Institute of Physics Publishing Journal of Physics: Conference Series **24** (2005) 87–110
doi:10.1088/1742-6596/24/1/011 VI Mexican School on Gravitation and Mathematical Physics

De Sitter Vacua and the Landscape of String theory

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Abstract. In this pedagogical lecture we explain some basic part of the standard cosmological model which is most relevant for the fundamental theoretical physics. We stress the common features and differences between early universe inflation and late-time acceleration. We than proceed with some recent attempts to address the issues of cosmology in string theory and higher dimensional supergravity with the emphasis on successes and still unsolved problems.

1. Introduction

Our universe presents an ultimate test for the fundamental physics: high-energy accelerators will probe the scale of energies way below GUT scales. Cosmology and astrophysics are the only known to us sources of data in the gravitational sector of the fundamental physics (above GUT, near Planck scale).

Starting from 1998 significant amount of cosmological data have been obtained which suggested an emergence of a standard cosmological model. Different independent cosmological observations are in agreement with each other and therefore the standard cosmological model is also called sometimes a cosmological concordance model. These observations suggest that the universe is spatially flat, contains dark matter and dark energy, only few percent of the total energy is in matter which we know. The primordial spectrum of fluctuations is approximately scale invariant and initial fluctuations are Gaussian and adiabatic. This standard cosmological model can be described in terms of only few parameters, which explain a large number of observations, such as the cosmic microwave background (CMB), galaxy clustering, supernova data, and weak lensing. The latest results from Wilkinson Microwave Anisotropy Probe (WMAP) CMB measurements, Sloan Digital Sky Survey (SDSS) and Two degree Field (2dF) galaxy clustering analysis, and from the latest Supernovae type Ia (SNIa) data have been analysed in [1] where also the properties of the standard cosmological models were presented.

The emergence of cosmological standard model affects all areas of fundamental theoretical physics, particularly M/string theory. To the extent to which one believes in the data supporting the standard cosmological model, one would expect that the fundamental physics should explain the main features of the cosmological observations. This includes the late-time acceleration of the universe as well as the early universe inflation. The recently discovered fact that 70% of the total energy of the universe is in mysterious dark energy requires an explanation since it is difficult to expect that this effect will go away when more observations will be performed.

The purpose of these lectures is to give a pedagogical explanation of some basic features of the standard cosmological model which are most relevant for the fundamental theoretical physics[1]. We will than proceed with the recent attempts to address the issues of cosmology in string theory. These will include the following String Theory–Cosmology topics.

(i) KKLT model of de Sitter space
(ii) Early Universe Inflation in String Theory
- Racetrack Inflation
- KKLMMT model of D3-anti-D3 Brane Inflation
- Hybrid Inflation in D3/D7 Brane System

We will conclude with some remarks on Landscape of String Theory.

2. Some Features of the Cosmological Concordance Model

All observations so far fit a four-dimensional Einstein general relativity. The background geometry of the expanding homogeneous isotropic universe of the Friedman-Robertson-Walker type is characterized by the scale factor of the universe $a(t)$ which grows in time.

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{1}$$

and $k = -1, 0, +1$ for the open, flat and closed FRW universe with the background metric (1). Quantum fluctuations over the background are responsible for the small anisotropy of the cosmic microwave background of the order 10^{-5}.

The common features and differences between early universe inflation (the period of *rapid acceleration* in the early universe is called inflation) and late-time acceleration are presented in the Table 1.

Table 1. COSMOLOGICAL CONCORDANCE MODEL DESCRIBES:

Early Universe Inflation	Late-time Acceleration
Near de Sitter space	Near de Sitter space
13.7 billion years ago	Now
During 10^{-35} sec	During few billion years
$V \sim H^2 M_P^2$	$V \sim H^2 M_P^2$
$H_{\rm infl} \leq 10^{-5} M_P$	$H_{\rm accel} \leq 10^{-60} M_P$

$$\frac{\dot a}{a} = H \approx \text{const}$$

$$\frac{\ddot a}{a} > 0\,, \qquad a(t) \sim e^{Ht}$$

During the early universe inflation as well as in the recent period of acceleration *the scale factor of the universe has a positive second derivative, i. e. the universe is accelerating*

[1] I will mostly describe the work in this direction in which I was involved or closely related work. There were many other developments in string cosmology which will not be covered in these lectures: it is a rapidly developing topic and it is difficult to follow all new studies.

(inflating). Note that during inflation when $\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3p) > 0$ the strong energy condition $\rho + 3p \geq 0$ is violated. Here ρ is the energy density and p is the pressure in FRW cosmology with the energy-momentum tensor of the perfect fluid form: $T_{\mu\nu} = (\rho + p)U_\mu U_\nu + pg_{\mu\nu}$.

The fundamental question is how to get this picture from the compactified ten-dimensional string theory or eleven-dimensional M-theory and supergravity. In addition, one would like to derive the values of the phenomenological parameters in cosmological standard model from string theory.

The most important cosmological parameters are defined as follows. The density parameter Ω is defined as a ratio of the energy density to the critical density

$$\Omega \equiv \frac{\rho}{\rho_{\rm crit}} = 1 + \frac{k}{a^2 H^2} \tag{2}$$

In inflationary universe the second term is negligible for all $k = -1, 0, +1$. A generic prediction that $\Omega = 1$ follows which is the property of the flat $k = 0$ universe.

A short very *simplified history of the universe* has the following important periods. One tends to start near the Big bang, the original $a = 0$ $t = 0$ singularity. In the simplest models during the first 10^{-35} seconds inflation with $a(t) \sim e^{Ht}$ makes the size of the universe growing from the Planckian size 10^{-33} cm to at least 10^{-4} cm or larger. The Hubble parameter H may be almost as large as $M_P = \sqrt{\frac{1}{8\pi G_N}}$ at the beginning of inflation and it is expected to be less or equal to $10^{-5} M_P$ during the last 50-60 e-foldings where the slow-roll regime takes place. This restriction follows from the fact that so far the gravitational waves produced during inflation have not been observed. During inflation the universe is accelerating. The reheating starts after inflation. Eventually the universe becomes dominated by the radiation with $a(t) \sim t^{1/2}$ and $\ddot{a} < 0$. This is changed to a matter dominated period with $a(t) \sim t^{2/3}$ and $\ddot{a} < 0$ at about 10^4 years. At 5×10^5 years the decoupling of radiation from matter takes place. The universe becomes transparent for the radiation and we observe it now as CMB. In both radiation and matter domination the universe is decelerating. At about 5 billion years ago dark energy began to accelerate the expansion of the universe and it keeps accelerating now, when the universe is about 13.7 billions year old.

Even if the universe will keep accelerating, the future of the Earth will be defined by the fact that the Sun will become a red giant at about 5 billions years from now and will toast the Earth. At about 150 billion years most galaxies will move away from the sight of our galaxy.

Dark energy equation of state

$$p = w\rho \tag{3}$$

has a parameter $w(t) = \frac{p}{\rho}$ which in principle may be time dependent. If it is constant, Friedmann eqs. can be integrated with the result that

$$\rho \sim a^{-3(1+w)} \tag{4}$$

At present the observational data are in agreement with $w = -1$ in which case $\rho = \text{const}$ and dark energy is due to cosmological constant term in four-dimensional general relativity, which explains why we need to find a de Sitter or near de Sitter space in string theory. One of the most important goals of future observations is to find out with great precision whether w is a constant or a function of time and whether this constant is really equal to -1. In any case, this is considered one of the prominent observational facts whose explanation may test fundamental physics.

The total energy density of the universe consists of the $\Omega_M \sim 0.3$ energy of pressure-less matter with equation of state $w = 0$ and $\Omega_\Lambda \sim 0.7$ dark energy with equation of state $w \sim -1$

$$\Omega = \Omega_m + \Omega_\Lambda = 1 \tag{5}$$

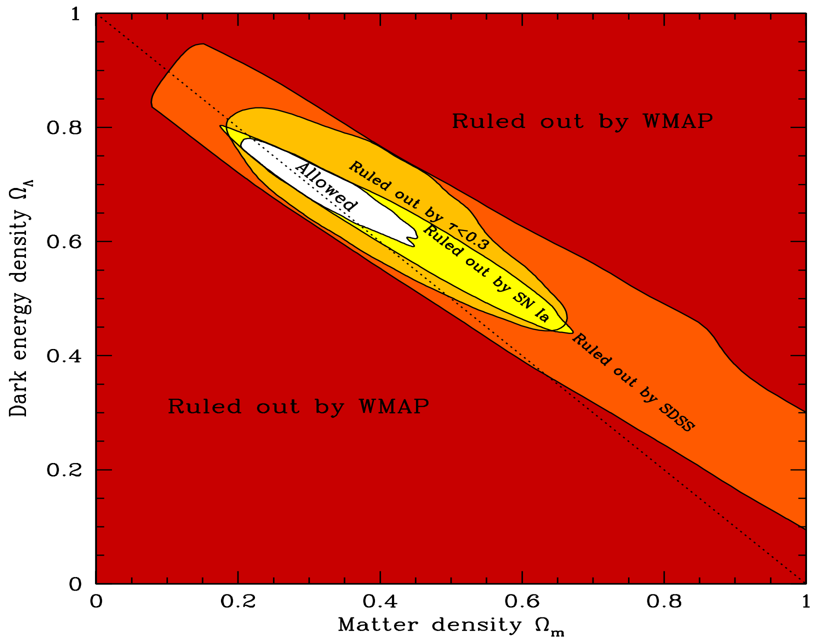

Figure 1. A plot of dark energy versus the energy of matter

One of the impressive recent plots of Ω_Λ versus Ω_m was presented in [2], we show it here in Fig. 1. It shows that with account of data from WMAP, SDSS and Supernovae it is practically impossible to avoid the fact that about 70% of the energy of the universe is dark and that the total Ω is very close to 1.

