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$(g - 2)_{e,\mu}$ and lepton flavor violating decays in a left-right model

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Abstract General expressions for one-loop contributions associated with lepton-flavor violating decays of the standard model-like Higgs boson $h \rightarrow e_b^\pm e_a^\mp$ and gauge boson $Z \rightarrow e_b^\pm e_a^\mp$ are introduced in the unitary gauge. The results are used to discuss these decays as new physics signals in a minimal left-right symmetric model containing only one bidoublet Higgs and a $SU(2)_R$ Higgs doublet accommodating data of neutrino oscillations and $(g - 2)_\mu$. The numerical investigation indicates that some of these decay rates can reach near future experimental sensitivities.

1 Introduction

Lepton flavor violating (LFV) decays, like those of charged leptons (cLFV) $e_b \rightarrow e_a \gamma$, the standard model-like Higgs boson (LFVh) $h \rightarrow e_b e_a$, and the neutral gauge boson (LFVZ) $Z \rightarrow e_b e_a$, are hot objects of experimental searches [1–6]. Although these decays do not appear in the standard model (SM), their existence is predicted by the LFV sources appearing in many models beyond the SM (BSM), such as minimal SM extensions guaranteeing neutrino oscillation data [7, 8]. Along with updates of the experimental data, LFV decays of h and Z , as promising signals of new physics, were indicated in various BSMs. On the other hand, the recent experimental results of charged lepton anomalies $a_{e_a} \equiv (g - 2)_{e_a}/2$ show a large deviation from the SM, which

often supports regions of parameter space predicting large LFV decay rates, especially for the BSM accommodating neutrino oscillation data. Therefore, a combination of simultaneous studies of the above LFV decays and the $(g - 2)_{e_a}$ data in BSMs accommodating the neutrino oscillation data is beneficial for searching for allowed regions of the parameter space of the models. To the best of our knowledge, in addition to the analytical formulas for one-loop contributions to cLFV decays and $(g - 2)_{e_a}$ anomalies [9, 10], which can generally be applied to a large class of BSMs, the one-loop contributions to LFVh and LFVZ decay amplitudes were introduced in specific SM extensions. Namely, various discussions on these LFV decays of h [11–28] and Z [29–43] as signals of new physics originating from loop contributions. On the other hand, BSMs consisting of both heavy seesaw neutrinos and right-handed gauge bosons, such as the left-right (LR) symmetric models [44–49], can predict complicated one-loop contributions to the LFV decay amplitudes. Phenomenology of these LR models including the $(g - 2)_{e,\mu}$ and cLFV decays has been discussed recently [50–54].

Therefore, concrete studies of LFVh and Z decays will be more useful for determining the allowed regions of parameter space that are not excluded by experimental constraints. Our main aim is to introduce a general complete class of one-loop contributions for LFV decay amplitudes calculated in the unitary gauge. We will use these results to discuss in detail the correlations among LFV decay rates and $(g - 2)_\mu$ in the LR model discussed in Ref. [54].

The paper is organized as follows. In Sect. 2, we review the one-loop contributions to decay amplitudes $e_b \rightarrow e_a \gamma$ and $(g - 2)_{e,\mu}$ and provide general one-loop contributions to decay amplitudes $h \rightarrow e_a^\pm e_b^\mp$ and $Z \rightarrow e_a^\pm e_b^\mp$, which are derived using the unitary gauge. In Sect. 3, we deter-

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mine analytic formulas for one-loop contributions to LFV decay rates, which are used to study the allowed regions of the parameter space that guarantee simultaneously the neutrino oscillation data, LFV constraints, and 1σ deviation of $(g-2)_\mu$ data from the SM prediction. Section 4 summarizes important results. Finally, three appendices are used to provide more detailed notations of Passarino–Veltman (PV) functions, precise expressions of relevant one-loop formulas, and the Higgs sector of the LRIS model.

2 General one-loop formulas

2.1 cLFV decays $e_b \rightarrow e_a \gamma$ and $(g-2)_{e_a}$

The one-loop contributions to cLFV decay amplitudes and a_{e_a} are available [9, 10, 55, 56], where calculations were performed in both gauges 't Hooft–Feynman and unitary for diagrams consisting of gauge boson exchanges. We adopt the following Lagrangian parts playing the roles of LFV sources discussed in this work [10]

$$\mathcal{L}_{FeS} = \sum_{F,S} \sum_{a=1}^3 \bar{F} (g_{aFS}^L P_L + g_{aFS}^R P_R) e_a S + \text{h.c.}, \quad (1)$$

$$\mathcal{L}_{FeV} = \sum_{F,V} \sum_{a=1}^3 \bar{F} \gamma^\mu (g_{aFV}^L P_L + g_{aFV}^R P_R) e_a V_\mu + \text{h.c.}, \quad (2)$$

where the fermion F and the boson $B = V_\mu, S$ have electric charges Q_F and Q_B , and masses m_F and m_B , respectively. This means that $\hat{Q}B = Q_B$ and $\hat{Q}B^* = -Q_B$, therefore the conventions $B \equiv B^{+Q_B}$ and $B^* \equiv B^{-Q_B}$ are used hereafter. Two Eqs. (1) and (2) are consistent with those introduced in Ref. [9]. Moreover, we adopt the Feynman rule that the photon always couples with two identical physical particles [9], as shown in Table 1,

where $\Gamma_{\mu\nu\lambda}(p_0, p_+, p_-) = g_{\mu\nu}(p_0 - p_+)_\lambda + g_{\nu\lambda}(p_+ - p_-)_\mu + g_{\lambda\mu}(p_- - p_0)_\nu$ is the standard form with $\partial_\mu = -ip_\mu$. Here, $p_\mu = p_{0,\pm}$ are incoming momenta into the relevant vertex consisting of a neutral, and two charged conjugated gauge bosons $V^0 (= A_\mu, Z_\mu)$, B , and B^* , respectively. The Ward Identity forbids the tree-level couplings of a photon with two different physical states [56]. In the unitary gauge, the one-loop contributions to the decay amplitudes $e_b \rightarrow e_a \gamma$ and a_{e_a} are shown in Fig. 1.

Previous calculations in the unitary gauge were performed, including approximate formulas with heavy gauge boson exchanges [10], or exact ones [56] expressed in terms of the PV functions [57], which are convenient for computation using numerical packages such as LoopTools [58]. They are also consistent with the calculation in the 't Hooft–

Feynman (HF) gauge based on the particular assumption of the Goldstone boson couplings [9], where the form factors are given in Appendix B. Useful transformations between different notations used in previous works are given in Ref. [56].

We note here that two Eqs. (1) and (2) contain general LFV sources generating 1-loop contributions to other LFV processes, including the LFV h and LFV Z we focus on in this work. The most attractive LFV source comes from the active neutrino oscillation data, the evidence of the LFV source confirmed by experiments [59–61]. Especially the heavy neutrinos generating active neutrino masses and mixing through the seesaw mechanism have couplings with forms given in two Eqs. (1) and (2). Usually, only the cLFV experimental data of $\mu \rightarrow e \gamma$ are considered as the strictest constraints on the allowed regions of the parameter space, successfully explaining the $(g-2)_{e,\mu}$ data [40, 62–64]. Recent discussions on ISS extensions of the 3-3-1 models suggest that the decay $\tau \rightarrow \mu \gamma$ may result in stricter constraints on $(g-2)_\mu$ data than that of the decay $\mu \rightarrow e \gamma$ [24, 40, 65]. This suggests that the regions of parameter space giving large one-loop contributions to $(g-2)_\mu$ may also be affected by the experimental data of LFV h and LFV Z decays. We will discuss this in detail in the LRIS model [54].

2.2 The LFV decays $Z \rightarrow e_b^\pm e_a^\mp$ and $h \rightarrow e_b^\pm e_a^\mp$

Unlike photon couplings, the gauge boson Z and neutral SM-like Higgs boson h can couple two different physical particles. In particular, the triple couplings of Z relating to one-loop contributions to the decay amplitude $Z \rightarrow e_b^\pm e_a^\mp$ are generally given in Table 2.

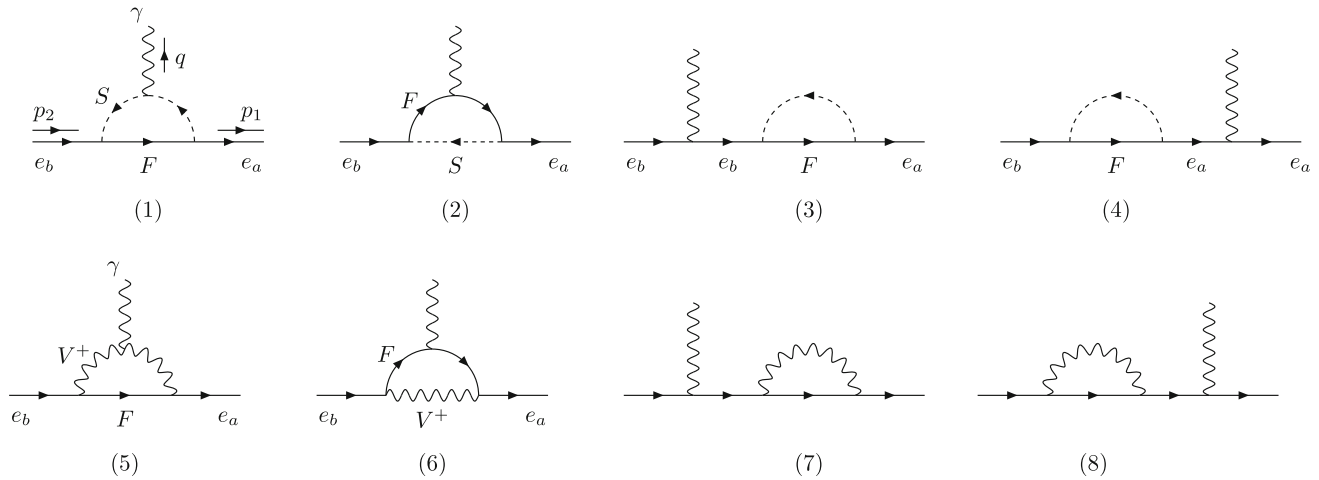
The triple self-couplings of Z are included in the kinetic parts of gauge bosons. The couplings of Z with fermions are included in the fermion kinetic Lagrangian. The triple couplings of Z with two bosons are included in the covariant kinetic Lagrangian of scalar multiplets R_S , $\mathcal{L}_{\text{kin}}^S = (D_\mu R_S)^\dagger (D^\mu R_S)$, where $D_\mu = \partial_\mu - i P_\mu$. If we denote that $(\partial_\mu R_S^\dagger) P^\mu R_S \equiv \sum_{V,S',S} g_{VS^*S'} (\partial_\mu S^*) V^\mu S'$, then it results in the following part for $V-S-S'$ couplings

$$\begin{aligned} \mathcal{L}_{VSS'} &= \sum_{V,S,S'} i g_{VS^*S'} [-(\partial_\mu S^*) S' + S^* (\partial_\mu S')] V^\mu + \text{H.c.} \\ &= \sum_{V,S,S'} [g_{VS^*S'} (p_{S'\mu} - p_{S^*\mu}) S^* S' V^\mu \\ &\quad + g_{VS^*S'}^* (p_{S\mu} - p_{S'^*\mu}) S S'^* V^{\mu*}], \end{aligned} \quad (3)$$

where the rules used to transform into the momentum space are $(\partial_\mu B) = -ip_{B\mu} B$ with $B = S, S', S^*, S'^*$, and all momenta are incoming vertex conventionally. The LFV Z and LFV h decays have couplings $Z S_k S_l$ and $V S h$, respectively. The notations show that $g_{VS^*S'}^* = g_{VS^*S}$. The cou-

Table 1 Feynman rules for cubic couplings of photon A^μ

Vertex	Coupling	Vertex	Couplings	Vertex	Couplings
$A^\mu(p_0)V^\nu(p_+)V^{*\lambda}(p_-)$	$-ieQ_V\Gamma_{\mu\nu\lambda}(p_0, p_+, p_-)$	$A^\mu S(p_+)S^*(p_-)$	$ieQ_S(p_+ - p_-)_\mu$	$A^\mu \bar{F}F$	$ieQ_F\gamma_\mu$

**Fig. 1** One-loop contributions to the decay amplitudes $e_b \rightarrow e_a \gamma$ and $(g-2)_{e_a}$ **Table 2** Feynman rules for cubic couplings of Z^μ with conventions defined in Table 1

Vertex	Coupling	Vertex	Couplings
$Z^\mu(p_0)V^\nu(p_+)V^{*\lambda}(p_-)$	$-ieg_{ZVV'}\Gamma_{\mu\nu\lambda}(p_0, p_+, p_-)$	$Z^\mu S^*(p_-)S(p_+)$	$ieg_{ZS^*S}(p_+ - p_-)_\mu$
$SV\mu^*Z^\nu$	$ieg_{SZV}g_{\mu\nu}$	$S^*V^\mu Z^\nu$	$ieg_{SZV}^*g_{\mu\nu}$
$Z^\mu \bar{F}F'$	$ie\gamma_\mu(g_{ZF'}^L P_L + g_{ZF'}^R P_R)$	$Z^\mu \bar{F}'F$	$ie\gamma_\mu(g_{ZF'}^{L*} P_L + g_{ZF'}^{R*} P_R)$

plings in the second row of Table 2 are derived from the covariant part containing at least one neutral Higgs component with non-zero vacuum expectation values (vev), namely $P_\mu R_S \rightarrow V_\mu \langle S^0 \rangle + V'_\mu S$. Such couplings do not contain momentum, hence it is easy to determine that

$$\mathcal{L}_{SVV'} = \sum_{S,V,V'} g_{SVV'} g_{\mu\nu} SV^\mu V'^{\nu*} + \text{H.c.},$$

which corresponds to the Feynman rules in the second row of Table 2 for $Z = V' \neq V$. We will identify $V'^\mu = W^{+\mu}$ for consistency with the SM notation when studying certain BSMs.

The one-loop Feynman diagrams for LFVZ decays are shown in Fig. 2.

