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Chiral Symmetry Breaking by Monopole Condensation

Aiichi IWAZAKI[1]

[1]*Nishogakusha University, 6-16 Sanbancho, Chiyoda-ku, Tokyo 102-8336, Japan*

E-mail: a_iwazaki@hotmail.com

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Under the assumption of Abelian dominance in QCD, we show that either color charge or chirality of quark is not conserved, when the low energy massless quark collides with QCD monopole. Because the color charge is conserved in reality, the chirality is not conserved. We show by using chiral $U_5(1)$ anomaly that chiral asymmetric quark pair production takes place when a classical color charge is putted in a vacuum with monopole condensation, while chiral symmetric pair production takes place in a vacuum with no monopole condensation. Our results indicate that the chiral $U_5(1)$ symmetry is broken by the monopole condensation.

KEYWORDS: chiral symmetry, monopole, QCD

1. Introduction

It has been shown with lattice gauge theories [1] that quark confinement and chiral symmetry breaking simultaneously arises in SU(3) gauge theory with massless quark color triplets. That is, the transition temperature between confinement and deconfinement phases almost coincides with the transition temperature between chiral symmetric and antisymmetric phases. Although extensive studies [2–8] have been performed, the explicit connection between the confinement and the chiral symmetry breaking has still not been clear. The confinement is caused by the monopole condensation [9–11] in the analysis with the use of maximal Abelian gauge [12]. On the other hand, the chiral symmetry breaking is caused by the chiral condensation of quark-antiquark pair. It has been discussed that the condensate arises with instanton effects according to chiral anomaly. No intimate relation between the monopole condensate and the chiral condensate was found, although there were numerical evidences [3] of the relation. But, it has recently been discussed [13, 14] that the monopoles induce the chiral condensate.

In this paper we show using chiral anomaly that when an external color charge is put in a vacuum with the monopole condensation, the vacuum expectation value $\langle dQ_5/dt\rangle$ does not vanish, while it vanishes when the vacuum has no monopole condensation. Here dQ_5/dt denotes the time derivative of the chirality Q_5; $Q_5 = N_R - N_L$ where N_R (N_L) denotes the number of the right (left) handed quarks. That is, $\langle dQ_5/dt\rangle$ represents the pair production of the massless quarks under the background electric field of the external charge. Therefore, the chiral asymmetric pair production of the massless quarks takes place in the vacuum with monopole condensation. It indicates that the chiral symmetry is broken by the monopole condensation. In contrast, the chiral asymmetric pair production does not take place when there are no magnetic excitations in vacuum. The vacuum with the monopole condensation is characterized such that it is not eigenstate of the magnetic charge oprerator $\hat{Q}_m$; $\hat{Q}_m|v\rangle \neq q_m|v\rangle$. Hereafter, we take SU(2) gauge theory for simplicity.

The positive charged fermion produced moves to the direction of the electric field, while the negative charged fermion does to the opposite direction of the electric field. Their spins can take arbitrary directions parallel or anti parallel to the electric field. There are no favorable directions. Therefore, the production is chiral symmetric; $\langle dQ_5/dt\rangle = 0$. On the other hand, the favorable directions of the spins spontaneously arise in the vacuum with the monopole condensation, even if there are no magnetic fields. This leads to the spontaneous chiral symmetry breaking, $\langle dQ_5/dt\rangle \neq 0$.

2. Chirality nonconservation in monople quark scattering

First, we explain that either the charge or the chirality of a quark is not conserved in the low energy massless quark scattering with a Dirac monopole. We need to impose an appropriate boundary condition on the quark at the location of the monopole, which determines the conserved quantity.

Our concern is massless quark doublet ($\begin{matrix}\Psi_+\\ \Psi_-\end{matrix}$) satisfying Dirac equation in the background gauge fields of the monopole,

$$\gamma_\mu(i\partial^\mu \mp \frac{g}{2}A^\mu)\Psi_\pm = 0 \tag{1}$$

where the gauge potentials A^μ denotes a Dirac monopole given by

$$A_\phi = g_m(1-\cos(\theta)), \quad A_0 = A_r = A_\theta = 0 \tag{2}$$

where $\vec{A}\cdot d\vec{x} = A_r dr + A_\theta d\theta + A_\phi d\phi$ with polar coordinates r, θ and $\phi = \arctan(y/x)$. g_m denotes a magnetic charge with which magnetic field is given by $\vec{B} = g_m\vec{r}/r^3$. The magnetic charge satisfies the Dirac quantization condition $g_m g = n/2$ with integer n where g denotes the U(1) gauge coupling. Hereafter, we assume the monopoles with the magnetic charge $g_m = 1/2g$.

