PLASMA ACCELERATION IN SLIGHTLY INHOMOGENEOUS HIGH-FREQUENCY FIELDS

M. A. Miller

Gorki University, Gorki, USSR

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1. INTRODUCTION

In the acceleration of clusters of like charged particles or of a quasi-neutral plasma, it is necessary either to create fields of force that ensure both acceleration and spatial stability of the clusters, or to confine oneself to brief pulse acceleration, during which time the unstabilized cluster would not have time to deform perceptibly.

In this connection, it is worthwhile distinguishing systems of two types: (1) accelerators, in which the same fields of force are utilized both for acceleration and for localization of the objects being accelerated, and (2) accelerators, which either do not at all ensure localization of the objects undergoing acceleration, or ensure it by means of other, supplementary, fields of force.

The mode of localization may serve as the starting point when considering accelerators of the first type, inasmuch as any system that localizes a motionless object converts into a system that accelerates it in generating appropriate movement of localized fields in space. A special version of this general principle is acceleration of charged particles localized inside moving wells of a high-frequency potential Φ [1, 2].

Great attention is presently being devoted to the study of different ways of containing plasma in connection with the well-known problem of controlled thermonuclear reactions. A possible method consists in the utilization of slightly inhomogeneous high-frequency electro-magnetic fields. The forces averaged during a period of oscillations and acting on a particle in such fields are independent of the sign of the charge. This, essentially, is what underlies the high-frequency methods of localizing plasma.[(*)]

The formation of an inhomogeneous field of required structure is achieved, both through the proper distribution of outside sources and at the expense of perturbations caused by the plasma itself. In certain cases it has been possible to obtain an electrodynamic self-consistent solution : a plasma sphere in a spherical resonator [4], a plasma cylinder in a circular waveguide [5], a two-dimensional plasma layer between ideal planes [6], a plane boundary of plasma retained by a plane wave normally incident on it [7]. In the general form, the problem has been solved only for highly rarefied plasma, the concentration of which is so small that we may ignore distortions introduced by the plasma into the external field, that is, we may actually deal with the localization of single charged particles [8, 9].

The present paper considers certain peculiarities of linear and circular high-frequency plasma accelerators of the first and, partially, of the second type; evaluations are given of the most important parameters of such accelerators.

(*) V. I. Veksler was the first to draw attention to this possibility. He suggested utilising the pressure of electromagnetic waves to accelerate plasma blobs[3].

2. THE MOTION AND LOCALIZATION OF SINGLE CHARGED PARTICLES

As is shown in other papers[8-10] (*), in a slightly inhomogeneous high-frequency field $\mathbf{E}(\mathbf{r})e^{i\omega t}$, $\mathbf{H}(\mathbf{r})e^{i\omega t}$, the non-relativistic motion of a charged particle may be represented as a superposition of a rapidly oscillating motion,

$$\mathbf{r}^{(1)}(t) = \mathbf{r}^{(1)}e^{i\omega t} = -\eta/\omega^2 \mathbf{E}e^{i\omega t} \quad (2.1)$$

and a smooth motion $\mathbf{r}^{(0)}(t)$ averaged over the period $2\pi/\omega$ and obeying equation.

$$\ddot{\mathbf{r}}^{(0)} = -\nabla\Phi(\mathbf{r}^{(0)}), \ \ i.e. \frac{\dot{\mathbf{r}}^{(0)2}}{2} + \Phi(\mathbf{r}^{(0)}) = \text{const} \quad (2.2)$$

Here, $\eta = e/m$ is the ratio of the charge of the particle e to its mass m, while Φ is the high-frequency potential equal to

$$\Phi = (\eta/2\omega)^2 |\mathbf{E}|^2 \quad (2.3)$$

This description is possible on the following assumptions

$$E \gtrsim H\,, \quad \dot{r}/c \ll 1$$
$$\frac{\dot{r}^{(0)}}{\omega L_E} \sim \frac{\dot{r}^{(1)}}{\omega L_E} \ll 1\,; \quad \frac{r^{(1)}}{L_E} \sim \frac{\eta E}{\omega^2 L_E} \ll 1 \quad (2.4)$$

where L_E is a characteristic dimension of the region of inhomogeneity of the electromagnetic field.

Thus, charged particles may be localized near absolute minima of the high-frequency potential.(**)

If the potential curve $\Phi(\mathbf{r})$ has the shape of a potential well, particles will be retained within it that possess, in the centre, (at the point of minimum Φ) velocities $|v_0| \leqslant \sqrt{2\Delta\Phi}$ where $\Delta\Phi$ is the minimum difference of potentials between the " edge " and the centre of the well.

Let us assume that such a potential well is formed in a frame of reference K' moving relative to the laboratory system K in the z direction with a velocity $v = \beta c$ and an acceleration w. Noting that :

$$w(1-\beta^2)^{-3/2} = \tilde{w} = \text{const} \quad (2.5)$$

Then, by introducing in place of (Φ) a certain effective potential (***) Φ'_{eff}, which includes a potential of the force of inertia

$$\Phi'_{eff} = \Phi' + \tilde{w}z' \quad (2.6)$$

we find that in order to retain the particle it is necessary to observe the inequality (****)

$$\left|\frac{\partial \Phi'}{\partial z'}\right| > \tilde{w} \quad (2.7)$$

Condition (2.7) is at the same time also a supplementary limitation of the value of the potential Φ itself, because by virtue of Eq. (2.4) one cannot utilize $\Phi(\mathbf{r})$ curves with very steep slopes.

3. THE MOTION AND LOCALIZATION OF PLASMA CLUSTERS

From Eq. (2.2) and Eq. (2.3) it follows that with the aid of slightly inhomogeneous high-frequency fields it is possible to control the motion of charged particles and ignore the dependence of their averaged trajectory on the phase of the field, and, consequently, to exercise control independently of the sign of their charge. For this reason, the majority of conclusions made with respect to systems with like charged particles is generalized also to systems with quasi-neutral plasma formations. However, one must bear in mind that in a real electron-ion plasma the high-frequency field actually acts only on electrons ($m_e \ll m_i$) as a result of which the effective value of a potential of an averaged force correspondingly diminishes.