We focus only on the unitary gauge so that particles in all diagrams are physical. The effective amplitude for the LFVZ decays $Z \rightarrow e_b^+(p_2)e_a^-(p_1)$ is written following the notations [30, 37, 40]:

$$i\mathcal{M}(Z \rightarrow e_b^+ e_a^-) = \frac{ie}{16\pi^2} \bar{u}_a [\not{\epsilon} (\bar{a}_l P_L + \bar{a}_r P_R) + (p_1 \cdot \epsilon) (\bar{b}_l P_L + \bar{b}_r P_R)] v_b, \quad (4)$$

where $\epsilon_\alpha(q)$ is the polarization of Z and $u_a(p_1)$, and $v_b(p_2)$ are Dirac spinors of e_a^- and e_b^+ . All form factors $\bar{a}_{l,r}$ and $\bar{b}_{l,r}$ receive contributions from one-loop corrections. The external on-shell gauge boson Z gives $q \cdot \epsilon = 0$, leading to $p_2 \cdot \epsilon = -p_1 \cdot \epsilon$. The on-shell conditions of the final leptons and Z boson are $p_1^2 = m_1^2 = m_a^2$, $p_2^2 = m_2^2 = m_b^2$, and $q^2 = m_Z^2$. The respective partial decay width is

$$\Gamma(Z \rightarrow e_b^+ e_a^-) = \frac{\sqrt{\lambda}}{16\pi m_Z^3} \times \left(\frac{e}{16\pi^2} \right)^2 \times \left(\frac{\lambda M_0}{12m_Z^2} + M_1 + \frac{M_2}{3m_Z^2} \right), \quad (5)$$

where $\lambda = m_Z^4 + m_b^4 + m_a^4 - 2(m_Z^2 m_a^2 + m_Z^2 m_b^2 + m_a^2 m_b^2)$, and

$$\begin{aligned} M_0 &= (m_Z^2 - m_a^2 - m_b^2) (|\bar{b}_l|^2 + |\bar{b}_r|^2) - 4m_a m_b \text{Re} [\bar{b}_l \bar{b}_r^*] \\ &\quad - 4m_b \text{Re} [\bar{a}_r^* \bar{b}_l + \bar{a}_l^* \bar{b}_r] - 4m_a \text{Re} [\bar{a}_l^* \bar{b}_l + \bar{a}_r^* \bar{b}_r], \\ M_1 &= 4m_a m_b \text{Re} [\bar{a}_l \bar{a}_r^*], \\ M_2 &= \left[2m_Z^4 - m_Z^2 (m_a^2 + m_b^2) - (m_a^2 - m_b^2)^2 \right] \end{aligned}$$

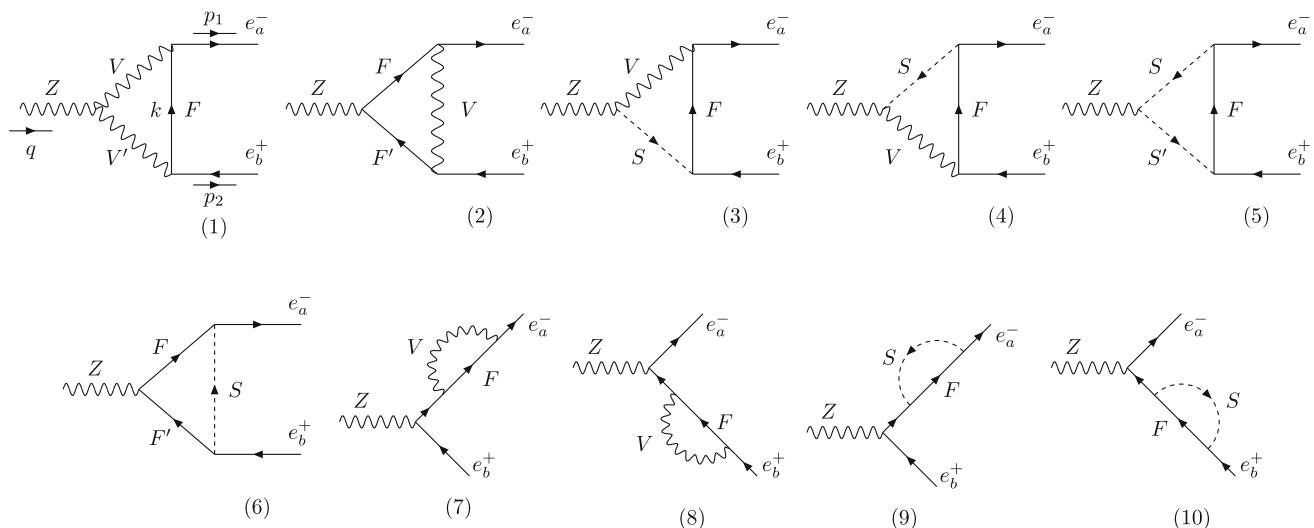


Fig. 2 One-loop contributions to the decay amplitudes $Z \rightarrow e_b^+ e_a^-$ in the unitary gauge

$$\times \left(|\bar{a}_l|^2 + |\bar{a}_r|^2 \right). \quad (6)$$

The total form factors consist of all the particular one-loop contributions originating from the diagrams given in Fig. 2. They are divided into three parts with different virtual particle exchanges in the loops: pure gauge bosons, pure scalars, and the appearance of both gauge bosons and scalars. The respective contributions are listed as follows

$$\begin{aligned} \bar{a}_{L,R} &= \bar{a}_{L,R}^{(1+2+7+8)} + \bar{a}_{L,R}^{(5+6+9+10)} + \bar{a}_{L,R}^{(3+4)}, \\ \bar{b}_{L,R} &= \bar{b}_{L,R}^{(1+2+7+8)} + \bar{b}_{L,R}^{(5+6+9+10)} + \bar{b}_{L,R}^{(3+4)}, \\ \bar{a}_{L,R}^{(1+2+7+8)} &= \sum_{V,V',F} \bar{a}_{L,R}^{FVV'} + \sum_{V,F,F'} \bar{a}_{L,R}^{VFF'} + \sum_{V,F} \bar{a}_{L,R}^{FV}, \\ \bar{b}_{L,R}^{(1+2+7+8)} &= \sum_{V,V',F} \bar{b}_{L,R}^{FVV'} + \sum_{V,F,F'} \bar{b}_{L,R}^{VFF'} + \sum_{V,F} \bar{b}_{L,R}^{FV}, \\ \bar{a}_{L,R}^{(5+6+9+10)} &= \sum_{S,S',F} \bar{a}_{L,R}^{FSS'} + \sum_{S,F,F'} \bar{a}_{L,R}^{SFF'} + \sum_{S,F} \bar{a}_{L,R}^{FS}, \\ \bar{b}_{L,R}^{(5+6+9+10)} &= \sum_{S,S',F} \bar{b}_{L,R}^{FSS'} + \sum_{S,F,F'} \bar{b}_{L,R}^{SFF'} + \sum_{S,F} \bar{b}_{L,R}^{FS}, \\ \bar{a}_{L,R}^{(3+4)} &= \sum_{S,V,F} \left(\bar{a}_{L,R}^{FSV} + \bar{a}_{L,R}^{FVS} \right), \\ \bar{b}_{L,R}^{(3+4)} &= \sum_{S,V,F} \left(\bar{b}_{L,R}^{FSV} + \bar{b}_{L,R}^{FVS} \right), \end{aligned} \quad (7)$$

where we omit the index (ab) for simplicity, namely $\Delta_{L,R}^X \equiv \Delta_{L,R}^{(ab)X}$ for all $X = FVV', \dots$. The analytical formulae of the form factors given in Eq. (7) were calculated by hand in the unitary gauge and cross-checked via the form package [66]. The results are collected in Appendix B, where all one-loop contributions are introduced in terms of the PV-functions consistent with those defined in LoopTools [58].

They will be used for the specific discussions in the LRIS model framework presented in Sect. 3. Apart from that, the analytic forms of the FV-functions in the limit $m_a = m_b = 0$ were introduced in Ref. [14], where it was shown that they could be used approximately without using LoopTools [67].

The couplings of the SM-like Higgs boson h related to one-loop contributions to the LFVh decays are generally given in Table 3.

Although the general one-loop formulas related to gauge boson exchanges have been determined in many works, the diagrams arising from the hSV couplings were first discussed in Ref. [14], for a 3-3-1 model including simple analytical forms used for numerical evaluations without the need for numerical packages such as LoopTools. These diagrams were also mentioned in the 2HDM, along with studying the cLFV and LFVZ decays, with or without $(g-2)_{e_a}$ anomalies [25, 40]. A class of general one-loop analytic formulas for LFVh decay amplitudes was given in Ref. [68], which are applicable only to the 't Hooft–Veltman gauge. On the other hand, the general LFV gauge couplings with non-zero $g_{aFV}^R \neq 0$ given in Eq. (2) have not been mentioned before. The one-loop Feynman diagrams for LFVh decays are shown in Fig. 3.

The effective Lagrangian of the LFVh decay $h \rightarrow e_a^\pm e_b^\mp$ is

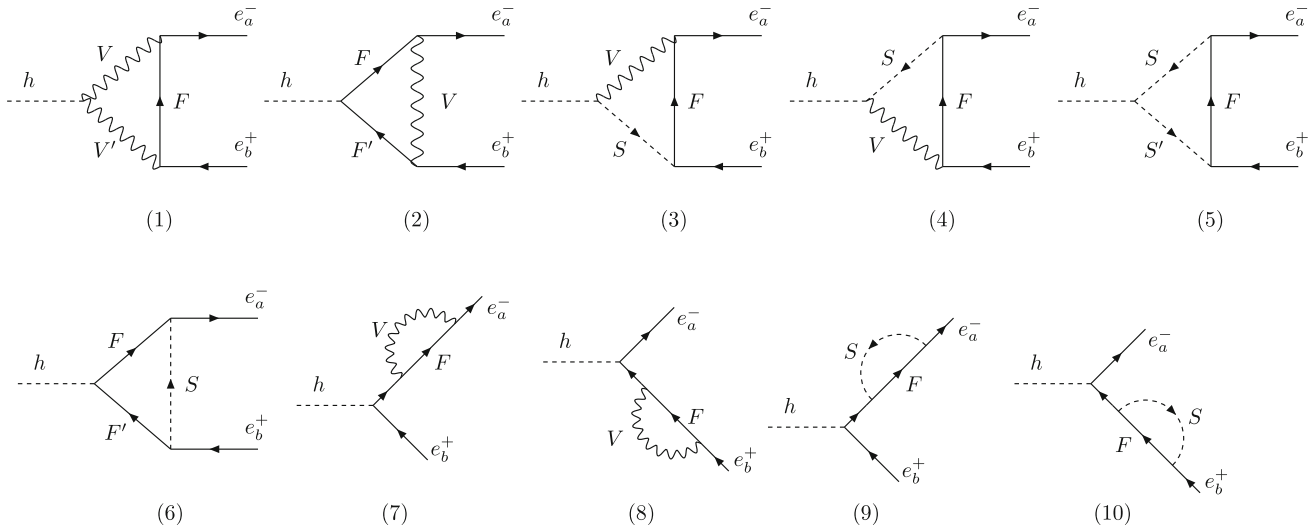
$$\mathcal{L}^{\text{LFVh}} = h \left(\Delta_L^{(ab)} \bar{e}_a P_L e_b + \Delta_R^{(ab)} \bar{e}_a P_R e_b \right) + \text{H.c.},$$

where $\Delta_{L,R}^{(ab)}$ arise from loop contributions. The respective partial decay width in the limit $m_h \gg m_{a,b}$ is [12]

$$\begin{aligned} \Gamma(h \rightarrow e_a e_b) &\equiv \Gamma(h \rightarrow e_a^- e_b^+) + \Gamma(h \rightarrow e_a^+ e_b^-) \\ &\simeq \frac{m_h}{8\pi} \left(|\Delta_L^{(ab)}|^2 + |\Delta_R^{(ab)}|^2 \right). \end{aligned} \quad (8)$$

Table 3 Feynman rules for cubic couplings of the SM-like Higgs boson h

Vertex	Coupling	Vertex	Couplings
$hV^\mu V'^{\mu\nu}$	$ig_{hVV'}g_{\mu\nu}$	hSS'^*	$-i\lambda_{hSS'}$
$V^\mu S^*(p_-)h(p_0)$	$ig_{VSh}(p_0 - p_-)_\mu$	$V^{*\mu}S(p_+)h(p_0)$	$ig_{VSh}^*(p_+ - p_0)_\mu$
$h\bar{F}F'$	$-i(g_{hFF'}^L P_L + g_{hFF'}^R P_R)$	$h\bar{F}'F$	$-i(g_{hFF'}^{L*} P_R + g_{hFF'}^{R*} P_L)$

**Fig. 3** One-loop contributions to the decay amplitudes $h \rightarrow e_b^+ e_a^-$ in the unitary gauge

The corresponding branching ratio is $\text{Br}(h \rightarrow e_a e_b) = \Gamma(h \rightarrow e_a e_b) / \Gamma_h^{\text{total}}$ where $\Gamma_h^{\text{total}} \simeq 4.1 \times 10^{-3} \text{ GeV}$ [69] with $q^2 \equiv (p_1 + p_2)^2 = m_h^2$. Similar to the case of discussion on the LFVZ decays, the particular formulas one-loop contributions to $\Delta_{L,R}^{(ab)}$ are given in Appendix B, namely

$$\begin{aligned}
 \Delta_{L,R}^{(ab)} &= \Delta_{L,R}^{(1+2+7+8)} + \Delta_{L,R}^{(5+6+9+10)} + \Delta_{L,R}^{(3+4)}, \\
 \Delta_{L,R}^{(1+2+7+8)} &= \sum_{F,V,V'} \Delta_{L,R}^{FVV'} + \sum_{F,F',V} \Delta_{L,R}^{VF'F'} + \sum_{F,V} \Delta_{L,R}^{FV}, \\
 \Delta_{L,R}^{(5+6+9+10)} &= \sum_{F,S,S'} \Delta_{L,R}^{FSS'} + \sum_{F,F',S} \Delta_{L,R}^{SFF'} + \sum_{F,S} \Delta_{L,R}^{FS}, \\
 \Delta_{L,R}^{(3+4)} &= \sum_{F,S,V} (\Delta_{L,R}^{FSV} + \Delta_{L,R}^{FVS}), \quad (9)
 \end{aligned}$$

where we omit the index (ab) for simplicity, namely $\Delta_{L,R}^X \equiv \Delta_{L,R}^{(ab)X}$ for all $X = FVV', \dots$. We note here important properties of the neutral boson couplings to fermions, namely, the two vertices $h\bar{F}F'$ and $Z\bar{F}F'$ may have different vertex factors from the respective Lagrangian parts. On the basis of the general Feynman rules for four-component spinors of fermions discussed in Ref. [70], two classes of the Lagrangian parts corresponding to the appearance of the Dirac or Majorana spinors are considered here. In particular,

the Lagrangian for at least one Dirac spinor is as follows:

$$\begin{aligned}
 \mathcal{L}_{Zff}^D &= e \sum_{F,F'} [\bar{F} \gamma_\mu (g_{ZFF'}^L P_L + g_{ZFF'}^R P_R) F' Z^\mu + \text{h.c.}], \\
 \mathcal{L}_{hff}^D &= - \sum_{F,F'} [\bar{F} (g_{hFF'}^L P_L + g_{hFF'}^R P_R) F' h + \text{h.c.}]. \quad (10)
 \end{aligned}$$

Various available BSMs consist of Dirac fermions and scalars to accommodate the $(g-2)_\mu$ anomaly data, such as vector-like leptons [10, 71] and SM quarks in leptoquark models [72]. Further studies to explain successfully simultaneous $(g-2)_e$ data with minimal Dirac fermion couplings with different charged leptons may result in promising LFVh and LFVZ decay signals.