In matter part again there is a puzzle, only few percent of the total energy are formed by matter which we know, the rest is a dark matter, which still has to be identified with something like lightest supersymmetric particles or axions or other objects.

Important cosmological observables describing inflation include the primordial slope $n_s = 1 + \frac{d\ln P(k)}{d\ln k}$ where $P(k)$ is the magnitude of the scalar power spectrum measured in CMB. The observational value of n_s is close to 0.98. The ratio between tensor fluctuations (primordial gravitational waves) and scalar fluctuations $r = \frac{T}{S}$ is restricted by the data to be $r < 0.36$. In the future a major effort will be dedicated towards the detection of the gravitational waves from inflation. This will give a significant information about inflationary models compatible with the data.

In theoretical models of inflation due to a scalar field (inflaton) with some potential V the value of the primordial slope n_s and tensor to scalar mode ratio r depends on the so-called slow-roll parameters, ϵ and η which are approximately constant:

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 , \qquad \eta = \frac{V''}{V} \tag{6}$$

and

$$r = \frac{T}{S} = 16\epsilon\,, \qquad r^{\text{obs}} < 0.36\,; \qquad n_s - 1 = -6\epsilon + 2\eta\,, \qquad n_s^{\text{obs}} = 0.98 \pm 0.02 \tag{7}$$

(8)

It is important to stress here that the deviation of the primordial slope n_s from the scale-invariant one with $n_s = 1$ is in the range of a percent, i. e. $\eta \sim 10^{-2}$, $\epsilon \leq 10^{-2}$. In case of dark energy, if it is not a cosmological constant but some model with a evolving scalar field, the relevant slow-roll parameters are also restricted by the data, in particular these parameters must be less than 1. Another important feature of the early universe inflation is the restriction

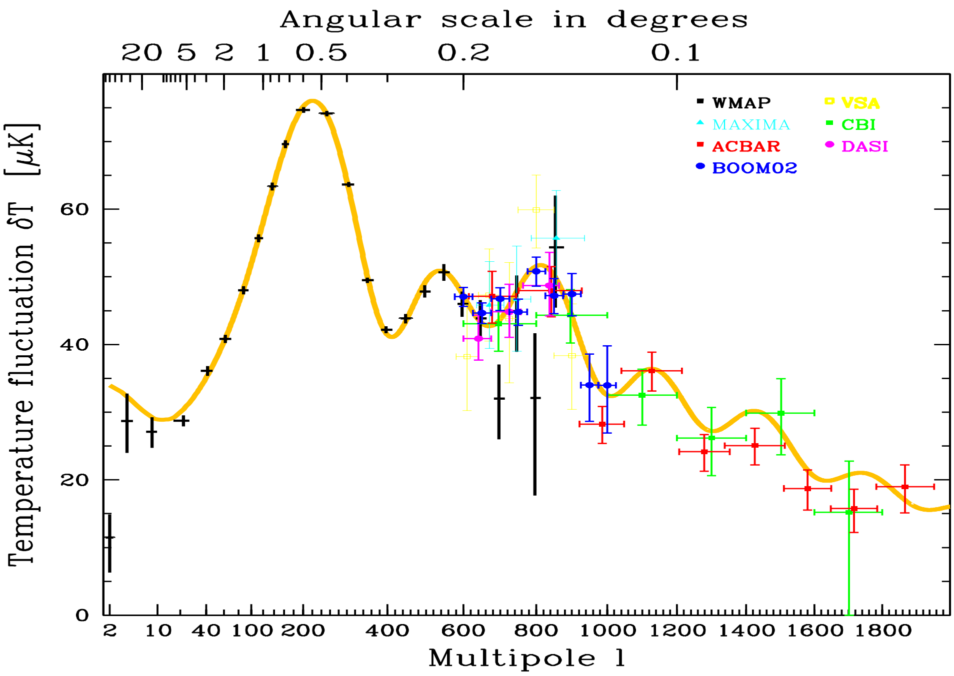

Figure 2. A plot of temperature fluctuations as a function of the multipole number l.

on the amount of e-foldings N, defined as

$$N(t) \equiv \ln \frac{a(t_{end})}{a(t)} \tag{9}$$

This measures an amount of inflation which still has to occur after time t before the end of inflation time t_{end}. At the end of inflation at t_{end}, $N(t_{end}) = 0$. For the slow-roll inflation models

$$N(t) \equiv \ln \frac{a(t_{end})}{a(t)} \approx \frac{1}{M_P^2} \int_{\phi_{end}}^{\phi} \frac{V}{V'} d\phi \tag{10}$$

The observational data require that the number of last e-foldings N_e during which a slow-roll inflation took place is at least of the order of 50-60, the actual number is somewhat model-dependent

$$N_e \geq 50 - 60 \tag{11}$$

Thus models of inflation which can provide 50-60 slow-roll e-foldings or any number of them which is larger than 50-60 are compatible with observations.

We present here in Fig. 2. one of the plots from [2] of temperature fluctuations as a function of the multipole number l with famous peak at about 200 and smaller peaks at larger values of l. The curve is given by the simplest "vanilla" models of inflation. The black points are from WMAP and they agree well with inflationary predictions, apart from the region of small l and few glitches now and there. Beyond $l \sim 600$ WMAP data are not expected to be valid, the data from less precise observations, like Maxima, Boomerang2, CBI, DASI etc. tend to agree with the set of smaller peaks predicted from inflation and the decrease of fluctuations at larger multipoles.

In conclusion, there is an agreement in cosmology community that the combined set of data suggests that the standard cosmological model is valid. Still much more data and much more precision will be required in the near future to fully confirm the cosmological model and to have more precise values of all cosmological parameters. Already at present the data are sufficient to justify any attempt to explain them from the fundamental physics.

3. Towards Cosmology in String theory

3.1. Impact of the discovery of the late-time acceleration of the universe

Until recently, string theory could not describe acceleration of the early universe (inflation). The discovery of current acceleration made the problem even more severe, but also helped to identify the root of the problem: compactified string theory under certain standard restrictions does not describe a four-dimensional de Sitter (dS) space. Moreover, until recently there was no example of a four-dimensional de Sitter space derived from string theory even with violation of these restrictions.

The set of no-go theorems on this starts from 1985 with the latest more “stringy” version of 2001, [3]. The theorems guarantee that four-dimensional dS solutions cannot be obtained in string or M-theory by using only the lowest order terms in the 10d or 11d supergravity action. One expects that corrections to the leading order Lagrangian in the g_s or α' expansion or inclusion of extended sources (branes) should improve the situation. Indeed, a careful discussion of how such additional sources (which are present in string theory) invalidate the no-go theorem for warped backgrounds and allow one to find highly warped compactifications appears in [4]. Additional sources which violate the assumptions of the theorem were shown to yield dS vacua in *noncritical* string theory in [5].

About two years ago a possibility of derivation of a four-dimensional dS space in the framework of compactified superstring theory was suggested in [6]. Independently of the cosmological applications one of the trends of string theory for the last few years has to do with the flux compactification, which allows to stabilize the axion-dilaton and the complex structure moduli of the Calabi-Yau space, as was shown for the case of IIB string theory in [4].

3.2. Can string theory afford runaway moduli: a dilaton and the volume?

No-scale effective 4d supergravity describing the type IIB axion-dilaton field $\tau = a + ie^{\phi}$ and the axion-volume field $\rho = \alpha + i\sigma$ has non-canonical kinetic terms. Here the volume modulus σ is related to a volume of the compact six-dimensional internal space.

$$-\frac{\partial\tau\partial\bar{\tau}}{(\tau-\bar{\tau})^2} - 3\frac{\partial\rho\partial\bar{\rho}}{(\rho-\bar{\rho})^2} \tag{12}$$

This corresponds to a logarithmic dependence of the effective Kähler potential on these fields of the form

$$K = -\ln(-i(\tau-\bar{\tau})) - 3\ln(-i(\rho-\bar{\rho})) \tag{13}$$

If there is a non-vanishing superpotential which, however, does not depend on these two fields but only on some other fields, one finds that the potential depends on τ and ρ via Kähler potential

$$V = e^K V_0 = \frac{1}{(\tau-\bar{\tau})(\rho-\bar{\rho})^3} V_0 \tag{14}$$

where V_0 is τ and ρ independent. Let us ignore axions for simplicity. To compare with observations one should switch to canonical kinetic terms for the dilaton and the volume:

$$-\frac{1}{2}(\partial\tilde{\phi})^2 - \frac{1}{2}(\partial u)^2 - e^{-\sqrt{2}\,\tilde{\phi}-\sqrt{6}\,\tilde{u}}V_0 \tag{15}$$

Here the relation between fields which have particular physical meaning in string theory and canonical scalars in 4d theory is the following: $\phi = \sqrt{2}\tilde{\phi}$ and $\sigma = e^{\sqrt{2/3}\,\tilde{u}}$. Now it is easy to study the slow-roll parameters for the dilaton and volume fields. We find that

$$\epsilon_{\tilde{\phi}} = 1 \qquad \eta_{\tilde{\phi}} = \sqrt{2} \qquad \epsilon_{\tilde{u}} = 3 \qquad \eta_{\tilde{u}} = 6 \tag{16}$$

where the slow-roll parameters are defined in (6) for each field. As explained in the previous section, we need $\sim 10^{-2}$ for the early universe inflation and ~ 1 for the late-time acceleration. None of these works for stringy dilaton and volume, particularly the volume whose runaway behaviour is the strongest.

Thus in a situation when both axions are irrelevant[2] and we are looking for a possibility to use the dilaton and/or volume as an evolving scalar in a four-dimensional cosmology we find that their potentials are too step and cannot work unless one can stabilize both of these fields by looking for corrections to the potential which will counterbalance the runaway due to the tree-level Kähler potential. This means that we have to find non-perturbative superpotentials depending on τ and ρ and possible corrections to tree-level Kähler potential, so that in the total potential there is a minimum at some finite values of these fields.

