

# HELLWEG IMPROVEMENTS FOR 3D TRAVELING WAVE LINAC DESIGN WITH BEAM LOADING \*

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## Abstract

The industrial, medical and homeland security markets for low-to-moderate energy electron linacs are growing rapidly, often requiring beam currents that strongly load the accelerating fields. The two-beam accelerator (TBA) is one concept for the structure wakefield acceleration approach to an electron-positron collider. Transient beam loading effects are a significant challenge for the drive beam in a TBA structure, where energy droop in high-charge bunch trains must be understood and compensated. The Hellweg code accurately models steady state beam loading for traveling wave RF structures with a fast reduced model. The Hellweg equations of motion have recently been generalized to include arbitrary charge-to-mass ratio and to use momentum as the dynamical variable. These and other recent developments are discussed, including a new browser-based GUI. Proposed future developments include support of standing wave RF structures and transient beam loading effects.

## INTRODUCTION

Industrial linac applications involve exposing large mass streams to kGy-class radiation fields, which requires: high average electron beam power from 0.5 to 10 MW; wall plug efficiency of 50% or more; operation in harsh industrial settings; and low capital and operating costs [1]. Traveling-wave (TW) RF structures with modest accelerating gradient have long been recognized as a solution for such applications [2]. The pulsed current in such linacs can be as high as tens of amperes, which means that beam loading (dynamic reduction of the accelerating cavity fields by intense electron bunches) must be correctly simulated. A reduced algorithm for accurate beam loading simulations [3–6] has been implemented in the Hellweg code [4–7]. Hellweg includes external magnetic fields and a reduced model for space charge forces [7, 8]. Users can specify the phase velocity and normalized electric field strength for each cell of a TW disc-loaded structure, and Hellweg will automatically determine the correct particle-field dynamics, with three orders of magnitude less computational effort.

Hellweg is an open source [9] and cross-platform C++ application, compiling on Microsoft Windows via Embarcadero [10] and on Linux via GNU g++. The Windows executable includes an interactive graphical user interface (GUI), which can be disabled for command-line execution

when doing parameter sweeps. On Linux, Hellweg is compiled to a C++ library that can be imported and executed from within Python, making it accessible via the command line and from Jupyter notebooks. Such notebooks can be developed and executed interactively via the JupyterHub server that is integrated into the free scientific gateway known as Sirepo [11]. Users can also run Hellweg simulations immediately from their browser via the Sirepo-Hellweg app [5]. Hellweg has been extensively benchmarked with CST Particle Studio and other codes [5], and it is actively used for the design of linacs. [12–18]

## NEW PHYSICS CAPABILITIES

### *Standing Wave Component in TW Coupler Cell*

Up to now Hellweg treated accelerating structure only through fundamental harmonic (or accelerating harmonic, if defined with STRUCT line), which is the one synchronous with the beam and the one that participates in the wave-to-beam energy transfer. However, it is known [8] that because of the geometric discontinuities in the iris diaphragms, these theoretical functions are modified by space harmonics. Typically, the effect of these harmonics is small in a normal linac section, and the beam parameters (energy) are not affected by their interaction. Nevertheless, some discrepancy can occur in the input (and output) coupler cells. The waves there are generally semi-stationary (the phase varies very little over the length of a half-cell in zones where the amplitude of the field is already high). The electrons, rather than being captured in this zone by the field, rapidly slip behind the wave and are sometimes slightly dispersed in phase. The intended perfect synchronism between the wave and the electrons is destroyed.

This effect was confirmed by 3D simulations in CST Microwave Studio. Since in Hellweg the wave phase is an independent parameter (through  $z$ ), in terms of beam dynamics this effect equivalent to the fact that the beam linearly slips through the phases in  $1/2$  of cells with a coupler, or half of the phase advance. For example, if the operating mode is 120 deg, the beam will slip by 60 degrees. The energy undergain in the coupling cell can then be estimated as  $W_0 \sin \theta$  (for 120 deg, the beam will undergain  $\sim 13\%$  of energy in the first cell). The corresponding correction modifications were introduced in Hellweg, according to these considerations. This effect can be turned on and off in Hellweg.ini file (default is “off”) or it can be turned off for a particular cell if the keyword HALF is used instead of CELL.

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## Thermal Emission of Low-energy Electrons

We improved Hellweg's automated handling of very low-energy electron beams from thermionic cathodes and developed an approach for correctly treating the RF fields in an initial half-cell cavity, as required for typical electron gun designs. Specifically, we added the ability to simulate particles with thermal energies, injected directly into high RF fields (as in RF guns). In general, the current equations support such capability but the simulation accuracy greatly depends on the integration (mesh) step. Therefore, we performed a study of simulation accuracy as a function of mesh step. In these studies we considered the field profiles, similar to those of 1.6-cell APS thermionic RF gun [15]. The injected electrons occupied full phase length (360 deg).