Let us demonstrate this with a simplified double-velocity model of a fully ionized quasi-neutral ($N_e \simeq N_i$) plasma. We shall assume the field E

(*) Our other paper " The motion of charged particles in slightly inhomogeneous high-frequency fields " is devoted to these problems.

(**) We have purposely simplified the situation by considering the case of a purely high-frequency field. However, in the presence of a constant magnetic field, localization is possible also in regions of maximum E, on the condition that $\omega_H > \omega$ where ω_H is the cyclotron frequency [9, 10].

(***) Quantities relating to the K' system are primed.

(****) Here and henceforth we assess only the possibility of retaining particles in the z-direction and it is assumed that localization in the cross-section z = const is accomplished either due to the suitable structure of the potential φ or by means of an additional longitudinal constant magnetic field.

to be given externally, that is, we assume the concentration of electrons in the plasma N_e to be sufficiently small (*)

$$\frac{\omega_{pl}^2}{\omega^2} \equiv \frac{4\pi e^2 N_e}{m_e \omega^2} \ll 1 \tag{3.1}$$

Combining this limitation with the condition of smallness of the Debye shielding radius

$$r_{Deb} = \sqrt{kT/4\pi e^2 N_e} = \sqrt{V_T/4\pi e N_e}\,,$$

[k is the Boltzmann constant, T the temperature in absolute units, $kT = eV_T$] in comparison with the dimensions of the plasma cluster L

$$L \sim L_E \sim \lambda\!\!\!^{-} = \lambda/2\pi$$

we obtain

$$\beta_T \ll \sqrt{2}\,\frac{\omega_{pl}}{\omega} \ll 1 \tag{3.2}$$

where

$$\beta_T = v_T/c = \sqrt{2\eta_e V_T}/c$$

Let the plane layer of such a plasma be incident on a monotonically increasing barrier of a high-frequency potential (2.3). Then we may write separate integrals of motion (Eq. (2.2)) for unlike charged particles with allowance made for the Coulomb field $E_q = -\nabla\phi_q$ which appears due to separation of the charges.

$$\begin{aligned} &\frac{v_e^2 - v_{0_e}^2}{2} + \Phi_e + \eta_e \phi_q = 0 \\ &\frac{v_i^2 - v_{0_i}^2}{2} + \Phi_i + \eta_i \phi_q = 0 \end{aligned} \tag{3.3}$$

Here, quantities relating to the electron have the subscript "e" and quantities relating to the ion have the subscript "i"; and in the general case it is assumed that

$$\eta_i = Z\frac{|e|}{m_i} = -Z\frac{m_e}{m_i}\eta_e$$

Subtracting the first equation (3.3) from the second, we find for the potential of the Coulomb field ϕ_q

$$\phi_q = \frac{1}{\eta_i - \eta_e}\left\{\Phi_e - \Phi_i + \frac{v_e^2 - v_{0_e}^2}{2} - \frac{v_i^2 - v_{0_i}^2}{2}\right\} \tag{3.4}$$

Let $v_{0_e} = v_{0_i} = v_0$; this occurs in the motion of a plasma layer as a single whole with a velocity greatly in excess of thermal velocities. Then, substituting Eq. (3.4) into Eq. (3.3) we obtain

$$\frac{v_e^2 - v_{0_e}^2}{2} + \frac{\eta_e \Phi_i - \eta_i \Phi_e}{\eta_i - \eta_e} = 0$$

that is, such a plasma layer behaves similarly to a single charged particle (e, m_i) in a field with potentials of averaged force :

$$\Phi_{eff} = \frac{\eta_e \Phi_i - \eta_i \Phi_e}{\eta_i - \eta_e} \simeq -\frac{\eta_i}{\eta_e}\Phi_e = Z\frac{m_e}{m_i}\Phi_e\,. \tag{3.5}$$

Consequently, the efficiency of all high-frequency devices for the control of the motion of plasma clusters is less at least by a factor of $Z\,m_e/m_i$ than in the case of electrons.

If in Eq. (3.3) and Eq. (3.4) we put $v_e^2/v_i^2 = Z|\eta_e/\eta_i|$ (this is the case in an equilibrium plasma with identical electron and ion temperatures)—then

$$\Phi_{eff} = \frac{Z}{Z+1}\left(\Phi_e - \frac{\eta_e}{\eta_i}\Phi_i\right) \simeq \frac{Z}{Z+1}\Phi_e \tag{3.6}$$

The ratio (3.6) refers to the problem on the retention of the plasma boundary by a high-frequency field [4-7, 11]. If we reject the two-velocity model and assume the existence of Maxwellian distributions around the averaged velocities, then the concentrations of electrons and ions will be determined by the Boltzmann formula.(**)

$$N_{e,i}(\mathbf{r}) = N_{e,i}(0)\exp\left\{-\frac{\Phi_{eff}^{(e,i)}}{|\eta_{e,i} V_T|}\right\} \tag{3.7}$$

In a particular case $Z = 1$ (henceforth we shall adhere to this case)

$$\eta_i^{-1}\Phi_{eff}^{(i)} = |\eta_e^{-1}|\Phi_{eff}^{(e)} \simeq \tfrac{1}{2}|\eta_e^{-1}|\Phi_e \tag{3.8}$$

(*) Although condition (3.1) is not a fundamental limitation of the principles of acceleration to be described, it permits a full analysis of the kinematic part of the problem.

(**) Further, we unconditionally assume that during the time of interaction of the cluster and the field no alterations occur in the distribution of particles over averaged velocities. Thus, we put aside problems associated with collisions and the heating of localized plasma. These problems are discussed in a study (now going to press) by A. V. Gaponov, M. I. Petelin, E. I. Yakubovich, in which the distribution functions for ions and electrons are obtained and the variation rate (different in the general case) of the electron and ion temperatures is determined on the basis of the solution of averaged kinetic equations with allowance made for collisions.

that is, the effective potential diminishes by only a factor of two as compared with the potential for electrons [5].