3.3. KKLT models of dS vacuum in string theory [6]

To stabilize the moduli in IIB string theory one starts with flux compactifications and include corrections. The reason to start with type IIB seems to be a technical simplicity, in other versions of string theory, like heterotic and type IIA the work in this direction is also performed.

We first describe the models of Giddings, Kachru, Polchinski [4] where stabilization of all but Kähler moduli can be achieved. Later we enumerate various quantum corrections which can modify the superpotential and Kähler potential. We show that incorporating the generic corrections can yield (supersymmetric) AdS minima with all moduli stabilized. And finally, we uplift the AdS supersymmetric minimum to a dS by adding to the theory some $\overline{D3}$ branes or some D7 branes with fluxes.

In Calabi-Yau orientifolds with flux and branes one has to satisfy a tadpole consistency condition

$$\frac{\chi(X)}{24} \;=\; N_{D3} + \frac{1}{2\kappa_{10}^2 T_3}\int_M H_3 \wedge F_3 \; . \tag{17}$$

Here T_3 is the tension of a $D3$ brane, N_{D3} is the net number of $(D3 - \overline{D3})$ branes one has inserted filling the noncompact dimensions, and H_3, F_3 are the three-form fluxes in the IIB theory which arise in the NS and RR sector, respectively. In the language of IIB orientifolds, M is the Calabi-Yau threefold which is orientifolded. In this language, the term $\frac{\chi(X)}{24}$ counts the negative D3-brane charge coming from the $O3$ planes and the induced D3 charge on $D7$ branes, while the terms on the right-hand side count the net D3 charge from transverse branes and fluxes in the CY manifold. As in [4], in KKLT construction it is assumed that we are working with a model having only one Kähler modulus, so $h^{1,1}(M) = 1$

In the presence of the nonzero fluxes, one generates a superpotential for the Calabi-Yau moduli

$$W_{\rm flux}(\tau, z) \;=\; \int_M G_3 \wedge \Omega \;=\; \int_M F_3 - \tau H_3 \wedge \Omega \tag{18}$$

[2] We will consider the case of racetrack inflation later when the evolution of the axion is dominant.

where τ is the IIB axion-dilaton. The holomorphic 3-form Ω depends on the complex structure moduli, z_α. Combining this with the tree-level Kähler potential

$$K = -3\ln[-i(\rho-\bar\rho)] - \ln[-i(\tau-\bar\tau)] - \ln[-i\int_M \Omega\wedge\overline{\Omega}] \tag{19}$$

where ρ is the single volume modulus ($\rho = \alpha + ie^{4u-\phi}$); our conventions are as in [4]), and using the standard $N=1$ supergravity formula for the potential, one finds

$$V_{\text{no-scale}} = e^K\left(\sum_{a,b} g^{a\bar b} D_a W \overline{D_b W} - 3|W|^2\right) \to e^K\left(\sum_{i,j} g^{i\bar j} D_i W \overline{D_j W}\right) \geq 0 \tag{20}$$

Here, a,b runs over all moduli fields, while i,j runs over all moduli fields except ρ; and we see that because ρ does not appear in (18), it cancels out of the potential energy (20), leaving the positive semi-definite potential[3] characteristic of no-scale models with $V \geq 0$.

One should use this potential as follows. Fix an integral choice of H_3, F_3 in $H^3(M,\mathbf{Z})$; then, the potential (20) fixes the moduli at values where the resulting G_3 is imaginary self-dual (ISD). Supersymmetric solutions furthermore require G_3 to be type (2,1) (more generally, G_3 would have a (0,3) piece). Thus in supersymmetric solutions $W=0$ on the vacuum, while in the nonsupersymmetric solutions, $W=W_0$, a constant which is determined by the (0,3) piece of G_3. In generic solutions, the complex structure moduli the complex structure moduli, the dilaton, and the moduli of D7 branes are completely fixed, leaving only the volume modulus ρ. The scale of the masses m for the moduli which are fixed is $m \sim \frac{\alpha'}{R^3}$ where R is the radius of the manifold ($\mathrm{Im}\,\rho$ scales like R^4). In this approximation, R is unfixed. By tuning flux quanta, it is possible (at least in some cases) to fix g_s at small values, though not arbitrarily small.

Such models with branes and flux are generically warped compactifications. One can write the Einstein frame metric of the compactification as

$$ds^2_{10} = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}\tilde g_{mn}(y)dy^m dy^n \tag{21}$$

with y are coordinates of the compact dimensions, and $\tilde g_{mn}$ the unwarped metric on M (so in the orientifold limit, it is a Calabi-Yau metric). Then it is shown in [4] (by compactifying the Klebanov-Strassler solution) that one can construct models parametrized by flux integers M, K such that $e^{A_{min}} \sim \exp[-(2\pi K)/(3g_s M)]$ with e^A being of order one at generic points. This means in particular that with reasonably small flux quanta, one can generate exponentially large ratios of scales in such models.

In the following, we will assume that g_s and the complex structure moduli have been fixed by a suitable choice of flux, and concentrate on an effective field theory for the volume modulus ρ. Typically the ρ modulus, which has a Planck scale suppressed mass, will be much lighter than these excitations and we can neglect them as well in the low energy theory.

There are two known sources of corrections to the no-scale models, both parametrize possible corrections to the superpotential (18).

1) Witten has argued in [10] that in type IIB compactifications of this type, there can be corrections to the superpotential coming from Euclidean D3 branes. This happens when the

[3] We would like to stress here that this clear separation of moduli which keeps the total volume at a different footing from other fields and leads to a no-scale supergravity, is a rather important starting point for cosmological applications of string theory. It has to be contrasted with "no-ghosts" dS supergravities associated with "non-compactified" solutions of M/string theory which typically have AdS and dS maxima and saddle points with potentials, unbounded from below [7,8]. In the most recent work on new Freund-Rubin and G_2 vacua in [9] also only tachyonic dS vacua have been found.

four-fold X used for F-theory compactification admits divisors of arithmetic genus one, which project to four-cycles in the base M. In the presence of such instantons, there is a correction to the superpotential which at large volume yields a new term $W_{\rm inst} = T(z_i)\exp(2\pi i\rho)$ where $T(z_i)$ is a complex structure dependent one-loop determinant, and the leading exponential dependence comes from the action of a Euclidean D3 brane wrapping a four-cycle in M. At the time when it was proved in [10] that the compactification four-fold must admit divisors of arithmetic genus one, $\chi_D \equiv \sum(-1)^n h^n = 1$, no background fluxes were considered. In such case it was possible to use a particular $U(1)$ symmetry of the fermionic M5 brane action when establishing this theorem. Later on it was suggested in [11] and argued for particular examples of compactification that in the presence of fluxes the $U(1)$ symmetry of the fermionic action of the M5 brane might be broken. This in turn leads to a possibility of generalizing non-perturbative superpotentials in models with divisors on a four–fold of an arithmetic genus $\chi_D \geq 1$. Quite recently the explicit Dirac action on M5 with background fluxes was derived [12] and indeed, fluxes tend to break the relevant $U(1)$ symmetry. However, it is still not clear what is the generalization of Witten's theorem in presence of fluxes and what are the necessary conditions for having instanton corrections. This is currently under investigation.

2) In general models of this sort, one finds non-Abelian gauge groups arising from stacks of N_c coincident D7 branes wrapping 4-cycles in M. The 4d gauge coupling of the $SU(N_c)$ Yang-Mills theory on such wrapped branes (we ignore the decoupled $U(1)$ factor) satisfies $\frac{8\pi^2}{g_{YM}^2} = 2\pi\frac{R^4}{g_s} = 2\pi\,{\rm Im}\,\rho$. Since the complex structure moduli of X are completely fixed, the D7 brane moduli (at least in cases where the 4-cycle being wrapped has vanishing h^1, which are easy to arrange) are also fixed. Therefore, any charged matter fields (which would create a Higgs branch for the D7 gauge theory) have also been given a mass at a high-scale; and the low-energy theory is pure $N=1$ supersymmetric $SU(N_c)$ gauge theory. This theory undergoes gluino condensation, which results in a nonperturbative superpotential $W_{\rm gauge} = \Lambda_{N_c}^3 = Ae^{\frac{2\pi i\rho}{N_c}}$ where Λ_{N_c} is the dynamical scale of the gauge theory, and the coefficient A is determined by the energy scale below which the the SQCD theory is valid. We see that this leads to an exponential superpotential for ρ similar to the one above (but with a fractional multiple of ρ in the exponent, since the gaugino condensate looks like a fractional instanton effect in W).

So effects 1) and 2) have rather similar consequences for our analysis; we will simply assume that there is an exponential superpotential for ρ at large volume.

Here, we show that the corrections to the superpotential considered above can stabilize the volume modulus, leading to a susy preserving AdS minimum. We perform an analysis of the vacuum structure just keeping the tree-level Kähler potential

$$K = -3\ln[-i(\rho-\overline{\rho})] \tag{22}$$

and a superpotential

$$W = W_0 + Ae^{ia\rho}\ . \tag{23}$$

W_0 is a tree level contribution which arises from the fluxes. The exponential term arises from either of the two sources above, and the coefficient a can be determined accordingly. In keeping with the fact that the complex structure moduli and the dilaton have received a mass, we have set them equal to their VEVs and consider only the low-energy theory of the volume modulus. To avoid the need to worry about additional open-string moduli, we assume the tadpole condition (17) has been solved by turning on only flux, i.e. with no additional $D3$ branes.

