$ is a real number and $\mathcal{A}_0$ a constant with dimensions of area. The values of $\mathcal{C}$ and $\mathcal{A}_0$ are model dependent. Considering the entanglement entropy interpretation allows us to calculate modified entropy even for causal horizons in a flat spacetime.[33] Interestingly, a logarithmic term then appears for spherical horizons,[50] but not for planar ones[33] (due to different Euler characteristics). This is the reason we consider GLCD's rather than local Rindler wedges to find effective low energy dynamics of quantum gravity from thermodynamics[11] (for arguments against the Rindler wedges even in the semiclassical setting see, e.g.[20,38]).

While the presence of a logarithmic term seems to be a very general prediction of quantum gravity, the value and even the sign of $\mathcal{C}$ differ in various approaches. Generally speaking, two types of corrections to Bekenstein entropy have been studied in the literature. On one hand, microcanonical corrections appear due to more precise counting of the microstates at fixed horizon area, which reduces our uncertainty, leading to negative contribution to entropy. On the other hand, canonical corrections stem from thermal fluctuations at fixed temperature, which are an additional source of uncertainty increasing the entropy. As we discuss in section 3, the sign of $\mathcal{C}$ determines the physical implications of the modified dynamics we propose, allowing the avoidance of spacetime singularities only for $\mathcal{C} > 0$. Therefore, the presently unknown overall sign of $\mathcal{C}$ is crucial for the interpretation of our results.

2.3. *Clausius entropy of the matter*

Apart from Bekenstein entropy, derivation of gravitational dynamics from thermodynamics also requires a way to account for the entropy of matter. The simplest approach is to consider the thermodynamic Clausius entropy, $\mathrm{d}S_C = \delta Q/T$. The heat flux δQ across an arbitrary timelike hypersurface Σ reads[18]

$$\delta Q = -\int_{\Sigma} T_{\mu\nu} V^{\nu} N^{\mu} \mathrm{d}^3\Sigma, \tag{5}$$

where V^{ν} and N^{μ} are timelike and spacelike unit normals of Σ. If V^{ν} corresponds to velocity of an eternal, uniformly accelerating observer with acceleration a, we can define the corresponding Unruh temperature, $T_U = \hbar a/2\pi k_B c$. This definition holds with sufficient precission even for observers with slowly varying acceleration or finite lifetime, as long as the proper length λ of the trajectory with an approximately constant acceleration is large enough, i.e. $\lambda \gg c^2/a$.[51] For the special case of uniformly accelerated observers travelling inside causal diamonds one also finds an equivalent result due to conformal mapping of a Rindler wedge to a causal diamond.[52,53] Therefore, as long as we consider a sufficiently large a, we can use the standard Unruh temperature together with the expression for heat flux to define Clausius entropy flux across a timelike hypersurface

$$S_C \equiv \frac{\delta Q}{T_U} = -\int_{\Sigma} \frac{2\pi k_B c}{\hbar a} T_{\mu\nu} V^{\nu} N^{\mu} \mathrm{d}^3\Sigma. \tag{6}$$

The timelike surface Σ coincides with the causal horizon perceived by the uniformly accelerating observer in the limit $a \to \infty$. Notably, this limit is well defined both for arbitrary causal horizons in flat spacetime and for sufficiently small horizons in curved spacetimes[18] (in that case, one simply considers Riemann normal coordinate expansion to the leading order). Before moving on to the special case of causal diamonds, several further remarks are in order. First, the definition of Clausius entropy flux across null surfaces is completely independent of gravitational dynamics or any requirements on symmetries of the spacetime. Second, the construction of the entropy is semiclassical as it explicitly depends on quantum field theory by invoking the Unruh effect. And, lastly, in contrast with the nature of Clausius entropy in non-relativistic thermodynamics, the entropy flux is manifestly observer dependent.[18]

For a GLCD, the resulting expression for Clausius entropy flux from $t = 0$ (geodesic ball Σ_0) to $t = l/c$ (future apex A_f) takes form[36]

$$\Delta S_C = \int_0^{l/c} \frac{2\pi k_B c}{\hbar} t \left(\int_{\mathcal{S}(t)} T_{\mu\nu} k^{\mu} k^{\nu} \mathrm{d}^2\mathcal{A} \right) \mathrm{d}t + O\left(l^5\right), \tag{7}$$

where k^{μ} denotes the future pointing null normal to the GLCD's boundary. Performing the integration and some straightforward manipulations then yields

$$\Delta S_C = -\frac{8\pi^2 k_B l^4}{9\hbar c} \left(T_{00}\left(P\right) + \frac{1}{4} T\left(P\right) \right) + O\left(l^5\right), \tag{8}$$

where we explicitly stress that the energy momentum tensor is evaluated at the diamond's origin, P.