It is interesting to note that by virtue of Eq. (3.7), Eq. (3.8) and Eq. (2.3)—provided the following conditions are satisfied :

$$\Phi^{(e)}_{eff} \simeq \frac{\Phi^{(e)}}{2} \sim \eta_e V_T\,, \quad i.e.\ \frac{E^2}{\omega^2} \sim 8\frac{V_T}{\eta_e}$$

the following ratio between the Debye radius and the amplitude of rapid oscillations should always be obeyed :

$$r_{Deb}/r^{(1)} \sim \frac{1}{2\sqrt{2}}\frac{\omega}{\omega_{pl}} \tag{3.9}$$

Hence, in the containment of a rarefied plasma there is fulfilled, in addition to Eq. (3.2), also the condition

$$\beta_T \ll \frac{r^{(1)}}{r_{Deb}} \ll 1 \tag{3.10}$$

Let us now determine certain peculiarities in the localization of plasma in non-inertial systems, in particular, inside a potential well Φ undergoing acceleration. In this case, we should introduce into Eq. (3.7) a potential similar to Eq. (2.6) but making allowance for a corresponding increase in the potential of the force of inertia

$$\Phi'_{eff} = \Phi'_e \frac{Z}{Z+1} + \frac{m_i}{m_e}\frac{\tilde{w}z'}{Z+1} \tag{3.11}$$

As a particular case, when $Z = 1$ (hydrogen plasma)

$$\Phi'_{eff} = \tfrac{1}{2}\Phi'_e + \tfrac{1}{2}\frac{m_i}{m_e}\tilde{w}z' \tag{3.12}$$

The restriction as to the steepness of the slope of the well becomes more strict than Eq. (2.7) :

$$\left|\frac{\partial \Phi'_e}{\partial z'}\right| > \frac{m_i \tilde{w}}{m_e Z} \tag{3.13}$$

By way of illustration, let us consider a purely sinusoidal (with respect to z) potential curve

$$\Phi'(z') = \Phi'_0 + \Phi'_1 \cos 2h'z' \tag{3.14}$$

Substituting Eq. (3.14) into Eq. (3.11) we obtain

$$\Phi'_{eff} = \frac{Z}{Z+1}\Phi'_o + \frac{Z}{Z+1}\Phi'_1[\cos 2h'z' + 2\alpha h'z'] \tag{3.15}$$

where

$$\alpha = \frac{m_i}{Zm_e}\frac{\tilde{w}}{2h'\Phi'_1} \tag{3.16}$$

Obviously, the potential wells are retained only when $\alpha \leq 1$, that is, in the interval $0 \leq \alpha \leq 1$. Hence, the maximum attainable acceleration is

$$\tilde{w}_{\max} = \frac{2h'\Phi'_1}{m_i} m_e Z \tag{3.17}$$

The position of the extrema of the potential curve Eq. (3.15) is likewise determined uniquely by means of the parameter α

$$\sin 2h'z'_{\max} = \sin 2h'z'_{\min} = \alpha\ ;$$
$$2h'z'_{\max} = 2h'z'_{\min} - \pi \tag{3.18}$$

For this reason, for the difference $\Delta\Phi'_{eff}$, which corresponds to the minimum difference of the potential between the edge and the centre of the well, we find

$$\Delta\Phi'_{eff} = 2\Phi'_1 \frac{Z}{Z+1}\psi(\alpha)$$

where

$$\psi(\alpha) = \sqrt{1-\alpha^2} - \alpha \arccos \alpha \tag{3.19}$$

The function $\psi(\alpha)$ may be called a factor of the inertial attenuation of the potential curve. The plasma is practically completely localized inside the moving potential well on the condition

$$m_e \Delta\Phi'_{eff} = 2m_e\Phi'_1 \frac{Z}{Z+1}\psi(\alpha) \geq kT = eV_T \tag{3.20}$$

In the limiting case Eq. (3.17) $\psi(\alpha) = 0$ and, consequently, it is possible to retain only an ideally "cold" plasma ($V_T = 0$), which does not correspond to the assumption of complete ionization.

4. MOVING POTENTIAL WELLS

As was pointed out elsewhere [1, 2], the simplest method of moving a potential bump consists in utilizing travelling waves of different frequencies. Let us consider a bump formed in the K' system by fields

$$\mathbf{E}' = \mathbf{E}'_+ e^{i\omega' t' - ih'z'} + \mathbf{E}'_- e^{i\omega' t' + ih'z'} \tag{4.1}$$

to which in system K there correspond two waves with different frequencies and propagation constants

$$\mathbf{E} = \mathbf{E}_+ e^{i\omega_+ t - ih_+ z} + \mathbf{E}_- e^{i\omega_- t + ih_- z} \tag{4.2}$$

The frequencies $\omega_\pm$ and wave numbers $h_\pm$ are connected with the corresponding quantities in the K' system by the ratios

$$k_\pm \equiv \frac{\omega_\pm}{c} = \frac{k' \pm h'\beta}{\sqrt{1-\beta^2}}, \quad h_\pm = \frac{h' \pm k'\beta}{\sqrt{1-\beta^2}} \tag{4.3}$$

where $\beta = v/c$, v is the velocity of motion of the K' system, and consequently also the velocity of motion of the potential bump formed by field Eq. (4.2) relative to the laboratory system K. By means of fields Eq. (4.2) it is possible to form the most diverse bumps of a high-frequency potential Φ[9]. For example, in a regular cylindrical line two propagating waves of the same mode create without losses a bump of type Eq. (14), in which

$$\begin{aligned} \Phi_0' &= (\eta/2\omega')^2\{|\mathbf{E}'_+|^2 + |\mathbf{E}'_-|^2\} \\ 2\Phi_1' &= (\eta/\omega')^2 |\mathbf{E}'_+ \mathbf{E}'_-| \end{aligned} \tag{4.4}$$