At a supersymmetric vacuum $D_\rho W = 0$. We simplify things by setting the axion in the ρ modulus to zero, and letting $\rho = i\sigma$. In addition we take A, a and W_0 to be all *real* and W_0 negative. The minimum then lies at

$$DW = 0 \quad \rightarrow \quad W_0 = -Ae^{-a\,\sigma_{cr}}(1+\frac{2}{3}\,a\,\sigma_{cr}) \tag{24}$$

The potential, $V = e^K \left(G^{\rho\bar{\rho}} D_\rho W \overline{D_\rho W} - 3|W|^2 \right)$, at the minimum is negative and equal to

$$V_{\rm AdS} = (-3e^K W^2)_{AdS} = -\frac{a^2 A^2 e^{-2\,a\,\sigma_{cr}}}{6\,\sigma_{cr}} \tag{25}$$

We see that we have stabilized the volume modulus while preserving supersymmetry. It is important to note that the AdS minimum is quite generic. A controlled calculation requires that $\sigma \gg 1$, this ensures that the supergravity approximation is valid and the α' corrections to the Kähler potential are under control. It also requires that $a\,\sigma > 1$ so that the contribution to the superpotential from a single (fractional) instanton is reliable. Generically, if the fluxes break supersymmetry, $W_0 \sim O(1)$, and these conditions will not be met. However it is reasonable to expect that by tuning fluxes one can arrange so that $W_0 \ll 1$. In these circumstances we see from (24) that $a\,\sigma > 1$. Taking $a < 1$, one can then ensure that $\sigma \gg 1$, as required.

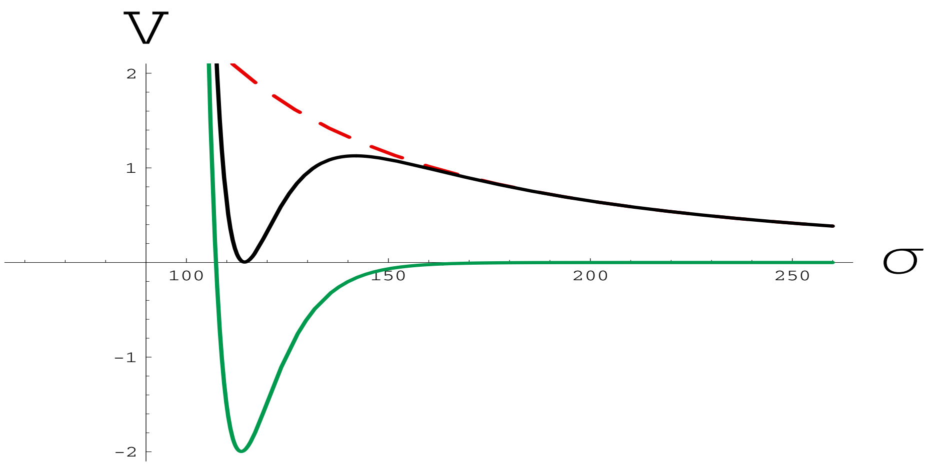

Figure 3. Thin green line corresponds to AdS stabilized potential for $W_0 = -10^{-4}$, $A = 1$, $a = 0.1$. Dashed line shows the additional term $\frac{C}{\sigma^2}$, which appears either due to the contribution of a $\overline{D3}$ brane or of a D7 brane. Thick black line shows the resulting potential including the $\frac{C}{\sigma^2}$ correction with $C = 2.6 \times 10^{-11}$, which uplifts the AdS minimum to a dS minimum. All potentials are shown multiplied by 10^{15}.

To uplifting the AdS vacua to dS vacua we proceed as follows. In the tadpole condition (17), there is a contribution from both localized $D3$ branes and from fluxes. We assume that in fact we turn on *too much* flux, so that (17) can only be satisfied by introducing one $\overline{D3}$ brane. Now, the tadpole is cancelled, but there is an extra bit of energy density from the "extra" flux and $\overline{D3}$ brane. In a geometry of the Klebanov-Strassler type throat, any anti-D3 branes are driven to the end of the throat, where the warp factor is minimized. In suitable models the inclusion of a $\overline{D3}$ adds to the potential an exponentially suppressed term. We get a term in the potential which goes like

$$\delta V = \frac{C}{(\mathrm{Im}\,\rho)^2} \tag{26}$$

The coefficient C depends on the number of $\overline{D3}$ branes and on the warp factor at the end of the throat. These parameters can be altered by discretely changing the total flux, and the fluxes which enter in (3.3), respectively. This allows us to vary the coefficient C and the susy breaking in the system, while still keeping them small We will see that by tuning the choice of C one can perturb the AdS vacua to produce dS vacua with a tunable cosmological constant. The vacua will clearly only be metastable, since all of the sources of energy we have introduced vanish as $\mathrm{Im}\,\rho \to \infty$.

We now add to the potential a term of the form C/σ^2, as explained above. For suitable choices of C, the AdS minimum will become a dS minimum, but the rest of the potential does not change too much. There is one new important feature, however: there is a dS maximum separating the dS minimum from the vanishing potential at infinity. The potential is:

$$V_{KKLT} = \frac{aAe^{-a\sigma}}{2\sigma^2}\left(\frac{1}{3}\sigma aAe^{-a\sigma} + W_0 + Ae^{-a\sigma}\right) + \frac{C}{\sigma^2} \tag{27}$$

By fine-tuning C, it is easy to have the dS minimum very close to zero. We plot an example in Fig. 3. An alternative to adding a $\overline{D3}$ brane is to add a D7 brane with fluxes on it [13]. In such case the additional terms has an interpretation as a D-term in N=1 supergravity.

In KKLT construction there is no claim that the cosmological constant is computable; only that the expected corrections in models with small g_s and W_0 are smaller than the barrier height stabilizing the vacuum. This means that it is now possible to derive from non-perturbative string theory the positive cosmological constant as the simplest form of dark energy describing the late-time acceleration of the universe. The actual value of this constant, $\Lambda \sim 10^{-120}M_P^2$, remains a mystery and requires additional assumptions like landscape of string theory and anthropic reasoning which will be discussed in the end of the lecture.

3.4. Lifetime of the known dS spaces in string/supergravity theory

KKLT model starts with a volume stabilization in an AdS minimum which exists due to non-perturbative effects. It can be uplifted to dS minimum with the barrier protecting it from the decay. If the value of dS minimum $\Lambda \sim 10^{-120}M_P^2$ is achieved in these models, this dS is metastable, but observationally indistinguishable from a cosmological constant. The lifetime is of the order $t \sim 10^{10^{120}}$ years, as explained in [6]. Thus if future observations will point out that the dark energy equation of state is $w = -1$, string theory is now capable of explaining it.

Exact solutions of 11d M/string-supergravity with fluxes, so-called ghost-free dS supergravities are unstable since dS is a saddle point. The lifetime for such dS spaces was evaluated in [14] and found to be of the order of 1-10 lifetime of the universe, i. e. of the order $t \sim 14 - 140 \times 10^9$ years, which is significantly smaller than the KKLT case. These models predict the future collapse of the universe and may lead to potentially observable consequences codified into equation of state function of time $w(t)$.

Over the last couple of years many other possibilities for stabilization of moduli in string theory leading to de Sitter space were suggested. They always require some non-perturbative effects and one may expect to learn more about this topic in the future.

3.5. Gravitino-Hubble relation and stabilization of moduli

It can be shown that in the simplest version of the KKLT model, the maximal value of the Hubble constant during inflation cannot exceed the present value of the gravitino mass, $H \le m_{3/2}$. This may have important implications for string cosmology and for the scale of the SUSY breaking in this model. If one wants to have inflation on high energy scale, one must develop phenomenological models with an extremely large gravitino mass. On the other hand, if one insists that the gravitino mass should be O(1 TeV), one will need to develop models with a very low scale of inflation. We will show here, following [15] that one can avoid these restrictions in a more general class of KKLT models based on the racetrack superpotential with more than one exponent. In this case one can combine a small gravitino mass and low scale of SUSY breaking with the high energy scale of inflation.

The simplest KKLT model has a minimum at some value of the field σ and at $\alpha = 0$. This minimum is separated from the Minkowski vacuum of Dine-Seiberg type at infinite volume of the internal space by a barrier, which makes the de Sitter minimum metastable with the lifetime $t \sim 10^{10^{120}}$ years.

Since $D_i W = 0$ in the AdS minimum, its depth is given by $V_{\rm AdS} = -3e^K|W|^2$. Here all functions are calculated at $\sigma = \sigma_{\rm cr}$, where $\sigma_{\rm cr}$ is the position of the minimum of the potential prior to the uplifting.

Before the uplifting, the potential has only one extremum, at $\sigma = \sigma_{\rm cr}$, and its absolute value exponentially decreases at $\sigma \gg \sigma_{\rm cr}$. When we add the term $\frac{C}{\sigma^2}$, the minimum shifts upward in such a way that the new dS minimum is positioned at $\sigma_0 \approx \sigma_{\rm cr}$. The gravitino mass in the uplifted dS minimum is given by

$$m^2_{3/2}(\sigma_0) \ = \ e^{K(\sigma_0)}|W(\sigma_0)|^2 \approx e^{K(\sigma_{\rm cr})}|W(\sigma_{\rm cr})|^2 = \frac{V_{\rm AdS}}{3} \ . \qquad (28)$$

The gravitino mass can be associated with the strength of supersymmetry breaking at the minimum where the total potential is approximately vanishing. Indeed, $V_{\rm KKLT}(\sigma_0) = V_F + V_D = |F|^2 - 3m^2_{3/2} + \frac{1}{2}D^2 \approx 0$. This yields $3m^2_{3/2} \approx \frac{1}{2}D^2 + |F|^2$.

Now let us discuss the height of the barrier $V_{\rm B}$ which stabilizes dS state after the uplifting. Since the uplifting is achieved by adding a slowly decreasing function C/σ^2 to a potential which rapidly approaches zero at large σ, the height of the barrier $V_{\rm B}$ is approximately equal (up to a factor $O(1)$) to the depth of the AdS minimum: $V_{\rm B} \sim |V_{\rm AdS}| \sim m^2_{3/2}$.