When one wishes to take into account low energy quantum gravity effects, defining the Clausius entropy flux becomes more complicated. The subtleties in its definition come from the need to consider the Unruh effect, which requires that the ground state of quantum fields is locally well approximated by Minkowski vacuum. This amounts to invoking Einstein equivalence principle:[26] "Fundamental non-gravitational test physics is not affected, locally and at any point of spacetime, by the presence of a gravitational field".[54] However, the status of the equivalence principle in the low energy quantum gravity regime is far from clear. For instance, possible violations of weak equivalence principle resulting from GUP phenomenology have been explored in a number of works with very different conclusions.[55–58]

A simple way to deal with possible influence of the equivalence principle violations on the Unruh effect is to consider heuristic modifications to Unruh temperature due to GUP, previously suggested in the literature.[59,60] All the proposals for modified Unruh temperature obey

$$T_{mod} = \frac{\hbar a\left(1+\psi\frac{l_P^2}{c^4}a^2\right)}{2\pi k_B c} + O\left(\frac{l_P^4 a^5}{c^8}\right), \tag{9}$$

where ψ denotes a model dependent real number that is expected to be of the order of unity.[59,60] A similar prescription has also been earlier proposed for modified Hawking temperature.[7,9,33] To our best knowledge, these modifications to temperature have not been confirmed by any rigorous method. However, since GUP phenomenologically implements effects of a minimal resolvable length that arises in a number of approaches to quantum gravity,[1,2] the idea of modified Unruh temperature is worth considering. Hence, to incorporate leading order quantum gravity corrections in thermodynamics of spacetime, we need to show that the previously outlined construction of Clausius entropy flux works with this modified temperature. Note that the previous reasoning does not apply directly, since in the limit $a \to \infty$ correction terms in the modified temperature formula become dominant. Instead, one must consider acceleration much larger than c^2/l but much smaller than $c^2/\sqrt{\psi}l_P$ (this can be satisfied for $l \gg l_P$, i.e., whenever the GLCD is much larger than the Planck scale). The construction of Clausius entropy flux under this assumption has been carried out by the authors,[11] yielding

$$\Delta S_{C,mod} = -\left(1-\psi\frac{l_P^2 a^2}{c^4}\right)\frac{2\pi k_B c}{\hbar}\int_0^{l/c} t\left(\int_{\mathcal{S}(t)} T_{\mu\nu}k^\mu k^\nu \mathrm{d}^2\mathcal{A}\right)\mathrm{d}t + O\left(l^5, \frac{l_P^4 a^4}{c^8}, \frac{1}{a^2}\right). \tag{10}$$

The only difference with respect to the semiclassical formula are the a-dependent sub-leading terms. We can recover it by setting $\psi = 0$ and then taking the limit $a \to \infty$. Let us note that while we assumed finite acceleration simply for mathematical convenience, the limit $a \lesssim c^2/\sqrt{\psi}l_P$ agrees quite well with the proposal of an upper limit to acceleration a_M set by quantum gravity effects, $a_M = c^2/l_P$.[61]

2.4. *Von Neumann entropy of the matter*

To be consistent with the entanglement interpretation of the entropy of GLCD's horizon, one should consider quantum von Neumann entropy of the matter rather than the thermodynamic Clausius one. For small variations from vacuum, this entropy can be obtained from the vacuum state density operator, $\rho = e^{-K/k_B T_\zeta}/Z$, where Z is the partition function, T_ζ denotes the previously defined temperature associated with conformal Killing vector ζ^μ and K is known as the modular Hamiltonian.[42] While variations of K can in general be non-local, for conformal fields they correspond to variations of the local matter Hamiltonian[42]

$$\delta H = \int_{\Sigma_0} \delta\langle T_{\mu\nu}\rangle \xi^\mu n^\nu \mathrm{d}^2\Omega \mathrm{d}r, \tag{11}$$

where $n = \partial/\partial t$. Then, the variation of entropy reads

$$\delta S_m = \frac{1}{T}\delta K = \frac{2\pi k_B}{\hbar c}\frac{4\pi l^4}{15}\delta\langle T_{00}\rangle + O\left(l^5\right). \tag{12}$$

For non-conformal field theories with a UV fixed point, the von Neumann entropy variation is modified only by the presence of an additional l-dependent spacetime scalar, δX,[19,62,63]

$$\delta S_m = \frac{2\pi k_B}{\hbar c}\frac{4\pi l^4}{15}\left(\delta\langle T_{00}\rangle + \delta X\right). \tag{13}$$

In our study concerning the quantum gravity corrections, we have gone beyond the semiclassical case just by including modifications to the temperature[11] (in a future work we plan to analyse possible corrections to the modular Hamiltonian). Implementing the leading order quantum gravity modification to temperature gives rise to the following modified variation of matter von Neumann entropy

$$\delta S_{m,mod} = \frac{2\pi k_B c}{\hbar}\frac{4\pi}{15}l^4\left(\delta\langle T_{00}\rangle + \delta X\right)\left(1 - \psi l_P^2\kappa^2\right) + O\left(l^5\right). \tag{14}$$

Note that for the particular case $\psi = 0$ we straightforwardly recover the semiclassical expression.