In contrast, the Lagrangian for vertices consisting of two Majorana spinors $F = (F)^c$ and $F' = (F')^c$ is

$$\begin{aligned}
 \mathcal{L}_{Zff}^M &= \frac{e}{2} \sum_{F,F'} \bar{F} \gamma_\mu (g_{ZFF'}^L P_L - g_{ZFF'}^{L*} P_R) F' Z^\mu, \\
 \mathcal{L}_{hff}^M &= -\frac{1}{2} \sum_{F,F'} \bar{F} (g_{hFF'}^L P_L + g_{hFF'}^{L*} P_R) F' h, \quad (11)
 \end{aligned}$$

where the Majorana conditions give $g_{hFF'}^R = g_{hFF'}^{L*}$ and $g_{ZFF'}^R = -g_{ZFF'}^{L*} = -g_{ZFF'}^L$. The usual conventions were

used previously $g_{ZF_i F_j}^L \propto q_{ij}, C_{ij}$ in Refs. [13,37] and $g_{hF_i F_j}^L \propto \lambda_{ij}^h, \lambda_{ij}$ [13], for example. These works considered $F, F' \equiv n_i, n_j$ as Majorana neutrinos explaining the experimental neutrino oscillation data through the standard seesaw (SS) or inverse seesaw (ISS) mechanism. After the Feynman rules are constructed, the same calculations are applicable to both Dirac and Majorana spinors using the conventions for four-component spinors [70].

At tree level, we adopt a Lagrangian for the SM-like couplings of Z and h to SM leptons $Z\bar{e}_b e_a$ and $h\bar{e}_b e_a$ in this model is:

$$\mathcal{L}_{nc} = eZ_\mu \sum_{a=1}^3 \bar{e}_a \gamma^\mu [t_L P_L + t_R P_R] e_a - \frac{gm_a}{2m_W} \delta_{hee} h \bar{e}_a e_a, \quad (12)$$

in which the SM limit results in the following relations

$$t_R = t_R^{\text{SM}} = \frac{s_W}{c_W}, \quad t_L = t_L^{\text{SM}} = \frac{s_W^2 - c_W^2}{2s_W c_W}, \quad \delta_{hee} = 1. \quad (13)$$

3 The LRIS model and Feynman rules for LFV decays

3.1 Brief review of the LRIS model

The LR symmetry models constructed based on the gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ have been widely studied, such as the minimal model [73] with a minimal seesaw (MSS) or linear seesaw (LSS) mechanism to generate neutrino mass and mixing. Another extension of the LR version (LRIS) was introduced [51–53] with inverse seesaw (ISS) neutrinos to successfully explain the R_D and R_{D^*} anomalies with a rather light scalar spectrum. The $(g-2)_{e,\mu}$ anomalies were also discussed in LRIS models [54] and the correlations with the cLFV decay $\text{Br}(\mu \rightarrow e\gamma)$. It is therefore interesting to study other LFVZ and LFVh decays in this model. We summarize here the particle content and Lagrangian discussed in the above works. The charged operator is defined as

$$Q = T_3^L + T_3^R + \frac{B-L}{2} = T_3^L + \frac{Y}{2}, \quad (14)$$

where $T_3^{L,R}$ are the generators of $SU(2)_{L,R}$, and B and L are the baryon and lepton numbers of the $U(1)_{B-L}$ group. The matching condition with SM gives $\frac{Y}{2} = T_3^R + \frac{B-L}{2}$.

The particle content bases corresponding to the gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times Z_2$ is:

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \sim \left(3, 2, 1, \frac{1}{3}, +\right),$$

$$\begin{aligned} L_L &= \begin{pmatrix} \nu \\ e \end{pmatrix}_L \sim (1, 2, 1, -1, +), \\ Q_R &= \begin{pmatrix} u \\ d \end{pmatrix}_R \sim \left(3, 1, 2, \frac{1}{3}, +\right), \\ L_R &= \begin{pmatrix} \nu \\ e \end{pmatrix}_R \sim (1, 1, 2, -1, +), \\ S_1 &\sim (1, 1, 1, 0, -), \quad S_2 \sim (1, 1, 1, 0, +), \\ \phi &= \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \sim (1, 2, 2, 0, +), \\ \chi_R &= \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix} \sim (1, 1, 2, 1, +). \end{aligned} \quad (15)$$

The $U(1)_{B-L}$ charges of $S_{1,2}$ are consistent with Ref. [50], guaranteeing the zero electric charges resulting from Eq. (14). The neutral Higgs components get the following non-zero vev:

$$\langle \phi \rangle = \text{diag} \left(\frac{v_1}{\sqrt{2}}, \frac{v_2}{\sqrt{2}} \right), \quad \langle \chi_R^0 \rangle = \frac{v_R}{\sqrt{2}}. \quad (16)$$

The parameter $t_\beta \equiv v_1/v_2$ and $v^2 \equiv v_1^2 + v_2^2$ will be used when matching with the SM. We focus on the Yukawa part of leptons as follows:

$$\begin{aligned} -\mathcal{L}_Y &= \sum_{i,j=1}^3 \left[y_{ij}^L \bar{L}_{Ri} \phi^\dagger L_{Lj} + \tilde{y}_{ij}^L \bar{L}_{Ri} \tilde{\phi}^\dagger L_{Lj} \right. \\ &\quad \left. + y_{ij}^S \bar{L}_{Ri} \tilde{\chi}_R (S_{2j})^c + \text{H.c.} \right], \end{aligned} \quad (17)$$

where $\tilde{\phi} = \sigma_2 \phi^* \sigma_2$, and $\tilde{\chi}_R = i\sigma_2 \chi^*$. This generates the charged lepton mass matrix $\mathcal{M}^\ell = (y^L c_\beta + \tilde{y}^L s_\beta) v / \sqrt{2}$, and the following neutrino mass matrix after symmetry breaking:

$$\begin{aligned} -\mathcal{L}_{mass}^v &= \bar{\nu}_R m_D \nu_L + \bar{\nu}_R M_R^T (S_2)^c + \frac{\mu_s}{2} \bar{S}_2 (S_2)^c + \text{h.c.} \\ &= \frac{1}{2} \begin{pmatrix} (\nu_L)^c, & \bar{\nu}_R, & \bar{S}_2 \end{pmatrix} \mathcal{M}^v \begin{pmatrix} \nu_L, & (\nu_R)^c, & (S_2)^c \end{pmatrix}^T + \text{h.c.}, \end{aligned} \quad (18)$$

where $\mathcal{M}^v$ is a symmetric 9×9 matrix having the following ISS form

$$\begin{aligned} \mathcal{M}^v &= \begin{pmatrix} \mathcal{O}_{3 \times 3} & m_D^T & \mathcal{O}_{3 \times 3} \\ m_D & \mathcal{O}_{3 \times 3} & M_R^T \\ \mathcal{O}_{3 \times 3} & M_R & \mu_s \end{pmatrix}, \quad m_D = \frac{v}{\sqrt{2}} (y^L s_\beta + \tilde{y}^L c_\beta), \\ M_R &= \frac{v^s T}{\sqrt{2}} \nu_R, \end{aligned} \quad (19)$$

where $\nu_L = (\nu_1, \nu_2, \nu_3)_L^T$, $\nu_R = (\nu_1, \nu_2, \nu_3)_R^T$, $S_2 = (S_{21}, S_{22}, S_{23})^T$. The total mixing matrix is defined as a 9×9

unitary matrix U^v satisfying

$$\begin{aligned} U^{vT} \mathcal{M}^v U^v &= \hat{\mathcal{M}}^v = \text{diag}(m_{n_1}, m_{n_2}, \dots, m_{n_9}) \\ &= \text{diag}(\hat{m}_v, \hat{M}_N), \\ n'_L &= U^v n_L, \quad n'_R = U^{v*} n_R = U^{v*} (n_L)^c, \end{aligned} \quad (20)$$

where the two left- and right-handed flavor base are $n'_L = (v_L, (v_R)^c, (S_2)^c)^T$, and $(n'_L)^c = n'_R = ((v_L)^c, v_R, S_2)^T$, $n_{L,R} = (n_1, n_2, \dots, n_9)_{L,R}$ are Majorana neutrino mass eigenstates $n_{iL,R} = (n_{iR,L})^c$. We will use the approximate form of U^v as follows [24, 41]

$$U^v \simeq \begin{pmatrix} \left(I_3 - \frac{R_0 R_0^\dagger}{2} \right) U_3^v & \frac{i R_0}{\sqrt{2}} & \frac{R_0}{\sqrt{2}} \\ \mathcal{O}_{3 \times 3} & -\frac{i I_3}{\sqrt{2}} & \frac{I_3}{\sqrt{2}} \\ -R_0^\dagger U_3^v & \frac{i}{\sqrt{2}} \left(I_3 - \frac{R_0^\dagger R_0}{2} \right) & \frac{1}{\sqrt{2}} \left(I_3 - \frac{R_0^\dagger R_0}{2} \right) \end{pmatrix}, \quad (21)$$

where

$$m_D = x_0^{\frac{1}{2}} M_R \hat{\mu}_s^{-\frac{1}{2}} \xi \hat{x}_v^{\frac{1}{2}} U_3^{v\dagger}, \quad (22)$$

$$R_0 \equiv x_0^{\frac{1}{2}} U_3^v \hat{x}_v^{\frac{1}{2}} \xi^\dagger \hat{\mu}_s^{-\frac{1}{2}}, \quad (23)$$

$$M_R = \hat{M}_R = \text{diag}(M_1, M_2, M_3), \quad \hat{M}_N = \text{diag}(\hat{M}_R, \hat{M}_R), \quad (24)$$

and new conventions are

$$\begin{aligned} x_0 &\equiv \frac{\max[m_{n_1}, m_{n_2}, m_{n_3}]}{|(\mu_s)_{22}|} \ll 1, \\ \hat{\mu}_s &\equiv \frac{\mu_s}{|(\mu_s)_{22}|}, \quad \hat{x}_v \equiv \frac{\hat{m}_v}{\max[m_{n_1}, m_{n_2}, m_{n_3}]}. \end{aligned} \quad (25)$$

The ISS condition $|\hat{m}_v| \ll |\mu_s| \ll |m_D| \ll M_{1,2,3}$ gives $x_0 \ll 1$ but non-zero. Note that can be considered as the non-unitary scale of the active neutrino mixing matrix.

Regarding the charged leptons, in general there are two left and right rotations $U_{L,R}^\ell$ diagonalize the lepton mass matrix $\mathcal{M}^\ell$:

$$\begin{aligned} U_R^{\ell\dagger} \mathcal{M}^\ell U_L^\ell &= \hat{\mathcal{M}}^\ell = \text{diag}(m_e, m_\mu, m_\tau), \\ e_{L,R} &\rightarrow U_{L,R}^\ell e_{L,R}. \end{aligned} \quad (26)$$

The Pontecorvo–MakiNakagawa–Sakata (PMNS) matrix U_{PMNS} relating to the neutrino oscillation data is defined as $U_{\text{PMNS}} = U_L^{\ell\dagger} U_3^v$ [74–76]. It can be seen that [54]:

$$\begin{aligned} y^L &= \frac{\sqrt{2}}{vc_{2\beta}} \left(\mathcal{M}^\ell c_\beta - m_D s_\beta \right), \\ \tilde{y}^L &= -\frac{\sqrt{2}}{vc_{2\beta}} \left(\mathcal{M}^\ell s_\beta - m_D c_\beta \right). \end{aligned} \quad (27)$$

The covariant derivative corresponding to bidoublet ϕ and doublets of the $SU(2)_{L,R}$ in the LRIS model are [77]

- For the bidoublet such as ϕ :

$$D_\mu \phi = \partial_\mu \phi + \sum_{a=1}^3 \left[-ig_L \frac{\sigma^a}{2} W_{L\mu}^a \phi + ig_R \phi \frac{\sigma^a}{2} W_R^a \right], \quad (28)$$

where σ^a is the Pauli matrix.

- For $SU(2)_{L,R}$ doublets such as $X_{L,R} = \chi_R, L_{iL,R}, Q_{iL,R}$:

$$\begin{aligned} D_\mu X_A &= \partial_\mu X_A - \sum_{a=1}^3 ig_A \frac{\sigma^a}{2} W_{A\mu}^a X_A \\ &\quad - ig_{BL} B'_\mu \frac{B-L}{2} X_A, \quad A = L, R. \end{aligned} \quad (29)$$

Consequently, the kinetic Lagrangian of Higgs multiplets generating gauge boson masses are:

$$\mathcal{L}_k^H = \text{Tr} \left[(D_\mu \phi)^\dagger (D^\mu \phi) \right] + (D_\mu \chi)^\dagger (D^\mu \chi). \quad (30)$$

Defining $W_{L,R\mu}^\pm \equiv (W_{L,R\mu}^1 \mp i W_{L,R\mu}^2)/\sqrt{2}$, the mixing angle of singly charged bosons $W - W'$ is θ determined as $t_{2\theta} = 2s_{2\beta} v^2/v_R^2$, leading to the following relations of $W_{L,R\mu}^\pm$ and physical states W_μ and W'_μ [52]

$$\begin{pmatrix} W_{L\mu}^\pm \\ W_{R\mu}^\pm \end{pmatrix} = \begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} W_\mu^\pm \\ W'^\pm_\mu \end{pmatrix}, \quad (31)$$

where we consider the simple case of $g_L = g_R = g_2 = e/s_W$ is the $SU(2)_L$ gauge coupling of the SM. In addition, W is identified with the SM charged gauge boson with mass $m_W \simeq gv/2$, where $v^2 = v_1^2 + v_2^2 = (246 \text{ GeV})^2$. The exact formulas of these two gauge boson masses are

$$m_W^2 = \frac{g^2 v^2}{4} \times (1 - t_\theta s_{2\beta}), \quad m_{W'}^2 = \frac{g^2 v_R^2}{4} \times \frac{c_\theta^2 (s_{2\beta} + t_\theta)}{c_{2\theta} s_{2\beta}}, \quad (32)$$

which result in the specific form of $\delta_{hee}^2 = 1 - t_\theta s_{2\beta}$, where δ_{hee} was defined in Eq. (12).

