

DSR-deformed relativistic symmetries in an expanding spacetime

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Abstract. We present here a perspective on results obtained in collaboration with Amelino-Camelia, Marcianò and Matassa, reported in Ref. [1]. We characterize a relativistic theory of worldlines of particles with 3 nontrivial relativistic invariants: a large speed scale (“speed-of-light scale”), a large distance scale (inverse of the “expansion-rate scale”), and a large momentum scale (“Planck scale”). This is particularly relevant in relation to the opportunities for testing Planck-scale-deformed Lorentz symmetry scenarios with analyses, from a signal-propagation perspective, of observations of bursts of particles from cosmological distances. We address some of the challenges that had obstructed success for previous attempts by exploiting the recent understanding of the connection between deformed Lorentz symmetry and relativity of spacetime locality.

1. Introduction

Besides the conceptual motivations¹, the interest attracted over the last decade by the possibility of Planck-scale-deformed relativistic kinematics, as conceived within the proposal “DSR” (doubly-special, or, for some authors, deformed-special relativity) introduced in Refs. [2, 3], comes from the opportunities in phenomenology. Some of the novel effects that can be accommodated within a DSR-relativistic kinematics can be tested through observations of bursts of particles from cosmological distances [10, 11]. The key for these analyses are the implications of DSR deformations for the propagation of signals, and the sought Planck-scale sensitivity is reached thanks to the huge amplification afforded by the cosmological distances.

DSR-deformed relativistic frameworks have been investigated so far only for flat (Minkowskian) spacetimes, and this is not the case relevant for the analysis of signals received from sources at cosmological distances, for which the curvature/expansion of spacetime is very tangible. With our work [1], we provide a significant step toward filling this gap by exhibiting an explicit example of Planck-scale-deformed relativistic symmetries of a spacetime with constant rate of expansion (deSitterian). This led us to introduce the first ever example² of a relativistic theory of worldlines of particles with 3 nontrivial relativistic invariants: a large speed scale

¹ Properties for the Planck length introduced as observer-independent laws [2, 3]; it reflects the content of results rigorously established for 3D quantum gravity properties (see, *e.g.*, Refs. [4, 5, 6]) and for some 4D noncommutative spacetimes (see, *e.g.*, Refs. [7, 8]); it fits the indications emerging from some compelling semiheuristic arguments based on 4D Loop Quantum Gravity [4, 9].

² Studies such as those in Refs. [12, 13, 14] did contemplate the possibility of 3 invariants, but did not go as far as giving a consistent relativistic picture of worldlines of particles.

(“speed-of-light scale”), a large distance scale (inverse of the “expansion-rate scale”), and a large momentum scale (“Planck scale”).

There had been previous attempts of investigating the interplay between DSR-type deformation scales and spacetime expansion (see, *e.g.*, Refs. [13, 14]), but without ever producing a fully satisfactory picture of how the worldlines of particles should be formalized and interpreted. In retrospect we can now see that these previous difficulties were due to the fact that the notion of relative locality had not yet been understood, and without that notion the interplay between DSR-deformation scale and expansion-rate scale remains unintelligible.

Relative locality is the spacetime counterpart of the DSR-deformation scale ℓ just in the same sense that relative simultaneity is the spacetime counterpart of the special-relativistic scale c (scale of deformation of Galilean Relativity into Special Relativity). This was understood only very recently, in studies such as the ones in Refs. [15, 16, 17]. Awareness of the possibility of relative locality is already very important in making sense of the implications of DSR-deformations in a flat/non-expanding spacetime. And, as we showed in [1], it plays an even more crucial role in the consistency of the spacetime picture emerging from the interplay between DSR-deformation scale and expansion-rate scale.

We work at leading order in the DSR-deformation scale ℓ , and in a 2D spacetime (one time and one spatial dimension).

2. DSR-deformed de-Sitter-relativistic symmetries and physical velocity

The de Sitter relativistic symmetries can themselves be viewed as a deformation of the special-relativistic symmetries of Minkowski spacetime such that the expansion-rate parameter H is an invariant. For our analysis of particles worldlines within ℓ, H -deformed relativistic symmetries, we take as starting point the analysis in Refs. [15, 22]. We start by specifying that our ℓ, H -deformed relativistic symmetries shall leave invariant the following combination of the energy E , momentum p and boost \mathcal{N} charges of particles:

$$\mathcal{C}_{H,\alpha,\beta} = E^2 - p^2 + 2H\mathcal{N}p + \ell(\alpha E^3 + \beta Ep^2) . \quad (1)$$

Evidently for $\ell \rightarrow 0$ this reproduces the standard invariant of de Sitter symmetries.

