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Proceeding Paper

The Future of Experimental Muon Physics

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MDPI

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The Future of Experimental Muon Physics †

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Abstract: In this talk, I discuss a possible future for the global muon physics program. I focus on the future of flavor studies, precision measurements and searches that can be pursued with a new class of muonium beam sources, and emerging practical applications of muons in the industrial, academic, and government sectors.

Keywords: muons; flavor physics; charged lepton flavor violation; muonium; anti-matter gravity; muon catalyzed fusion; muon tomography

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1. Introduction

The future of experimental muon physics is indeed very bright. There are so many experiments recently completed, ongoing, or proposed for the future that I only have space to discuss a few of the highlights. Of course, any overview is heavily influenced by the experiences and prejudices of the author, and this one will be no exception. In the pages I have available here, I intend to discuss a few potential projects exploring the physics of flavor; precision measurements of muonium spectroscopy, flavor violation, and anti-matter gravity; and a few potential practical applications of muon physics. While I discuss one possible future for our field, for a detailed overview of the current state of the field, I refer the interested reader to Angela Papa's excellent contribution to this conference.

2. Flavor

What distinguishes an electron from a muon from a tau? In the standard model of particle physics with massless neutrinoes, the identity or *flavor* of the charged leptons is distinguised only by their mass; they are otherwise identical. This version of the Standard Model has an explicit symmetry that protects, or conserves, individual lepton flavors in all interactions. That is, we assign the quantum numbers "electron number", "muon number", etc to each particle, and in any given interaction, this number can not change from early to late times. The confirmation of neutrino mass via observation of neutrino oscillations indicates that nature does not respect these flavor symmetries, a symmetry failure known as lepton flavor violation. Despite this, *Charged Lepton Flavor Violation* (CLFV)—for example, the $\mu \to e\gamma$, or MEG, decay of a muon into an electron and a photon—has never been observed, and very strict experimental upper limits exist on this and many other branching ratios. Despite these flavor symmetry violations, the most straightforward modifications of the Standard Model to describe neutrino oscillations still predict an overall lepton number preserving symmetry that so far has held in all interactions; non-minimal extensions may predict lepton number violation (LNV), but there are very severe experimental constraints on such extensions.

With a minimal extension of the Standard Model including neutrino oscillations, we can calculate the branching ratio for the CLFV $\mu \to e\gamma$ process [1]:

$$\mathrm{Br}(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_{k=2,3} U^*_{\mu k} U_{ek} \frac{\Delta m^2_{1k}}{M^2_W}\right|^2 < 10^{-54}\,. \quad (1)$$

Similarly tiny branching ratios are predicted for any other CLFV process. While non-zero, these are so small that observation is inconceivable; therefore, any confirmed observation must be the result of new physics! This is certainly not a new insight: searches in numerous muon and tau-based CLFV observables have been pursued—all with negative observation—since the 1947 experience of Pontecorvo and Hincks ruled out MEG at the 10% level [2]. Exclusion bounds have improved by nearly fourteen orders of magnitude since, with five or six additional orders of sensitivity conceivable in current or future projects; see Figure 1.

