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There is only one set of the correct values of f^F, f^D and f^S coupling constants in SU(3) invariant Lagrangian of the vector-meson-baryon interactions

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Abstract. One can prove, there is generally eight various $\omega - \phi$ mixings forms in elementary particle physics, which on one side give different forms of the vector-meson-nucleon coupling constants through f^F, f^D and f^S in SU(3) invariant Lagrangian of the vector-meson-baryon interactions, and on the other side different signs of the universal vector-meson coupling constants f_ρ, f_ω and f_ϕ. Identical set of numerical values of f^F, f^D and f^S is evaluated only in that case, if the same $\omega - \phi$ mixing is applied to a derivation of the vector-meson-nucleon coupling constant forms and also to the signs of the universal vector-meson coupling constants f_ρ, f_ω and f_ϕ.

1 Quark content of the ω and ϕ mesons

The $\omega-\phi$ mixing [1] is well established concept and it is an integral part of many textbooks of the particle physics. However, one can find out that many textbooks have different definitions of the $\omega - \phi$ mixing and so, there are at least three different incompatible definitions of them. We can demonstrate the correct application of $\omega-\phi$ mixing on the obtaining an explicit quark representation of the ϕ and ω mesons.

Under the application of SU(3) group the vector mesons are classified by the singlet ω_0 and the octet matrix

$$V = \begin{pmatrix} \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{2}}\rho^0 & \rho^+ & K^{*+} \\ \rho^- & \frac{1}{\sqrt{6}}\omega_8 - \frac{1}{\sqrt{2}}\rho^0 & \overline{K}^{*0} \\ K^{*-} & K^{*0} & -\frac{2}{\sqrt{6}}\omega_8 \end{pmatrix} \equiv q^a \bar{q}_b - \frac{1}{3}\delta^a_b q^c \bar{q}_c, \tag{1}$$

where quark representation $q^a = (u, d, s)$ of the corresponding vector mesons is given as

$$\rho^0 = \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d), \quad \rho^+ = u\bar{d}, \quad \rho^- = d\bar{u}, \quad K^{*+} = u\bar{s}, \quad K^{*-} = s\bar{u}, \quad K^{*0} = s\bar{d}, \quad \overline{K}^{*0} = d\bar{s},$$
$$\omega_8 = \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s), \quad \omega_0 = \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{s}s). \tag{2}$$

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The ω_8, ω_0 states do not correspond to the known physical states $\omega(782)$ and $\phi(1020)$, as one can verify by the evaluation of the ω_8 mass by the Gell-Mann-Okubo quadratic mass formula [3]

$$m^2_{\omega_8} = \frac{4\frac{m^2_{K^{*0}}+m^2_{\bar{K}^{*0}}}{2} - m^2_\rho}{3} \cong 933\,\mathrm{MeV}^2, \tag{3}$$

and one must to introduce $\omega - \phi$ mixing mechanism, with some value of the mixing angle θ, in order to get the eigenstates $\omega(782)$ and $\phi(1020)$. The procedure of a diagonalization of mass matrix of the phenomenological lagrangian

$$\mathcal{M} = \begin{pmatrix} m^2_{\omega_8} & (m^2_{08} + m^2_{80})/2 \\ (m^2_{08} + m^2_{80})/2 & m^2_{\omega_0} \end{pmatrix} \tag{4}$$

by the orthogonal matrices $R = (R_1, R_2)$

$$\begin{pmatrix} m^2_\phi & 0 \\ 0 & m^2_\omega \end{pmatrix} = R^{-1}\mathcal{M}R, \qquad R^{-1}\begin{pmatrix}\omega_8 \\ \omega_0\end{pmatrix} = \begin{pmatrix}\phi \\ \omega\end{pmatrix}, \qquad (\omega_8, \omega_0)R = (\phi, \omega) \tag{5}$$

with the most general form

$$R_1 = \begin{pmatrix} -p & q \\ q & p \end{pmatrix} \qquad R_2 = \begin{pmatrix} p & q \\ -q & p \end{pmatrix} \tag{6}$$

where $p^2 + q^2 = 1$, $\det R_1 = -1$ and $\det R_2 = +1$, leads to a quadratic mass relation