The following ℓ -deformed (2D) de Sitter algebra of charges is compatible with the invariance of $\mathcal{C}_{H,\alpha,\beta}$

$$\begin{aligned} \{E, p\} &= Hp - \ell\alpha HEp , \\ \{\mathcal{N}, E\} &= p + H\mathcal{N} - \ell\alpha E(p + H\mathcal{N}) - \ell\beta Ep , \\ \{\mathcal{N}, p\} &= E + \frac{1}{2}\ell\alpha E^2 + \frac{1}{2}\ell\beta p^2 . \end{aligned} \quad (2)$$

One easily sees that for $\ell \rightarrow 0$ this reproduces the standard properties [25] of the classical de Sitter algebra of charges while for $H \rightarrow 0$ it reduces to a ℓ -deformation of the Poincaré algebra (see Ref. [1]).

We introduce spacetime coordinates η, x with Poisson bracket³

$$\{\eta, x\} = -\ell x , \quad (3)$$

³ This choice of coordinates is motivated by studies of quantum-spacetime pictures that can be analyzed in relation to such DSR scenarios. The most notable case is “ κ -Minkowski” (see, *e.g.*, Ref. [18, 19]) non-commutative spacetime, with $[\hat{x}, \hat{t}] = i\ell\hat{x}$. Some authors (see, *e.g.*, Ref. [21, 20]) have suggested that in cases in which the deformations of Lorentz symmetry is inspired by non-commutative spacetime scenarios such as the one of “ κ -Minkowski”, one should adopt spacetime coordinates with non-vanishing Poisson brackets $\{t, x\} = -\ell x$. It is worth noticing however, as shown in Refs. [22, 23, 1], that the results of analyses of travel times give the same results both assuming $\{t, x\} = -\ell x$ and assuming $\{\tilde{t}, \tilde{x}\} = 0$. This is a simple consequence of the fact that one can obtain coordinates $\{\tilde{t}, \tilde{x}\} = 0$, from our coordinates ($\{t, x\} = -\ell x$) by posing $\tilde{t} = t$, $\tilde{x} = x - \ell Ex$. (The point is [22, 23] that x and $x - \ell Ex$ coincide in the origin, so their difference can never affect the determination of when a particle reaches (or is emitted) from the origin of the spacetime coordinates of an observer.

and we define the ℓ, H -deformed phase space

$$\begin{aligned}\{E, \eta\} &= 1 - H\eta - \ell\alpha(1 - H\eta)E, \\ \{E, x\} &= -Hx - \ell(1 - \beta + H\eta)p + \ell\alpha HxE \\ \{p, \eta\} &= 0, \quad \{p, x\} = -1 - \ell \frac{E - Hxp}{1 - H\eta}.\end{aligned}\tag{4}$$

In these coordinates the charge associated to boost transformations (i.e. the boost transformations generator) can be represented as

$$\begin{aligned}\mathcal{N} &= -\eta p + xE + \frac{1}{2}H(\eta^2 - x^2)p \\ &+ \ell \left(\left(\frac{1}{2}\alpha - \frac{1}{1 - H\eta} \right) xE^2 + \left(\eta + \frac{Hx^2}{1 - H\eta} \right) Ep + \left(\frac{1}{2}\beta - H\eta \right) xp^2 \right)\end{aligned}\tag{5}$$

The set of Poisson brackets defined by (2-5) between $E, p, \mathcal{N}, \eta, x$ satisfy the Jacobi identities.