One may achieve inflation by considering dynamics of branes in the compactified space. This involves a second uplifting, which corresponds to a nearly dS (inflationary) potential added to the KKLT potential V_{KKLT}. The added potential should be flat in the inflaton direction. It has been studied in [15] in the context of D3/D7 brane inflation. Figure 4 shows that the vacuum stabilization is possible in this model only for sufficiently small values of the inflaton potential, $V^{\rm infl}_{\rm tot} \leq c\, V_{\rm B} \sim c\, |V_{\rm AdS}| \sim c\, m^2_{3/2}$, where $c \approx 3$ for the original version of the KKLT model. The key reason for the vacuum destabilization is the σ^{-n} dependence of the inflaton potential, with $n > 0$.

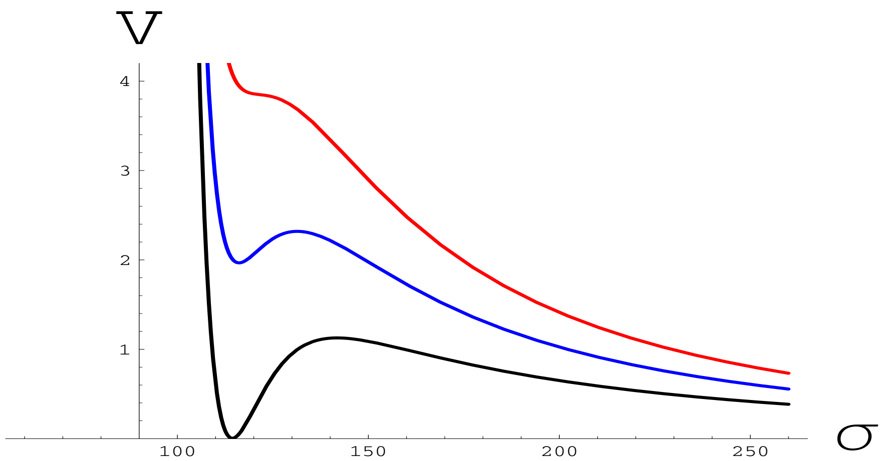

Figure 4. The lowest curve with dS minimum is the one from the KKLT model. The second one describes, e.g., the D3/D7 inflationary potential with the term $V_{\rm infl} = \frac{V(\phi)}{\sigma^3}$ added to the KKLT potential; it originates from fluxes on D7 brane. The top curve shows that when the inflationary potential becomes too large, the barrier disappears, and the internal space decompactifies. This explains the constraint $H \leq m_{3/2}$.

One should note that there could be many stages of inflation in the early universe, some of which could happen in a vicinity of a different minimum of the effective potential in stringy landscape, with much higher barriers surrounding it. Thus it is quite possible that at some stage of the evolution of the universe the Hubble constant was much greater than $m_{3/2}$. However, this could not be the last stage of inflation. We cannot simply jump to the KKLT minimum after the tunneling with bubble formation following some previous stage of inflation, because such tunneling would create an open universe. After such tunneling, we will still need to have a long stage of inflation, which should make the universe flat, form the large scale structure of the observable part of the universe, and end by a slow roll to the KKLT minimum. Our results imply that the Hubble constant H at this last and most important stage of inflation should be smaller than the present value of the gravitino mass.

The simplest KKLT potential has only one minimum, and this minimum occurs at large negative values of the effective potential. Therefore we can look for a possibility to stabilize the volume modulus in a supersymmetric Minkowski minimum. We perform an analysis of the vacuum structure[4] keeping the tree-level Kähler potential $K = -3\ln[(\rho+\overline{\rho})]$ and a racetrack superpotential similar to the one recently used in the racetrack inflation scenario [17]

$$W = W_0 + Ae^{-a\rho} + Be^{-b\rho} \ . \tag{29}$$

Here W_0 is a tree level contribution which arises from the fluxes. The exponential terms arise either from Euclidean D3 branes of from gaugino condensation on D7 branes.

At a supersymmetric vacuum $D_\rho W = 0$. The supersymmetric Minkowski minimum then lies at

$$W(\sigma_{cr}) = 0 \ , \qquad DW(\sigma_{cr}) = 0 \ . \tag{30}$$

As in KKLT, we simplify things by setting the imaginary part of the ρ modulus (the axion field α) to zero, and letting $\rho = \overline{\rho} = \sigma$. (Even though in some models the condition $\alpha = 0$ is not satisfied at the minimum of $V(\rho)$ [17], we have verified that it is satisfied in the model which we are going to propose). In addition we take A, a, B, b and W_0 to be all real and the sign of A and B opposite.

We find a simple relation between the critical value of the volume modulus and parameters of the superpotential

$$\sigma_{cr} = \frac{1}{a-b} \ln \left| \frac{a\,A}{b\,B} \right| \ . \tag{31}$$

Equations (30) require also a particular relation between the parameters of the superpotential:

$$-W_0 = A \left| \frac{a\,A}{b\,B} \right|^{\frac{a}{b-a}} + B \left| \frac{a\,A}{b\,B} \right|^{\frac{b}{b-a}} \tag{32}$$

Note that only solutions with non-vanishing value of W_0 are possible in this model; these solutions disappear if we put A or B equal to zero, as in the original version of the KKLT model.

The potential, $V = e^K \left(G^{\rho\overline{\rho}} D_\rho W \overline{D_\rho W} - 3|W|^2 \right)$, as the function of the real field $\rho = \overline{\rho} = \sigma$ is given by

$$V = \frac{e^{-2(a+b)\sigma}}{6\sigma^2} (bBe^{a\sigma} + aAe^{b\sigma}) \times \left[Be^{a\sigma}(3+b\sigma) + e^{b\sigma}(A(3+a\sigma) + 3e^{a\sigma}W_0) \right] \tag{33}$$

It vanishes at the minimum which corresponds to Minkowski space:

$$V_{\rm Mink}(\sigma_{cr}) = 0 \ , \qquad \frac{\partial V}{\partial \sigma}(\sigma_{cr}) = 0 \ . \tag{34}$$

[4] We performed the calculations and we plot the corresponding potentials using the “SuperCosmology” code [16].

Thus it is possible to stabilize the volume modulus while preserving Minkowski supersymmetry. The gravitino mass in this minimum vanishes.

An example of the model where the vacuum stabilization occurs in the supersymmetric Minkowski vacuum is given by the theory with the superpotential (29) with $A = 1$, $B = -1.03$, $a = 2\pi/100$, $b = 2\pi/99$, $W_0 = -2 \times 10^{-4}$. The resulting potential is shown in Fig. 5. The vacuum stabilization occurs at $\sigma \approx 62 \gg 1$, which suggests that the effective 4D supergravity approach used in our calculations should be valid.

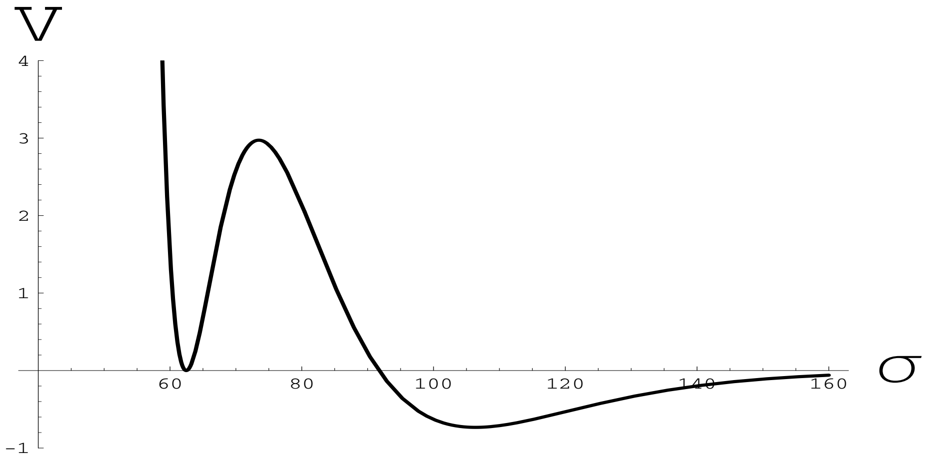

Figure 5. The F-term potential (33), multiplied by 10^{14}, for the values of the parameters $A = 1$, $B = -1.03$, $a = 2\pi/100$, $b = 2\pi/99$, $W_0 = -2 \times 10^{-4}$. A Minkowski minimum at $V = 0$ stabilizes the volume at $\sigma_{cr} \approx 62$. AdS vacuum at $V < 0$ stabilizes the volume at $\sigma_{cr} \approx 106$. There is a barrier protecting the Minkowski minimum. The height of the barrier is not correlated with the gravitino mass, which vanishes if the system is trapped in Minkowski vacuum.

We have found the supersymmetric Minkowski vacuum prior to adding any nonperturbative terms $\sim C/\sigma^2$ related to $\overline{\rm D3}$ brane or D7 branes. We assume, as usual, that by changing the parameters and by adding the term C/σ^2 one can fine-tune the value of the potential in its minimum to be equal to the observed small constant $\Lambda \sim 10^{-120}$. What is important for us is that in the first approximation one can make the gravitino mass vanish as compared to all other parameters of the superpotential. As a result, the value of $m_{3/2}$ in our model does not have any relation to the height of the potential, and, correspondingly, to the Hubble constant during inflation.

The same model may also have AdS vacua defined by

$$W(\sigma) \neq 0 \ , \qquad DW(\sigma) = 0 \ . \tag{35}$$

At the AdS minimum one has

$$-W_0 = Ae^{-a\sigma}(1 + \frac{2}{3}a\sigma) + Be^{-b\sigma}(1 + \frac{2}{3}b\sigma) \ . \tag{36}$$

The vacuum energy in this minimum is negative,

$$V(\sigma) = -3e^K |W|^2 = -\frac{(aAe^{-a\sigma} + bBe^{-b\sigma})^2}{6\sigma} \ . \tag{37}$$

The supersymmetric Minkowski vacuum is absolutely stable with respect to the tunneling to the vacuum with a negative cosmological constant. Indeed, tunneling from a supersymmetric

Minkowski vacuum would require creation of bubbles of a new phase with vanishing total energy, which is impossible because of the positive energy theorems.