2.5. *Entropy equivalence*

Assuming the entanglement interpretation of Bekenstein entropy, the natural equilibrium condition for the GLCD involves von Neumann entropy of the matter. To consider the Clausius entropy instead, its equivalence with the von Neumann entropy is necessary. This condition requires that fluxes of Clausius and von Neumann entropy of matter across the horizon are equal with sufficient precission. Since the von Neumann entropy is defined for the geodesic ball Σ_0 corresponding to $t = 0$, the meaningful quantities to compare are time derivatives of both entropies evaluated at $t \ll l/c$ (at precisely $t = 0$ the entropy fluxes vanish). We start by considering conformal fields. For the time derivative Clausius entropy we have[36]

$$\frac{\mathrm{d}S_C}{\mathrm{d}t}(t) \approx -\frac{32\pi^2 k_B c}{3\hbar} t l^2 T_{00}, \tag{15}$$

while the time derivative of the von Neumann entropy reads[36]

$$\frac{\mathrm{d}S_m}{\mathrm{d}t} \approx -\frac{32\pi^2 k_B c}{3\hbar} t l^2 \delta\langle T_{00}\rangle. \tag{16}$$

Both expressions are indeed equivalent, making the use of Clausius entropy in thermodynamics of spacetime justified for conformal fields. Note that the argument in no way involves gravitational dynamics.

The situation becomes more complicated for non-conformal matter as the previously outlined approach cannot be directly applied to formulas for the von Neumann entropy.[62,63] Nevertheless, semiclassical gravitational dynamics derived from Clausius and von Neumann entropy are equivalent even in this case,[17,19,36] suggesting that both entropies are at least strongly related.

When low energy quantum gravity effects on the temperature are taken into account, we find for conformal fields

$$\frac{\mathrm{d}S_C}{\mathrm{d}t}(t) \approx -\frac{32\pi^2 k_B c}{3\hbar}\left(1-\psi\frac{l_P^2}{c^4}a^2\right)tl^2T_{00}, \tag{17}$$

and

$$\frac{\mathrm{d}S_m}{\mathrm{d}t} \approx -\frac{32\pi^2 k_B c}{3\hbar}\left(1-\psi l_P^2\kappa^2\right)tl^2\delta\langle T_{00}\rangle. \tag{18}$$

Both formulas are equivalent only for a special value of the surface gravity, $\kappa = a/c^2$. Since we are not aware of any way to motivate a specific choice of κ, and a needs to be very large, $a \gg c^2/l$, we cannot say anything conclusive about the equivalence of both entropies. We will return to this question after we introduce the modified gravitational dynamics.

3. Effective low energy quantum gravity dynamics

Upon presenting the various notions of entropy involved, we proceed to discuss our proposal for the effective low energy quantum gravitational dynamics. Details of the derivation can be found in the original paper of the authors;[11] here we just briefly recall the main points. For the sake of comparison, we employed two different thermodynamic derivations. The first one, known as the physical process approach, keeps track of the Clausius entropy flux across the horizon and the corresponding changes in its Bekenstein entropy. Equilibrium condition for this process reads $\Delta S_{e,q}+\Delta S_C = 0$, where the Clausius entropy flux contribution, ΔS_C, was discussed in the previous section. The Bekenstein entropy part, $\Delta S_{e,q}$, can be evaluated from the formula for area of GLCD's boundary cross-section introduced in subsection 2.1. After a series of calculations (see[11] for details), we obtain the modified equations of motion

$$S_{\mu\nu} - \frac{\mathcal{C}l_P^2}{18\pi}S_{\mu\lambda}S^{\lambda}{}_{\nu} + \frac{\mathcal{C}l_P^2}{72\pi}\left(R_{\kappa\lambda}R^{\kappa\lambda}-\frac{1}{4}R^2\right)g_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu}-\frac{1}{4}Tg_{\mu\nu}\right), \tag{19}$$

where $S_{\mu\nu} = R_{\mu\nu} - Rg_{\mu\nu}/4$ denotes the traceless part of Ricci tensor. The derivation works for GLCD's length scale l much larger than the Planck length but much smaller than the curvature length scale (square root of the inverse of the largest eigenvalue of the Riemann tensor).

