Error analysis of statistical model for fission

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Predictive power of a model is tested by experimental measurements, which testify its credibility to be used as a benchmark. However, phenomenological models with a few parameters employed to reproduce a set of experimental data are not free of errors. It is often the case that an objective function χ^2 is minimized to optimize the set of parameters, which is defined as

$$\chi^2(\mathbf{p}) = \frac{1}{N_d - N_p} \sum_{i=1}^{N_d} \left(\frac{\mathcal{O}_i^{exp} - \mathcal{O}_i^{th}(\mathbf{p})}{\Delta \mathcal{O}_i} \right)^2, \quad (1)$$

where, N_d and N_p are the number of experimental data points and the number of fitted parameters, respectively. $\mathcal{O}_i^{exp}$ and $\mathcal{O}_i^{th}(\mathbf{p})$ are the experimental and the corresponding theoretical values for a given observable. $\Delta\mathcal{O}_i$ is the adopted error which is given by

$$\Delta\mathcal{O}_i = \Delta\mathcal{O}_i^{th} + \Delta\mathcal{O}_i^{exp} + \Delta\mathcal{O}_i^{num}. \quad (2)$$

The terms in the right hand side are theoretical, experimental and numerical errors respectively. Once the minimum value of the χ^2 $(= \chi_0^2)$ corresponding to the optimized parameter set $\mathbf{p}$ $(= \mathbf{p_0})$ is obtained, one can calculate errors on different parmaters as well as observables by employing covariance analysis [1]. We have employed this method of covariance analysis to examine the merits of statistical model for fission.

In its primitive form, a standard statistical model (SM) could reproduce the experimental observables like evaporation residue ($\sigma_{\rm ER}$), fission cross sections ($\sigma_{\rm fiss}$), neutron multiplicity ($\nu_{\rm pre}$) data by tuning its parameters (*viz.* level density parameters at ground state and saddle, a scaling factor for the fission barrier, a pre-saddle delay and saddle-to-scission transition time) on an *ad-hoc* basis [2, 3]. But, eventually after incorporation of parameters like shell correction (in level density and fission barrier), orientation degree of freedom ($K_{\rm or}$), collective enhancement in level density ($K_{\rm coll}$) and a suitable dissipation, the $\sigma_{\rm ER}$, $\sigma_{\rm fiss}$ and $\nu_{\rm pre}$ are simultaneously reproduced for asymmetric reactions populating compound nucleus (CN) of mass $A_{\rm CN}$ up to ~ 200 [4], which had been hitherto uncomprehended. The pre-saddle dissipation strength (β) was the only free parameter in that analysis. There were several other parameters (e.g. parameters that describe the damping of shell correction ($E_{\rm D}$) and collective modes ($E_{\rm cr}$)) which were taken from independent studies assuming that those values would be same in the CN mass region ($A_{\rm CN} \sim 170 - 224$) and different excitation energies (of CN).

So, it was of paramount importance to do an independent error analysis treating β, $E_{\rm D}$, $E_{\rm cr}$ and ΔE (width parameter of the Fermi function defining the collective mode) as free parameters and check whether the values kept fixed, would remain similar in the CN mass $A_{\rm CN} \sim 200$ region at all and they correspond to a global minima or not and to put error bars to the predicted values of the observables.

In the present analysis, the model-I does not take the effects of $K_{\rm or}$ and $K_{\rm coll}$ into account whereas model-II does. So, while only β and $E_{\rm D}$ are varied in case of model-I, β, $E_{\rm D}$, $E_{\rm cr}$ and ΔE are treated as free parame-

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TABLE I: Observables $\mathcal{O}$ of different nuclei, adopted errors on them ($\Delta\mathcal{O}_i$), their experimental values ($\mathcal{O}_i^{exp}$) and the ones ($\mathcal{O}_i^{th}$) obtained for model-I and II.

Reactions	CN	E^*	$\mathcal{O}$	$\mathcal{O}_i^{exp}$	Ref.	$\Delta\mathcal{O}_i$	$\mathcal{O}_i^{th}$ model-I	$\mathcal{O}_i^{th}$ model-II
^{16}O+^{154}Sm	^{170}Yb	107.0	$\sigma_{\rm ER}$	1260±200	[5]	200.0	1316.50±11.30	1324.87±7.49
		107.0	$\sigma_{\rm fiss}$	40±4	[5]	15.0	38.32±9.52	21.32±2.91
		120.8	$\nu_{\rm pre}$	4.4±0.15	[6]	1.0	3.66±0.28	4.40±0.66
^{16}O+^{176}Yb	^{192}Pt	72.97	$\sigma_{\rm ER}$	927±129	[7]	129.0	1034.14±2.73	1036.87±3.54
		99.98	$\nu_{\rm pre}$	4.4±0.5	[8]	1.0	3.37±0.15	4.35±0.40
^{16}O+^{184}W	^{200}Pb	72.41	$\sigma_{\rm ER}$	557±21	[9]	200.0	812.53±30.99	611.46±67.89
		72.41	$\sigma_{\rm fiss}$	398±6	[9]	100.0	190.01±28.69	394.50±72.84
		195.8	$\nu_{\rm pre}$	7.7±0.3	[10]	2.0	7.16±0.84	9.16±0.69

TABLE II: The optimised parameters in different models and their correlated errors. χ^2 per degree of freedom is also mentioned.

Models (χ^2)	β ($\times 10^{21} s^{-1}$)	$E_{\rm D}$ (MeV)	$E_{\rm cr}$ (MeV)	ΔE (MeV)
I (1.19)	1.21±0.40	42.62±1.55	-	-
II (1.30)	1.98±0.43	51.08±36.59	80.10±19.00	10.37±6.95
others	2-4 [11–13]	18.5 [14] 28.57 [15]	40-60 [16, 17]	10 [16]

ters in model-II, to investigate the correlations among them and their influences on the observables. The different experimental observables and adopted errors on them along with their theoretically obtained values for the reactions (^{16}O+^{154}Sm, ^{16}O+^{176}Yb, ^{16}O+^{184}W) populating CNs (^{170}Yb, ^{192}Pt, ^{200}Pb) used for analysis are mentioned in Table I. The optimized parameters, their associated errors, the χ^2 values and their optimized values reported elsewhere are mentioned in Table II for comparison.

The damping of collective enhancement ($E_{\rm cr}$) with excitation energy, being comparable to the damping of the shell effects *i.e.* $E_{\rm D}$, has definitely an influence on the production cross section [16]. Apparently, from Table I and II, the optimized parameters are found to be within their reasonable limits. Model-II gives a better agreement with the measured data. A detailed covariance analysis with more data points, more SM parameters and more events, is underway.

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