In conclusion, we have explained here a modification of the original KKLT scenario where the volume stabilization does not require an uplifting of a deep AdS minimum, and where the large scale of inflation is compatible with the small gravitino mass.

4. Towards Inflation in String Theory

There are few types of string inflation models known at present:

- Modular Inflation with volume stabilization: this is the simplest class of models. It uses only the fields that are already present in the KKLT model. The first working case of modular inflation is the so-called racetrack inflation [17]
- Brane inflation with volume stabilization: the inflaton field corresponds to the distance between branes in Calabi-Yau space [18]. Known examples of brane inflation with volume stabilization taken into account are the KKLMMT model [19] with modifications [20,21] and D3/D7 model [22-27].
- Dirac-Born-Infeld inflationary model [28]: this model has particular features very different from other ones. It is not a slow-roll inflationary model, it predicts significant non-Gaussianity and detectable gravitational waves.

4.1. Racetrack Inflation

Having found classes of string models where the geometrical moduli of the compactification manifold can be stabilized, one can then discuss possible inflationary scenarios. One could not do it in absence of moduli stabilization. For instance, the known sources of energy density in string theory (including any possible source of inflationary energy) scale like inverse powers of the compactification radius R. So in absence of stabilization of the radius of compactification, one decompactifies instead of inflating.

The simplest way to obtain inflation would be to use the radius of compactification, or the corresponding axion field, as an inflaton. For a long time it did not seem possible to do it because the curvature of the potential with respect to these fields typically is too large. Therefore most of the models of string theory inflation developed after the discovery of the KKLT mechanism were based on the investigation of additional degrees of freedom, associated with motion of branes. We will discuss these models in the next sections.

However, very recently a class of models named "Racetrack inflation" was found in [17], which does not require the presence of moving branes. The corresponding potential has the KKLT structure $V = V_F + \frac{D}{\sigma^2}$, where the superpotential has two exponents and the flux contribution W_0.

$$W = A\,e^{-aT} + B\,e^{-bT} + W_0\,. \tag{38}$$

$$K = -3\log(T + T^*)\,, \tag{39}$$

where $T = -i\rho = X + iY$. The inflaton field is the imaginary part of the Kähler structure modulus which is an axion-like field in the 4D effective field theory. In this example the structure of the potential allows for the existence of saddle points between two degenerate local minima for which the slow-roll conditions can be satisfied in a particular range of parameter space. We plot the potential in Fig. 6.

The calculation of potentials of N=1 supergravity from the Kähler and superpotential can be sometimes quite involved, it is useful therefore to perform a computer calculation. A published version of calculation of such (and more complicated, with many moduli) potentials using "Mathematica" can be looked at [16]. It seems to be practically impossible to find interesting

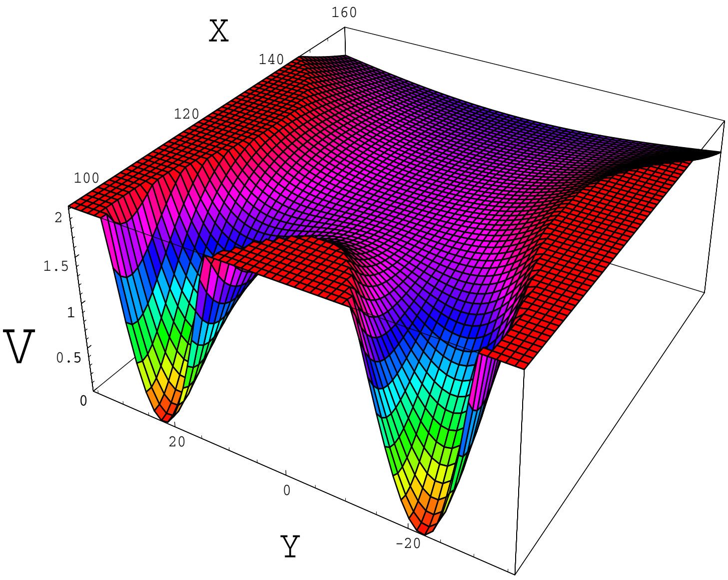

Figure 6. A plot of a racetrack potential. It has a saddle point at $X \sim 115$ and at $Y = 0$ where the systems spends time and inflates. Later on an instability develops and there is a waterfall towards one of the two KKLT-type minima at $X = 90$ and $Y = \pm 23$.

features in the potentials, like a flat saddle point, without using computers and fine-tuning parameters of the potential.

This model is most economical with respect to the number of fields/structures involved, so the discovery of this class of inflationary models is rather encouraging. However, in order to achieve inflation in this scenario one should find the models exhibiting the saddle point of the potential in the axion direction, and then ensure that the curvature of the potential near the saddle point is sufficiently small. This requires fine-tuning of the parameters. Having in mind enormous amount of possible stringy vacua, one may not necessarily consider it a problem. Still it would be interesting to find a model where such fine-tuning can be either avoided or replaced by a requirement of a symmetry.

Inflationary predictions in this model include a flat spectrum of metric perturbations with $n_s = 0.95$ and no gravitational waves. This number at present is marginally compatible with the data. Future precision experiments will tell us if this model will remain a viable model of inflation in string theory.

4.2. Inflation in KKLMMT and related models

The most popular approach to inflationary model building in string theory involves D-brane inflation [18] – the idea that the inflaton is an interbrane separation, and inflation is the macroscopic result of a microscopic Coulomb attraction between branes in a higher dimensional space. While a host of possible models exist in the various string theories, the two types which are most commonly considered in the context of models with volume stabilization involve either D3 and anti-D3 branes, or D3 and D7 branes.

A standard problem with D3/anti-D3 inflation is that the Coulomb attraction between the brane and anti-brane is in fact too strong to yield small η in a standard (unwarped) compactification. This is reviewed in e.g. §2 of [19]. Happily, however, one can involve significant warping. So it is natural to consider a scenario where an anti-D3 brane at the end of a KS throat in the compactification manifold (where is naturally driven by the background fluxes, attracts a D3 brane down the throat. The resulting Coulomb attraction is naively diluted so severely by the warping that beautiful models of slow-roll inflation should result as the D3 rolls down to annihilate with the anti-D3 [19]. The warping also plays an important role in keeping V during inflation small enough to avoid moduli de-stabilization – in fact, these models have $V << (10^{16}\ \mathrm{GeV})^4$.

However, this is too naive. The same warping which flattens the brane/anti-brane potential also produces a conformal coupling on the D3 brane worldvolume [19]. One can understand this as either a reflection of AdS/CFT, or through explicit computations involving the detailed form of the D3 Kähler potential in these models. Using the effective Kähler potential and superpotential for the volume modulus ρ and the position of the D3 brane along the throat ϕ one finds that

$$K = -3 \ln \left(-i(\rho - \bar{\rho}) - \bar{\phi}\phi \right) , \qquad W = W_0 + A e^{-i\rho} \tag{40}$$

From this perspective, the problem is just a version of the standard supergravity eta problem. The conformal coupling makes a contribution to the inflaton mass of order H^2,

$$m_\phi^2 \sim V_{cr} \sim H^2 \tag{41}$$

This must be cancelled to 1 part in 100 to obtain a working model. The cancellation can plausibly happen in models with appropriate superpotential dependence on the D3 brane position [19], but the resulting predictions are non-Universal (depending on rather global details of the model, not just local properties of brane/anti-brane dynamics). The mechanism of fine-tuning is based on the fact that the potential V depends on the Kähler invariant combination $\mathcal{G}$ of the Kähler potential K and superpotential W

$$\mathcal{G} = -(K + \ln |W|^2) \ . \tag{42}$$

This combination can be fine-tuned which leads to a small inflaton mass. An example of calculations of string theory corrections which may, in principle, provide such cancellation, were presented in [20]. The calculation is valid under condition that D3 brane in the throat slowly approaches the anti-D3 brane. A stuck of D7 branes which is responsible for the gaugino condensation and volume stabilization must coincide with the anti-D3 brane. In such situation the inflaton field which is the distance between D3 and anti-D3 brane becomes also a distance between a D3 brane and a stuck of D7 branes. The value of the string corrections to the superpotentials presented in [20] depends on the values of some additional moduli, beyond the overall volume and position of D3 brane. Therefore they may be large and may help to cancel the large inflaton mass for the $\mathbb{K}$L$\mathbb{M}$T model.

Several modifications of this basic scenario have been proposed which circumvent this issue in various ways, see [20,21]. So clearly, there are many ideas for obtaining inflation using the dynamics of D3 and/or anti-D3 branes in the class of compactifications described in [4]. However, it is fair to say that none of these scenarios is particularly compelling, and it is certainly worth exploring other brane inflation models which also fit into scenarios for stabilized moduli and inflate at low enough scales to avoid moduli destabilization.

An interesting recent development related to cosmic strings took place in the context of $\mathbb{K}$L$\mathbb{M}$T model, assuming that the necessary fine-tuning has been achieved. Cosmic strings may be produced after inflation. These cosmic strings may be observable. If their tension is too high, they would contradict the data on CMB spectrum. The interesting feature of $\mathbb{K}$L$\mathbb{M}$T model which was first observed in [19] and developed later in a number of papers is that the effect of warping may allow us to produce models which at present are still below the level of detection and do not contradict the CMB data. However, with the increased precision of cosmological observations one may expect that these cosmic strings may be detected. We refer the reader to the lecture on this in [29].