$$m^2_{\omega_8} = p^2 m^2_\phi + q^2 m^2_\omega. \tag{7}$$

The unknown constants can be numerically determined with the masses from [4], which gives the results $p = \pm 0.63192$ and $q = \pm 0.77500$. One can use a correspondence $0.63192 = \sin\theta$, $0.77500 = \cos\theta$ with the angle $\theta = 39.19^\circ$ and together with the equations (5),(6) produce exactly eight possible different forms of $\omega - \phi$ mixing forms. Taking into account (2) one gets an explicit quark representation of the ω and ϕ mesons as follows

$$\begin{aligned}
&1.\quad \phi = +s\bar{s} && \omega = +\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \\
&2.\quad \phi = -\frac{2}{3}(u\bar{u} + d\bar{d}) + \frac{1}{3}s\bar{s} && \omega = +\frac{1}{3\sqrt{2}}(u\bar{u} + d\bar{d}) + \frac{4}{3\sqrt{2}}s\bar{s} \\
&3.\quad \phi = +\frac{2}{3}(u\bar{u} + d\bar{d}) - \frac{1}{3}s\bar{s} && \omega = -\frac{1}{3\sqrt{2}}(u\bar{u} + d\bar{d}) - \frac{4}{3\sqrt{2}}s\bar{s} \\
&4.\quad \phi = -s\bar{s} && \omega = -\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \\
&5.\quad \phi = -s\bar{s} && \omega = +\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \\
&6.\quad \phi = +\frac{2}{3}(u\bar{u} + d\bar{d}) - \frac{1}{3}s\bar{s} && \omega = +\frac{1}{3\sqrt{2}}(u\bar{u} + d\bar{d}) + \frac{4}{3\sqrt{2}}s\bar{s} \\
&7.\quad \phi = -\frac{2}{3}(u\bar{u} + d\bar{d}) + \frac{1}{3}s\bar{s} && \omega = -\frac{1}{3\sqrt{2}}(u\bar{u} + d\bar{d}) - \frac{4}{3\sqrt{2}}s\bar{s} \\
&8.\quad \phi = +s\bar{s} && \omega = -\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})
\end{aligned} \tag{8}$$

from where it is transparent that only solutions 1., 4., 5., 8. are physically acceptable $\omega - \phi$ mixing versions. The observed decays of ω and ϕ vector mesons confirm that the decays of ω mesons are dominantly into states with the negligible strange quark content and the decays of ϕ mesons are dominantly into K vector mesons with the strange quark content. As a result the strange quark content of the ω meson should be negligible and the strange quark content of the ϕ meson should be dominant.

2 Vector meson coupling constants

The quadratic values of the universal vector meson coupling constants $f_V = f_\rho, f_\omega, f_\phi$ are estimated from the experimental values the vector-meson lepton widths $\Gamma(V \to e^+e^-) = \frac{\alpha^2 m_V}{3}(f_V^2/4\pi)^{-1}$, therefore their signs are not satisfactory determined and they depend on the applied $\omega-\phi$ mixing. Actually, the hadronic electromagnetic (EM) current in the quark form is

$$J_\mu^h = \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) + \frac{1}{6}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) - \frac{1}{3}\bar{s}\gamma_\mu s = \frac{1}{\sqrt{2}}J_\mu^{\rho^0} + S_\omega \frac{1}{3\sqrt{2}}J_\mu^\omega - S_\phi \frac{1}{3}J_\mu^\phi, \quad (9)$$

where S_ω, S_ϕ are the current signs ± 1. If one defines the ω_8 and ω_0 meson EM currents as

$$J_\mu^{\omega_8} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d - 2\bar{s}\gamma_\mu s), \qquad J_\mu^{\omega_0} = \frac{1}{\sqrt{3}}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d + \bar{s}\gamma_\mu s), \quad (10)$$

and applying directly the acceptable $\omega - \phi$ mixing versions (8), one finds that

$$\begin{aligned}
1.\quad J_\mu^h &= \frac{1}{\sqrt{2}}J_\mu^{\rho^0} + \frac{1}{3\sqrt{2}}J_\mu^\omega - \frac{1}{3}J_\mu^\phi,\\
4.\quad J_\mu^h &= \frac{1}{\sqrt{2}}J_\mu^{\rho^0} - \frac{1}{3\sqrt{2}}J_\mu^\omega + \frac{1}{3}J_\mu^\phi,\\
5.\quad J_\mu^h &= \frac{1}{\sqrt{2}}J_\mu^{\rho^0} - \frac{1}{3\sqrt{2}}J_\mu^\omega - \frac{1}{3}J_\mu^\phi,\\
8.\quad J_\mu^h &= \frac{1}{\sqrt{2}}J_\mu^{\rho^0} + \frac{1}{3\sqrt{2}}J_\mu^\omega + \frac{1}{3}J_\mu^\phi. \qquad (11)
\end{aligned}$$