As done in Ref. [15, 22, 1], we derive the worldlines adopting a covariant formulation of classical relativistic mechanics. Evolution is coded in a pure-constraint Hamiltonian; specifically, the evolution in the auxiliary parameter on the worldline is governed by a Hamiltonian constraint: $\mathcal{C}_{H,\alpha,\beta} - m^2 = 0$. Of course the fact that $\mathcal{C}_{H,\alpha,\beta}$ is an invariant of the (deformed-)relativistic symmetries implies that the charges that generate the symmetry transformations are conserved over this evolution:

$$\dot{E} = \{\mathcal{C}_{H,\alpha,\beta}, E\} = 0, \quad \dot{p} = \{\mathcal{C}_{H,\alpha,\beta}, p\} = 0, \quad \dot{\mathcal{N}} = \{\mathcal{C}_{H,\alpha,\beta}, \mathcal{N}\} = 0.\tag{6}$$

One finds [15, 22, 1], for the worldlines of massless particles (specifying $p > 0$)

$$x_{m=0, p>0}(\eta) = x_0 + \eta - \eta_0.\tag{7}$$

By construction these worldlines are covariant under the (deformed-)relativistic transformations generated by the charges E, p, \mathcal{N} , as one can also easily verify explicitly.

As already established in previous studies of the case without spacetime expansion [15, 22] the DSR-deformed relativistic symmetries introduce (as most significant among many other novel features) a dependence on energy of the travel time of a massless particle from a given source to a given detector. At this stage of the analysis the physical content of these worldlines is still hidden behind the relativity of locality. A warning that this might be the case is seen in the fact that we are analyzing a case where the on-shell relation, in light of (1), is ℓ , and α/β , dependent, whereas our worldlines are independent of ℓ (and α/β). Most importantly the coordinate velocity one infers from those worldlines is independent of ℓ (and α/β). But one of the main known manifestations of relative locality is a mismatch [15, 22, 24] between coordinate velocity⁴ and physical velocity of particles.

For what concerns specifically the analysis so far reported, the main challenge resides in the fact that we are used to read velocities off the formulas for worldlines, but this implicitly

⁴ Examples of velocity artifacts were of course known well before the understanding of relative locality for theories with deformed Lorentz symmetry: for example in de Sitter spacetime (and any expanding spacetime) the coordinate velocity of a particle distant from the observer can be described by the observer as a velocity greater than the speed of light, even though in classical de Sitter spacetime the physical velocity measured by an observer close to the particle is of course always no greater than the speed of light. Since these previously known velocity artifacts are connected with spacetime curvature, the recent realization that some of our currently investigated theories formulating (one form or another of) “deformed Minkowski spacetime” are subject to relative locality, with some associated velocity artifacts [15, 22, 24], was largely unexpected.

assumes that translation transformations are trivial. Essentially we take the worldline written by a certain observer Alice to describe both the emission of a particle “at Alice” (in Alice’s origin) and the detection of the particle far away from Alice. The observer/detector Bob that actually detects the particle, since he is distant from Alice, should be properly described by acting with a corresponding translation on Alice’s worldline. And the determination of the “arrival time at Bob” (crucial for determining the physical velocity [22]) should be based on Bob’s description of the worldline, just as much as the “emission time at Alice” should be based on Alice’s description of the worldline. When translations are trivial (translation generators conjugate to the spacetime coordinates) we can go by without worrying about this more careful level of discussion, since the naive argument based solely on Alice’s worldline gives the same result as the more careful analysis using Alice’s worldline for the emission and Bob’s description of that same worldline for the detection. But when translations are nontrivial, and one has associated features of relativity of locality, this luxury is lost. One way to have “relative locality” is indeed the case here of interest, with the translation generators E, p acting on spacetime coordinates by the Poisson brackets (4).

Following Ref. [22], let us probe the difference between coordinate velocity and physical velocity considering the simultaneous emission “at Alice” of two massless particles, one “soft” (with momentum p_s small enough that ℓ -deformed terms in formulas fall below the experimental sensitivity available) and one “hard” (with momentum p_h big enough that at least the leading ℓ -deformed terms in formulas fall within the experimental sensitivity available). Alice describes the two particles according to

$$x_{p_s}^A(\eta^A) = \eta^A, \quad (8)$$

$$x_{p_h}^A(\eta^A) = \eta^A \quad (9)$$

where we specified $x_0^A = \eta_0^A = 0$, so that the emission is at $(0, 0)^A$. Since translations are a relativistic symmetry of our novel framework, we already know that the same two worldlines will be described by the distant observer Bob in the following way