The mixing parameters and masses of neutral gauge bosons are shown as follows:

$$\begin{pmatrix} W_{R\mu}^3 \\ B'_\mu \\ W_{L\mu}^3 \end{pmatrix} = \begin{pmatrix} c_\zeta c_\varphi - s_W s_\zeta s_\varphi & -c_\varphi s_\zeta - c_\zeta s_W s_\varphi & c_W s_\varphi \\ -c_\varphi s_W s_\zeta - c_\zeta s_\varphi & s_\zeta s_\varphi - c_\zeta c_\varphi s_W & c_W c_\varphi \\ c_W s_\zeta & c_W c_\zeta & s_W \end{pmatrix}$$

$$\begin{pmatrix} Z'_\mu \\ Z_\mu \\ A_\mu \end{pmatrix}, \quad (33)$$

where $t_\varphi \equiv g_{BL}/g_2$, $s_\varphi = t_W$ is the condition matching to the SM gauge couplings to guarantee the massless photon, and

$$\begin{aligned} t_{2\xi} &= \frac{2c_\varphi^3 c_W v^2}{c_\varphi^2 v^2 (c_\varphi^2 c_W^2 - 1) + c_W^2 v_R^2} \\ &= \frac{2(2c_W^2 - 1)^{3/2} v^2}{c_W^4 v_R^2 + (4c_W^4 - 6c_W^2 + 2)v^2}, \\ m_Z^2 &\simeq \frac{m_W^2}{c_W^2}, \quad m_{Z'}^2 \simeq \frac{m_{W'}^2}{c_\varphi^2}. \end{aligned} \quad (34)$$

The Higgs potential is [52]:

$$\begin{aligned} V_h &= \mu_1^2 \text{Tr}(\phi^\dagger \phi) + \mu_2^2 [\text{Tr}(\phi^\dagger \tilde{\phi}) + \text{Tr}(\phi \tilde{\phi}^\dagger)] \\ &\quad + \lambda_1 [\text{Tr}(\phi^\dagger \phi)]^2 \\ &\quad + \lambda_2 \left[(\text{Tr}(\phi^\dagger \tilde{\phi}))^2 + (\text{Tr}(\phi \tilde{\phi}^\dagger))^2 \right] \\ &\quad + \lambda_3 \text{Tr}(\phi^\dagger \tilde{\phi}) \text{Tr}(\phi \tilde{\phi}^\dagger) + \lambda_4 \text{Tr}(\phi^\dagger \phi) [\text{Tr}(\phi^\dagger \tilde{\phi}) + \text{Tr}(\phi \tilde{\phi}^\dagger)] \\ &\quad + \mu_3^2 (\chi_R^\dagger \chi_R) + \lambda_5 (\chi_R^\dagger \chi_R)^2 \\ &\quad + \alpha_1 \text{Tr}(\phi^\dagger \phi) (\chi_R^\dagger \chi_R) + \alpha_2 (\chi_R^\dagger \phi^\dagger \phi \chi_R) \\ &\quad + \alpha_3 (\chi_R^\dagger \tilde{\phi}^\dagger \tilde{\phi} \chi_R) + \alpha_4 [(\chi_R^\dagger \phi^\dagger \tilde{\phi} \chi_R) + \text{H.c.}]. \end{aligned} \quad (35)$$

The physical states corresponding masses and mixing parameters of the model were shown previously [52]; therefore, we do not repeat this in this work. The main results and notations used in this work are summarized in Appendix C. The SM-like Higgs boson was also indicated to be consistent with the experimental results.

3.2 Couplings and Feynman rules for LFV decays in LRIS

The above ingredients lead to the LFV couplings as follows:

$$\begin{aligned} \mathcal{L}^{\ell\ell H} &= \left[-\frac{h}{2v} \bar{n}_i [\lambda_{ij} P_L + \lambda_{ij}^* P_R] n_j \right. \\ &\quad \left. - \left(1 + \frac{h}{v} \right) \bar{e}_R \mathcal{M}^\ell e_L + \text{h.c.} \right] \\ &\quad - \sum_{i=1}^9 \bar{n}_i \left\{ \left[U_{ai}^{v*} (y^{L\dagger} s_\beta - \tilde{y}^{L\dagger} c_\beta) \right]_{ab} c_\xi \right. \\ &\quad \left. + U_{(a+6)i}^{v*} y_{ab}^{s\dagger} s_\xi \right] P_R \\ &\quad \left. + U_{(a+3)i}^v (y^L c_\beta - \tilde{y}^L s_\beta) \right\}_{ab} c_\xi P_L \Big\} e_b H^+ \end{aligned}$$

$$\begin{aligned} &+ \text{h.c.} + \dots, \quad (36) \\ \mathcal{L}^{\ell\ell V} &= -e A_\mu \bar{e}_a \gamma^\mu e_a \\ &+ \left[\frac{g_2}{\sqrt{2}} W^{+\mu} \sum_{i=1}^9 \bar{n}_i \gamma^\mu (c_\theta U_{ai}^{v*} P_L \right. \\ &\quad \left. + s_\theta U_{(a+3)i}^v P_R) e_a + \text{h.c.} \right] \\ &+ \left[\frac{g_2}{\sqrt{2}} W'^{+\mu} \sum_{i=1}^9 \bar{n}_i \gamma^\mu (-s_\theta U_{ai}^{v*} P_L \right. \\ &\quad \left. + c_\theta U_{(a+3)i}^v P_R) e_a + \text{h.c.} \right] \\ &+ e Z^\mu \bar{e}_a \left[\left(c_\xi t_L^{\text{SM}} - \frac{s_\xi s_W}{2c_\varphi c_W^2} \right) P_L \right. \\ &\quad \left. + \left(c_\xi t_R^{\text{SM}} + \frac{s_\xi (c_\varphi^2 c_W^2 - s_W^2)}{2s_W c_\varphi c_W^2} \right) P_R \right] e_a \\ &+ \frac{e}{2} Z^\mu \sum_{i,j=1}^2 \bar{n}_i [g_{Zij}^L P_L - g_{Zji}^L P_R] n_j + \dots, \end{aligned} \quad (37)$$

where we have used $U_{L,R}^\ell = I_3$,

$$\begin{aligned} \lambda_{ij} &= \lambda_{ji} = \sum_{c=1}^3 (m_{n_i} U_{ci}^{v*} U_{cj}^v + m_{n_j} U_{cj}^{v*} U_{ci}^v) \rightarrow g_{hi}^{R*} \\ &= g_{hij}^L = \frac{g\delta_{hee}}{2m_W} \lambda_{ij}, \\ g_{Zij}^L &= \sum_{c=1}^3 \left[U_{ci}^{v*} U_{cj}^v \left(\frac{c_\xi}{2c_W s_W} - \frac{s_\xi s_W}{2c_\varphi c_W^2} \right) \right. \\ &\quad \left. + U_{(c+3)i}^{v*} U_{(c+3)j}^v \frac{s_\xi (c_\varphi^2 c_W^2 + s_W^2)}{2c_\varphi c_W^2 s_W} \right], \end{aligned} \quad (38)$$

and $t_\xi = v s_{2\beta}/v_R$ relating to the singly charge Higgs mixing defined in Appendix C.

It is seen easily that the SM-like Higgs couplings $h\bar{e}_a e_b$ are LFV conservative, therefore LFVh decays are loop-induced. In the numerical investigation, we focus on the case of $\mathcal{M}^\ell = \hat{\mathcal{M}}^\ell$ for simplicity. In addition, the $h\bar{n}n$ is the same as those in the previous simple ISS extension of the SM. The same conclusion holds for the $Z\bar{n}n$ coupling in the limit $s_\xi = 0$. In the LRIS model, the particular couplings corresponding to Lagrangian parts given in Eqs. (1) and (2), are $g_{aFS}^{L,R} = g_{aiH^+}^{L,R}$ and $g_{aFV}^{L,R} = g_{aiW}^{L,R}, g_{aiW'}^{L,R}$, where

$$\begin{aligned} g_{aiH^+}^L &= \frac{\sqrt{2}c_\xi}{vc_{2\beta}} \sum_{c=1}^3 [U_{(c+3)i}^v (\hat{\mathcal{M}}^\ell - m_{DS_{2\beta}})_{ca}], \\ g_{aiH^+}^R &= \frac{\sqrt{2}c_\xi}{vc_{2\beta}} \sum_{c=1}^3 [U_{ci}^{v*} (\hat{\mathcal{M}}^{\ell\dagger} s_{2\beta} - m_D^\dagger)_{ca} \end{aligned}$$

Table 4 Vertex factors for SM-like Higgs couplings to charged Higgs and gauge bosons in the LRIS model

Vertex	Coupling	Vertex	Coupling
$g_{hW^+W^-}$	$gm_W(1 - s_{2\beta}s_{2\theta})\delta_{hee}^{-1}$	$g_{hW'^+W'^-}$	$gm_W(s_{2\beta}s_{2\theta} + 1)\delta_{hee}^{-1}$
$g_{hW^+W'^-}$	$gm_W s_{2\beta}(s_\beta^2 - c_\beta^2)\delta_{hee}^{-1}$		
$g_{W^+H^-h}$	$\frac{g}{2}c_\xi s_\theta(c_\beta^2 - s_\beta^2)$	$g_{W'^+H^-h}$	$\frac{g}{2}c_\theta c_\xi(c_\beta^2 - s_\beta^2)$

$$\begin{aligned}
& + U_{(c+6)i}^{v*}(\hat{M}_R)_{cat} t_\xi^2 \Big], \\
g_{aiW}^L &= \frac{g_2}{\sqrt{2}} c_\theta U_{ai}^{v*}, \quad g_{aiW}^R = \frac{g_2}{\sqrt{2}} s_\theta U_{(a+3)i}^v, \\
g_{aiW'}^L &= -\frac{g_2}{\sqrt{2}} s_\theta U_{ai}^{v*}, \quad g_{aiW'}^R = \frac{g_2}{\sqrt{2}} c_\theta U_{(a+3)i}^v, \quad (39)
\end{aligned}$$

where formulas of y^L and $\tilde{y}^L$ given in Eq. (27) were used. In the numerical investigation, we will consider the simple case of m_D and R_0 with $U_N = I_3 = \xi$ and the diagonal $\hat{\mu}_s$, which is enough to guarantee the $(g-2)_\mu$ data. The form factors relating to the one-loop contributions to Δa_{e_a} and $\text{Br}(e_b \rightarrow e_a \gamma)$ predicted by the LRIS model are shown in Appendices B1 and C2. The main one-loop contributions to $(g-2)_{e_a}$ anomalies and LFV decay rates predicted by the LRIS model originate from the couplings given in Eq. (39), namely

$$c_{(ab)R}^{\text{LRIS}} = c_{(ab)R}(H^+) + c_{(ab)R}(W) + c_{(ab)R}(W'). \quad (40)$$

The respective one-loop contributions to a_{e_a} and $\text{Br}(e_b \rightarrow e_a \gamma)$ originating from the LRIS model are:

$$\begin{aligned}
a_{e_a}^{\text{LRIS}} &= -\frac{4m_a}{e} \text{Re} \left[c_{(aa)R}^{\text{LRIS}} \right] - a_{e_a}^{\text{SM}}(W), \\
\text{Br}(e_b \rightarrow e_a \gamma)^{\text{LRIS}} &= \frac{48\pi^2}{G_F m_b^2} \left(|c_{(ab)R}^{\text{LRIS}}|^2 + |c_{(ba)R}^{\text{LRIS}}|^2 \right) \\
&\quad \times \text{Br}(e_b \rightarrow e_a \bar{\nu}_a \nu_b), \quad (41)
\end{aligned}$$

where $a_{e_a}^{\text{SM}}(W)$ is the one-loop contribution from W exchange predicted by the SM.

Feynman rules for couplings of the SM-like Higgs boson with bosons relating to LFVh decays are shown in Table 4.

The triple coupling $\lambda_{hH^+H^-}$ of the SM-like Higgs boson derived from the Higgs potential is:

$$\begin{aligned}
\lambda_{hH^+H^-} &= v \left[\left(-2s_{2\beta}^2(2\lambda_2 + \lambda_3) + 2\lambda_1 \right) s_\xi^2 \right. \\
&\quad \left. + c_\xi^2 \left(\alpha_1 + \alpha_3 + \alpha_4 s_{2\beta} + (\alpha_2 - \alpha_3) (s_{2\beta} + s_\beta^2) \right) \right].
\end{aligned}$$

Feynman rules for the couplings of the gauge boson Z to charged Higgs and gauge bosons associated with one-loop contributions to the decay amplitude $Z \rightarrow e_b^+ e_a^-$ are collected in Table 5.

Table 5 Feynman rules for couplings of Z to charged Higgs and gauge bosons

Vertex	Coupling
$g_{ZH^+H^-}$	$-\frac{c_\xi(s_\xi^2 + 2s_W^2 - 1)}{2s_W c_W} + \frac{s_\xi((s_\xi^2 + 2)s_W^2 - 1)}{2s_W c_W \sqrt{1 - 2s_W^2}}$
$g_{H^-W^+Z}$	$\frac{c_{2\beta} c_\xi m_W s_\theta}{s_W c_W} \left(c_\xi - \frac{s_\xi s_W^2}{\sqrt{1 - 2s_W^2}} \right)$
$g_{H^-W'^+Z}$	$\frac{c_{2\beta} c_\theta c_\xi m_W}{s_W c_W} \left(c_\xi - \frac{s_\xi s_W^2}{\sqrt{1 - 2s_W^2}} \right)$
$g_{ZW^+W^-}$	$\frac{c_\xi c_\theta^2}{t_W} - s_\theta^2 \left(c_\xi t_W + s_\xi \sqrt{1 - t_W^2/s_W} \right)$
$g_{ZW'^+W'^-}$	$c_\xi s_\theta^2 t_W^{-1} - c_\theta^2 \left(c_\xi t_W + s_\xi \sqrt{1 - t_W^2/s_W} \right)$
$g_{ZW'^+W^-}, g_{ZW^+W'^-}$	$-c_\theta s_\theta \left(\frac{c_\xi}{s_W c_W} + s_\xi \sqrt{1 - t_W^2/s_W} \right)$

The above results for couplings of the SM-like Higgs h and Z show that our results are consistent with the SM results in the limit $\theta, \zeta, \xi \propto v/v_R \rightarrow 0$, corresponding to the condition that $v_R \gg v$. Consequently, a number of couplings are suppressed, namely, $g_{W^+H^-h}$, $g_{H^-W^+Z}$, $g_{ZW'^+W^-} = g_{ZW^+W'^-} \rightarrow 0$, leading to the weak constraint by experiments searching for decays $H^\pm \rightarrow W^\pm h$, $H^\pm \rightarrow W^\pm Z$, and $W'^\pm \rightarrow W^\pm Z$. On the other hand, the nonzero values of these mixing parameters result in nonzero factors of divergent parts in one-loop contributions to the LFVh and LFVZ decay amplitudes. Therefore, these one-loop contributions must be included to guarantee the divergent cancellation requirements in the total decay amplitudes, even if the finite parts may be tiny. In numerical estimation, the one-loop contributions from diagrams containing very heavy gauge boson exchanges such as W' are ignored, even when the couplings $hW'^+W'^-$, $hW^\pm W'^\mp$, and $ZW'^+W'^-$ are orders of magnitude of the SM couplings even when $\theta = \zeta = \xi = 0$. However, we can easily see that the divergent parts are generally nonzero, therefore these one-loop contributions are still useful for checking the overall divergent cancellation in the final results.