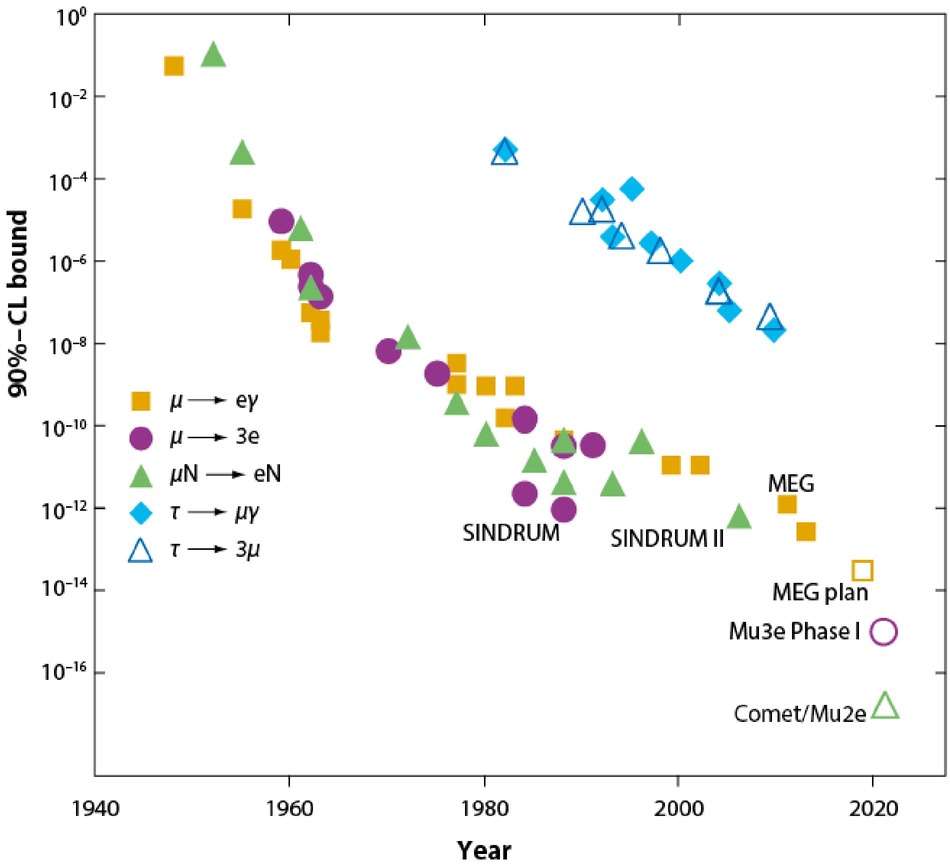

Figure 1. The historical progression of CLFV measurements in the muon and tau sectors over the previous 70 years. Sensitivities have improved by seventeen orders of magnitude, with an addition three orders achievable with plausible technical advancements in the coming years. Image produced by the Mu2e Collaboration.

Why this continued interest despite null results for over seven decades? We know that the Standard Model is incomplete, and attempts to fix it generically introduce flavor violations that must be constrained to avoid experimental bounds. Due to their favorable experimental properties, muons are significantly more sensitive tools to search for flavor violation sources than other particles. Furthermore, in many channels, we know how to build significantly more sensitive experiments in the future than we do today. Some of these experiments can only be completed with negative muons, while the rest are most sensitively completed with positive muons from surface beams. Of the surface muon beam experiments, the leading contenders for significant improvement are $\mu^+ \rightarrow e^+\gamma$, $\mu^+ \rightarrow e^+e^+e^-$, and $\mu^+e^- \rightarrow \mu^-e^+$. MEG-II [3] is currently searching for the first channel, while the Mu3e experiment is under construction to improve limits on the second; see contributions by Palo and Perrevoort at this conference. The third channel is very interesting as it would be realized as the oscillation of muonium—the lightest leptonic atom—into antimuonium and represents CLFV by two units—double CLFV. Performing this experiment requires the development of high-intensity muonium sources, with R&D going on at PSI (Paul Scherrer Institut) and Fermilab. Depending on the choice of stopping target material, the surface muon experiments can enhance or suppress the fraction of muonium in the observation, but the lifetime of the muon is not altered compared with the free muon lifetime by more than a few parts in 10^{10} [4]. For the negative muon experiments, this is not the case; negative muons participate as heavy electrons in chemical processes, forming "muonic atoms" whose lifetimes decrease rapidly with increasing atomic number due to capture of the muon on the nucleus. Despite this, the coherent, neutrinoless conversion channel $\mu^- A(Z, N) \to e^- A(Z, N)$ remains the most sensitive CLFV search channel due to the clean kinematics: the electron in the final state is monochromatic, with an energy just slightly less than the muon mass. This is well above the peak energy of the bulk of the electrons from normal decay in orbit, at half the muon mass. The coherent, neutrinoless conversion channel is currently being pursued by the operating DeeMee [5] and two experiments under construction: COMET [6] (Coherent Muon to Electron Transition) at J-PARC (Japan Proton Accelerator Research Complex), and Mu2e [7] at Fermilab. Mu2e (but not COMET, given their detector design choices) will also be able to search for the kinematically similar $\mu^- A(Z, N) \to e^+ A(Z-2, N)$ reaction, which is both CLFV and LNV. Finally, the possibility exists to observe $\mu^- e^- \to e^- e^-$ interactions of the bound muon with the atomic electrons, which are Coulomb enhanced in high-Z muonic atoms, although with less favorable kinematics [8,9].