The waves (Eq. (4.2)) in the K system may propagate in the same ($h_+ h_- < 0$) or in opposite ($h_+ h_- > 0$) directions. Quantities that refer to antiparallel waves will have the subscript "$\rightleftarrows$", while those that refer to waves of the same direction (parallel waves) have he subscript "$\rightrightarrows$". In addition, we shall distinguish systems with slow ($h > k$) and fast ($h < k$) waves. Though in both cases the dispersion equation has the form

$$h_\pm^2 = K_\pm^2 - x_\pm^2 \tag{4.5}$$

for fast waves (if we are speaking of waveguide systems without dissipation of energy and with isotropic boundaries) the transverse wave number x is purely real and does not necessarily depend on the frequency (regular waveguides). Now for slow waves, x is a quantity which is imaginary (or complex) and fundamentally dependent on the frequency (*)

$$h_\pm^2 = K_\pm^2 - x_\pm^2(K_\pm) = K_\pm^2 + \tilde{x}_\pm^2(K_\pm) \geq K_\pm^2 \tag{4.7}$$

where

$$\tilde{x}_\pm = ix_\pm$$

Let us first examine the case of fast waves, the study of which, thanks to Eq. (3.5), may be carried out in the general form without specifying the shape of the cross-section of the wave-guide and the type of field in it.

Two very important parameters determine the quality of the bump (Eq. (3.14)) from the point of view of suitability for accelerating charged particles : first, the propagation constant h', which characterizes the steepness of the slopes of the potential well, and second, the quantity $\beta = v/c$ which determines the rate of motion of potential wells. Taking into consideration Eq. (4.6) and introducing dimensionless designations

$$\gamma = K_+/K_-, \quad P_\pm = x_\pm/K_-, \quad \rho = h'/K_- \tag{4.8}$$

directly from Eq. (4.3), we obtain

$$\beta_{\rightleftarrows}(\gamma,p) = \frac{K_+ - K_-}{|h_+| + |h_-|} = \frac{\gamma - 1}{\sqrt{\gamma^2 - p^2} + \sqrt{1-p^2}} \tag{4.9}$$

$$\beta_{\rightrightarrows}(\gamma,p) = \frac{K_+ - K_-}{|h_+| - |h_-|} = \frac{\gamma - 1}{\sqrt{\gamma^2 - p^2} - \sqrt{1-p^2}} \tag{4.10}$$

$$\rho_{\rightleftarrows}(\gamma,p) = \frac{1}{\sqrt{2}}\sqrt{\gamma - p^2 + \sqrt{(\gamma^2-p^2)(1-p^2)}} \tag{4.11}$$

$$\rho_{\rightrightarrows}(\gamma\, p) = \frac{1}{\sqrt{2}}\sqrt{\gamma - p^2 - \sqrt{(\gamma^2-p^2)(1-p^2)}} \tag{4.12}$$

Since the case of exponentially diminishing waves is excluded from consideration, γ varies within the limits $p \leq \gamma < \infty$ and $0 \leq p \leq 1$. The precise equality $\gamma = p$ or $p = 1$ corresponds to excitation of the waveguide on one of the critical frequencies, that is, to independence, of one of the fields Eq. (4.2) of the coordinate z. And from Eq. (4.9) - Eq. (4.10) we have

$$\begin{aligned} \beta_{\rightleftarrows}(\gamma,1) &= \beta_{\rightrightarrows}(\gamma,1) = \sqrt{(\gamma-1)/(\gamma+1)} \\ |\beta_{\rightleftarrows}(\gamma,\gamma)| &= |\beta_{\rightrightarrows}(\gamma,\gamma)| = |\sqrt{(1-\gamma)/(1+\gamma)}| \end{aligned} \tag{4.13}$$

Thus it is precisely the curves $\sqrt{(\gamma-1)/(\gamma+1)}$ and $\sqrt{(1-\gamma)/(1+\gamma)}$ that divide the regions of antiparallel and parallel waves. Similarly,

$$\begin{aligned} \rho_{\rightleftarrows}(\gamma,1) &= \rho_{\rightrightarrows}(\gamma,1) = \frac{1}{\sqrt{2}}\sqrt{\gamma-1} \\ \rho_{\rightleftarrows}(\gamma,\gamma) &= \rho_{\rightrightarrows}(\gamma,\gamma) = \frac{1}{\sqrt{2}}\sqrt{\gamma-\gamma^2} \end{aligned} \tag{4.14}$$

(*) Since the transverse wave number x remains invariant with respect to transformation Eq. (4.3), the slow waves Eq. (4.1) in the accompanying frames of reference should also, generally speaking, have different wave numbers. Note that the wave number h', which enters into all subsequent formulae, should be put equal to $h' = \frac{1}{2}(h'_+ + h'_-)$.

Obviously,

$$\rho_{\rightleftarrows}(\gamma,p) \geq \rho_{\rightrightarrows}(\gamma,d); \; |\beta_{\rightleftarrows}(\gamma,p)| \leq |\beta_{\rightrightarrows}(\gamma,p)| \quad (4.15)$$

higher velocities of motion of localized fields are always achieved in the use of waves of a single direction, but on the other hand, antiparallel waves form potential wells with steeper slopes. Bumps with maximum steep slopes are formed (from fast waves) by antiparallel TEM waves that propagate with the velocity of light ($p = 0$, $h = k$); according to Eq. (4.11), we have for them

$$(\rho_{\rightleftarrows})_{\max} = \sqrt{\gamma} \quad (4.16)$$

For waves of a single direction, the most suitable in this respect are regimes with maximum values of p

$$(\rho_{\rightrightarrows})_{\max} = \frac{1}{\sqrt{2}} \sqrt{\gamma - 1} \quad (4.17)$$

If the frequencies ω_+ and ω_- coincide ($\gamma = 1$), then the bump $\phi'_{\rightleftarrows}$ stops ($\beta_{\rightleftarrows} = 0$); as for bump $\phi'_{\rightrightarrows}$, though (according to Eq. (4.10)) it should nominally move with the group velocity of the wave

$$\beta_{\rightrightarrows} \to \sqrt{1-\beta^2} \quad \text{if} \quad \gamma \to 1 \quad (4.18)$$

the potential wells disappear in it because $\rho_{\rightrightarrows}(1,p) = 0$.