4.3. D3/D7 brane inflation

Here we will explain the class of D3/D7 brane constructions and related inflationary models which were developed in [22]-[27]. We will assume here, as it was done in KKLMMT model that there is one total volume modulus[5]. The D3/D7 brane inflation model has a shift symmetry which protects the inflaton from a large mass. This symmetry originates from the choice of the compactification manifold which has isometries. We will discuss it later in details.

The resulting four-dimensional effective model of D3/D7 brane inflation at the stabilized total volume is a stringy version of a hybrid D-term inflation [29,30]. The original D-term inflationary model[31] was recently revisited and improved in [32] with regard to the distinction between the case of constant FI terms and field-dependent D-terms in effective supergravity.

Hybrid inflation [30] can be naturally implemented in the context of supersymmetric theories. The basic feature of these inflationary models is the existence of two phases in the evolution of the universe: a slow-roll inflation in the de Sitter valley of the potential (the Coulomb phase of the gauge theory) and a tachyon condensation phase, or 'waterfall stage', towards the ground state Minkowski vacuum (a Higgs phase in gauge theory).

In $\mathcal{N}=1$ supersymmetric theories, hybrid inflation may arise as F-term inflation [33,34] or D-term inflation [31], [35]. In $\mathcal{N}=2$ supersymmetric theories there is a triplet of Fayet-Iliopoulos (FI) terms, ξ^r, where $r=1,2,3$. Choosing the orientation of the triplet of FI terms, ξ^r, in directions 1,2, F-term inflation is promoted to $\mathcal{N}=2$ supersymmetry [36]. The more general case with $\mathcal{N}=2$, when all 3 components of the FI terms are present, is called *P-term inflation*[37]. When ξ^3 is non-vanishing, a special case of D-term inflation with Yukawa coupling related to gauge coupling is recovered. In this fashion, the two supersymmetric formulations of hybrid inflation are unified in the framework of $\mathcal{N}=2$ P-term inflation. This N=2 gauge theory has the potential[37]

$$V=\frac{g^2}{2}\Phi^\dagger\Phi(A^2+B^2)-\left[\frac{1}{2}(P^r)^2+gP^r\left(\frac{1}{2}\Phi^\dagger\sigma^r\Phi+\xi^r\right)\right] , \tag{43}$$

where P^r is a triplet of auxiliary fields of the $\mathcal{N}=2$ vector multiplet, $\Phi^\dagger,\Phi$ are 2 complex scalars forming a charged hypermultiplet, A,B are scalars from the $\mathcal{N}=2$ vector multiplet and g is the gauge coupling. The auxiliary field satisfies the equation $P^r=-g(\Phi^\dagger\sigma^r\Phi/2+\xi^r)$ and the potential simplifies to

$$V=\frac{g^2}{2}\left[\Phi^\dagger\Phi(A^2+B^2)+\left(\frac{1}{2}\Phi^\dagger\sigma^r\Phi+\xi^r\right)^2\right] . \tag{44}$$

An additional advantage of using $\mathcal{N}=2$ supersymmetric models for inflation is the possibility to link it to M/string theory where cases with $\mathcal{N}=2$ supersymmetry are simpler and less arbitrary than the cases with $\mathcal{N}=1$ supersymmetry.

One can link open string theory to a gauge model with a hybrid potential either using a system with a D4-brane attached to NS5-branes and having a small angle relative to a D6-brane [38] or a D3/D7 system of branes [22]. This setup reproduces accurately the properties of the non-supersymmetric de Sitter vacuum of P-term inflation, for which $(P^r)_{\rm deSit}=-\xi^r$ and $(V)_{\rm deSit}=\vec{\xi}^2/2$. In particular, the mass splitting of the scalars in the hypermultiplet (e.g. for the case of $\xi^3=\xi\neq 0$)

$$M^2_{\rm hyper}=g^2[(A^2+B^2)\pm\xi] , \tag{45}$$

[5] We hope that the details of stabilization of moduli in D3/D7 model will be better understood in future. The model is defined in the context of $K3\times T^2/Z^2$ compactification. The stabilization of the volume of $K3$ is well understood in presence of D7 branes, whereas the stabilization of the volume of the torus requires a contribution from Euclidean D3 branes.

is reproduced by the low-lying string states; the attractive force between the D4 and D6-brane or a D3 and D7 branes is a one loop effect from the open string channel, and correctly reproduces the one-loop gauge theory potential

$$\Delta V = \frac{\xi^2 g^4}{16\pi^2} \ln \frac{|A^2 + B^2|}{|A^2 + B^2|_c} \ , \tag{46}$$

for large values of the inflaton field. Notice the inflaton is the distance between branes.

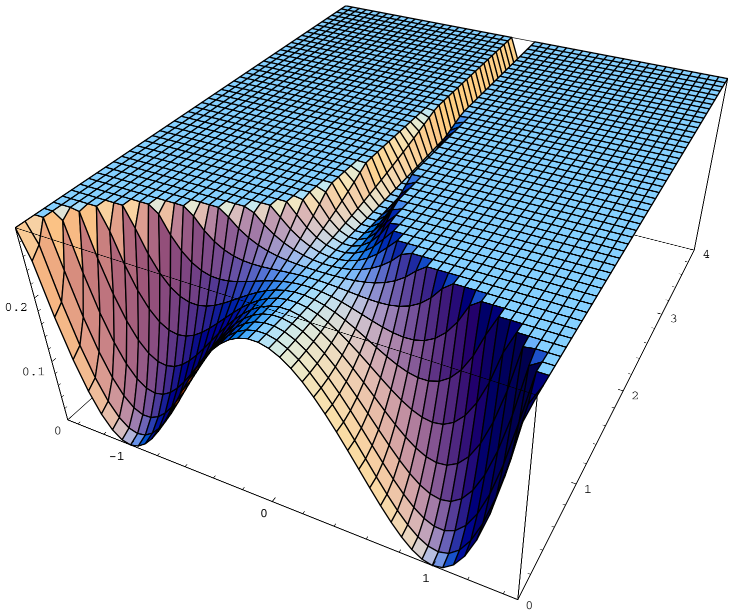

Figure 7. A hybrid potential representing an effective D3/D7 model with de Sitter valley where the system inflates and approaches the critical point where the local de Sitter minimum turns into a de Sitter maximum. At this point system waterfalls to the minimum. The minimum should have Mexican hat $U(1)$ structure, not a Z_2 but this is difficult to plot simultaneously showing a de Sitter valley.

When the distance between the branes becomes smaller than the critical distance

$$|A^2 + B^2|_c = \xi \ , \tag{47}$$

the spectrum of 3-7 strings develops a tachyon. The tachyon condensation is associated to a phase transition. A final Higgs phase with unbroken $\mathcal{N} = 2$ supersymmetry is described in D4/D6 model by a reconfiguration of branes: D6 cuts D4 into two disconnected parts, so that $\mathcal{N} = 2$ supersymmetry is restored. From the viewpoint of the gauge theory it is described by a vanishing auxiliary field $(P^r)_{\rm susy} = 0$, a vanishing vev of the inflaton field $A = B = 0$, and a vanishing potential $(V)_{\rm susy} = 0$. The hypermultiplet vev is not vanishing and has to satisfy

$$\frac{1}{2}\Phi^\dagger \sigma^r \Phi = -\xi^r \ . \tag{48}$$

D3/D7 model is partially dual to D4/D6 model and gives a better description of the Higgs branch (exit from inflation) keeping the nice properties of the Coulomb branch (inflation).

The model consists of a D3-brane parallel to a D7-brane at some distance, which again is the inflaton field. The supersymmetry breaking parameter is related to the presence of the antisymmetric $\mathcal{F}_{mn}$ field on the worldvolume of the D7-brane, but transverse to the D3-brane. When this field is not self-dual in this four dimensional space, the supersymmetry of the combined system is broken.

Equation (48) can be associated with the Atiyah-Drinfeld-Hitchin-Manin construction of instantons with gauge group $U(N)$. The moduli space of one instanton is the moduli space of vacua of a $U(1)$ gauge theory with N hypermultiplets and (48) is the corresponding ADHM equation. This suggests that in the brane construction of the cosmological model we may look for some instantons on the worldvolume of the brane in the Higgs phase of the theory. One can find an abelian non-linear instanton solution with associated ADHM equation (48). Moreover, the presence of a cosmological constant will translate as the resolution of the small size instanton singularity. One other nice feature of the D3/D7 cosmological model is that it is well explained in terms of κ-symmetry of the D7-brane action, both in Coulomb phase as well as in the Higgs phase.

Unbroken supersymmetry of bosonic configurations in supergravity has already proved to be an important tool in our understanding of M/string theory. To obtain BPS solutions of supergravity one has to find Killing spinors, satisfying a condition $\delta_{susy}\psi = 0$ for all fermions and find out how many non-vanishing spinors solve this equation for a given solution of bosonic equations. This allows to find configurations with some fraction of unbroken supersymmetry in supergravity. Unbroken supersymmetry of the bosonic configurations on the worldvolume of the κ-symmetric branes embedded into some curved space (on shell superspace) for all cases of D-branes, M2 and M5 κ-symmetric branes, a universal equation for the BPS configurations on the worldvolume is given by

$$(1-\Gamma)\,\epsilon = 0. \tag{49}$$

Here $\Gamma(X^\mu(\sigma), \theta(\sigma), A_i(\sigma))$ is a generator of κ-symmetry and it should be introduced into the equation for unbroken supersymmetry with vanishing value of the fermionic worldvolume field $\theta(\sigma)$. The existence or absence of solutions to these equations fits naturally in the two stages of our cosmological model. We refer to the details of how κ-symmetry controls D3/D7 system to the original paper [22].