COMET and Mu2e anticipate similar single event sensitivities of a few times 10^{-17}, a four order of magnitude improvement on the best limits from SINDRUM-II [10,11]. Both achieve their similar performance with designs inspired by the MELC (the Moscow Muon to Electron Conversion experiment) effort of the 1990s [12]. I will use Mu2e as an illustration of the concept; see Figure 2. A pulsed proton beam is delivered to a target mounted inside a high-field, superconducting Production Solenoid (PS). Backward muons (direction chosen to minimize forward collision debris) are collected into a curved Transport Solenoid (TS) channel designed to both sign-separate particles and eliminate line of sight between the production target and detectors. Muons that survive the TS enter a Detector Solenoid (DS) that houses a stopping target and detector systems optimized to observe the conversion electrons at 105 MeV while minimizing the number of accepted normal decay electrons.

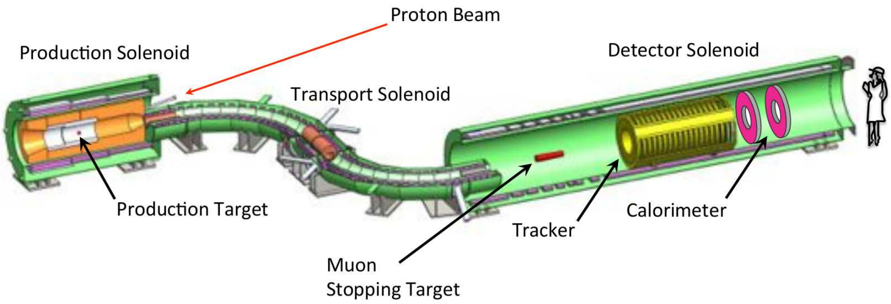

Figure 2. The Mu2e apparatus, one of two under-construction realizations of the MELC experimental concept for muon conversion searches. The full experiment is split into a production section housing the production target, a transport section that sign selects the muons of interest, and a detector section that includes a muon stopping target and detectors optimized to observe the conversion electron signal. More details can be found in the text. Image produced by the Mu2e Collaboration.

Improvements over the COMET and Mu2e sensitivities are possible with conceptually modest improvements to this basic design. In particular, the Mu2e-II collaboration [13] proposes an evolution of the Mu2e experiment that will enable an additional order of magnitude sensitivity improvement during the 2030s. While Mu2e will utilize a slow-extracted, 8 kW, 8 GeV pulsed proton beam, the key enabler for Mu2e-II will be a much higher-power 100 kW, 800 MeV proton beam delivered directly from the new PIP-II (Proton Improvement Plan—II) linear accelerator currently under construction at Fermilab. Mu2e-II will reuse as many components of the Mu2e complex as possible but will require substantial upgrades in a few areas:

- A new, actively cooled production target designed to survive the 30 kW of deposited thermal power and the significant associated radiation damage.
- Upgraded detector systems with improved energy and momentum resolution.
- A data acquisition and storage system able to handle higher peak and sustained transfer and storage rates.

Whether or not Mu2e sees a signal, Mu2e-II will provide a valuable contribution to the global muon physics program. If Mu2e does not see a signal, the improved sensitivity will enable a more extensive search of the parameter space. If Mu2e does see a signal, however, Mu2e-II will be able to refine the measurement of rates and also study the relative contributions of various CLFV operators by utilizing different stopping target materials. Together with limits or signals measured by, for example, MEG-II or Mu3e, the Mu2e-II results will make a valuable contribution to piecing together the details of flavor physics in the muon sector.

Mu2e-II will likely be the ultimate version of the MELC concept, and further improvements on sensitivity for conversion-like effects will require a new approach. There are two main constraints on further improvements: proton beam pulse lengths and the associated beam backgrounds. The key prompt background is electrons at the conversion energy caused by radiative pion capture in the stopping target. In the MELC-based design, the pions arrive at roughly the same time and via the same transport mechanism as the muons. To reduce this background, it requires vetoing the signal until the pion background has been reduced by decay to less than one expected event over the lifetime of the experiment. As sensitivity increases, this veto duration grows, reducing the effective muon rate due to the longer veto window length. Additionally, when trying to utilize higher-Z stopping target materials, the duration of the muon pulse arriving at the target (which is always longer than the proton pulse length) must be less than the lifetime of the muonic atom. The MELC design can only effectively scale to target materials around Titanium, leaving heavier targets like gold or lead out of reach. Extension beyond Mu2e-II requires a new concept.