The second method to derive the modified equations of motion starts from the entanglement equilibrium hypothesis: "When the geometry and quantum fields are simultaneously varied from maximal symmetry, the entanglement entropy in a small geodesic ball is maximal at fixed volume".[19] For a small variation from a maximally symmetric spacetime the hypothesis implies $\delta S_{e,q}+\delta S_m = 0$, where δS_m denotes the variation of matter von Neumann entropy. From here, calculations proceed similarly as for the physical process approach, yielding[11]

$$S_{\mu\nu} - \frac{\mathcal{C}l_{Pl}^2}{30\pi} S_{\mu\lambda} S^{\lambda}{}_{\nu} + \frac{\mathcal{C}l_{Pl}^2}{120\pi}\left(R_{\kappa\lambda}R^{\kappa\lambda} - \frac{1}{4}R^2\right) g_{\mu\nu} = \frac{8\pi G}{c^4}\left(\delta\langle T_{\mu\nu}\rangle - \frac{1}{4}\delta\langle T\rangle g_{\mu\nu}\right). \tag{20}$$

Notably, in neither derivation does the form of modified temperature affect the resulting equations. This is expected as definitions of both Hawking and Unruh temperature (even modified ones) are purely kinematic with no dependence on gravitational dynamics.[64] The correction terms are thus fully determined by the logarithmic contribution to Bekenstein entropy and they depend only on one presently unknown parameter, $\mathcal{C}$.

Comparing the results of both approaches, we can see that the equations have the same form, although coefficients in front of the terms quadratic in curvature differ. We can write both equations in a common form

$$S_{\mu\nu} - Dl_P^2 S_{\mu\lambda} S^{\lambda}{}_{\nu} + \frac{D}{4}l_P^2\left(R_{\kappa\lambda}R^{\kappa\lambda} - \frac{1}{4}R^2\right) g_{\mu\nu} = \frac{8\pi G}{c^4}\left(\langle T_{\mu\nu}\rangle - \frac{1}{4}\langle T\rangle g_{\mu\nu}\right), \tag{21}$$

with $D = \mathcal{C}/18\pi$ for the physical process approach and $D = \mathcal{C}/30\pi$ for the entanglement equilibrium hypothesis one. In both cases, the coefficient D has the same sign and differs only by a factor 3/5. The difference might be due to the details of the variation considered in the entanglement equilibrium hypothesis derivation, which is performed at fixed spatial volume.[11,19] Since, for modified theories of gravity, a more complicated quantity known as generalised volume needs to be held fixed instead,[21] finding the appropriate generalised volume in our case could account for the difference in D between both approaches. Alternatively, it might indicate a failure on the equivalence between Clausius and von Neumann entropy of the matter at this level (although, given the similarity of the results, both entropies would have to remain closely related).

Upon reviewing the two derivations of the modified equations of motion, we proceed to discus their properties (for a more detailed treatment, see[11]). First, note that while the value and even the sign of $\mathcal{C}$ is model dependent, any possible value can be viewed as being of the order of unity compared to the squared Planck length, $l_P^2 \approx 2.6 \times 10^{-70}\,\mathrm{m}^2$. Therefore, the correction terms become relevant only when the curvature length scale nears the Planck length (although it needs to remain significantly larger than l_P, otherwise the assumptions of our derivation will break down).

As a second property, it is easy to check that the modified equations are traceless. Hence, they do not imply local energy-momentum conservation. If we want to assume it, we must impose divergence-free energy-momentum tensor, $\langle T_{\mu}{}^{\nu}\rangle_{;\nu} = 0$, as an additional condition. Satisfying it requires

$$\frac{1}{4}R_{;\mu} - Dl_P^2\left(S^{\lambda\nu}S_{\mu\lambda}\right)_{;\nu} + \frac{Dl_P^2}{2}\left(R^{\kappa\lambda}R_{\kappa\lambda;\mu} - \frac{1}{4}RR_{;\mu}\right) = -\frac{2\pi G}{c^4}\langle T\rangle_{;\mu}. \tag{22}$$

This condition does not have a general solution for T. Therefore, in general, the equations of motion cannot be recast in a form directly implying a divergence-free energy-momentum tensor. Nevertheless, the condition can be solved for spacetimes with a vanishing Weyl tensor. In that case, the cosmological constant, absent in the traceless equations of motion, appears as an arbitrary integration constant.

Third, while any terms containing higher than fourth derivatives of the metric or more than quadratic in curvature tensors are likely suppressed by higher powers of l_P, the higher derivative terms known from quadratic gravity should appear at the same order as the corrections we propose. They implicitly appear on the right hand side of modified equations in the quantum expectation value of the energy-momentum tensor.[65] In principle, one might also find higher derivative contributions to the left hand side by considering higher orders in the Riemann normal coordinate expansion of the metric. However, these corrections are ambiguous as they depend on shape deformations of the GLCD's horizon.[40] Without a physically motivated way to resolve these ambiguities, it is not possible to determine higher derivative terms contributing to the left hand side of the equations. Since these terms are anyway contained with undetermined coefficients on the right hand side, in the energy-momentum tensor expectation value, their omission on the left hand side does not change the resulting dynamics in any significant way.