$$x_{p_s}^B(\eta^B) = x_{0;s}^B + \eta^B - \eta_{0;s}^B, \quad (10)$$

$$x_{p_h}^B(\eta^B) = x_{0;h}^B + \eta^B - \eta_{0;h}^B, \quad (11)$$

i.e. the same type of worldlines but with a difference of parameters here codified in $x_{0;s}^B, \eta_{0;s}^B, x_{0;h}^B, \eta_{0;h}^B$. Indeed, Alice’s worldlines (8)-(9) and Bob’s worldlines (10)-(11) have exactly the same form, but for the ones of Alice we had by construction (by having specified simultaneous emission at Alice) that $x_{0;h}^A = \eta_{0;h}^A = x_{0;s}^A = \eta_{0;s}^A = 0$ whereas Bob’s values of the parameters, $x_{0;s}^B, \eta_{0;s}^B, x_{0;h}^B, \eta_{0;h}^B$, should be determined by establishing which (ℓ -deformed) translation transformation connects Alice to Bob.

This is the same task performed in some of the previous studies (such as Ref. [22]) involving relative locality in DSR-deformed relativistic theories without spacetime expansion. But here we have to deal with spacetime expansion. Let us then proceed determining the translation transformation that connects Alice to Bob. As usual we shall give the action on phase space coordinates $k \equiv (E, p, \eta, x)$ of a worldline by a finite transformation $T_{G;a}$ generated by the generic element G of the relativistic-symmetry algebra in terms of its exponential representation as

$$T_{G;a} \triangleright k = e^{-aG} \triangleright k \equiv \sum_{n=0}^{\infty} \frac{(-a)^n}{n!} \{G, k\}_n, \quad (12)$$

where a is the transformation parameter and $\{G, k\}_n$ is the n -nested Poisson bracket defined by the relation

$$\{G, k\}_n = \{G, \{G, k\}_{n-1}\}, \quad \{G, k\}_0 = k. \quad (13)$$

Before proceeding, we must also implement something else that can be specified about observer Bob. The worldline parameters $x_{0;h}^A, \eta_{0;h}^A, x_{0;s}^A, \eta_{0;s}^A$ of observer Alice are fully specified ($x_{0;h}^A = \eta_{0;h}^A = x_{0;s}^A = \eta_{0;s}^A = 0$) by its being at the point of simultaneous emission of the two particles. We have introduced observer Bob as one that detects the particles (the particles worldlines should cross Bob spatial origin) but we have so far left completely unspecified its worldline parameters $x_{0;s}^B, \eta_{0;s}^B, x_{0;h}^B, \eta_{0;h}^B$. We assume, without loss of generality, that the soft particle reaches Bob in his spacetime origin so that $x_{0;s}^B = \eta_{0;s}^B = 0$. This is particularly convenient because it involves the “soft particle”, *i.e.* the one whose momentum p_s has been chosen to be small enough to render the ℓ -deformed effects inappreciable within the experimental sensitivities available to Alice and Bob.

The fact that both $x_{0;s}^A = \eta_{0;s}^A = 0$ and $x_{0;s}^B = \eta_{0;s}^B = 0$ for a soft particle leads us to focus on the case of the observer Bob connected to Alice by the following transformation

$$X^B = (e^{a_x p} \triangleright e^{-a_\eta E} \triangleright X)^A, \quad (14)$$

with

$$a_x = \frac{1 - e^{-Ha_\eta}}{H}. \quad (15)$$

Through this we are essentially exploiting the fact that the deformation is ineffective on the soft particle as a way for us to focus on a distant observer Bob whose relationship to Alice (translation parameters connecting Alice to Bob) can be specified using only known results on the undeformed/standard relativistic properties⁵. Indeed in classical (relativistically undeformed) de Sitter spacetime one easily finds that a massless particle emitted in the origin of some observer Alice will cross the origin of all observers connected to Alice by a spatial translation of parameter a_x followed by a conformal-time translation of parameter a_η with the request that $a_x = H^{-1}[1 - e^{-Ha_\eta}]$.