The quark doublet coupled with the Dirac monopole arises in SU(2) gauge theory under the assumption of the Abelian dominance [15,16]. The maximal Abelian group is described by the diagonal component (σ_3) of the SU(2) gauge fields. The Dirac monopole is represented by the Abelian gauge fields. Thus, the quark doublet $q = (q^+, q^-)$ carry the charges $\pm g/2$ associated with the diagonal component.

We will explain why either the charge or the chirality is not conserved when the quark collides with the monopole. As is well known, the conserved angular momentum [17] of the quark under the magnetic monopole located at $r = 0$ is given by $\vec{J} = \vec{L} + \vec{S} \mp gg_m\vec{r}/r$, where $\vec{L}$ ($\vec{S}$) denotes orbital (spin) angular momentum. The last term is peculiar to the particle under the background field of the monopole. Owing to the term we can show that either the charge or the chirality is not conserved in the scattering. In order to see it we note the conserved quantity $\vec{J}\cdot\vec{r} = \vec{S}\cdot\vec{r} \pm gg_m r$. When the chirality (or helicity $\sim \vec{p}\cdot\vec{S}/|\vec{p}||\vec{S}|$) is conserved, the spin must flip $\vec{S} \to -\vec{S}$ after the scattering because the momentum flips after the scattering; $\vec{p} \to -\vec{p}$. Then, the charge must flip $g \to -g$ because of the conservation of $\vec{J}\cdot\vec{r}$, i.e. $\Delta(\vec{J}\cdot\vec{r}) = \Delta(\vec{S}\cdot\vec{r}) + \Delta(gg_m r) = 0$. ($\Delta(Q)$ denotes the change of the value Q after the scattering.) On the other hand, when the charge is conserved ($0 = \Delta(\vec{J}\cdot\vec{r}) = \Delta(\vec{S}\cdot\vec{r})$), the chirality $\vec{p}\cdot\vec{S}/|\vec{p}||\vec{S}|$ must flip because the spin does not flip $\vec{S} \to \vec{S}$. In this way, either the charge or the chirality conservation is lost in the scattering.

To appropriately define the scatterings, it has been discussed [18] that we need to imposed a boundary condition for the quarks at the location of the monopoles. It is either of charged conserved but chirality non conserved boundary condition or chirality conserved but charge non conserved one. The charge conservation is strictly guaranteed by the gauge symmetry. Therefore, we need to impose the boundary conditions $q_R^\pm(r=0) = q_L^\mp(r=0)$ at the location of the monopole, which breaks the chiral symmetry. The boundary conditions effectively describe monopole quark interactions such as $(\bar{u}u + \bar{d}d)|\Phi|^2$ with up and down quarks u, d, where Φ denotes the monopole field. Obviously the term explicitly breaks the chiral $U_5(1)$ as well as flavor $SU_A(2)$ symmetries.

3. Chiral asymmetric pair production in monopole condensed vacuum

We give another explanation of the chiral nonconservation using chiral anomaly. The anomaly equation describing the evolution of the chirality Q_5 is given by

$$\frac{dQ_5}{dt} = \frac{g^2}{4\pi^2}\int d^3r\vec{E}\cdot\vec{B} = \frac{g^2}{4\pi^2}\int d^3r\vec{E}\cdot\frac{g_m\vec{r}}{r^3} = \frac{g^2}{4\pi^2}\int d^3r\frac{g(\vec{r}-\vec{x}(t))}{4\pi|\vec{r}-\vec{x}(t)|^3}\cdot\frac{g_m\vec{r}}{r^3} = \frac{g^3g_m}{4\pi^2|\vec{x}(t)|}, \quad (3)$$

where the electric field $\vec{E} = g(\vec{r}-\vec{x}(t))/(4\pi|\vec{r}-\vec{x}(t)|^3)$ is produced by a quark located at $\vec{x}(t)$. The magnetic field $\vec{B} = g_m\vec{r}/r^3$ is produced by a monopole located at $\vec{r} = 0$. The anomaly equation describes how the chirality of the classical quark changes with time t, depending on its coordinate $\vec{x}(t)$. Here, we consider the scattering of the quark such that it goes from $\vec{x}(t=-\infty) = -\infty$ to $\vec{x}(t=\infty) = +\infty$, passing $\vec{x}(t=0) = \vec{x}_0 \neq 0$ at $t = 0$. When the quark does not flip its color charge, the quantity dQ_5/dt does not change its sign. Thus, $Q_5(+\infty) - Q_5(-\infty) = \int_{-\infty}^{+\infty} dt\, dQ_5/dt$ does not vanish. The chirality is not conserved, when the charge is conserved.