The hadronic EM current can be written as the linear combination of the vector meson fields ρ^0, ω, ϕ and dimensionless coupling constants f_V as

$$J_\mu^h = -\frac{m_{\rho^0}^2}{f_\rho}\rho^0{}_\mu - \frac{m_\omega^2}{f_\omega}\omega_\mu - \frac{m_\phi^2}{f_\phi}\phi_\mu. \quad (12)$$

By the means of the relations (8) and (11), the ratios of the universal vector meson coupling constants can be represented with the help of mixing angle θ as

$$\begin{aligned}
1.\quad \frac{1}{f_\rho} : \frac{1}{f_\omega} : \frac{1}{f_\phi} &= \sqrt{3} : +\sin\theta : -\cos\theta\\
4.\quad \frac{1}{f_\rho} : \frac{1}{f_\omega} : \frac{1}{f_\phi} &= \sqrt{3} : -\sin\theta : +\cos\theta\\
5.\quad \frac{1}{f_\rho} : \frac{1}{f_\omega} : \frac{1}{f_\phi} &= \sqrt{3} : -\sin\theta : -\cos\theta\\
8.\quad \frac{1}{f_\rho} : \frac{1}{f_\omega} : \frac{1}{f_\phi} &= \sqrt{3} : +\sin\theta : +\cos\theta. \qquad (13)
\end{aligned}$$

The SU(3) invariant interaction Lagrangian $\mathcal{L}_{V\bar{B}B}$ for the strong interaction of the baryon octet with the vector meson nonet is described by

$$\mathcal{L}_{V\bar{B}B} = \frac{\mathrm{i}}{\sqrt{2}} f^F \Big[\bar{B}^\alpha_\beta \gamma_\mu B^\beta_\gamma - \bar{B}^\beta_\gamma \gamma_\mu B^\alpha_\beta\Big](V_\mu)^\gamma_\alpha + \frac{\mathrm{i}}{\sqrt{2}} f^D \Big[\bar{B}^\beta_\gamma \gamma_\mu B^\alpha_\beta + \bar{B}^\alpha_\beta \gamma_\mu B^\beta_\gamma\Big](V_\mu)^\gamma_\alpha + \frac{\mathrm{i}}{\sqrt{2}} f^S \bar{B}^\alpha_\beta \gamma_\mu B^\beta_\alpha \omega^0_\mu, \quad (14)$$

where the f^F, f^D, f^S are unknown coupling constants of the Lagrangian, B, $\bar{B}$ are baryon, anti-baryon octet matrices. By the schema presented in [5], one can find four different sets of the expressions for f^F, f^D, f^S coupling constants. They can be evaluated with the help of fitted values of the ratios of coupling constants f_{VNN}/f_V and previously received relations (13). It can be shown that in all cases of different sets, one receive the same values of f^F, f^D, f^S as follows

$$f_1^F = 5.414, \qquad f_1^D = -1.699, \qquad f_1^S = 42.916\,. \qquad (15)$$

By means of a similar procedure one can find also numerical values of all other coupling constants under consideration

$$\begin{aligned} f_2^F &= 7.626, & f_2^D &= 21.088, & f_2^S &= -7.111, \\ f_1^{F'} &= 8.343, & f_1^{D'} &= 12.498, & f_1^{S'} &= -7.858; \\ f_2^{F'} &= -30.450, & f_2^{D'} &= -5.271, & f_2^{S'} &= -18.614\,, \end{aligned} \qquad (16)$$

needed for the prediction of octet hyperon EM form factors.

3 Conclusions

We revisited $\omega - \phi$ mixing configurations satisfying Gell-Mann-Okubo quadratic mass formula and quark representation of ω_8, ω_0 mesons. We have specified the signs of universal vector meson coupling constants f_ρ, f_ω, f_ϕ. We have derived and evaluated the f^F, f^D, f^S coupling constants in SU(3) invariant lagrangian. Their values do not depend on the configuration of ω-ϕ mixing. Universal vector-meson coupling constants f_V's play an essential role in a prediction of $1/2^+$ octet hyperon electromagnetic form factor behaviors.

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