4.4. De Sitter valley: separated D3/D7 system with fluxes

Consider a type IIB system with $D3$ and $D7$-branes plus a constant worldvolume gauge field $\mathcal{F}$ field along directions

	0	1	2	3	4	5	6	7	8	9
$D3$	×	×	×	×						
$D7$	×	×	×	×			×	×	×	×
$\mathcal{F}$							×	×	×	×

Table 2

This is illustrated in figure 7. It describes a de Sitter stage of a hybrid inflation and it is T-dual to the type IIA $D4/D6$ model of branes at an angle (without the NS5-branes), considered in [38].

We place D7 at $(x^4)^2 + (x^5)^2 = 0$ and D3 is initially at some $d^2 = (x^4)^2 + (x^5)^2 >> d_c^2$, where d_c is defined below in (54). There is a constant worldvolume gauge field $\mathcal{F} = dA - B$ present on D7:

$$\mathcal{F}_{67} = \tan\theta_1, \qquad \mathcal{F}_{89} = \tan\theta_2 \ , \tag{50}$$

responsible for spontaneous breaking of supersymmetry. For example we may have $B = 0$ in the bulk and the following vector fields on the brane:

$$A_6 = -\frac{1}{2}\tan\theta_1 x^7 \ , \quad A_7 = \frac{1}{2}\tan\theta_1 x^6 \ , \quad A_8 = -\frac{1}{2}\tan\theta_2 x^9 \ , \quad A_9 = \frac{1}{2}\tan\theta_2 x^8 \ . \tag{51}$$

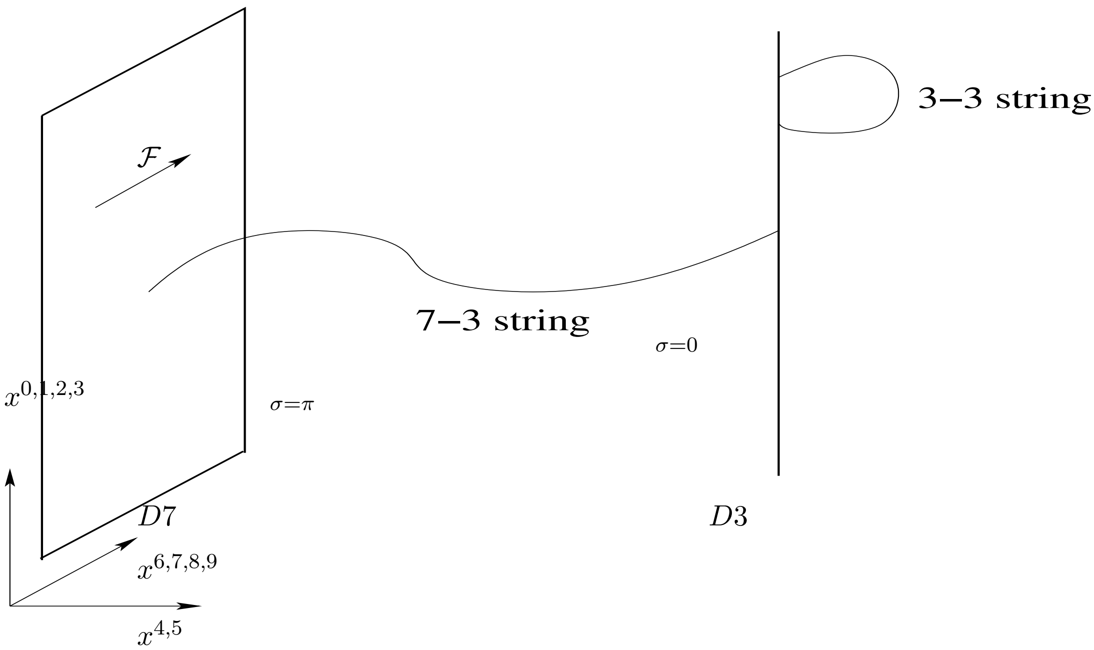

Figure 8. The D3/D7 "cosmological" system. The 3-3 strings give rise to the $\mathcal{N}=2$ vector multiplet, the 7-3 strings to the hypermultiplet and the worldvolume gauge field $\mathcal{F}$ to the FI terms of the $D=4$ gauge theory.

Note that if $\mathcal{F}$ is self-dual, supersymmetry would be unbroken, which can be explained via κ-symmetry. For future reference we define $\mathcal{F}^{\pm}=\mathcal{F}\pm\star\mathcal{F}$, and similarly for B. We can choose the GSO even states to be either 1,2 or 3,4. We will choose the former. The standard calculation zero point energy with the choice of the GSO even states gives

$$E=\pm\frac{1}{2}(\nu_1-\nu_2)=\pm\frac{\theta_1-\theta_2}{2\pi}\ . \tag{52}$$

Therefore the lowest lying multiplet of states of open strings consist of bosons whose masses are given by

$$M^2_{\pm}=\frac{d^2}{(\pi\alpha')^2}\pm\frac{\theta_1-\theta_2}{2\pi\alpha'}\ , \tag{53}$$

where we observe that the boson of mass M^2_{-} becomes tachyonic at the critical distance[6]

$$d_c^2\equiv\pi\alpha'\Big(\frac{\theta_1-\theta_2}{2}\Big)\ , \tag{54}$$

and the other boson remains at positive mass. The fermion in the multiplet has a mass

$$M^2_{\psi}=\frac{d^2}{(\pi\alpha')^2}\ , \tag{55}$$

as the contribution from the zero point energy to the Ramond sector is zero.

The $\mathcal{F}$ field plays the role of the Fayet-Illiopoulos term, from the viewpoint of the field theory living on the $D3$-brane. It creates an instability in the system driving the $D3$-brane into

[6] We are assuming $\theta_1>\theta_2$.

the $D7$-brane; this is the de Sitter stage. The evolution follows the description given in [38]. In particular, a tachyon will form and the system will end in a supersymmetric configuration describing a bound state of D3 and D7, a D3 resolved in D7.

The shift symmetry in which we are interested here is the independence of the Kähler potential K on the $\mathrm{Re}\, y$ where y is a complex field representing a distance between a D3 brane and a D7 brane in an isolated D3/D7 system. This symmetry takes place even with account of the superpotential: the combination $\mathcal{G}$ of K and W shown in eq. (42) is independent on $\mathrm{Re}\, y$ [26]. Only loop corrections due to a flux on D7 presented in eq. (46) break this symmetry as a result of a splitting of a mass in a supersymmetric multiplet. The procedure of stabilization of moduli in this model requires that the values of other fields, like the volume of $K3$, the volume of T^2/Z^2, the axion-dilaton field, the complex structure of the torus and the position of D7 branes are all fixed via fluxes or non-perturbative corrections to the superpotential. For a successful model of inflation we need to keep a flat direction associated with the position of a single D3 brane. In such situation, after volume stabilization we expect an inflaton trench potential presented in fig. 9.

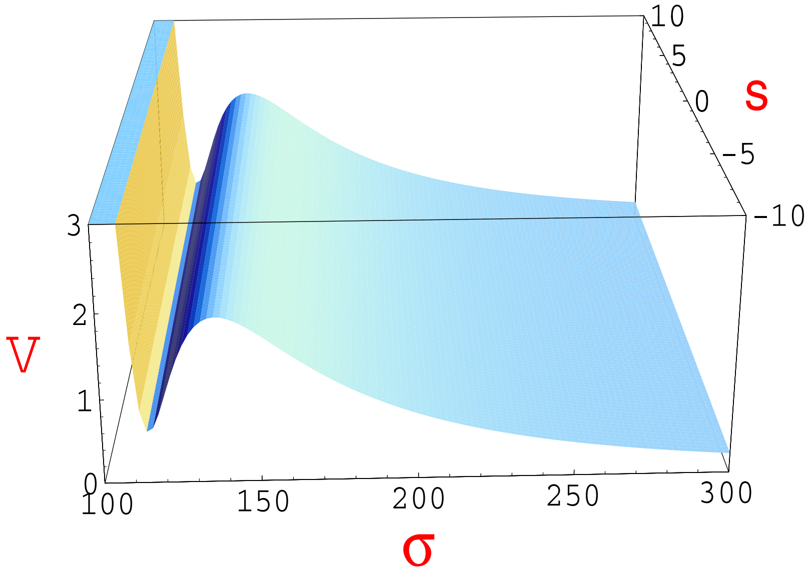

Figure 9. A potential which shows an inflaton trench: whereas the distance between D3 and D7 changes along the flat direction $s = \mathrm{Re}\, y$, the volume moduli σ is not destabilized.

5. Stringy Theory Landscape

It is known for about 20 years that it is extremely difficult to derive an effective cosmological model from string theory/10-11-dimensional supergravity. Over the last few years observational data seem to point out that development of the string cosmology becomes more and more important.

A new concept of a stringy landscape have emerged as the result of these attempts and was formulated by Susskind. It has some roots in the ideas of eternal inflation developed by Linde and Vilenkin and Bousso-Polchinski ideas of a relation between a cosmological constant and flux compactification. It is also based on the fact that de Sitter space is possible in string theory when non-perturbative effects are taken into account. It is extremely difficult at present to explain the scale of the cosmological constant. Stringy landscape suggests to use the fact that there is a huge amount of stringy vacua, due to an enormous amount of combination of fluxes.It is believed that string theory may have $10^{100} - 10^{1000}$ different vacua.

With account of loop corrections each vacuum will change. However, the total landscape picture with many vacua will survive. There are many vacua with negative, vanishing and positive energies. Somewhere there is our vacuum with $\Lambda \sim 1/N$ where N, the number of vacua, is required to be $N > 10^{120}$. The number of phenomenologically (or anthropically) acceptable vacua is smaller than the number of total vacua, so the picture may, in principle, explain the data.

5.1. Acknowledgments

I am grateful to my collaborators who helped me to understand the issues of string cosmology and to the organizers and participants of the VI Mexican School (DGFM) "Approaches to Quantum Gravity," November 21-27, 2004, for an interesting and stimulating meeting. This work was supported by NSF grant PHY-0244728.

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