Eliminating pion backgrounds in the experiment just requires a sufficiently long transport channel to give the pions time to decay. This can be most compactly achieved with a storage ring; this is already completed, for instance, at the Fermilab Muon $g-2$ experiment, where the complex's Delivery Ring is used to eliminate proton and pion contamination in the muon beam. A future conversion effort would most likely use the PRISM concept [14] to not only eliminate pions but also monochromate the muons. PRISM (Phase Rotated Intense Source of Muons) is a Fixed Field, Alternating Gradient (FFA) storage ring with a wide momentum acceptance. During injection, a muon pulse with a narrow time spread (produced from an even narrower proton pulse) but a wide momentum spread is rotated in phase space; low energy muons are accelerated while high-energy muons are decelerated. At extraction time, the circulating bunch has a very narrow energy spread and zero pion contamination. This narrow energy spread would allow the use of a single-layer, thin stopping target, reducing brehmsstrahlung, multiple scattering, and energy straggling in the target, leading to better resolution and lower systematics in the detector system. An FFA similar in concept to what is required for a conversion experiment has been demonstrated at Osaka [15].

A concept for a facility leveraging the PRISM scheme is being advanced for the Fermilab site following the completion of Mu2e-II. Dubbed the *Advanced Muon Facility*—or AMF—the facility would be able to provide a long-lived facility for the needs of a wide range of muon physics users. It should be able to:

- utilize the available proton beam enabled by PIP-II that will not be used by the LBNF/DUNE (Long Baseline Neutrino Facility/Deep Underground Neutrino Experiment) program—1 MW or more at 800 MeV,

- provide a flexible facility for future experiments after the current muon program has run its course, and
- build on synergies with the dark matter and muon collider communities.

An early cartoon of the AMF concept is shown in Figure 3. The key enabling technologies:

- PIP-II: will provide up to 1 MW of proton beam power at 800 MeV.
- Proton compressor ring: (not shown) It will be necessary to convert the CW beam from PIP-II into very short, highly intense proton pulses. A compressor ring will be needed to store and manipulate beam, and the design will share many similarities with the needs of the accelerator dark matter community.
- Production Solenoid and target systems: As in COMET and Mu2e, a target in the solenoid will be required to efficiently collect muons produced by the high-power target system. The target system is the key technical challenge for AMF. An undemonstrated concept for a 100 kW target exists for Mu2e-II, but a compact megawatt scale target is still a true R&D effort, with significant synergies with the muon collider. Discussions between these communities on how to support each other are ongoing.
- Muon Transport: A muon transport system is required to eliminate line-of-sight from the target to the experiment, and to match the beam dynamics between the Production Solenoid and the FFA.
- FFA Ring: The key enabling technology for AMF.
- Experiments: Conceptually, AMF can provide muon beams of both signs, suitable for multiple experiments run either serially or in parallel.

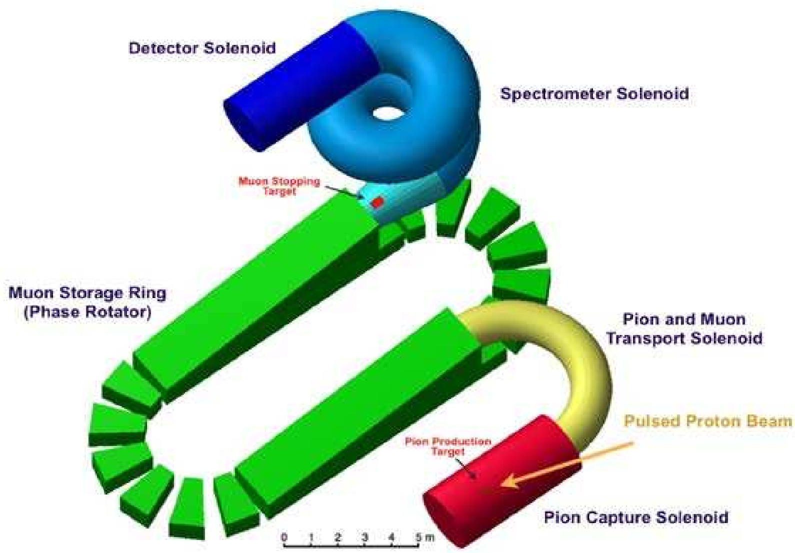

Figure 3. A cartoon of the components of the Advanced Muon Facility proposed for Fermilab. The system would require a series of new components—a proton compressor ring (not shown), a new production solenoid and target system, a muon transport line, and a Fixed Field Alternating (FFA) Gradient storage ring. Additional effort will be required to build and operate experiments that could be attached to the output of the FFA; shown here is a conversion search experiment. Image produced by the AMF Collaboration.