Beginning with certain values of γ, the velocities of motion of both profiles $\phi'_{\rightleftarrows}$ and $\phi'_{\rightrightarrows}$ increase with γ, and approach the velocity of light. If we disregard for the time being purely technical limitations, the fundamentally attainable velocity is determined by the conditions of localization of the plasma inside the potential well. For ρ (when $\gamma \gg p$) we have, approximately,

$$\rho_{\rightleftarrows} \simeq [\gamma/2(1+\sqrt{1-p^2})]^{1/2}$$
$$\rho_{\rightrightarrows} \simeq [\gamma/2(1-\sqrt{1-p^2})]^{1/2} \quad (4.19)$$

Consequently, ρ increases in proportion to $\sqrt{\gamma}$ asymptotically, and beginning with a certain velocity, the condition of localization of particles inside the well (Eq. (2.4)) proves to be violated due to the fact that the amplitude of oscillation of the electrons turns out to be comparable with the thickness of the potential barrier

$$L \sim 1/h' = 1/k_- \rho \sim v_o/\omega$$

Hence,

$$(\rho_{\rightleftarrows})_{\lim} = \left[\frac{\gamma_{\lim}}{2}(1 \pm \sqrt{1-p^2})\right]^{1/2} \simeq 1/\beta'_o \quad (4.20)$$

where $\beta'_o = v'_o/c$ is the mean velocity of the contained particles in the accompanying system K'.

Taking into consideration also the fact that when $\gamma \gg p$, according to Eq. (4.9) and Eq. (4.10), the velocities of the profiles are asymptotically equal to

$$\beta_{\rightleftarrows} \simeq 1 - \gamma^{-1}(1+\sqrt{1-p^2})$$
$$\beta_{\rightrightarrows} \simeq 1 - \gamma^{-1}(1-\sqrt{1-p^2}) \quad (4.21)$$

and substituting Eq. (4.10) into Eq. (4.11) we obtain formulae that permit evaluating the order of magnitude of the limiting velocity

$$(\beta_{\rightleftarrows})_{\lim} \simeq 1 - \frac{\beta_o'^2}{2}[1+\sqrt{1-p^2}]^2 \quad (4.22)$$
$$(\beta_{\rightrightarrows})_{\lim} \simeq 1 - \frac{\beta_o'^2}{2}[1-\sqrt{1-p^2}]^2$$

Of course, when applying Eq. (4.12) one should also take into account the methods of attaining these limiting velocities: they may turn out to be either impossible due to too large values of γ or, on the contrary, to be trivially attainable, as was the case for $p = 0$, when $\beta_{\rightrightarrows} = 1$.

In conclusion we should like to make several remarks as to the utilization of slow electromagnetic waves.

Quite naturally, in systems with slow waves it is possible to attain lower velocities of motion of profiles than in analogous systems with fast waves. But on the other hand, accelerators with slow waves have a number of unquestionable merits. These have first of all to do with the possibility of raising the initial acceleration without additional consumption of high-frequency power, or the possibility of a corresponding reduction in the power for the same accelerations. Thus, given a fixed Φ_1 in Eq. (3.17), the maximum attainable acceleration is increased by a factor of $\rho = h'/k'$. In actual decelerating systems it has not been possible to increase the retardation while retaining the distribution Φ invariable: as a rule, with increasing ρ, there is an increase in the degree of localization of the field near the directing surface and a corresponding decrease in the value of E on the axis of the waveguide line. This undesirable effect may be reduced by shifting the centre of the localized cluster closer to the directing surface; this is achieved by a slight curvature of the trajectory of the cluster (*).

5. MODES OF ACCELERATED MOTION OF POTENTIAL PROFILES

As is clear directly from Eq. (4.9) or Eq. (4.10), there are two possible ways for the accelerated motion of potential bumps formed by fields Eq. (4.1). The first consists in varying the frequency $\omega_{\pm}$ of one of the waves Eq. (4.2) as a function of time, and the second, in varying the propagation constants h_+ and h_- as functions of the coordinates. To retain the stationary state of the bump Φ' (otherwise the potential description will itself lose all meaning), it is necessary to demand that these variations should occur slowly in the scale of period $2\pi/h$ or of the wavelength

$$\left|\frac{dk}{dt}\right| \ll k^2 c\,, \quad \left|\frac{dh}{dz}\right| \ll \frac{h^2}{\beta} \tag{5.1}$$

In addition, it is natural that the transit time of the potential well through the accelerator should not be much less than the characteristic time of variation of the frequency or the propagation constant. (**)

Let us consider certain peculiarities of each of these methods. Let the frequency ω_- be fixed, and $\omega_+ = \omega_+(t)$; this is equivalent to a variation of γ in Eq. (4.9) and Eq. (4.10) for $p = \text{const}$. In the systems $\beta_{\rightleftarrows}$ it is possible to begin acceleration from zero velocity ($\gamma = 1$, $\beta_{\rightleftarrows} = 0$) ; in systems $\beta_{\rightrightarrows}$ this is impossible, i.e., an obligatory step is the injection of a preliminarily accelerated cluster. The magnitude of the initial velocity is determined by the conditions of formation of the potential well at the input of the accelerator ($\rho_{\rightleftarrows} > 0$), but in any case

$$(\beta_{\rightrightarrows})_{\text{start}} = \beta(\gamma,\gamma) > \sqrt{(1-p)/(1+p)}\,.$$

The acceleration values are found by differentiating Eq. (4.9) and Eq. (4.10). When $\gamma \to p$ the derivative $d\beta/d\gamma$ increases without limit; this excludes the selection of a variable frequency near one of the critical frequencies of the waveguide, but generally speaking, a fixed frequency may be set equal to the critical frequency ($p = 1$). If at the initial instant $t = 0$ the frequencies coincide ($\gamma = 1$) and in the course of the whole cycle $t\dfrac{d\gamma}{dt} \ll 0$, then according to Eq. (4.9) and Eq. (4.10), the acceleration is determined by the ratios

$$\frac{d\beta_{\rightleftarrows}}{dt} = \frac{(1-p^2)^{-1/2}}{2}\frac{d\gamma}{dt}\,;$$
$$\frac{d\beta_{\rightrightarrows}}{dt} = \frac{p^2(1-p^2)^{-1/2}}{2}\frac{d\gamma}{dt}\,. \tag{5.2}$$

Hence it follows, incidentally, that the linear variation of frequency corresponds (at the initial period of acceleration) to a uniformly accelerated motion of the profile.