We would like to mention the another meaning of the anomaly equation. The equation shows the quantum production of the chirality when a classical charged particle is put in a vacuum with the monopole. Actually, it has been shown [19] that the anomaly equation can describe quark pair production under the classical homogeneous fields $\vec{E}$ and $\vec{B}$. The result is coincident with the one obtained by the analysis in the standard Schwinger mechanism. In our case we have a classical charged particle and a monopole which produce the electric field $\frac{g(\vec{r}-\vec{x}(t))}{4\pi|\vec{r}-\vec{x}(t)|^3}$ and the magnetic field $\frac{g_m\vec{r}}{r^3}$. Then, the equation shows that the chiral asymmetric pair production takes place under these external fields.

Now, we show using the anomaly equation that $\langle dQ_5/dt\rangle \neq 0$ when an external charge is put in the vacuum with the monopole condensation, while $\langle dQ_5/dt\rangle = 0$ without the monopole condensation. The nonvanishing of $\langle dQ_5/dt\rangle$ implies that the chiral asymmetric pair production of the massless quarks arises.

When there are monopoles with their magnetic charges $g_m\eta_i$ at $\vec{x}_i$ ($i = 1,,,$) with $\eta_i = \pm 1$, the anomaly equation is given by

$$\frac{dQ_5(\vec{x})}{dt} = \sum_{i=1,,,}\frac{g^3g_m\eta_i}{4\pi^2|\vec{x}-\vec{x}_i|} = \int d^3y\frac{g^3\rho_m(\vec{y})}{4\pi^2|\vec{x}-\vec{y}|} \quad \text{with} \quad \rho_m(\vec{y}) \equiv \sum_{i=1,,,}\eta_i g_m\delta(\vec{y}-\vec{x}_i), \quad (4)$$

where ρ_m denotes the magnetic charge density of the monopoles.

In order to discuss the quantum effects of the monopoles, we replace the classical magnetic charge density ρ_m with the quantum operator $\hat{\rho}_m$ of the magnetic charge. For instance, it is given such that $\hat{\rho}_m = g_m\Phi^\dagger iD_t\Phi + h.c.$ in terms of monopole field Φ. Because we wish to obtain the vacuum expectation value $\langle dQ_5/dt\rangle$, we estimate it in the following

$$\lim_{|\vec{x}|\to\infty}\langle\frac{dQ_5(\vec{x})}{dt}\frac{dQ_5(0)}{dt}\rangle = \lim_{|\vec{x}|\to\infty}\int d^3yd^3y'\frac{(g^3)^2\langle\hat{\rho}_m(\vec{y})\hat{\rho}_m(\vec{y}')\rangle}{(4\pi^2)^2|\vec{x}-\vec{y}||\vec{y}'|} = \langle dQ_5/dt\rangle^2, \quad (5)$$

where the expectation value is taken in the vacuum of the monopoles. Noting the translational invariance, we rewrite the equation(5) with the use of the function $f(\frac{\vec{y}-\vec{y}'}{\sqrt{2}}) \equiv \langle\hat{\rho}_m(\vec{y})\hat{\rho}_m(\vec{y}')\rangle$ and the variables $\vec{y}_\pm \equiv (\vec{y}\pm\vec{y}')/\sqrt{2}$,

$$\lim_{|\vec{x}|\to\infty}\langle\frac{dQ_5(\vec{x})}{dt}\frac{dQ_5(0)}{dt}\rangle = \lim_{|\vec{x}|\to\infty}\int d^3y_+d^3y_-\frac{2(g^3)^2f(\vec{y}_-)}{(4\pi^2)^2|\vec{y}_+ + 2\vec{y}_- - \sqrt{2}\vec{x}||\vec{y}_+|}$$

$$
\begin{aligned}
&= \lim_{|\vec{x}|\to\infty} 4\pi(g^3)^2 \int_0^\infty dy_+ \int d^3y_- \frac{\exp(-y_+/L)\big(y_+ + |2\vec{y}_- - \sqrt{2}\vec{x}| - |y_+ - |2\vec{y}_- - \sqrt{2}\vec{x}||\big) f(\vec{y}_-)}{(4\pi^2)^2|2\vec{y}_- - \sqrt{2}\vec{x}|} \\
&= \lim_{|\vec{x}|\to\infty} L^2(g^3)^2 \int d^3y_- \frac{\big(1-\exp(-|2\vec{y}_- - \sqrt{2}\vec{x}|/L)\big) f(\vec{y}_-)}{2\pi^3|2\vec{y}_- - \sqrt{2}\vec{x}|} \\
&\simeq \frac{L(g^3)^2}{2\pi^3}\int d^3y_- f(\vec{y}_-) = \frac{L(g^3)^2}{2\pi^3}\int d^3y_- \langle\hat{\rho}_m(\sqrt{2}\vec{y}_-)\hat{\rho}_m(0)\rangle = \frac{L(g^3)^2}{4\sqrt{2}\pi^3}\langle Q_m\hat{\rho}_m(0)\rangle, \qquad (6)
\end{aligned}
$$