3.3 Numerical discussion

The numerical values of experimental data were taken from Ref. [76], including the neutrino oscillation data, masses of charged leptons; masses of two gauge bosons W , Z , and the SM-like Higgs bosons, namely

$$\begin{aligned} g &= 0.652, \quad G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}, \\ \alpha_e &= \frac{1}{137} = \frac{e^2}{4\pi}, \quad s_W^2 = 0.231, \\ m_W &= 80.377 \text{ GeV}, \quad m_Z = 91.1876 \text{ GeV}, \\ m_h &= 125.25 \text{ GeV}, \quad \Gamma_Z = 2.4955 \text{ GeV}, \\ m_e &= 5 \times 10^{-4} \text{ GeV}, \quad m_\mu = 0.105 \text{ GeV}, \\ m_\tau &= 1.776 \text{ GeV}. \end{aligned} \quad (42)$$

We will focus on the best-fit values of the neutrino oscillation data [76] corresponding to the normal order (NO) scheme with $m_{n_1} < m_{n_2} < m_{n_3}$, namely

$$\begin{aligned} s_{12}^2 &= 0.32, \quad s_{23}^2 = 0.547, \quad s_{13}^2 = 0.0216, \\ \Delta m_{21}^2 &\equiv m_{n_2}^2 - m_{n_1}^2 = 7.55 \times 10^{-5} [\text{eV}^2], \\ \Delta m_{32}^2 &\equiv m_{n_3}^2 - m_{n_2}^2 = 2.424 \times 10^{-3} [\text{eV}^2]. \end{aligned} \quad (43)$$

On the other hand, we fix $\delta = 180$ [Deg] for simplicity in numerical investigation. Consequently, the neutrino masses and mixing matrix are fixed as follows:

$$\begin{aligned} \hat{m}_\nu &= (\hat{m}_\nu^2)^{1/2} = \text{diag}(m_{n_1}, \\ &\quad \times \sqrt{m_{n_1}^2 + \Delta m_{21}^2}, \sqrt{m_{n_1}^2 + \Delta m_{21}^2 + \Delta m_{32}^2}), \\ U_{\text{PMNS}} &= f(\theta_{12}, \theta_{13}, \theta_{23}, \delta) \\ &= \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta} & c_{13}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{23}s_{12}e^{i\delta}s_{13} - c_{13}s_{23} & c_{13}c_{23} \end{pmatrix}, \end{aligned} \quad (44)$$

where $m_{n_1} \leq 0.035$ eV in order to guarantee the data of Planck 2018 [78].

Experimental data for $(g - 2)_{e,\mu}$ anomalies have been updated from Ref. [79] showing a 5.1σ standard deviation from the SM prediction [80–107] that: $\Delta a_\mu^{\text{NP}} \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (2.49 \pm 0.48) \times 10^{-9}$ [108]. The experimental a_e data was reported from different groups [109–112], predict the same order of $|\Delta a_e^{\text{NP}}| = \mathcal{O}(10^{-13})$ defined as the deviation between experiments and the SM prediction [113–118].

The cLFV rates are constrained from recent experiments as follows [119–122]: $\text{Br}(\mu \rightarrow e\gamma) < 3.1 \times 10^{-13}$, $\text{Br}(\tau \rightarrow e\gamma) < 3.3 \times 10^{-8}$, and $\text{Br}(\tau \rightarrow \mu\gamma) < 4.2 \times 10^{-8}$. The latest experimental constraints for LFVh decay rates are $\text{Br}(h \rightarrow \tau\mu) < 1.5 \times 10^{-3}$, $\text{Br}(h \rightarrow \tau e) < 2 \times 10^{-3}$,

and $\text{Br}(h \rightarrow \mu e) < 4.4 \times 10^{-5}$ [1–4]. The latest experimental constraints for LFVZ decay rates are $\text{Br}(Z \rightarrow \tau^\pm \mu^\mp) < 6.5 \times 10^{-6}$, $\text{Br}(Z \rightarrow \tau^\pm e^\mp) < 5.0 \times 10^{-6}$, and $\text{Br}(Z \rightarrow \mu^\pm e^\mp) < 2.62 \times 10^{-7}$ [5,6]. In the following numerical investigation, we emphasize that all allowed points we collect for illustrations simultaneously satisfy the 1σ experimental range of $(g - 2)_\mu$ data, $2.01 \times 10^{-9} \leq \Delta a_\mu^{\text{LRIS}} \equiv \Delta a_\mu^{\text{NP}} \leq 2.97 \times 10^{-9}$, and all recent experimental constraints of cLFV, LFVh and LFVZ decays mentioned above. The maximal value of $\Delta a_e^{\text{LRIS}} \leq \mathcal{O}(10^{-14})$ predicted by our numerical result is smaller than that in recent experimental data.

The unknown parameters of the LRIS model will be scanned in the following ranges:

$$\begin{aligned} v_R &\in [10, 100] [\text{TeV}]; \quad m_{H^\pm} \in [0.3, 5] [\text{TeV}]; \\ M_{1,2,3} &\in [0.1, 10] [\text{TeV}]; \\ m_{n_1} &\in [10^{-3}, 0.035] [\text{eV}]; \quad t_\beta \in [0.02, 0.8]; \\ x_0 &\in [10^{-6}, 5 \times 10^{-4}]; \quad (\hat{\mu}_s)_{11,33} \in [0.2, 50], \end{aligned} \quad (46)$$

and the matrix ξ given in Eq. (22) is parameterized as $\xi = -f(\xi_1, \xi_2, \xi_3, 0)$ with scanning ranges $|\xi_i| \leq \pi$. The lower bound of v_R is chosen based on the searches for the heavy gauge boson W' at the High Luminosity Large Hadron Collider (HL-LHC) [50]. The upper bound of x_0 is constrained from the data of non-unitarity of the active neutrino mixing matrix [123–125].

The correlations of Δa_μ with all LFV decay rates are illustrated numerically in Fig. 4.

There are three decay rates that reach the experimental bounds, namely $\text{Br}(\mu \rightarrow e\gamma)$, and $\text{Br}(\tau \rightarrow \mu\gamma)$ and $\text{Br}(h \rightarrow \tau\mu)$. Fortunately, they are still allowed for future sensitivities of $\text{Br}(\mu \rightarrow e\gamma) \simeq 6 \times 10^{-14}$ [126], $\text{Br}(\tau \rightarrow \mu\gamma) \simeq \mathcal{O}(10^{-9})$ [127], and $\text{Br}(h \rightarrow \tau\mu) \simeq \mathcal{O}(10^{-4})$ [128–131]. We conclude that the allowed regions of parameters of the LRIS model will be changed strongly by the incoming experimental results of the three mentioned LFV decays.

The upper bounds of the remaining LFV decay rates predicted by the LRIS model are:

$$\begin{aligned} \text{Br}(\tau \rightarrow e\gamma) &\leq 9 \times 10^{-13}, \quad \text{Br}(h \rightarrow \tau e) \leq 1.77 \times 10^{-7}, \\ \text{Br}(h \rightarrow \mu e) &\leq 1.3 \times 10^{-5}, \\ \text{Br}(Z \rightarrow \mu^+ e^-) &\leq 2.8 \times 10^{-12}, \\ \text{Br}(Z \rightarrow \tau^+ e^-) &\leq 1.1 \times 10^{-11}, \\ \text{Br}(Z \rightarrow \tau^+ \mu^-) &\leq 3.8 \times 10^{-7}. \end{aligned} \quad (47)$$

Therefore, two decays $h \rightarrow \mu e$ and $Z \rightarrow \mu^+ \tau^-$ are close to incoming sensitivities [132,133].

The allowed regions of the parameter space obtained in our investigation have stricter constraints than the scanning ranges chosen in Eq. (46). In particular, the singly charged

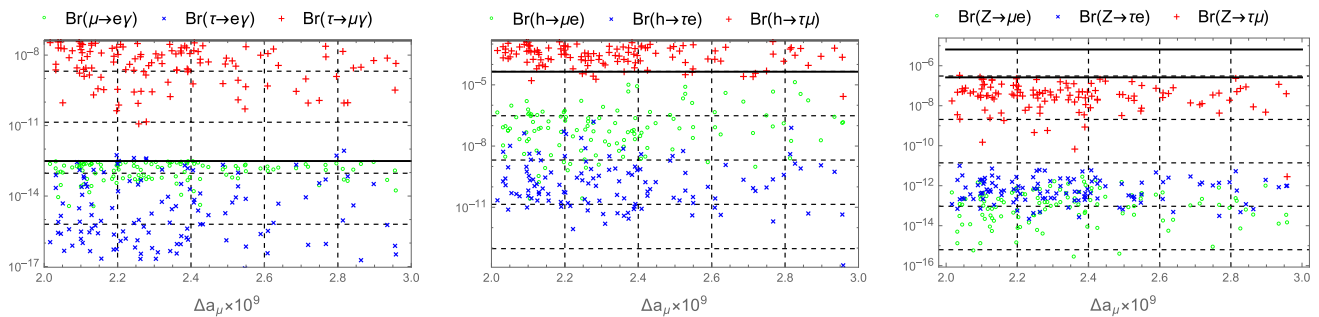


Fig. 4 The dependence of LFV decay rates on Δa_μ . Two black lines in the left, second, and right-panels show the upper bounds of LFV decay rates for cLFV ($\tau \rightarrow \mu\gamma$, $\mu \rightarrow e\gamma$), LFVh ($h \rightarrow \tau\mu$, μe), and LFVZ ($Z \rightarrow \tau\mu$, μe), respectively

Higgs mass has an upper bound $m_{H^\pm} < 4.2$ TeV, but it is still heavier than the values of interest in the popular 2HDMs [134–137]. A hierarchy of heavy neutrino masses appears, namely $m_{n_5} > 2.8$ TeV, whereas the two remaining masses have small upper bounds such that $m_{n_4}, m_{n_6} < 380$ GeV.

We note here that one-loop contributions from heavy charged gauge bosons W' are suppressed with a fixed value $C_{UV} = 0$, as in LoopTools. Therefore, total calculations, at least the divergent parts of all one-loop diagrams, including those relating to W' exchanges, must be considered.

We confirm here a property indicated in Ref. [54] that the dominant one-loop contributions to $\Delta a_\mu^{\text{LRIS}}$ come from charged Higgs exchanges with the appearance of the chiral enhancement term proportional to $c_{(aa)R}^{\text{LRIS}} \propto g_{aiH}^{L*} g_{aiH}^R$ [10]. Consequently, the values of $|c_{(ab)R}^{\text{LRIS}}|$ with $a \neq b$ are large too. The numerical investigation showed that $c_{(ab)R}^{\text{LRIS}}$ requires strong destructive correlations between the Higgs and gauge boson contributions to ensure that all the LFV decay rates mentioned in this work satisfy the experimental constraints. Therefore, if the SM result for $(g-2)_\mu$ in Ref. [138] is accepted, implying smaller values of $\Delta a_\mu^{\text{LRIS}}$ than those chosen for our numerical illustration, the cancellation requirements for $c_{(ab)R}^{\text{LRIS}}$ will be relaxed, but the qualitative conclusions for LFV decay rates are unchanged.

There are indirect sensitivities from other LFV processes such as the LFV decays $\mu \rightarrow 3e$, and $\mu - e$ conversion in nuclei, as discussed in Ref. [139] for the seesaw models. In the LRIS framework, the $\mu - e$ conversion in nuclei was shown to be invisible with recent experimental sensitivities [54]. A similar conclusion is derived for the decay $\mu \rightarrow 3e$, because they have the similar one-loop contributions from the diagrams with Z exchanges. Another LFV indirect sensitivity is the LFV Higgs $\mu - e$ couplings, which give two-loop contributions from Barr-Zee diagrams [140] to the decay amplitude $\mu \rightarrow e\gamma$ [141–143] and tree-level decays $e_b \rightarrow 3e_a$. As we indicated in Eq. (36), the second term implies that the two matrices expressing the SM-like Higgs couplings and charged lepton masses are proportional to each other. Consequently, the tree-level LFV couplings of

the SM-like Higgs boson do not appear in the LRIS model under consideration, implying the absence of two-loop Barr-Zee and tree-level diagrams with this Higgs exchange. In addition, the similar contributions from heavy neutral Higgs exchange do not qualitatively change the numerical results presented in this work.

4 Conclusions

In this work, we completely introduce two classes of general master formulas expressing one-loop contributions to the LFVh and LFVZ decay amplitudes in the BSMs. The calculations were performed in the unitary gauge, independent of the couplings of nonphysical states such as Goldstone bosons. Analytical formulas are expressed in terms of the PV functions consistent with the notations introduced by LoopTools. The results show that the formulas corresponding to private one-loop diagrams generally contain divergent parts, which must vanish in the final finite amplitudes. Therefore, all diagrams containing divergences must be considered before ignoring them when their finite parts are estimated to be small with fixed divergence parts. The results introduced in this work are sufficient to estimate these divergences thoroughly. We numerically investigated the LFVh and LFVZ decay rates in the LRIS model, which accommodates all the data of neutrino oscillation, the cLFV decays, and the $(g-2)_\mu$ anomaly. The results show that some of these decays are promising signals for incoming experimental searches. More importantly, the allowed regions of the parameter space are strongly affected by the recent experimental data from searches for three LFV decays $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$, and $h \rightarrow \tau\mu$. This implies that the allowed regions predicted by the LRIS model change strongly once new LFV upper bounds are established.