An important distinguishing feature of the second method of acceleration is the use of waves with fixed frequencies; this permits accomplishing resonance excitation of the electrodynamic system without the use of special non-linear frequency-retuning elements. The smooth variation of the propagation constant $h(z)$ is, for example, realized in quasi-cyclindrical waveguides or in resonators, the cross-section of which and consequently the transverse wave number x, slowly varies in the z-direction. For these purposes one may also invoke metal-plate systems with slowly varying surfaces of impedances or waveguides with inhomogeneous dielectric filling in the z-direction. The appropriate velocities and accelerations are easily determined from Equations (4.9) and (4.10), noting that $\dfrac{d\beta}{dt} = \beta c \dfrac{d\beta}{dp}\dfrac{dp}{dz}$. Inasmuch as the parameter p should be situated within the interval $0 \le p \le 1$ for rapidly propagating waves, the maximum ranges of velocities, that is, the differences between the velocities at the input and at the output of the accelerator, are equal to

(*) A more detailed discussion of these questions is given in an article written by I. G. Kondratiev and the author and submitted to Izv. vys. ucheb. zav.; radiofiz., entitled : " On the Utilization of slow electromagnetic waves for accelerating plasma ".

(**) In the excitation of a line in a single cross-section (this is particularly convenient in the systems $\beta_{\rightrightarrows}$), it is necessary to add the requirement that the propagation velocity of the perturbations $\partial\omega/\partial h$ should exceed the velocity of motion of the potential well, $\partial\omega/\partial h > \beta_0 C$. However, this requirement may be reduced to Eq. (5.1) in the distributed excitation of a waveguide, which excitation performs the transmission of the perturbations with the velocity of light.

$$(\Delta\beta_{\rightleftarrows})_{\max} = \sqrt{\frac{\gamma-1}{\gamma+1}} - \frac{\gamma-1}{\gamma+1}$$
$$(\Delta\beta_{\rightrightarrows})_{\max} = |1-\sqrt{(\gamma-1)/(\gamma+1)} \tag{5.3}$$

and the initial velocities differ from zero. This means that preliminary acceleration is required.

The use of retardation systems or combination systems, in which the propagation constants vary from $h_{\max} > k$ to $h_{\min} < k$ permit extending the limits Eq. (5.3). Apparently, in a number of cases it is expedient to make use also of a supplementary frequency modulation.

We may note that in waveguides with a variable cross-section the potential profile always deforms as it moves along z, since the acceleration principle itself is based on the variation of $h_\pm$ and h'. However, the factor of inertial attenuation (Eq. (3.19)) may remain constant during an entire cycle, if $h'\Phi' = \text{const}$ or, according to Eq. (2.3),

$$h'E'^2 = \text{const}$$

6. CHOICE OF PARAMETERS OF THE ACCELERATOR

Here we explain certain peculiarities in the selection of parameters of plasma accelerators (*).

The point of departure is observance of the conditions of localization. Taking into account Eq. (3.20) and Eq. (2.4) we have

$$L\omega^2/|\eta_e| \gg |E_\pm| \gg \sqrt{2}\,\omega(V_T/\eta_e\Psi(\alpha))^{1/2} \tag{6.1}$$

Assuming $L = \lambda/4 = \pi c/2\omega$ and substituting $|\eta_e| = 5.3\times10^7$ we may rewrite Eq. (6.1) as

$$5\times10^8(\Psi/V_T)^{1/2}|E_\pm| \gg \omega \gg 10^7|E_\pm| \tag{6.2}$$

Hence it is clear that the conditions of localization fix rather rigidly the possible frequency range of operation of the accelerator; it is worthwhile selecting ω closer to the left-hand limit in Eq. (6.2), since limitation Eq. (3.20) is weaker than Eq. (2.4); and in turn the dimensions of the region of localization predetermine the permissible values of the electric field strength E: for example, when $L = 5$ cm ($\omega/2\pi = 10^{10}{}_{Hz}$), $V_T = 3.3$ CGSE; $\psi = 0.5$ the field strength should be $E = 10^2$ CGSE (3×10^4 V/cm) and when $E = 0.1$ CGSE (30 V/cm) $\omega = 10^7$ that is $L \simeq 5\times10^3$ cm. Thus, in order to contain the cluster in relatively small spatial regions one has to resort to fields with rather large amplitudes. Satisfaction of condition (6.2), i.e., the choice of parameters V_T, ψ, E, ω almost completely determines the regime of the accelerator.

The permissible values of $\widetilde{w}$ are found from Equation (3.16) if allowance is made for the fact that according to Eq. (4.4) the quantity $\Phi' \sim \frac{1}{2}\eta_e^2/\omega^2(E)_{av}$. One gets

$$\widetilde{w}_{av} = \alpha_{av}\frac{m_e}{m_i}\left(\eta_e/(\omega)_{av}\right)^2 k_{av}\rho_{av}(E_{av})^2 \simeq$$
$$\simeq 5.2\times10^{21}\left(\frac{\alpha\rho}{\omega}\right)_{av}(E_{av})^2\,. \tag{6.3}$$

the subscript $av.$ denoting the average in time.

The actually produced accelerations and also the maximum attainable velocities depend on the mode of moving the localizing field and the electrodynamic characteristics of the system; as an example, if we demand the existence in the waveguide of propagating waves of only one type (**), it is necessary to vary the frequency of the generators within the limits between the first and second critical frequencies of the waveguide

$$cx_2 \geq \omega \geq cx_1 \tag{6.4}$$

which, in addition to Eq. (6.2), is an additional restriction of the range ω and determines the maximum possible velocity at the output of the accelerator (***). From the point of view of Eq. (6.4) the most favourable is the use of TEM-waves in multi-wire lines, for which $x = 0$, and ω is limited below only by the condition (6.2) or the use of slow waves, for which

(*) All evaluations in this section refer to the case of hydrogen plasma.