Traceless equations of motion and the status of cosmological constant as an arbitrary integration constant both point out to the modified dynamics being a generalisation of unimodular gravity (or, more precisely, Weyl transverse gravity[66]). In fact, even the semiclassical gravitational dynamics derived from thermodynamics has more in common with unimodular gravity than general relativity.[25,36,67] We can partially understand the emergence of unimodular gravity from thermodynamics by noting that only the difference between entropy of two states is relevant for deriving the gravitational dynamics. In other words, vacuum contribution to matter entanglement entropy does not affect the conditions for thermodynamic equilibrium. Consequently, vacuum energy naturally does not couple to gravity, leading to the behaviour of the cosmological constant characteristic for unimodular gravity. Furthermore, thermodynamics of spacetime do not imply local energy-momentum conservation, which always needs to be postulated as an additional condition, again in agreement with unimodular gravity.

3.1. *Application to a cosmological toy model*

To illustrate the physical consequences of the modified dynamics, we examine a simple cosmological toy model, a homogeneous, isotropic, spatially flat universe filled with a perfect fluid with the equation of state $p = (\gamma - 1)\, c^2 \rho$ for some $\gamma \in [1, 2]$ (the limits correspond to dust and stiff matter, respectively). Due to previously noted unimodular character of the equations, we consider a unimodular form of the

Friedmann-Lemaître-Robertson-Walker (FLRW) metric

$$\mathrm{d}s^2 = -\frac{c^2}{a\left(t\right)^6}\mathrm{d}t^2 + a\left(t\right)^2\left(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right), \tag{23}$$

where $a\left(t\right)$ denotes the scale factor. Because of the symmetries of the metric, the modified equations of the motion yield only one non-trivial condition

$$\dot{H} - D\frac{l_P^2\dot{H}^2}{c^2} = -4\pi\gamma G\rho, \tag{24}$$

where $H = \dot{a}/a$ is the Hubble parameter and the dot denotes a time derivative. This condition corresponds to a modified Raychaudhuri equation, with a correction term, $-Dl_P^2\dot{H}^2/c^2$, non-linear in second time derivatives of the scale factor. To simplify it, we assume that the Hubble parameter corresponds to its classical value H_0 up to $O\left(l_P^2\right)$ terms (validity of this assumption is extensively discussed in[11]),

$$H = H_0 + l_P^2 H_1 + O\left(l_P^4\right), \tag{25}$$

and rewrite the modified Raychaudhuri equations in the following way

$$\dot{H} = -4\pi\gamma G\rho\left(1 - 4\pi D\gamma\frac{\rho}{\rho_P}\right), \tag{26}$$

where $\rho_P = \sqrt{c^5/G^2\hbar}$ denotes the Planck density. If we further assume local energy-momentum conservation, we can integrate the modified Raychaudhuri equation to obtain the modified Friedmann equation

$$H^2 = \frac{8\pi G\rho}{3}\left(1 - \frac{2\pi D\gamma\rho}{\rho_P}\right) + \tilde{\Lambda}, \tag{27}$$

where $\tilde{\Lambda}$ is an arbitrary integration constant corresponding to the cosmological term ($\tilde{\Lambda} = \Lambda c^2/3$). We see that the appearance of cosmological constant is indeed consistent with unimodular gravity.

Notably, for $D > 0$, our results correspond to the effective description of loop quantum cosmology (LQC).[68] Since effective dynamics of LQC replaces the Big Bang singularity by a non-singular quantum bounce, the same conclusion holds for our modified equations of motion assuming $D > 0$. The bounce corresponds to a critical density, $\rho_{crit} = \rho_P/2\pi D\gamma$. In other words, if the logarithmic correction to Bekenstein entropy of a GLCD is positive, the resulting modifications of the dynamics are already sufficient to remove the cosmological singularity. On the other hand, the case of $D < 0$ appears to not only preserve the singularity, but actually strengthen the gravitational attraction responsible for it. These preliminary results warrant a more detailed analysis of cosmological models and possible values for parameter D.

4. Discussion

In the present work we have first reviewed the issue of entropy equivalence in thermodynamics of spacetime. For conformal fields in the semiclassical regime, Clausius and von Neumann entropy turn out to be equivalent in the semiclassical regime independently of the gravitational dynamics, although a similar clear result is not available for non-conformal fields.

Then, we proceeded to review our proposal for a new phenomenological perspective on the effects of quantum gravity at low energies. The effective low energy quantum gravitational dynamics we have proposed is based on a single assumption: leading order quantum gravity correction to entropy associated with spherical local causal horizons is logarithmic in the horizon area. Since a number of conceptually different approaches predict such a logarithmic term (at the very least in the case of black hole entropy), our conclusions are fairly robust and relevant to many candidate theories of quantum gravity, e.g. LQG, string theory, AdS/CFT correspondence and GUP phenomenology.

Let us remark that the equations we found are a generalisation of the classical equations of motion of unimodular gravity and cannot even be restated as some generalised Einstein equations. This agrees with our previous results concerning the relation between thermodynamics of spacetime and unimodular gravity in the semiclassical setting. It does appear that the corrections we propose break the equivalence of unimodular gravity and general relativity that holds on the level of classical dynamics.