The primary motivation for AMF is CLFV physics, potentially enabling both improved surface muon decay experiments ($\mu \to 3e$, and $\mu \to e\gamma$)—improving by a factor of 100 over the current generation of PSI experiments—and conversion searches with a factor of 100–1000 over Mu2e and COMET, with the unique capability to exploit high-Z stopping targets. An extremely intense source could enable other possibilities on the Fermilab site, including a robust muonium physics program, a new muon MDM/EDM (Magnetic and Electric Dipole Moment) source, support for MuSR beamlines for industrial uses, and perhaps even intense pion and kaon sources. With appropriate resourcing, AMF could even be designed to feed multiple experiments simultaneously. A staged approach to reaching an ultimate power of 1 MW or more is likely, with the ability to utilize high-Z targets in conversion searches even at 100 kW adding a valuable capability with significant physics impact.

3. Muonium Physics

Muonium (or Mu) is the bound state of an antimuon and an electron—μ^+e^-. Muonium is nearly the simplest atomic species—a purely leptonic hydrogen isotope like positronium, but with a "heavy" nucleus. Muonium has a strong advantage as an experimental probe—a lifetime of 2 µs compared with the 100 ns lifetime of positronium. Muonium has a rich structure and phenomenology

- It is readily formed, with the muonium fraction dependent on the microstructural properties of the material.
- With minimal hadronic physics contamination compared with even hydrogen, the spectrum is extremely well calculated and understood.
- It participates in chemistry and forms molecules.
- It decays with the free muon lifetime.

High-quality spectroscopic measurements combined with high-order QED calculations allow the extraction of Standard Model parameters with unparalleled precision. For instance, the $1s \leftrightarrow 2s$ transition frequency, $\Delta\nu_{1s-2s}$, is predicted in QED to 0.6 ppb due to minimal hadronic contributions; there is a similar story for the $1s$ hyperfine splitting, $\Delta\nu_{\mathrm{HFS},n=1}$. Recent advances in experimental technique and apparatus will enable order-of-magnitude improvements in the corresponding experimental measurements of these parameters. MuSEUM (Muonium Spectroscope Experiment Using Microwaves) at J-PARC [16] will improve the measurement of the $1s$ hyperfine transition by an order of magnitude, to 1 ppb, improving the extraction of the Bohr magneton to a similar level. The Mu-MASS (Muonium lASer Spectroscopy experiment) at PSI [17] will make a dramatic improvement of three orders of magnitude on the $1s \leftrightarrow 2s$ transition frequency, improving the muon mass determination to 1 ppb.

As I mentioned before, muonium forms readily in many materials, but extracting that muonium for experiments can be a challenge. Future improvements in muonium-based measurements will require the availability of slow muonium beams in a vacuum. Forming these muonium beams requires nearly stopping a positive muon beam in a target that is mounted in a vacuum; the goal is to have some of the formed muonium ejected into the vacuum space with suitable properties. The J-PARC Muon $g-2$ experiment (see the talks by Nakazawa and Zhang in this conference) will generate a cold, slow muon beam for injection into their experiment from laser-ionized muonium; muonium formation will use a vacuum-mounted, laser-ablated silica aerogel target. This should yield muonium emerging into the vacuum at about 1% of the incoming muon rate, with a thermal distribution centered around 7 km/s.

There are both PSI and Fermilab-based efforts to develop a novel, next-generation superfluid helium muonium production target first envisioned by Taqqu [18]. Hydrogenic species (including Mu) are immiscible in superfluid helium. If a thin layer of superfluid is maintained on the inside wall of a vacuum vessel, muonium stopped within the liquid will be ejected normal to the surface with an extremely narrow (non-thermal) energy spread determined by the chemical potential of the system. The ejected muonium beam will then be naturally cooled, with a mean velocity of 6.3 km/s normal to the surface. The same chemical properties can be used as a mirror for slow muonium beams [19].

Cold muonium beams could be used in multiple rare event and precision measurement efforts, two of which I will discuss here: CLFV searches in $\mathrm{Mu}-\overline{\mathrm{Mu}}$ oscillations and antimatter gravity measurements. I mentioned the doubly CLFV $\mathrm{Mu}-\overline{\mathrm{Mu}}$ oscillation in Section 2. The current branching limit of 8.3×10^{-11} on this process comes from the MACS (Muonium-Antimuonium Conversion Spectrometer) effort at PSI [20]. When the anti-muon in the Mu decays, the decay positron is emitted with a Michel spectrum—hence fast—

while the atomic electron is ejected with only the binding energy—hence slow. If the Mu oscillates to $\overline{\text{Mu}}$, a fast Michel electron will be emitted in the decay, in coincidence with a slow positron. This is the signal that can be searched for. There is a group actively seeking R&D support to mount a new effort at Fermilab with increased sensitivity.