with the magnetic charge $\hat{Q}_m \equiv \int d^3x\hat{\rho}_m(\vec{x})$, where we have introduced a cut off L in the integration of y_+ and have taken a limit $L \to \infty$ before taking the limit $|\vec{x}| \to \infty$. Therefore, the vacuum expectation value $\langle dQ_5/dt\rangle$ is given by

$$
\langle v|\frac{dQ_5}{dt}|v\rangle = \pm\sqrt{\frac{L(g^3)^2}{4\pi^3\sqrt{2}}\langle v|\hat{Q}_m\hat{\rho}_m(0)|v\rangle}, \qquad (7)
$$

where we denote the vacuum as $|v\rangle$.

The equation (7) implies that $\langle v=0|dQ_5/dt|v=0\rangle = 0$ when the vacuum is an eigenstate of the magnetic charge $\hat{Q}_m$, i.e. $\hat{Q}_m|v\rangle = 0$. On the other hand, $\langle v|dQ_5/dt|v\rangle \neq 0$ when it is not eigenstate, $\hat{Q}_m|v\rangle \neq 0$ (the monopole condensed state $\langle v|\Phi|v\rangle = v \neq 0$.) Therefore, we find that the chiral asymmetric pair production takes place in the monopole condensed vacuum.

4. Conclusion

By analyzing the monopole quark interactions, we have shown that the chiralities are not conserved in the monopole quark scattering. Furthermore, the chiral asymmetric pair production of the massess quarks arises when a classical color charge is put in the vacuum with the monopole condensation. On the other hand, the chiral symmetric pair production arises in the vacuum with no monopole condensation. (We have recently explicitly shown [14] by using the effective monopole quark interactions $(\bar{u}u + \bar{d}d)|\Phi|^2$ that the chiral condensate $\langle\bar{q}q\rangle \neq 0$ arises only when the monopole condensation $\langle\Phi\rangle \neq 0$ takes place; q denotes light quarks u or d.) Therefore, we find that the chiral $U_5(1)$ (as well as flavor $SU_A(2)$) symmetry breaks simultaneously when the quark confinement takes place owing to the monopole condensation.

References

[1] F. Karsch, Lect. Notes Phys. 358 (2002) 209, see its references.
[2] H. Suganuma, S. Sasaki, and H. Toki, Nucl. Phys. B435 (1995) 207.
[3] O. Miyamura, Phys. Lett. B353 (1995) 91.
[4] R.M. Woloshyn, Phys. Rev. D51 (1995) 6411.
[5] Y. Hatta and K. Fukushima, Phys. Rev. D69 (2004) 097502.
[6] C. Gattringer, Phys. Rev. Lett. 97 (2006) 032003.
[7] C.B. Lang and M. Schrock, Phys. Rev. D84 (2011) 087704.
[8] S. Gongyo, T. Iritani and H. Suganuma, Phys. Rev. D86 (2012) 034510; T. Iritani and H. Suganuma, Prog. Theor. Exp. Phys. 2014 (2014) 033B03.
[9] Y. Nambu, Phys. Rev. D10 (1974) 4262.
[10] S. Mandelstam, Phys. Lett. B53 (1975) 476; Phys. Rep. 23 (1976) 245.
[11] G. 'tHooft, Nucl. Phys. B190 (1981) 455.
[12] A.S. Kronfeld, G. Schierholz, and U.-J. Wiese, Nucl. Phys. B293 (1987) 461.
[13] A. DiGiacomo and M. Hasegawa, Phys. Rev. D91 (2015) 054512.
[14] A. Iwazaki, arXiv:1901.03045.
[15] Z.F. Ezawa and A. Iwazaki, Phys. Rev. D25 (1982) 2681; Phys. Rev. D26 (1982) 631.

[16] T. Suzuki and I. Yotsuyanagi, Phys. Rev. D42 (1990) 4257.
[17] S. Coleman, "the magnetic monopole 50 years later" in The Unify of the Fundamental Interaction (1983), A. Zichichi, editor.
[18] Y. Kazama, C. N. Yang and A. S. Goldhaber, Phys. Rev. D15 (1977) 2287.
[19] A. Iwazaki, Phys. Rev. C80 (2009) 052202.