Our results also extend the semiclassical equivalence between Clausius and entanglement entropies. While they suggest a possible breaking of the exact equivalence on the quantum level, both entropies remain strongly related. Their precise relation should be analysed carefully in a future work. Importantly, a more systematic inclusion of quantum gravity effects on matter entropy will be necessary to completely resolve this issue.

Note that our approach still requires further development. First, issues of diffeomorphism invariance, equivalence principle and locality of the modified dynamics previously discussed by the authors[11] need to be addressed in greater detail. Moreover, the apparent breaking of local energy conservation requires a physically motivated explanation. Relating our proposal to other modified gravity theories might shed some light on all these questions. In particular, our effective equations of motion resemble the 4-dimensional version of Einstein-Gauss-Bonnet gravity[69,70] and non-local effective field theory of gravity.[71] The latter approach even leads to logarithmic term in Bekenstein entropy consistent with our assumptions.[32]

Another possible further development of our results lies in a careful inclusion of higher order Riemann normal coordinate corrections in the analysis in order to obtain higher derivative terms on the left hand side. Similarly, accounting for low energy quantum gravity effects on the entropy of matter might lead to interesting conclusions.

Furthermore, we have seen that the modified dynamics resolve the Big Bang singularity only if the logarithmic correction to entropy of a GLCD is positive, i.e., $D > 0$. To find which approaches to quantum gravity support this sign and what are the bounds on the magnitude of D they imply would help to constrain the physical implications of our proposal.

Lastly, the emergence of unimodular (Weyl transverse) gravity from thermodynamics needs to be better understood even in the semiclassical case. The first steps in this direction include developing rigorous formalism for thermodynamics of spacetime in the Weyl transverse gravity and connecting Weyl invariance of gravitational dynamics with properties of entropy. All these issues will be addressed in future works.

Acknowledgments

AA-S is supported by the ERC Advanced Grant No. 740209. ML is supported by the Charles University Grant Agency project GAUK 297721.

References

1. L. J. Garay, "Quantum gravity and minimum length," Int. J. Mod. Phys. A **10** (1995) 145 [gr-qc/9403008].
2. S. Hossenfelder, "Minimal Length Scale Scenarios for Quantum Gravity," Living Rev. Relativ. **16** (2013) 2 [gr-qc/1203.6191].
3. L. Smolin, "Four principles for quantum gravity," Fundam. Theor. Phys. **187** (2017) 427 [gr-qc/1610.01968].
4. S. Chakraborty, D. Kothawala and A. Pesci, "Raychaudhuri equation with zero point length," Phys. Lett. B **797** (2019) 134877 [gr-qc/1904.09053].
5. l. Buoninfante, G. Luciano, L. Petruzziello and F. Scardigli, "Bekenstein bound and uncertainty relations," [hep-th/2009.12530].
6. J. D. Bekenstein and V. F. Mukhanov, "Spectroscopy of the quantum black hole," Phys. Lett. B **360** (1995) 7 [gr-qc/9505012].
7. R. J. Adler, P. Chen and D. I. Santiago, "The Generalized Uncertainty Principle and Black Hole Remnants," Gen. Rel. Grav. **33** (2001) 2101 [gr-qc/0106080].
8. A. Awad and A. F. Ali, "Minimal Length, Friedmann Equations and Maximum Density," JHEP **06** (2014) 093 [gr-qc/1404.7825].
9. A. Alonso-Serrano, M. P. Dabrowski and H. Gohar, "Generalized uncertainty principle impact onto the black holes information flux and the sparsity of Hawking radiation," Phys. Rev. D **97** (2018) 044029 [gr-qc/1801.09660].
10. R. Casadio, "Quantum black holes and resolution of the singularity," [gr-qc/2103.00183].
11. A. Alonso-Serrano and M. Liška, "Quantum phenomenological gravitational dynamics: A general view from thermodynamics of spacetime," JHEP **2020** (2020) [gr-qc/2009.03826].
12. J. D. Bekenstein, "Black Holes and Entropy," Phys. Rev. D **7** (1973) 2333.
13. J. D. Bardeen, B. Carter and S. W. Hawking, "The Four Laws of Black Hole Mechanics," Commun. Math. Phys. **31** (1973) 161.
14. S. W. Hawking, "Particle Creation by Black Holes," Commun. Math. Phys. **43** (1975) 199.