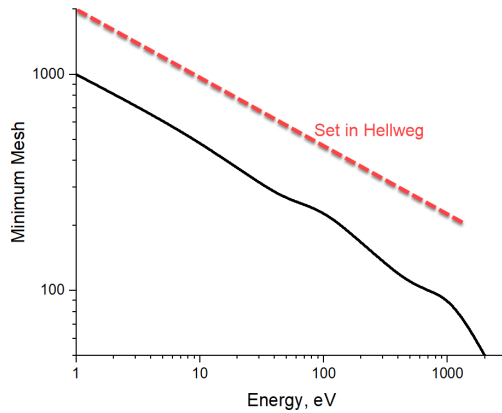


Figure 1: Minimum number of mesh points in the first cell, required to produce a converged result (black) and a threshold value, automatically used in Hellweg (red).

The results of these simulations for different initial energies and number of mesh points in the first cell show convergence for the number of mesh points  $\sim 1000$  for any initial energy. Figure 1 plots the minimal number of mesh points required to produce a converged solution for the beams with different initial energies. We can see that this number has a dependence, which we approximate as  $2000E^{-1/2}$  (red line). This threshold can be automatically applied in Hellweg to simulate RF guns. If enabled, the code will increase the number of mesh point in the first RF element and any drift elements before it, depending on the injection energy  $E$ .

## Arbitrary Particle Species (Modeling Ion Linacs)

We have fundamentally generalized the Hellweg equations of motion in two ways. First, the charge-to-mass ratio is now arbitrary, so that ion linacs can now be simulated. Second, as described in the next section, the normalized velocity vector  $\beta$  has been replaced as a dynamical variable with the momentum  $\gamma\beta$ , enabling Hellweg to be used with Energy Frontier electron and positron linacs.

A particle's charge and mass enter the equations that govern the evolution of the EM field and particle phase space coordinates in Hellweg only through the ratio of charge  $q$  and the rest energy  $W_0$ , via the normalized field compo-

nents  $q\lambda E_\alpha/W_0$ ,  $q\lambda cB_\alpha/W_0$ , and the normalized current  $qJ_0R_b/2W_0$ . Therefore, in order to simulate ions in Hellweg, we have added the possibility to change the normalization of the above parameters, according to the user-defined particle species. The user can now specify the particle type (ELECTRONS, PROTONS, or IONS), as well as two optional parameters,  $A$  (atomic mass in a.m.u.) and  $Q$  (charge number), to simulate ions with arbitrary charge-to-mass ratio. Electrons are still the default. Because Hellweg was originally designed to simulate electrons in disk-loaded-waveguide-like structures, the user must understand the applicability limits of ion beam dynamics simulation.

We benchmarked the code against CST Particle Studio and the TRACK code, by simulating  $^{12}\text{C}^{6+}$  ion beam dynamics in a 15-cell constant-impedance backward traveling wave disk-loaded structure with phase velocity  $\beta = 0.3$ , operated on  $-1^{\text{st}}$  harmonic, that is proposed for a carbon therapy linac [14]. We found similar energy gain (84 MeV in CST and 84.5 MeV in Hellweg), and phase shift ( $-30 \dots 0 \dots -30$  deg in TRACK, and  $-30 \dots -28.5$  deg in Hellweg). We should note that difference in the results is due to the fact that Hellweg cannot directly simulate higher spatial harmonics, so the simulated structure was approximated as fundamental harmonic with phase advance of 210 deg and  $\beta = 0.3$  to replicate the same length (actual  $-1^{\text{st}}$  harmonic operates at 150 deg and  $\beta = 0.3$ ).

## Normalized Momenta as Dynamical Variables

We have refactored the code to use the normalized particle momentum as a dynamical variable in place of the particle velocity. The corresponding equations of motion in cylindrical coordinates are given by

$$\begin{aligned} \frac{d(\gamma\beta_r)}{d(z/\lambda)} &= \frac{(\gamma\beta_\theta)^2}{(r/\lambda)\gamma\beta_z} \\ &+ \frac{Z}{M} \frac{1}{\gamma\beta_z} [\gamma\tilde{E}_r + (\gamma\beta_\theta)\tilde{B}_z - (\gamma\beta_z)\tilde{B}_\theta], \\ \frac{d(\gamma\beta_\theta)}{d(z/\lambda)} &= -\frac{(\gamma\beta_r)(\gamma\beta_\theta)}{(r/\lambda)\gamma\beta_z} \\ &+ \frac{Z}{M} \frac{1}{\gamma\beta_z} [\gamma\tilde{E}_\theta + (\gamma\beta_z)\tilde{B}_r - (\gamma\beta_r)\tilde{B}_z], \\ \frac{d(\gamma\beta_z)}{d(z/\lambda)} &= \frac{Z}{M} \frac{1}{\gamma\beta_z} [\gamma\tilde{E}_z + (\gamma\beta_r)\tilde{B}_\theta - (\gamma\beta_\theta)\tilde{B}_r], \end{aligned} \quad (1)$$

where  $\gamma = (1 + (\gamma\beta_r)^2 + (\gamma\beta_\theta)^2 + (\gamma\beta_z)^2)^{1/2}$ ,  $Z \equiv q/|e|$ ,  $M \equiv m/m_e$ , and the normalized electric and magnetic field components  $\tilde{E}_\alpha$  and  $\tilde{B}_\alpha$  (in MKS) are given by

$$\tilde{E}_\alpha = \frac{|e|\lambda}{m_e c^2} E_\alpha \quad \text{and} \quad \tilde{B}_\alpha = \frac{|e|\lambda}{m_e c^2} c B_\alpha. \quad (2)$$

We have implemented the above equations of motion and modified all of the Hellweg algorithms (including space charge) to work with the new dynamical variables and user-specified particle species. The I/O and numerous diagnostics routines have been converted to work with the new variables, as well.