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Appendix A: Notations for Passarino-Veltman functions

A1: General notations

The PV-functions [144] used here to compute all form factors that give one-loop contributions to the LFVh and LFVZ decay amplitudes were listed in ref. [55], namely

$$\begin{aligned} A_0(m^2) &= \frac{(2\pi\mu)^{4-d}}{i\pi^2} \int \frac{d^d k}{k^2 - m^2 + i\delta}, \\ B_{\{0,\mu\}}(p_i^2, M_0^2, M_i^2) &= \frac{(2\pi\mu)^{4-d}}{i\pi^2} \int \frac{d^d k \times \{1, k_\mu\}}{D_0 D_i}, \\ i &= 1, 2, \\ C_{0,\mu,\mu\nu} &= \frac{(2\pi\mu)^{4-d}}{i\pi^2} \int \frac{d^d k \{1, k_\mu, k_\mu k_\nu\}}{D_0 D_1 D_2}, \end{aligned} \quad (A1)$$

where $D_0 \equiv k^2 - M_0^2 + i\delta$, $D_1 \equiv (k - p_1)^2 - M_1^2 + i\delta$, $D_2 \equiv (k + p_2)^2 - M_2^2 + i\delta$, $C_{0,\mu,\mu\nu} = C_{0,\mu,\mu\nu}(p_1^2, q^2, p_2^2; M_0^2, M_1^2, M_2^2)$ with $q = p_1 + p_2$, and μ is an arbitrary mass parameter introduced via dimensional regularization [144]. The scalar PV-functions are defined consistent with LoopTools [58], namely:

$$\begin{aligned} A_0(m^2) &= m^2(C_{UV} - \ln(m^2) + 1), \\ B_\mu(p_i^2, M_0^2, M_1^2) &= (-1)^i p_{1\mu} B_1(p_i^2, M_0^2, M_1^2), \quad i = 1, 2; \end{aligned}$$

$$\begin{aligned} C_\mu &= (-p_{1\mu}) C_1 + p_{2\mu} C_2, \\ C_{\mu\nu} &= g_{\mu\nu} C_{00} + p_{1\mu} p_{1\nu} C_{11} + p_{2\mu} p_{2\nu} C_{22} \\ &\quad - (p_{1\mu} p_{2\nu} + p_{2\mu} p_{1\nu}) C_{12}, \end{aligned} \quad (A2)$$

where $C_{UV} = 2/(4-d) - \gamma_E + \ln(4\pi\mu^2)$ are the divergent part. In our work, new reduced notations will be used are $B_0^{(i)} \equiv B_0(p_i^2, M_0^2, M_i^2)$ and $B_1^{(i)} \equiv B_1(p_i^2, M_0^2, M_i^2)$. The scalar functions $A_0, B_0, C_0, C_{00}, C_i, C_{ij}$ ($i, j = 1, 2$) are well-known PV functions consistent with notations introduced in LoopTools [58]. The scalar functions A_0, B_0, C_0 can be calculated using the techniques given in Ref. [57]. Other PV functions needed in this work are

$$\begin{aligned} B_{0,\mu}(q^2; M_1, M_2) &= \frac{(2\pi\mu)^{4-d}}{i\pi^2} \int \frac{d^d k \{1, k_\mu\}}{D_1 D_2}, \\ B_0^{(12)} \equiv B_0(q^2; M_1^2, M_2^2) &= \frac{(2\pi\mu)^{4-d}}{i\pi^2} \int \frac{d^d k}{D_1 D_2} \\ &= \frac{(2\pi\mu)^{4-d}}{i\pi^2} \int \frac{d^d k}{D'_1 D'_2}, \\ B_\mu^{(12)} \equiv B_\mu(q^2; M_1^2, M_2^2) &= \frac{(2\pi\mu)^{4-d}}{i\pi^2} \int \frac{d^d k \times k_\mu}{D_1 D_2} \\ &= \frac{(2\pi\mu)^{4-d}}{i\pi^2} \int \frac{d^d k \times (k + p_1)_\mu}{D'_1 D'_2} \\ &= B_1^{(12)} q_\mu + B_0^{(12)} p_{1\mu}, \end{aligned} \quad (A3)$$

where $B_1^{(12)} \equiv B_1(q^2; M_1^2, M_2^2)$, $D'_1 \equiv k^2 - M_1^2 + i\delta$, $D'_2 \equiv (k + q)^2 - M_2^2 + i\delta$.

For simplicity, we define the following notations appearing in many important formulas:

$$\begin{aligned} X_0 &\equiv C_0 + C_1 + C_2, \\ X_1 &\equiv C_{11} + C_{12} + C_1 \\ X_2 &\equiv C_{12} + C_{22} + C_2, \\ X_3 &\equiv C_1 + C_2 = X_0 - C_0, \\ X_{012} &\equiv X_0 + X_1 + X_2, \quad X_{ij} = X_i + X_j. \end{aligned} \quad (A4)$$

The divergent parts of the PV-functions are:

$$\begin{aligned} \text{div}[C_0] &= \text{div}[C_i] = \text{div}[C_{ij}] = 0; \quad i, j = 1, 2, \\ \text{div}[C_{00}] &= \frac{C_{UV}}{4}, \quad \text{div}[B_0^{(1)}] = \text{div}[B_0^{(2)}] \\ &= \text{div}[B_0^{(12)}] = C_{UV}, \\ \text{div}[B_1^{(1)}] &= \text{div}[B_1^{(2)}] = \text{div}[B_1^{(12)}] = -\frac{C_{UV}}{2}. \end{aligned} \quad (A5)$$

The Feynman rules for propagators of any gauge boson V_μ and their goldstone bosons in the unitary gauge are as follows

$$\Delta_V^{(u)\mu\nu} = \frac{-i}{k^2 - m_V^2} \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_V^2} \right), \quad \Delta_{G_V}^{(u)} = 0. \quad (A6)$$

Before going to the details of the calculation. We list here important well-known results such as the on-shell conditions gives $p_1^2 = m_a^2$, $p_2^2 = m_b^2$, and $q^2 = m_Z^2$, where m_a , m_b , and m_Z are the masses of leptons a , b ($a, b = 1, 2, 3$), and gauge boson Z . The momentum conservation gives $q = p_1 + p_2$. Two internal momenta $k_1 \equiv k - p_1$ and $k_2 \equiv k + p_2$ with $i = 1, 2$ are denoted in the diagram (1) of Fig. 2.

Then we take $d = 4$ for all finite integrals. For all divergent integrals, after changing into the expressions in terms of the PV-functions, we take $d = 4 - 2\epsilon$, then determining the final finite results before fixing $\epsilon = 0$. In addition, we will use the following transformation to change from integral to the notations of the PV-functions:

$$\int \frac{d^4 k}{(2\pi)^4} \rightarrow \frac{i}{16\pi^2} \times \frac{(2\pi\mu)^{4-d}}{i\pi^2} \int d^d k z$$

$$= \frac{i}{16\pi^2} \times (\text{PV} - \text{functions}).$$

In practice, the overall factor $i/(16\pi^2)$ will be added in the final results. Some intermediate steps used in this work were presented precisely in Refs. [14, 40, 41].

Appendix B: General One-loop contributions to LFBZ and LFBh amplitudes in the unitary gauge

B1: Decays $e_b \rightarrow e_a \gamma$ and $(g - 2)_{e_a}$

We use the analytic formulas for computing one-loop contributions to cLFV decay amplitudes and $(g - 2)_{e_a}$ given in Ref. [10], which consistent with previous results [9, 55]. From the couplings given by two Eqs. (1) and (2), the form factors $c_{(ab)R}$ corresponding to the one-loop contribution of a boson X coupling with a fermion F and a charged lepton e_a are:

$$c_{(ab)R}^X = \frac{e}{16\pi^2 m_X^2} \left\{ g_{aFX}^{L*} g_{bFX}^R m_F [f_X(t_X) + Q_F g_X(t_X)] \right.$$

$$\left. + [m_b g_{aFX}^{L*} g_{bFX}^L + m_a g_{aFX}^{R*} g_{bFX}^R] \right.$$

$$\left. [\tilde{f}_X(t_X) + Q_F \tilde{g}_X(t_X)] \right\}, \quad (\text{B1})$$

where $X = S, V_\mu, t_X \equiv m_F^2/m_X^2$, Q_F is the electric charge of the fermion F , and the master functions are

$$f_\Phi(x) = 2\tilde{g}_\Phi(x) = \frac{x^2 - 1 - 2x \ln x}{4(x-1)^3},$$

$$g_\Phi = \frac{x - 1 - \ln x}{2(x-1)^2},$$

$$\tilde{f}_\Phi(x) = \frac{2x^3 + 3x^2 - 6x + 1 - 6x^2 \ln x}{24(x-1)^4},$$

$$f_V(x) = \frac{x^3 - 12x^2 + 15x - 4 + 6x^2 \ln x}{4(x-1)^3},$$

$$g_V(x) = \frac{x^2 - 5x + 4 + 3x \ln x}{2(x-1)^2},$$

$$\tilde{f}_V(x) = \frac{-4x^4 + 49x^3 - 78x^2 + 43x - 10 - 18x^3 \ln x}{24(x-1)^4},$$

$$\tilde{g}_V(x) = \frac{-3(x^3 - 6x^2 + 7x - 2 + 2x^2 \ln x)}{(x-1)^3}. \quad (\text{B2})$$

Formulas for one-loop contributions to $a_{e,\mu}$ and cLFV decay rates are:

$$a_{e_a}(X) = -\frac{4m_a}{e} \text{Re} \left[c_{(aa)R}^X \right], \quad (\text{B3})$$

$$\text{Br}(e_b \rightarrow e_a \gamma) = \frac{48\pi^2}{G_F^2 m_b^2} \left(|c_{(ab)R}|^2 + |c_{(ba)R}|^2 \right)$$

$$\text{Br}(e_b \rightarrow e_a \bar{\nu}_a \nu_b), \quad (\text{B4})$$

where $G_F = g^2/(4\sqrt{2}m_W^2)$ is the Fermi constant, $\text{Br}(\mu \rightarrow e \bar{\nu}_e \nu_\mu) \simeq 1$, $\text{Br}(\tau \rightarrow e \bar{\nu}_e \nu_\tau) \simeq 0.1782$, $\text{Br}(\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau) \simeq 0.1739$, and $c_{(ab)R} = \sum_X c_{(ab)R}(X)$ with X being all relevant bosons predicted by the particular BSM.

In the LRIS model, precise expressions of $c_{(ab)R}$ with $U_N = I_3$ are

$$c_{(ab)}(H^+) = \frac{e}{16\pi^2 m_{H^+}^2} \sum_{i=1}^9 \left[g_{aiH^+}^{L*} g_{biH^+}^R m_{n_i} f_\Phi(x_{i,H}) \right.$$

$$\left. + (m_b g_{aiH^+}^{L*} g_{biH^+}^L + m_a g_{aiH^+}^{R*} g_{biH^+}^R) \right.$$

$$\left. \times \tilde{f}_\Phi(x_{i,H}) \right],$$

$$c_{(ab)R}(W^+) = \frac{eg_2^2}{32\pi^2 m_W^2} \sum_{i=1}^9 \left[s_\theta c_\theta U_{ai}^v U_{(b+3)i}^v m_{n_i} f_V(x_{i,W}) \right.$$

$$\left. + (m_b c_\theta^2 U_{ai}^v U_{bi}^{v*} + m_a s_\theta^2 U_{(a+3)i}^v U_{(b+3)i}^{v*}) \right.$$

$$\left. \times \tilde{f}_V(x_{i,W}) \right],$$

$$c_{(ab)R}(W'^+) = \frac{eg_2^2}{32\pi^2 m_{W'}^2} \sum_{i=1}^9 \left[-s_\theta c_\theta U_{ai}^v U_{(a+3)i}^v m_{n_i} \right.$$

$$\left. \times f_V(x_{i,W'}) \right.$$

$$\left. + (m_b s_\theta^2 U_{ai}^v U_{ai}^{v*} + m_a c_\theta^2 U_{(a+3)i}^v U_{(a+3)i}^{v*}) \right.$$

$$\left. \times \tilde{f}_V(x_{i,W'}) \right], \quad (\text{B5})$$

where $x_{i,B} \equiv m_{n_i}^2/m_B^2$ with $B = H^\pm, W, W'$.

B2: Decays $Z \rightarrow e_a^\pm e_b^\mp$

For simplicity, we will use new notations of products of two LFV couplings introducing in Eqs. (1) and (2) that $g_{FBB'}^{XY} \equiv$

$g_{aFB}^{X*} g_{bFB'}^Y$ with $X, Y = L, R$ and $B, B' = V, V', S, S'$ being charged Higgs and gauge boson exchanges in particular diagrams. The corresponding arguments for PV-functions are $(m_a^2, q^2, m_b^2, m_F^2, m_B^2, m_{B'}^2)$, which is used to identify with the LoopTools notations that $C_x = C_x(m_a^2, q^2, m_b^2, m_F^2, m_B^2, m_{B'}^2)$ for $x = 0, i, 00, ij$ ($i, j = 1, 2$), $B_{0,1}^{(1)} = B_{0,1}(m_a^2, m_F^2, m_B^2)$, $B_{0,1}^{(2)} = B_{0,1}(m_b^2, m_F^2, m_{B'}^2)$, and $B_{0,1}^{(12)} = B_{0,1}(q^2, m_B^2, m_{B'}^2)$. The second notation is $g_{BFF'}^{XY} \equiv g_{aFB}^{X*} g_{bFB'}^Y$ with $B = S, V$ corresponding the argument $(m_a^2, q^2, m_b^2, m_F^2, m_B^2, m_{B'}^2)$ for PV-functions used in LoopTools that $C_x = C_x(m_a^2, q^2, m_b^2, m_F^2, m_{B'}^2)$, $B_{0,1}^{(1)} = B_{0,1}(m_a^2, m_B^2, m_{B'}^2)$, $B_{0,1}^{(2)} = B_{0,1}(m_b^2, m_B^2, m_{B'}^2)$, and $B_{0,1}^{(12)} = B_{0,1}(q^2, m_F^2, m_{B'}^2)$. In particular LFVh or LFVZ decays with $q^2 = m_h^2, m_Z^2$, we will pay attention to the last three parameters in the arguments under consideration $(m_F^2, m_B^2, m_{B'}^2)$ or $(m_B^2, m_F^2, m_{B'}^2)$.