(**) As a result of the appearance of propagating waves of higher types, the efficiency of the accelerator may be reduced and there may also occur a de-localization of the cluster. It should be noted that even when fundamental waves are excited by external sources, they may transform into waves of higher modes directly on the plasma clusters if their dielectric permeability is noticeably different from unity.

(***) Thus, in a waveguide of circular cross-section (the TE_{11} wave $x_2/x_1 = 1.3$) when using antiparallel waves $\beta_{\max} = 0.36$, and in a waveguide of square cross-section (the TE_{01} wave $x_2/x_1 = 1.41$) $\beta_{\max} = 0.41$. Actually, however, the maximum velocity is even slightly lower if we keep to limitation (6.4), because in order to avoid infinitely large accelerations at the initial instant of motion of the cluster one has to select $\omega_- \neq cx_1$, i.e. $p \neq 1$ then $\beta_{\max} = (x_2p - x_1)/[p\sqrt{x_2^2 - x_1^2} + x_1\sqrt{1-p_2}]$. For example, when $p = 0.9$ for a circular waveguide $\beta_{\max} = 0.14$, and for a waveguide with a square cross-section $\beta_{\max} = 0.2$.

the limits (6.4) may be extended by means of the function $x(\omega)$. If one is discussing accelerators with fixed frequencies, then Eq. (6.4) determines the limits of variation of the transverse wave number. In this case apparently, one may move into the region of the simultaneous existence of oscillations of several types (not too far, of course), because the resonance character of the excitation can ensure sufficient excess of the necessary type of field over the de-localizing fields of higher modes.

From the technical viewpoint, the power spent in forming the localizing fields is probably the most important factor limiting the possibilities of high-frequency plasma accelerators. In the case of non-resonance excitation, this power is roughly equal to the mean flux of the poynting vector in a travelling wave and may be evaluated from the equations

$$P_{TM} = \frac{c}{8\pi}\frac{h}{k}\int |\mathbf{E}_{tr}|^2 dS \simeq \frac{c}{8\pi}\frac{h}{k} E_{av}^2 \Delta S$$

$$P_{TE} = \frac{c}{8\pi}\frac{k}{h}\int |\mathbf{E}_{tr}|^2 dS \simeq \frac{c}{8\pi}\frac{k}{h} E_{av}^2 \Delta S \qquad (6.5)$$

where ΔS is the effective cross-section of the wave-guide.

In the case of resonance excitation of an appropriate cylindrical (or quasi-cylindrical) cavity of length l, evaluation of P should be carried out with account taken of the quality factor of the system (Q)

$$P_Q = \frac{\omega}{8\pi Q}\int |\mathbf{E}|^2 dV \simeq \frac{c}{16\pi Q}(kl)^2 E_{av}^2 \Delta S \qquad (6.6)$$

The following table will give some idea about the most important orders of magnitude that enter into relations (6.2), (6.3), (6.5) and (6.6). It is easy to find the principal parameters V_T, $\tilde{w}$, P.

TABLE I

ω	E_{av}	$\frac{m_e \Delta\varphi_{eff}}{\Psi(\alpha)}$	$\tilde{W}_{av}/\alpha\varrho$	P/ϱ
1/sec	CGSE	eV	cm/sec	W
6.3×10^{10}	10	1.9	8.2×10^{1}	2.8×10^{4}
	10^2	1.9×10^{2}	8.2×10^{12}	2.8×10^{6}
($\lambda = 3$ cm)	10^3	1.9×10^{4}	8.2×10^{16}	2.8×10^{8}
1.9×10^{10}	10	22	2.8×10^{13}	3×10^{5}
	10^2	2.2×10^{3}	2.8×10^{15}	3×10^{7}
($\lambda = 10$ cm)	10^3	2.2×10^{5}	2.8×10^{17}	3×10^{9}
1.9×10^{9}	1	22	2.8×10^{14}	3×10^{5}
	10	2.2×10^{3}	2.8×10^{16}	3×10^{7}
($\lambda = 10^2$ cm)	10^2	2.2×10^{5}	2.8×10^{18}	3×10^{9}

The power in the right-hand column is evaluated for a travelling TM wave in a waveguide with an effective cross-section $\Delta S = \lambda^2/4$.

When using resonance circuits in accelerators with fixed frequencies one may gain by a factor of $Q\rho/2kl$, since, according to Eq. (6.5) and Eq. (6.6),

$$2P_{TM}/\rho \sim P_Q Q/kl$$

This gain is appreciable in the case of resonators whose dimensions are comparable to the wavelength, and is practically absent for very long resonators ($kl \gg 1$).

Another way of reducing P is to reduce the cross-section of the waveguide. But in this case there are always added joule losses. Besides, one must bear in mind that reflecting potential barriers must fit inside the waveguide. The thickness of these barriers cannot be less than the amplitude of rapidly oscillating vibrations (see condition (6.1)), so that one cannot hope for an essential reduction in the power in this case either.

But if power sources of the order of $10^6 - 10^7$ W are required for the containment and acceleration of a plasma cluster heated to a temperature of the order of 10 – 100 eV., it is obvious that, at least for the present, one can speak only of operation with pulses of short ($\sim 10^{-6} - 10^{-7}$ sec) duration. Hence, it follows that only clusters with non-relativistic velocities can be obtained practically at the output of the accelerator in the case of a single interaction with the field. From the Table it is clearly seen that the magnitude of acceleration increases with diminishing wavelength—a thing that is due to an increase in the value of the effective potential $\Phi_{eff} \sim E^2/\omega^2$. However, one must bear in mind the limitation (6.2), which intimately connects the permissible values of E with the frequency ω. If, for example, we put $E = \text{const}\ \omega$ then the acceleration will diminish with diminishing ω. To illustrate this, we give the following Table for accelerators with antiparallel waves.