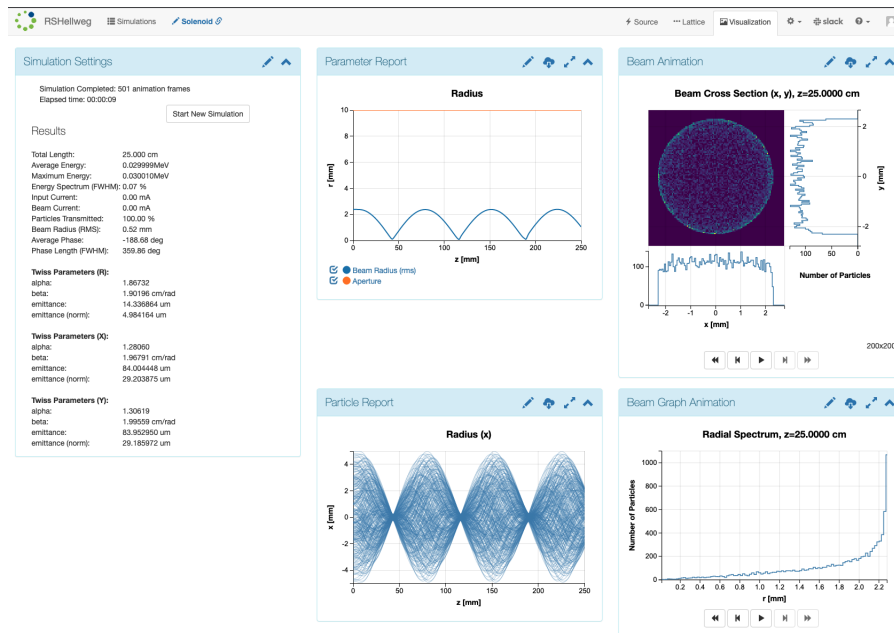


Figure 2: Sirepo-Hellweg screenshot of betatron oscillations within an extended solenoid.

## RF Cavities with Phase Velocity $> c$

Waves in disk-loaded RF structures can have any value of phase velocity  $\beta_w$ . The original Hellweg equations for the field distribution in a disk loaded waveguide involve terms of the form  $I_0(x)$  and  $I_1(x)/x$ , where  $I_0$  and  $I_1$  are the modified Bessel functions of the first kind, with  $x \propto \sqrt{1 - \beta_w^2}$ . In order to permit modeling with  $\beta_w > 1$ , we express the  $I_0(x)$  and  $I_1(x)$  of a purely imaginary argument in terms of regular Bessel functions  $J_k(ix)$  of the first kind of a real argument, and retain the first few terms in the Taylor series expansion. With this change, Hellweg can now perform simulations with the phase velocity  $v_w > c$ .

## ONGOING SOFTWARE DEVELOPMENT

The Hellweg code is actively developed. Bug fixes and user requests are routinely addressed. Our vision is for the code to evolve into an open source environment for rapid self-consistent simulation of electron and ion beams, with reduced models for both space charge and transient beam loading, in both TW and SW structures. The emerging software architecture will allow for incremental expansion of capabilities, without jeopardizing the original premise of rapid simulations, ease of use, and interactivity. The Python-based command line interface, on both Windows and Linux, will support multi-parameter optimizations and machine learning model development.

Over the years, Hellweg users have developed Windows-based command-line workflows for linac design. Hence, we recently established a code development methodology that enables rapid development of Hellweg on both Windows and Linux environments, using GitHub for quality control [9], with a mix of automated and manual testing on the two platforms. The Sirepo-Hellweg app [11] will

support students and industry engineers, while also enabling instantaneous collaboration for distributed teams. [19–21] Figure 2 illustrates the interface to the Sirepo-Hellweg app. The Jupyter notebook interface, also enabled by Sirepo, provides more flexibility than the GUI and greater ease-of-use than command-line execution.

## FUTURE PLANS

Hellweg support for standing wave (SW) RF structures is planned for the future. The TW particle-field dynamics is largely applicable to SW linacs [17]. However, changes will be required to include the discrete jump in phase between adjacent cells, and the RF transit-time factor must be included. Power attenuation and beam loading effects are also very different in SW and TW structures. Support for transient beam loading effects will also be developed, as this is essential for the two-beam accelerator (TBA) energy frontier concept [22] and for some industrial applications. The present algorithms apply only to steady-state beam loading. Another planned addition is calculation of beam break-up (BBU) instability growth rates.

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