Using our translation generators (4) one finds that for points on the worldline of the hard particle the map from Alice to Bob is such that

$$\begin{aligned} \eta_h^B &= \frac{1 - e^{Ha_\eta}}{H} + e^{Ha_\eta} \eta_h^A + \ell \alpha a_\eta e^{Ha_\eta} (1 - H\eta_h^A) (E_h^A - Ha_x p_h^A), \\ x_h^B &= e^{Ha_\eta} (x_h^A - a_x) - \ell \beta \frac{\sinh(Ha_\eta)}{H} p_h^A - \ell \alpha a_\eta e^{Ha_\eta} H (x_h^A - a_x) (E_h^A - Ha_x p_h^A) \\ &\quad + \ell \frac{(1 - e^{-Ha_\eta}) (1 + e^{Ha_\eta} H\eta_h^A)}{H} p_h^A - \ell (a_x - (1 - e^{Ha_\eta}) x_h^A) \frac{E_h^A - Hx_h^A p_h^A}{1 - H\eta_h^A}, \\ p_h^B &= e^{-Ha_\eta} (p_h^A + \ell \alpha H a_\eta p_h^A (E_h^A - Ha_x p_h^A)). \end{aligned}$$

Crucial for us is the fact that, in light of this result for the laws of transformation from Alice’s η_h^A, x_h^A, p_h^A to Bob’s η_h^B, x_h^B, p_h^B , we can deduce that the worldline of the hard particle emitted at Alice is described by Bob as follows

$$x_{p_h}^B(\eta^B) = \eta^B - \ell |p_h^B| \left(\alpha a_\eta + \beta \frac{e^{2Ha_\eta} - 1}{2H} \right), \quad (16)$$

where we made use of all the specifications discussed above, including $a_x = \frac{1 - e^{-Ha_\eta}}{H}$.

In turn this allows us to obtain the sought result for the dependence on energy(/momentum) of the travel times of massless particles: by construction of the worldlines and of the Alice→Bob

⁵ This also implicitly requires [22] that the clocks at Alice and Bob are synchronized by exchanging soft massless particles.

transformation the soft massless particle emitted in Alice's spacetime origin reaches Bob's spacetime origin, whereas from (16) we see that the hard massless particle also emitted in Alice's spacetime origin reaches Bob at a nonzero conformal time. Specifically the difference in conformal travel times derivable from (16) is

$$\Delta\eta^B = \eta^B \Big|_{x_{ph}^B=0} = \ell|p^B| \left(\alpha a_\eta + \beta \frac{e^{2H a_\eta} - 1}{2H} \right). \quad (17)$$

We summarize the relativistic properties of this travel-time analysis, in conformal coordinates, in Fig. 1.