Regarding to the LFVZ decay, form factors for one-loop contribution from diagram (1) in Fig. 2 are:

$$\begin{aligned} \bar{a}_L^{FVV'} &= g_{ZVV'} \left\{ g^{LL} \left[(2(2-d) + m_F^2 f) C_{00} \right. \right. \\ &\quad + 2(m_Z^2 - m_a^2 - m_b^2) X_3 \\ &\quad - (f(m_V^2 + m_{V'}^2) + 4) (B_0^{(12)} + m_F^2 C_0) \\ &\quad + \frac{1}{m_V^2} (A_0(m_V) + m_F^2 B_0^{(1)} + m_a^2 B_1^{(1)} \\ &\quad - (m_V^2 - m_{V'}^2 + m_Z^2) m_a^2 C_1) \\ &\quad + \frac{1}{m_{V'}^2} (A_0(m_{V'}) + m_F^2 B_0^{(2)} + m_b^2 B_1^{(2)} \\ &\quad - (-m_V^2 + m_{V'}^2 + m_Z^2) m_b^2 C_2) \Big] \\ &\quad + g^{RR} m_a m_b \left[f (C_{00} + m_{V'}^2 C_2 + m_V^2 C_1) - 2X_3 \right] \\ &\quad - g^{RL} m_a m_F \left[f C_{00} + (2 - f m_{V'}^2) C_0 + f (m_V^2 \right. \\ &\quad \left. - m_{V'}^2) C_1 + \frac{B_0^{(1)} + B_1^{(1)}}{m_V^2} \right] \\ &\quad \left. - g^{LR} m_b m_F \left[f C_{00} + (2 - f m_V^2) C_0 - f (m_V^2 \right. \right. \\ &\quad \left. \left. - m_{V'}^2) C_2 + \frac{B_0^{(2)} + B_1^{(2)}}{m_{V'}^2} \right] \right\}, \end{aligned} \quad (B6)$$

$$\bar{a}_R^{FVV'} = \bar{a}_L^{FVV'} \left[g^{LL} \rightarrow g^{RR}, g^{RL} \rightarrow g^{LR}, g^{LR} \rightarrow g^{RL} \right], \quad (B7)$$

$$\bar{b}_L^{FVV'} = g_{ZVV'} \left\{ g^{LL} m_a \left[4(X_3 - X_1) + f(m_F^2 X_{01} + m_b^2 X_2) \right. \right. \\ \left. \left. - 2(m_{V'}^2 f + 2) C_2 \right] \right.$$

$$\left. + g^{RR} m_b \left[4(X_3 - X_2) + f(m_F^2 X_{02} + m_a^2 X_1) \right. \right. \\ \left. \left. - 2(m_V^2 f + 2) C_1 \right] \right. \\ \left. - g^{RL} m_F \left[f (m_F^2 X_0 + m_a^2 X_1 + m_b^2 X_2 \right. \right. \\ \left. \left. - 2m_V^2 C_1 - 2m_{V'}^2 C_2) + 4X_3 \right] \right. \\ \left. - g^{LR} m_a m_b m_F f X_{012} \right\}, \quad (B8)$$

$$\begin{aligned} \bar{b}_R^{FVV'} &= \bar{b}_L^{FVV'} \left[g^{LL} \rightarrow g^{RR}, g^{RR} \rightarrow g^{LL}, \right. \\ &\quad \left. g^{RL} \rightarrow g^{LR}, g^{LR} \rightarrow g^{RL} \right], \end{aligned} \quad (B9)$$

where $g^{XY} \equiv g_{FVV'}^{XY}$, the arguments for PV-functions are $(m_F^2, m_V^2, m_{V'}^2)$, and

$$f = \frac{(m_Z^2 - m_V^2 - m_{V'}^2)}{m_V^2 m_{V'}^2}.$$

The diagram (2) in Fig. 2 corresponding to the following form factors:

$$\begin{aligned} \bar{a}_L^{VFF'} &= \frac{g^{LL}}{m_V^2} \left\{ g_{ZFF'}^L \left[m_V^2 \left((2-d)^2 C_{00} + 2m_a^2 X_{01} \right. \right. \right. \\ &\quad + 2m_b^2 X_{02} - 2m_Z^2 (C_{12} + X_0) \Big) \\ &\quad - A_0(m_V) - (m_F^2 - m_a^2) B_0^{(1)} - (m_{F'}^2 - m_b^2) B_0^{(2)} \\ &\quad + m_a^2 B_1^{(1)} + m_b^2 B_1^{(2)} \\ &\quad - m_a^2 m_b^2 X_0 - m_F^2 m_{F'}^2 C_0 + m_a^2 m_F^2 (C_0 + C_1) \\ &\quad + m_b^2 m_{F'}^2 (C_0 + C_2) \Big] \\ &\quad - g_{ZFF'}^R m_F m_{F'} \left[2m_V^2 C_0 + (2-d) C_{00} \right. \\ &\quad \left. - m_a^2 (X_1 - C_1) - m_b^2 (X_2 - C_2) + m_Z^2 C_{12} \right] \Big\} \\ &\quad + \frac{g^{RR} g_{ZFF'}^R m_a m_b}{m_V^2} \\ &\quad \times \left[2m_V^2 X_0 - (2-d) C_{00} + m_a^2 X_1 + m_b^2 X_2 \right. \\ &\quad \left. - m_Z^2 C_{12} - m_F^2 C_1 - m_{F'}^2 C_2 \right], \end{aligned} \quad (B10)$$

$$\begin{aligned} \bar{a}_R^{VFF'} &= \bar{a}_L^{VFF'} \left[g^{LL} \rightarrow g^{RR}, g^{RR} \rightarrow g^{LL}, \right. \\ &\quad \left. g_{ZFF'}^L \rightarrow g_{ZFF'}^R, g_{ZFF'}^R \rightarrow g_{ZFF'}^L \right], \end{aligned} \quad (B11)$$

$$\begin{aligned} \bar{b}_L^{VFF'} &= \frac{2g^{LL} m_a}{m_V^2} \left[g_{ZFF'}^L \left(-2m_V^2 X_{01} - m_b^2 X_2 + m_{F'}^2 C_2 \right) \right. \\ &\quad \left. - g_{ZFF'}^R m_F m_{F'} (X_1 - C_1) \right] \\ &\quad + \frac{2g^{RR} m_b}{m_V^2} \left[g_{ZFF'}^R \left(-2m_V^2 X_{02} - m_a^2 X_1 + m_F^2 C_1 \right) \right. \end{aligned}$$

$$-g_{ZF'F'}^L m_F m_{F'} (X_2 - C_2) \Big], \quad (\text{B12})$$

$$\begin{aligned} \bar{b}_R^{VFF'} &= \bar{b}_L^{VFF'} \left[g^{LL} \rightarrow g^{RR}, g^{RR} \rightarrow g^{LL}, \right. \\ &\quad \left. g_{ZF'F'}^L \rightarrow g_{ZF'F'}^R, g_{ZF'F'}^R \rightarrow g_{ZF'F'}^L \right], \end{aligned} \quad (\text{B13})$$

where $g^{XY} \equiv g_{VFF'}^{XY}$ and arguments for PV-functions are $(m_V^2, m_F^2, m_{F'}^2)$.

The sum of two diagrams (7) and (8) gives the following form factors:

$$\begin{aligned} \bar{a}_L^{FV} &= \frac{t_L}{(m_a^2 - m_b^2)m_V^2} \left\{ g^{LL} \left[\left((d-2)m_V^2 + m_F^2 \right) \right. \right. \\ &\quad \times \left(m_a^2 B_1^{(1)} - m_b^2 B_1^{(2)} \right) + m_a^4 B_1^{(1)} - m_b^4 B_1^{(2)} \\ &\quad \left. + (m_a^2 - m_b^2) A_0(m_V) + 2m_F^2 \left(m_a^2 B_0^{(1)} - m_b^2 B_0^{(2)} \right) \right] \\ &\quad + g^{RR} m_a m_b \left[\left(2m_V^2 + m_F^2 \right) \left(B_1^{(1)} - B_1^{(2)} \right) \right. \\ &\quad \left. + m_a^2 B_1^{(1)} - m_b^2 B_1^{(2)} + 2m_F^2 \left(B_0^{(1)} - B_0^{(2)} \right) \right] \\ &\quad \left. + 3 \left(m_a g^{RL} + m_b g^{LR} \right) m_F m_V^2 \left(B_0^{(1)} - B_0^{(2)} \right) \right\}, \\ \bar{a}_R^{FV} &= \bar{a}_L^{FV} \left[t_L \rightarrow t_R, g^{LL} \rightarrow g^{RR}, g^{RR} \rightarrow g^{LL}, \right. \\ &\quad \left. g^{RL} \rightarrow g^{LR}, g^{LR} \rightarrow g^{RL} \right], \\ \bar{b}_L^{FV} &= \bar{b}_R^{FV} = 0, \end{aligned} \quad (\text{B14})$$

where $g^{XY} \equiv g_{FVV}^{XY}$, and $B_{0,1}^{(k)} = B_{0,1}(p_k^2; m_F^2, m_V^2)$.

One-loop form factors from diagram (3) in Fig. 2 are:

$$\begin{aligned} \bar{a}_L^{FVS} &= \frac{g_{SVZ}^*}{m_V^2} \left[g^{LL} m_F (m_V^2 C_0 - C_{00}) \right. \\ &\quad \left. + g^{RL} m_a (m_V^2 C_1 + C_{00}) - g^{LR} m_b m_V^2 C_2 \right], \end{aligned} \quad (\text{B15})$$

$$\bar{a}_R^{FVS} = \bar{a}_L^{FVS} \left[g^{LL} \rightarrow g^{RR}, g^{RL} \rightarrow g^{LR}, g^{LR} \rightarrow g^{RL} \right], \quad (\text{B16})$$

$$\begin{aligned} \bar{b}_L^{FVS} &= \frac{g_{SVZ}^*}{m_V^2} \left[-m_F \left(g^{LL} m_a X_{01} + g^{RR} m_b X_2 \right) \right. \\ &\quad \left. + g^{RL} \left(-2m_V^2 C_1 + m_a^2 X_1 + m_F^2 X_0 \right) \right. \\ &\quad \left. + g^{LR} m_a m_b X_2 \right], \end{aligned} \quad (\text{B17})$$

$$\begin{aligned} \bar{b}_R^{FVS} &= \bar{b}_L^{FVS} \left[g^{LL} \rightarrow g^{RR}, g^{RR} \rightarrow g^{LL}, g^{RL} \rightarrow g^{LR}, \right. \\ &\quad \left. g^{LR} \rightarrow g^{RL} \right], \end{aligned} \quad (\text{B18})$$

where $g^{XY} \equiv g_{FVS}^{XY}$ and arguments for PV-functions are (m_F^2, m_V^2, m_S^2) .

One-loop form factors from diagram (4) in Fig. 2 are:

$$\bar{a}_L^{FSV} = \frac{g_{SVZ}}{m_V^2} \left[g^{LL} m_F (m_V^2 C_0 - C_{00}) \right.$$

$$\left. + g^{RL} m_b (m_V^2 C_2 + C_{00}) - g^{LR} m_a m_V^2 C_1 \right], \quad (\text{B19})$$

$$\bar{a}_R^{FSV} = \bar{a}_L^{FSV} \left[g^{LL} \rightarrow g^{RR}, g^{RL} \rightarrow g^{LR}, g^{LR} \rightarrow g^{RL} \right], \quad (\text{B20})$$

$$\begin{aligned} \bar{b}_L^{FSV} &= \frac{g_{SVZ}}{m_V^2} \left[-m_F \left(g^{RR} m_b X_{02} + g^{LL} m_a X_1 \right) \right. \\ &\quad \left. + g^{LR} \left(-2m_V^2 C_2 + m_b^2 X_2 + m_F^2 X_0 \right) \right. \\ &\quad \left. + g^{RL} m_a m_b X_2 \right], \end{aligned} \quad (\text{B21})$$

$$\begin{aligned} \bar{b}_R^{FSV} &= \bar{b}_L^{FSV} \left[g^{LL} \rightarrow g^{RR}, g^{RR} \rightarrow g^{LL}, g^{RL} \rightarrow g^{LR}, \right. \\ &\quad \left. g^{LR} \rightarrow g^{RL} \right], \end{aligned} \quad (\text{B22})$$

where $g^{XY} \equiv g_{FSV}^{XY}$ and arguments for PV-functions are (m_F^2, m_S^2, m_V^2) .

The contributions from diagrams with pure scalar exchanges were shown previously in Ref. [37, 40]. Particular formulas of the amplitudes are written as follows. Final results for form factors corresponding to diagram (5) are:

$$\begin{aligned} \bar{a}_L^{FSS'} &= -2g_{ZS'^*S} g^{LL} C_{00}, \\ \bar{a}_R^{FSS'} &= -2g_{ZS'^*S} g^{RR} C_{00}, \\ \bar{b}_L^{FSS'} &= -2g_{ZS'^*S} \left[m_a g^{LL} X_1 + m_b g^{RR} X_2 - m_F g^{RL} X_0 \right], \\ \bar{b}_R^{FSS'} &= -2g_{ZS'^*S} \left[m_a g^{RR} X_1 + m_b g^{LL} X_2 - m_F g^{LR} X_0 \right], \end{aligned} \quad (\text{B23})$$

where arguments for PV-functions are $(m_F^2, m_S^2, m_{S'}^2)$, and $g^{XY} = g_{FSS'}^{XY}$.

Forms factors corresponding to diagram (6) are

$$\begin{aligned} \bar{a}_L^{SFF'} &= - \left\{ g_{ZFF'}^L \left[g^{LL} m_F m_{F'} C_0 + g^{RL} m_a m_{F'} (C_0 + C_1) \right. \right. \\ &\quad \left. + g^{LR} m_b m_F (C_0 + C_2) + g^{RR} m_a m_b X_0 \right] \\ &\quad - g_{ZFF'}^R \left[g^{LL} \left((d-2) C_{00} + m_a^2 X_1 \right. \right. \\ &\quad \left. \left. + m_b^2 X_2 - m_Z^2 C_{12} \right) \right. \\ &\quad \left. \left. + m_a m_F g^{RL} C_1 + m_b m_{F'} g^{LR} C_2 \right] \right\} \end{aligned}$$

$$\begin{aligned} \bar{a}_R^{SFF'} &= - \left\{ -g_{ZFF'}^L \left[g^{RR} \left((d-2) C_{00} + m_a^2 X_1 \right. \right. \right. \\ &\quad \left. \left. + m_b^2 X_2 - m_Z^2 C_{12} \right) \right. \\ &\quad \left. + g^{LR} m_a m_{F_1} C_1 + g^{RL} m_b m_{F_2} C_2 \right] \\ &\quad \left. + g_{ZFF'}^R \left[g^{RR} m_F m_{F'} C_0 + g^{LR} m_a m_{F'} (C_0 + C_1) \right. \right. \\ &\quad \left. \left. + g^{RL} m_b m_F (C_0 + C_2) + g^{LL} m_a m_b X_0 \right] \right\}, \end{aligned}$$

$$\bar{b}_L^{SFF'} = -2 \left[g_{ZFF'}^L \left(g^{RL} m_{F'} C_2 + g^{RR} m_b X_2 \right) \right.$$

$$\begin{aligned}
& +g_{ZFF'}^R \left(g^{RL} m_F C_1 + g^{LL} m_a X_1 \right) \Big], \\
\bar{b}_R^{SFF'} = & -2 \left[g_{ZFF'}^L \left(g^{LR} m_F C_1 + g^{RR} m_a X_1 \right) \right. \\
& \left. + g_{ZFF'}^R \left(g^{LR} m_{F'} C_2 + g^{LL} m_b X_2 \right) \right], \quad (B24)
\end{aligned}$$

where $g^{XY} \equiv g_{SFF'}^{XY}$ and arguments for PV-functions are $(m_S^2, m_F^2, m_{F'}^2)$.