Anti-matter interactions with gravity are another area ripe for exploitation of a cold muonium source. Of course, while there are no theoretical expectations that matter and anti-matter will interact differently with gravity, it is also true that the effect of gravity on anti-matter has never been directly measured. We do not even have a reliable measurement of the *sign* for g for anti-matter. Long-lived muonium provides a perfect laboratory for pursuing such a measurement [21,22]. Both Leming at PSI [23] and a new effort at Fermilab aim to address this question using similar approaches. Using the cold muonium beam from a superfluid-based source, both efforts will send a horizontal beam of muons through a three-grating Mach-Zehnder interferometer with 100 nm scale grating line spacing. The gratings will be separated by approximately one muon lifetime, with the third able to scan vertically to measure the vertical displacement of the muonium beam through the grating. Muonium decay will trigger the detectors downstream of the third grating, measuring the vertical interference pattern as it is shifted by gravitational interaction and grating displacement. With a 100 kHz muonium source rate, the sign of g could be measured to 5σ in one day, with a precision measurement of the g value in about a month of beam operations.

4. Muon Catalyzed Fusion

There are a number of practical applications of muon physics that will only become more important in the future; for instance, cosmic ray muon tomography has been used for geological, archaeological, structural, and homeland security remote sensing applications. Muon-catalyzed fusion was a formerly extremely active field of investigation that is seeing a small-scale renaissance in recent times [24–26]. Traditional thermonuclear fusion efforts focus on confining very high-temperature (millions of degrees) deuterium-tritium plasma—either via magnetic or inertial means—to overcome the coulomb barrier between the nuclei. Muon-catalyzed fusion, on the other hand, is a low-temperature (nearly room temperature) process with a cross section nearly a million times larger than the peak thermonuclear cross section.

Negative muons are injected into a "container" with a mixture of deuterium and tritium fuel, where the muon is atomically captured and transferred rapidly to a tritium atom. The muonic tritium forms a molecule with a deuterium atom, a $dt\mu$. Because of the large mass of the muon compared with the electron, these muonic molecules are much smaller than the corresponding electronic molecules, and the nuclear wave-functions of the two nuclei overlap to a sufficient extent that fusion occurs rapidly. The fusion products are an alpha, a neutron, 14 MeV of released energy, and the original spectator muon, which can go on to catalyze additional fusions. This cycling rate is a key parameter that controls scientific and economic breakeven. In about 1 fusion in a hundred, the muon "sticks" to the alpha particle, becoming unavailable for further reactions. Collisional or excitational processes (such as microwave or optical pumping) can release, or "reactivate", the muons before they reach the muonic helium ground state, freeing them to reenter the catalytic chain.

The key parameters that control breakeven then are the muon production energy budget, the cycling rate, and the final sticking fraction. Production costs have fallen significantly since the 1980s, when there was widespread active research in this field. Cycling rates are known to rise with increasing temperature and density. Most intriguingly, there is a long-standing discrepancy between theory and data on the rate with which sticking fractions fall with increasing density, with the data showing more favorable trends. Given the exponential impact on fusion gain from linear changes in the sticking fraction, even a small improvement could make muon-catalyzed fusion a viable commercial alternative to thermonuclear fusion. To resolve the data-theory discrepancy, an industry-academic collaboration has been funded in the US by the ARPA-E BETHE (Breakthroughs

Enabling Thermonuclear-fusion Energy) Novel Fusion Concepts program to study the trends in cycling rate and sticking fraction in diamond anvil cells at higher temperatures and densities than previously explored. Studies are being carried out on both DD and DT fusion channels at both PSI and Fermilab. The work at Fermilab will result in a new low-energy muon beamline that will be available for other muon physics work, including the development of the muonium experiments discussed above.

5. Summary

As I said at the start, the future of muon physics is bright! From flavor physics to precision measurements to practical applications, I have only had time and space here to scratch the surface of what is possible. Left out are major efforts such as the new Muon $g-2$ experiment at J-PARC, renewed efforts on the muon collider front, and EDM measurements proposed at Fermilab. What time to be a muon physicist!

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