15. T. Jacobson and R. Parentani, "Horizon Entropy," Found. Phys. **33** (2003) 323 [gr-qc/0302099].
16. R. M. Wald, "Black Hole Entropy is Noether Charge," Phys. Rev. D **48** (1993) 3427 [gr-qc/9307038].
17. T. Jacobson, "Thermodynamics of space-time: The Einstein equation of state," Phys. Rev. Lett. **75** (1995) 1260 [gr-qc/9504004].
18. V. Baccetti and M. Visser, "Clausius entropy for arbitrary bifurcate null surfaces," Class. Quant. Grav. **31** (2014) 035009 [gr-qc/1303.3185].
19. T. Jacobson, "Entanglement Equilibrium and the Einstein Equation," Phys. Rev. Lett. **116** (2016) 201101 [gr-qc/1505.04753].
20. M. Parikh and A. Svesko, "Einstein's Equations from the Stretched Future Light Cone," Phys. Rev. D **98** (2018) 026018 [hep-th/1712.08475].
21. P. Bueno, V. S. Min, A. J. Speranza and M. R. Visser, "Entanglement equilibrium for higher order gravity," Phys. Rev. D **95** (2017) 046003 [hep-th/1612.04374].
22. A. Svesko, "Equilibrium to Einstein: Entanglement, Thermodynamics, and Gravity," Phys. Rev. D **99** (2019) 086006 [hep-th/1810.12236].
23. R. Guedens, T. Jacobson and S. Sarkar, "Horizon entropy and higher curvature equations of state," Phys. Rev. D **85** (2012) 064017 [gr-qc/1112.6215].
24. C. Eling, R. Guedens and T. Jacobson "Non-equilibrium Thermodynamics of Spacetime," Phys. Rev. Lett. **96** (2006) 121301 [gr-qc/0602001].
25. T. Padmanabhan, "Thermodynamical Aspects of Gravity: New insights," Rep. Prog. Phys. **73** (2010) 046901 [gr-qc/0911.5004].
26. G. Chirco, S. Liberati, "Non-equilibrium Thermodynamics of Spacetime: the Role of Gravitational Dissipation," Phys. Rev. D **81** (2010) 024016 [gr-qc/0909.4194v1].
27. R. K. Kaul and P. Majumdar, "Logarithmic Correction to the Bekenstein-Hawking Entropy," Phys. Rev. Lett. **84** (2000) 5255 [gr-qc/0002040].
28. K. A. Meissner, "Black hole entropy in Loop Quantum Gravity," Class. Quant. Grav. **21** (2004) 5245 [gr-qc/0407052].
29. A. Sen, "Logarithmic Corrections to Schwarzschild and Other Non-extremal Black Hole Entropy in Different Dimensions," JHEP **04** (2013) 156 [hep-th/1205.0971].
30. S. Banerjee, R. K. Gupta, I. Mandal and A. Sen, "Logarithmic Corrections to N=4 and N=8 Black Hole Entropy: A One Loop Test of Quantum Gravity," JHEP **11** (2011) 143 [hep-th/1106.0080].
31. T. Faulkner, A. Lewkowycz and J. Maldacena, "Quantum corrections to holographic entanglement entropy," JHEP **2013** (2013) 074 [hep-th/1307.2892].
32. Y. Xiao and Y. Tian, [gr-qc/2104.14902].
33. S. Solodukhin, "Entanglement entropy of black holes," Living Rev. Rel. **14** (2011) 8 [hep-th/1104.3712].
34. S. Das, P. Majumdar and R. K. Bhaduri, "General Logarithmic Corrections to Black Hole Entropy," Class. Quant. Grav. **19** (2002) 2355 [hep-th/0111001].
35. G. Gourd and A. J. M. Medved, "Thermal Fluctuations and Black Hole Entropy," Class. Quant. Grav. **20** (2003) 3307 [gr-qc/0305018].
36. A. Alonso-Serrano and M. Liška, "New Perspective on thermodynamics of spacetime: The emergence of unimodular gravity and the equivalence of entropies," Phys. Rev. D **102** (2020) 104056 [gr-qc/2008.04805].
37. C. W. Misner, K. S. Thorne and J. A. Wheeler, "Gravitation," Princeton University Press, Princeton, NJ, U.S.A. (2017).
38. S. M. Carroll and G. N. Remmen, "What is the Entropy in Entropic Gravity?," Phys Rev D **93** (2016) 124052 [hep-th/1601.07558].