TABLE II

$E_+ = 10^{-8}\omega$, $\alpha = 0.6$, $\Psi = 0.25$
$P_{TE} \sim 3 \times 10^7 W$, $V_T = 10V$

N	ω sec^{-1}	λ cm	W_{av} cm sec^{-1}	$l_{\beta=0.1}$ M	$t_{\beta=0.1}$ η sec
1.	10^9	189	2×10^{14}	190	13
2.	10^{10}	18.9	2×10^{15}	19	1.3
3.	6.3×10^{10}	3.0	1.5×10^{16}	3.1	0.2

7. ON THE POSSIBILITY OF CIRCULAR ACCELERATION OF PLASMA

The principal difficulty in the circular acceleration of plasma is connected with the necessity of maintaining a quasi-neutral cluster in a definite stable closed path. Although the use, for these purposes, of slightly inhomogeneous high-frequency fields is fundamentally feasible, it is possible, as will be shown below, only for low velocities. To take an example, let it be required to establish a cluster in stable motion along a circular path of radius $r_\perp = r_o$ (naturally, considerably in excess of the dimensions of the cluster itself). We build a toroidal channel of a high-frequency potential $\Phi(r_\perp)$ so that the circle $r_\perp = r_o$ is situated inside this toroidal potential well (*). To compensate the centrifugal forces, it is necessary for the slopes of the potential well to be sufficiently steep. In a non-relativistic approximation, this restriction, analogous in meaning to Eq. (2.7), is written as

$$\left|\frac{\partial\phi}{\partial r_\perp}\right|_{r_\perp = r_o} > \frac{m_i}{m_e}\frac{v_\theta^2}{r_o} \tag{7.1}$$

Here, $v_\theta = \beta_\theta c$ is the mean linear velocity of the cluster. Considering that $\dfrac{\partial\phi}{\partial r_\perp} \sim \phi/L$ and substituting in Eq. (7.1) the value Φ from Eq. (2.3), we find the condition that restricts from below the field strength E which forms the outer (with respect to $r_\perp$) potential barrier

$$\frac{E}{\beta_\theta} > \left(\frac{L}{r_o}\right)^{1/2}\left(\frac{m_i}{m_e}\right)^{1/2}\frac{2\omega c}{|\eta_e|} \tag{7.2}$$

By way of illustration, let us take $\omega = 2\times10^{10}\ \mathrm{sec}^{-1}$, $m_i/m_e = 1.8\times10^3$, $L/r_o \simeq 10^{-2}$. Substitution of these values into Eq. (7.2) yields $|E|/\beta_\theta > 10^4$ CGSE; consequently, even in the case of velocities $\beta_\theta \sim 10^{-1}$ fields are required with strengths $E \sim 10^3$ CGSE (3×10^5 V/cm) and by virtue of Eq. (6.5) and Eq. (6.6), sources are required with powers of the order of $10^7 - 10^9$ W. Hence it follows that in circular accelerators of plasma with radial high-frequency focusing, lower speeds may be achieved than in corresponding linear accelerators; and the principal power consumption of the high-frequency field is by fields that maintain the cluster in the radial direction.

Cluster acceleration may be accomplished by means of moving additional potential wells created by waves revolving azimuthally. It is interesting to note in this connection that if two standing waves of different frequencies are excited inside a self-closed waveguide system, in such a field one can distinguish four potential profiles revolving towards each other in pairs with velocities

$$\beta_{\rightleftarrows} = \pm\frac{k_+ - k_-}{h_+ + h_-}, \quad \beta_{\rightrightarrows} = \pm\frac{k_+ - k_-}{h_+ - h_-}.$$

This permits the simultaneous moving and even acceleration of clusters in opposite directions.

8. CONCLUSION

Our entire attention has been devoted to accelerators of plasma localized inside high-frequency potential wells. It is precisely such systems that fortunately combine the possibilities of simultaneous stabilization and acceleration of the cluster. Of course fundamentally, as was pointed out in paragraph 1, accelerating and localizing fields of force can be unlike in character. To accelerate such objects it is possible to utilize a single stationary potential barrier. For example, a cluster of hydrogen plasma sliding from a single stationary pure high-frequency potential barrier (Eq. (2.3)) acquires, according to Eq. (3.3) and Eq. (3.5) a velocity

$$\beta_{\max} = \frac{e}{\omega c\sqrt{2m_e m_i}}|E_{\max}|$$

When $\omega/2\pi = 10^{10}$Hz and $\beta = 0.1$ this yields $E = 6.5\times10^6$ V/cm. There exist several ways of reducing $E_{\max}$, but they are all to some degree, connected with an increase in the dimensions of the system. Thus it is possible to form a barrier by means of a low-frequency field, for instance, when

$$\omega/2\pi = 10^7\mathrm{Hz}, \quad L_E \sim \lambda \sim 4.5\mathrm{m},$$

$$\beta = 0.1, \quad E_{\max} = 6.5\times10^3\mathrm{V/cm}$$

(*) Such a toroidal channel may be created, for example, by means of TM_{01} type waves in a corrugated metal torus of circular cross-section, by means of TE_{01} waves (common relief) or TE_{11} waves (inverse relief) in a smooth-walled torus of circular cross-section; by means of symmetrical waves in a spirally conducting torus etc. One way of constructing a two-dimensional annular potential well in a cylindrical resonator is considered, for instance, by Veksler and Kovrizhnykh [12].

or to utilize potential barriers formed by a magnetostatic homogeneous H_o and high-frequency fields. When ω approaches the cyclotron frequency

$$\omega_H = \frac{|\eta_e| H_o}{c}$$

the value of E_{max} may be slightly reduced within limits, however, permitting the fulfilment of the inequality $\dot{r}^{(o)} \ll (\omega - \omega_H) L_E$ which is necessary for averaging over the period of difference frequency $2\pi/\omega - \omega_H$.

The possibility of accelerating plasma in the field of a standing wave [13] also belongs to a similar class. However, all these systems (in contrast to the foregoing) are fundamentally non-relativistic for the reason that restriction (2.4) underlies the applicability of the method of averaged description of particle motion.

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DISCUSSION

OHKAWA: Did you take the non-linear effect into account in the second paper where the two waves exist at a very high power level?

RODIONOV: No.