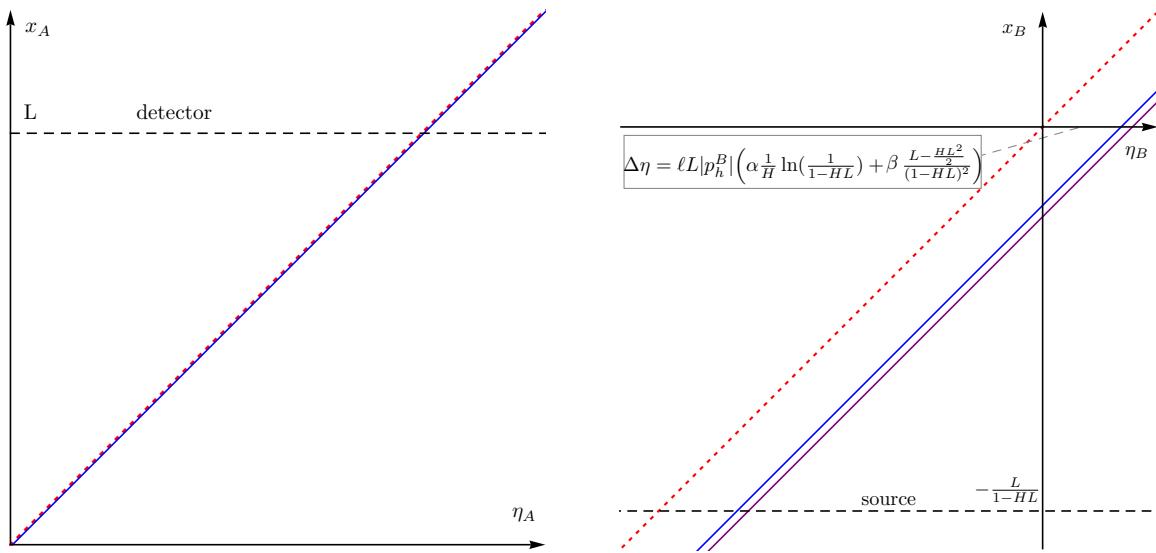


Figure 1. We illustrate the results for travel times of massless particles derived in this section, adopting conformal coordinates. We consider the case of two distant observers, Alice and Bob, connected by a pure translation, and two massless particles, one soft (dashed red) and one hard (solid blue and solid violet), emitted simultaneously at Alice. And we consider two combinations of values of α and β producing the same $\alpha + \beta$: the case $\alpha = 2/3, \beta = 2/3$ (Bob's hard worldline in blue) and the case $\alpha = 1/3, \beta = 1$ (Bob's hard worldline in violet). We have here that in Bob's coordinatization (right panel) the emission of the particles at Alice appears not to be simultaneous. Similarly for the difference in times of arrival at the distant detector Bob Alice finds in her coordinatization (left panel) that the photons arrive at the detector simultaneously. Comparison of the blue and violet worldlines shows that in the case with spacetime expansion the travel time does depend individually on α and β (not just on $\alpha + \beta$ as in the case without spacetime expansion). For visibility we assumed here unrealistic values for the scales involved.

Differently to the non expanding case [1], with spacetime expansion, there are tangible differences between $E p^2$ deformations and E^3 deformations.

3. Implications for phenomenology

We close this manuscript mentioning some results we stumbled upon during our work [1], which are of rather sizable significance for phenomenology. A more detailed discussion can be found in [1]. The results we derived in the previous section are easily reformulated as the following prediction for the differences in detection times of photons of different energies(/momenta)

emitted simultaneously by a source at redshift z

$$\Delta t = \ell|p| \left(\alpha \frac{\ln(1+z)}{H} + \beta \frac{z + \frac{z^2}{2}}{H} \right). \quad (18)$$

Here again Δt is the difference in detection times⁶ between a hard gamma-ray photon of momentum p and a reference ultrasoft photon emitted simultaneously to the hard photon at the distant source.

A first aspect of phenomenological relevance which must be noticed in our result (18) is the dependence on the parameters α and β , *i.e.* the difference between $E p^2$ deformations and E^3 deformations. Related to this observation is the comparison with analogous studies of scenarios where relativistic symmetries are actually “broken” (allowing for a preferred/“aether” frame), rather than DSR-deformed. Of course, there is a crucial difference between the properties of ℓ and the properties of λ_{LIV} , the characteristic scale of Lorentz symmetry breaking: ℓ , as characteristic scale of a deformed-symmetry picture, takes the same value for all observers, while λ_{LIV} , as characteristic scale of a broken-symmetry picture, takes a certain value in the preferred frame and different values in frames boosted with respect to the preferred frame. Setting momentarily these differences aside we can compare our results for DSR-deformed symmetries with constant rate of expansion, with the special case of the broken-symmetry formula, on which a rather universal consensus has been achieved throughout several years of investigations [27, 28, 29, 29, 30, 33], obtained for constant rate of expansion (more details are given in [1]).

We find that fixing $\alpha = 0$ in (18), one gets a formula (valid in all reference frames) which is the same as the formula of the broken-symmetry case in the preferred frame. So if $\alpha = 0$ the differences between deformed-symmetry and broken-symmetry cases would be tangible only by comparing studies of travel times of massless particles between two telescopes with a relative boost: the difference there would be indeed that ℓ takes the same value for studies conducted by the two telescopes whereas for λ_{LIV} the two telescopes should give different values.

We must stress however that within our deformed-symmetry analysis we found no reason to focus specifically on the choice $\alpha = 0$. And if $\alpha \neq 0$ in the deformed symmetry case even studies conducted by a single telescope could distinguish between the case of symmetry deformation and the case of symmetry breakdown.

In Fig. 2 we compare the dependence on redshift of our deformed-symmetry effect among two limiting cases of balance between α and β , and we also compare these results to bounds on travel-time anomalies [31, 32, 33] obtained in studies of sources at redshift smaller than 1 (where the assumption of a constant rate of expansion is not completely misleading).

In Fig. 3 we illustrate the “constraining power” of our results and of foreseeable generalizations of our analysis gaining access to cases with non-constant rate of expansion. Analyzing data from sources at redshift of $\simeq 1$ assuming a picture with constant rate of expansion cannot produce conservative experimental bounds, but the content of the right panel of Fig. 3 serves the purpose of providing evidence of the fact that full Planck-scale sensitivity will be within reach of improved versions of our analysis, extending our results to the case of expansion at non-constant rate.