39. G. W. Gibbons and S. Solodukhin, "The Geometry of Small Causal Diamonds," Phys. Lett. B **649** (2007) 317 [hep-th/0703098].
40. T. Jacobson, J. M. M. Senovilla and A. J. Speranza, "Area deficits and the Bel-Robinson tensor," Class. Quant. Grav. **35** (2018) 085005 [gr-qc/1710.07379].
41. J. Wang, "Geometry of small causal diamonds," Phys. Rev. D **100** (2019) 064020 [gr-qc/1904.01034].
42. T. Jacobson and M. R. Visser, "Gravitational Thermodynamics of Causal Diamonds in (A)dS," SciPost Phys **7** (2019) 079 [hep-th/1812.01596].
43. L. Bombelli, R. K. Koul, J. Lee, R. D. Sorkin, "Quantum source of entropy for black holes," Phys. Rev. D **34** 373 (1986).
44. M. Srednicki, "Entropy and area," Phys. Rev. Lett. **71** (1993) 666 [hep-th/9303048v2].
45. T. Jacobson, "Black Hole Entropy and Induced Gravity," [gr-qc/9404039].
46. A. Davidson, "From Planck area to graph theory: Topologically distinct black hole microstates," Phys Rev D **100** (2019) 081502 [hep-th/1907.03090].
47. S. Hod, "High-Order Corrections to the Entropy and Area of Quantum Black Holes," Class. Quant. Grav. **21** (2004) L97 [hep-th/0405235].
48. A. J. M. Medved and E. C. Vagenas, "When conceptual worlds collide: The GUP and the BH entropy," Phys. Rev. D **70** (2004) 124021 [hep-th/0411022].
49. A. J. M. Medved, "A follow-up to 'Does Nature abhor a logarithm?' (and apparently she doesn't)," Class. Quant. Grav. **22** (2005) 5195 [gr-qc/0411065].
50. S. Solodukhin, "Entanglement entropy of round spheres," Phys. Lett. B **693** (2010) 605 [hep-th/1008.4314].
51. L. C. Barbado and M. Visser, "Unruh-DeWitt detector event rate for trajectories with time-dependent acceleration," Phys. Rev. D **86** (2012) 084011 [gr-qc/1207.5525].
52. P. Martinetti and C. Rovelli, "Diamonds's Temperature: Unruh effect for bounded trajectories and thermal time hypothesis," Class. Quant. Grav. **20** (2003) 4919-4932 [gr-qc/0212074].
53. M. Arzano, "Conformal quantum mechanics of causal diamonds," JHEP **2020** (2020) [gr-qc/2002.01836].
54. E. di Casola, S. Liberati, S. Sonego, "Nonequivalence of equivalence principles," Am. J. Phys. **83** (2015) 39 [gr-qc/1310.7426].
55. S. Ghosh, "Quantum Gravity Effects in Geodesic Motion and Predictions of Equivalence Principle Violation," Class. Quant. Grav. **31** (2014) 025025 [gr-qc/1303.1256].
56. V. M. Tkachuk, "Effect of the generalized uncertainty principle on Galilean and Lorentz transformations," Found. Phys. **46** (2016) 1666 [quant-ph/1310.6243].
57. S. Pramanik, "Implication of Geodesic Equation in Generalized Uncertainty Principle framework," Phys. Rev. D **90** (2014) 024023 [hep-th/1404.2567].
58. R. Casadio and F. Scardigli, "Generalized Uncertainty Principle, Classical Mechanics, and General Relativity," Phys. Lett. B **598** (2020) 135558 [gr-qc/2004.04076].
59. F. Scardigli, M. Blasone, G. Luciano and R. Casadio, "Modified Unruh effect from Generalized Uncertainty Principle," Eur. Phys. J. C **78** (2018) 728 [hep-th/1804.05282].
60. G. G. Luciano and L. Petruzziello, "GUP parameter from Maximal Acceleration," Eur. Phys. J. C **79** (2019) 283 [hep-th/1902.07059].
61. E. R. Caianiello, "Maximal acceleration as a consequence of Heisenberg's uncertainty relations," Lett. Nuovo Cimento **41** (1984) 370.
62. A. J. Speranza, "Entanglement entropy of excited states in conformal perturbation theory and the Einstein equation," JHEP **2016** (2016) [hep-th/1602.01380].
63. H. Casini, D. A. Galante, R. C. Myers, "Comments on Jacobson's "Entanglement equilibrium and the Einstein equation"," JHEP **2016** (2016) [hep-th/1601.00528].

64. M. Visser, "Essential and inessential features of Hawking radiation," Int. J. Mod. Phys. D **12** (2003) 649 [hep-th/0106111].
65. R. M. Wald, "Quantum field theory in curved spacetime and black hole thermodynamics," The University of Chicago Press, Chicago, IL, U.S.A. and London, U.K. (1994).
66. C. Barceló, L. J. Garay and R. Carballo-Rubio, "Absence of cosmological constant problem in special relativistic field theory of gravity," Ann. Phys. **398** (2018) 9 [gr-qc/1406.7713].
67. S. C. Tiwari, "Thermodynamics of Spacetime and Unimodular Relativity," (2006) [gr-qc/0612099].
68. A. Ashtekar and B. Gupt, "Generalized effective description of loop quantum cosmology," Phys. Rev. D **92** (2015) 084060 [gr-qc/1509.08899].
69. D. Glavan and C. Lin, "Einstein-Gauss-Bonnet gravity in 4-dimensional space-time," Phys. Rev. Lett. **124** (2019) 081301 [gr-qc/1905.03601].
70. J. Arrechea, A. Delhom and A. Jiménez-Cano, "Inconsistencies in four-dimensional Einstein-Gauss-Bonnet gravity," CPC **45** (2021) 013107 [gr-qc/2004.12998].
71. J. F. Donoghue, "General relativity as an effective field theory: The leading quantum corrections," Phys. Rev. D **50** (1994) 3784 [gr-qc/405057].