⁶ Notice that with Δt we denote the comoving time but in the relevant timing sequences of detections at telescopes the difference between comoving and conformal time is intangible (see [1]).

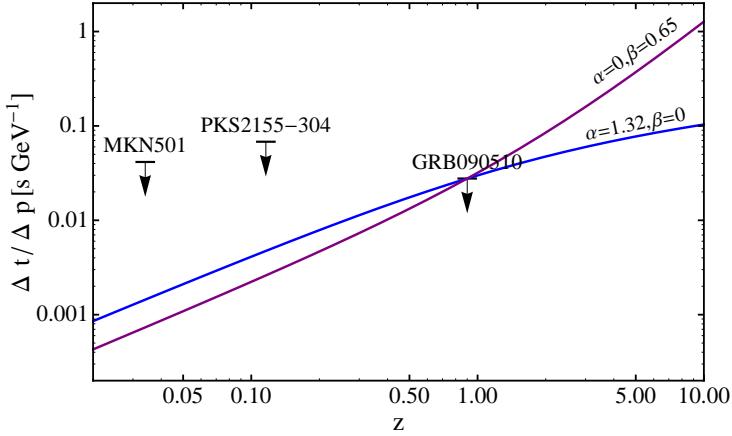


Figure 2. Here we show the dependence on redshift of the expected time-of-arrival difference divided by the difference of energy of the two massless particles. Two such functions are shown, one for the case $\alpha = 0, \beta = 0.65$ (violet) and one for the case $\alpha = 1.3, \beta = 0$ (blue). We also show the upper limits that can be derived from data reported in Refs. [31, 32, 33], setting momentarily aside the fact that our analysis adopted the simplifying assumption of a constant rate of expansion (whereas a rigorous analysis of the data reported in Refs. [31, 32, 33] should take into account the non-constancy of the expansion rate). The values $\alpha = 0, \beta = 0.65$ and $\alpha = 1.3, \beta = 0$ have been chosen so that we have consistency with the tightest upper bound, the one established in Ref. [33]. The main message is coded in the fact that at small values of redshift the blue and the violet lines are rather close, but at large values of redshift they are significantly different (this is a log-log plot). In turn this implies that at high redshift the difference between adding correction terms of form Ep^2 and adding correction terms of form E^3 can be very tangible.

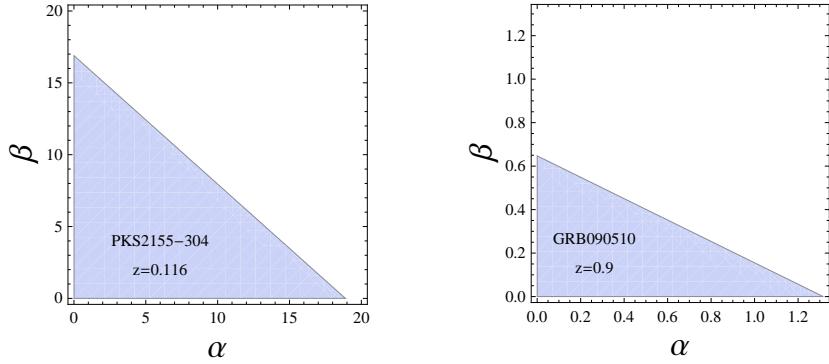


Figure 3. Here in the left panel we show the constraint on the α, β parameter space that can be obtained from Ref. [32], concerning a source at the relatively small redshift of $z = 0.116$, where we can confidently apply our results for constant rate of expansion as a reliable first approximation. Through this we show that even within the confines of our analysis Planck-scale sensitivity (values of $|\alpha|$ and $|\beta|$ smaller or comparable to 1) is not far. In the right panel we show the much tighter (indeed “Planckian”) constraint on the α, β parameter space which would be within our reach if we could assume our analysis to apply also to redshifts close to 1, as for GRB090510 observed by the Fermi telescope [33]. By comparing the left and the right panel one also finds additional evidence of how the difference between adding correction terms of form Ep^2 and adding correction terms of form E^3 becomes more significant at higher redshifts: in the left panel (data on source at small redshift of $z = 0.116$) the bound on the α parameter is nearly as strong as on the β parameter, whereas in the right panel (data on source at redshift of $z \simeq 0.9$) the constraint on the alpha parameter is significantly weaker than the constraint on the β parameter.

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