Sum of two diagrams (9) and (10) gives the following form factors

$$\begin{aligned}
\bar{a}_L^{FS} = & -\frac{t_L}{m_a^2 - m_b^2} \left[m_F \left(m_a g^{RL} + m_b g^{LR} \right) \left(B_0^{(1)} - B_0^{(2)} \right) \right. \\
& - m_a m_b g^{RR} \left(B_1^{(1)} - B_1^{(2)} \right) \\
& \times \left. - g^{LL} \left(m_a^2 B_1^{(1)} - m_b^2 B_1^{(2)} \right) \right], \\
\bar{a}_R^{FS} = & \bar{a}_L^{FS} \left[t_L \rightarrow t_R, g^{LL} \rightarrow g^{RR}, g^{RR} \rightarrow g^{LL}, \right. \\
& \left. g^{RL} \rightarrow g^{LR}, g^{LR} \rightarrow g^{RL} \right], \\
\bar{b}_L^{FS} = & \bar{a}_R^{FS} = 0, \quad (B25)
\end{aligned}$$

where $g^{XY} \equiv g_{FSS}^{XY}$ and $B_{0,1}^{(k)} = B_{0,1}(p_k^2; m_F^2, m_S^2)$.

B3: One-loop contributions to decays $h \rightarrow e_a^\pm e_b^\mp$

The diagram (1) in Fig. 3 corresponding to the following amplitude:

$$\begin{aligned}
\Delta_L^{FVV'} = & -\frac{g_{hVV'} g^{LL} m_a}{16\pi^2} \left\{ 2C_1 - \frac{1}{m_V^2} \left[B_1^{(12)} \right. \right. \\
& - (m_F^2 - m_a^2)(C_0 + C_1) - (B_0^{(2)} + m_V^2 C_0) \Big] \\
& - \frac{1}{m_{V'}^2} \left[B_1^{(12)} + B_0^{(12)} - (m_F^2 - m_b^2) C_1 \right] \\
& - \frac{1}{2m_V^2 m_{V'}^2} \left[A_0(m_{V'}) + m_F^2 \left(B_0^{(1)} + B_0^{(2)} + B_1^{(1)} \right) \right. \\
& + m_b^2 B_1^{(2)} + (m_V^2 + m_{V'}^2 - q^2) \\
& \left. \left(m_F^2(C_0 + C_1) + m_b^2 C_2 - B_1^{(12)} \right) \right] \Big\} \\
& - \frac{g_{hVV'} g^{RR} m_b}{16\pi^2} \left\{ 2C_2 + \frac{1}{m_V^2} \left[B_1^{(12)} \right. \right. \\
& + (m_F^2 - m_a^2) C_2 \Big] \\
& + \frac{1}{m_{V'}^2} \left[B_1^{(12)} + B_0^{(12)} + (m_F^2 - m_b^2)(C_0 + C_2) \right. \\
& + B_0^{(1)} + m_{V'}^2 C_0 \Big] \\
& \left. - \frac{1}{2m_V^2 m_{V'}^2} \left[A_0(m_V) + m_F^2 \left(B_0^{(1)} + B_0^{(2)} + B_1^{(2)} \right) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& + m_a^2 B_1^{(1)} + (m_V^2 + m_{V'}^2 - q^2) \\
& \left. \left(m_F^2(C_0 + C_2) + m_a^2 C_1 + B_1^{(12)} + B_0^{(12)} \right) \right] \Big\} \\
& - \frac{g_{hVV'} g^{RL} m_F}{32\pi^2 m_V^2 m_{V'}^2} \left\{ 4m_V^2 m_{V'}^2 C_0 + A_0(m_V) \right. \\
& + A_0(m_{V'}) + (m_F^2 - 2m_V^2) B_0^{(1)} \\
& + (m_F^2 - 2m_{V'}^2) B_0^{(2)} + m_a^2 B_1^{(1)} + m_b^2 B_1^{(2)} \\
& + (m_V^2 + m_{V'}^2 - q^2)(B_0^{(12)} + m_a^2 C_1 \\
& + m_b^2 C_2 + m_F^2 C_0) \Big\} \\
& - \frac{g_{hVV'} g^{LR} m_a m_b m_F}{32\pi^2 m_V^2 m_{V'}^2} \left[B_0^{(1)} + B_0^{(2)} + B_1^{(1)} \right. \\
& + B_1^{(2)} + (m_V^2 + m_{V'}^2 - q^2) X_0 \Big], \quad (B26)
\end{aligned}$$

$$\Delta_R^{FVV'} = \Delta_L^{FVV'} \left[g^{LL} \rightarrow g^{RR}, g^{RR} \rightarrow g^{LL}, g^{RL} \rightarrow g^{LR}, \right. \\
\left. g^{LR} \rightarrow g^{RL} \right], \quad (B27)$$

where the coupling notations are $g^{XY} = g_{FVV'}^{XY}$, $q^2 = m_h^2$, and the arguments of PV-functions is $(m_F^2, m_V^2, m_{V'}^2)$.

The diagram (2) in Fig. 3 corresponding to the following form factors:

$$\begin{aligned}
\Delta_L^{VFF'} = & \frac{1}{16\pi^2 m_V^2} \left\{ g^{LL} m_a \left[g_{hFF'}^L m_F \left(B_1^{(1)} \right. \right. \right. \\
& + (2m_V^2 + m_{F'}^2 - m_b^2) C_1) \\
& + g_{hFF'}^R m_{F'} \left(m_V^2 C_0 - B_0^{(12)} + (2m_V^2 + m_F^2 - m_a^2) C_1 \right) \\
& + g^{RR} m_b \left[g_{hFF'}^L m_{F'} \left(B_1^{(2)} + (2m_V^2 + m_F^2 - m_a^2) C_2 \right) \right. \\
& + g_{hFF'}^R m_F \left(m_V^2 C_0 - B_0^{(12)} + (2m_V^2 + m_{F'}^2 - m_b^2) C_2 \right) \\
& - g^{RL} \left[g_{hFF'}^L \left(d \times m_V^2 B_0^{(12)} + 4m_V^2 (m_V^2 C_0 \right. \right. \\
& + m_a^2 C_1 + m_b^2 C_2) \\
& - 2m_V^2 (m_h^2 - m_a^2 - m_b^2) X_0 - A_0(m_V) \\
& - (m_F^2 - m_a^2)(B_0^{(1)} - m_b^2 C_2) - (m_{F'}^2 - m_b^2)(B_0^{(2)} - m_a^2 C_1) \\
& + m_a^2 B_1^{(1)} + m_b^2 B_1^{(2)} - (m_F^2 - m_a^2)(m_{F'}^2 - m_b^2) C_0) \\
& + g_{hFF'}^R m_F m_{F'} \left(3m_V^2 C_0 - B_0^{(12)} \right) \\
& - g^{LR} g_{hFF'}^R m_a m_b \left[(m_F^2 - m_a^2) C_1 \right. \\
& + (m_{F'}^2 - m_b^2) C_2 - m_V^2 C_0 - B_0^{(12)} \Big] \Big], \\
\Delta_R^{VFF'} = & \Delta_L^{VFF'} \left[g_{hFF'}^L \leftrightarrow g_{hFF'}^R, g^{LL} \leftrightarrow g^{RR}, g^{LR} \leftrightarrow g^{RL} \right], \quad (B28)
\end{aligned}$$

where $g^{XY} = g_{VFF'}^{XY}$ and the argument of PV-functions is $(m_V^2, m_F^2, m_{F'}^2)$.

Sum of two diagrams (7) and (8) gives the following form factors:

$$\begin{aligned}\Delta_L^{FV} &= \frac{g\delta_{hee}}{32\pi^2 m_W m_V^2 (m_a^2 - m_b^2)} \\ &\times \left\{ \left(g^{LL} m_b + g^{RR} m_a \right) m_a m_b \right. \\ &\times \left[2m_F^2 \left(B_0^{(2)} - B_0^{(1)} \right) + \left(2m_V^2 + m_F^2 \right) \left(B_1^{(2)} - B_1^{(1)} \right) \right. \\ &\left. + m_b^2 B_1^{(2)} - m_a^2 B_1^{(1)} \right] \\ &+ g^{RL} m_F \left[3m_V^2 \left(m_a^2 B_0^{(2)} - m_b^2 B_0^{(1)} \right) \right. \\ &\left. - \left(m_a^2 - m_b^2 \right) A_0(m_F) \right] \\ &\left. + g^{LR} m_a m_b m_F \times 3m_V^2 \left(B_0^{(2)} - B_0^{(1)} \right) \right\}, \\ \Delta_R^{FV} &= \Delta_L^{FV} \left[g^{LL} \rightarrow g^{RR}, g^{RR} \rightarrow g^{LL}, g^{RL} \rightarrow g^{LR}, \right. \\ &\left. g^{LR} \rightarrow g^{RL} \right],\end{aligned}\quad (B29)$$

where $g^{XY} = g_{FVV}^{XY}$ and $B_{0,1}^{(k)} = B_{0,1}(p_k^2; m_F^2, m_V^2)$ with $k = 1, 2$.

The form factors corresponding to diagram (3) in Fig. 2 are:

$$\begin{aligned}\Delta_L^{FVS} &= \frac{g_V S_h}{16\pi^2 m_V^2} \left\{ m_F \left[g^{LL} m_a \left(B_0^{(1)} + B_1^{(1)} \right) \right. \right. \\ &+ (m_V^2 + m_S^2 - m_h^2) C_0 - (m_V^2 - m_S^2 + m_h^2) C_1 \\ &- g^{RR} m_b \left(2m_V^2 C_0 + (m_V^2 - m_S^2 + m_h^2) C_2 \right) \\ &+ g^{RL} \left[(m_V^2 - m_S^2 + m_h^2) \left(B_0^{(12)} + m_F^2 C_0 \right) \right. \\ &- A_0(m_V) - m_F^2 B_0^{(1)} - m_a^2 B_1^{(1)} \\ &+ \left(m_a^2 (m_V^2 - m_S^2 + m_h^2) - 2m_V^2 (m_h^2 - m_b^2) \right) C_1 \\ &\left. \left. + 2m_V^2 m_b^2 C_2 \right] \right. \\ &\left. - g^{LR} m_a m_b \left[2m_V^2 C_1 + (m_V^2 + m_S^2 - m_h^2) C_2 \right] \right\}, \\ \Delta_R^{FVS} &= \Delta_L^{FVS} \left[g^{LL} \rightarrow g^{RR}, g^{RR} \rightarrow g^{LL}, g^{RL} \rightarrow g^{LR}, \right. \\ &\left. g^{LR} \rightarrow g^{RL} \right],\end{aligned}\quad (B30)$$

where $g^{XY} = g_{FVS}^{XY}$ and the arguments of PV-functions is (m_F^2, m_V^2, m_S^2) .

Final results for form factors corresponding to Diagram (4) in Fig. 3:

$$\begin{aligned}\Delta_L^{FSV} &= \frac{g_V^* S_h}{16\pi^2 m_V^2} \left\{ m_F \left[-g^{LL} m_a \left(2m_V^2 C_0 + (m_V^2 \right. \right. \right. \\ &\left. \left. - m_S^2 + m_h^2) C_1 \right) + g^{RR} m_b \left(B_0^{(2)} + B_1^{(2)} \right) \right. \right. \\ &\left. \left. + g^{RL} m_F \left(m_b^2 B_0^{(1)} - m_a^2 B_0^{(2)} \right) \right] \right. \\ &\left. + g^{LR} m_a m_b m_F \left(B_0^{(2)} - B_0^{(1)} \right) \right\},\end{aligned}$$

$$\begin{aligned}&+ (m_V^2 + m_S^2 - m_h^2) C_0 - (m_V^2 - m_S^2 + m_h^2) C_2 \Big) \Big] \\ &+ g^{RL} \left[(m_V^2 - m_S^2 + m_h^2) (B_0^{(12)} + m_F^2 C_0) \right. \\ &- A_0(m_V) - m_F^2 B_0^{(2)} - m_b^2 B_1^{(2)} \\ &+ 2m_a^2 m_V^2 C_1 + \left(m_b^2 (m_V^2 - m_S^2 + m_h^2) \right. \\ &\left. - 2m_V^2 (m_h^2 - m_a^2) \right) C_2 \Big] \\ &\left. - g^{LR} m_a m_b \left(2m_V^2 C_2 + (m_V^2 + m_S^2 - m_h^2) C_1 \right) \right\}, \\ \Delta_R^{FSV} &= \Delta_L^{FSV} \left[g^{LL} \rightarrow g^{RR}, g^{RR} \rightarrow g^{LL}, g^{RL} \rightarrow g^{LR}, \right. \\ &\left. g^{LR} \rightarrow g^{RL} \right],\end{aligned}\quad (B31)$$

where $g^{XY} = g_{FSV}^{XY}$ and the arguments of PV-functions is (m_F^2, m_S^2, m_V^2) .

Final results for form factors corresponding to Diagram (5) in Fig. 3:

$$\begin{aligned}\Delta_L^{FSS'} &= \frac{\lambda_{hSS'}}{16\pi^2} \left[g^{RL} m_F C_0 - (g^{LL} m_a C_1 + g^{RR} m_b C_2) \right], \\ \Delta_R^{FSS'} &= \Delta_L^{FSS'} \left[g^{LL} \rightarrow g^{RR}, g^{RR} \rightarrow g^{LL}, g^{RL} \rightarrow g^{LR}, \right. \\ &\left. g^{LR} \rightarrow g^{RL} \right],\end{aligned}\quad (B32)$$

where $g^{XY} = g_{FSS'}^{XY}$ and the arguments $(m_F^2, m_S^2, m_{S'}^2)$.

Final results for form factors corresponding to Diagram (6) in Fig. 3:

$$\begin{aligned}\Delta_L^{SFF'} &= \frac{1}{16\pi^2} \\ &\times \left\{ g_{hFF'}^{LL} \left[g^{RL} m_F m_{F'} C_0 + g^{RR} m_F m_b (C_0 + C_2) \right. \right. \\ &+ g^{LL} m_{F'} m_a (C_0 + C_1) + g^{LR} m_a m_b X_0 \Big] \\ &+ g_{hFF'}^{RR} \left[g^{RL} \left(B_0^{(12)} + m_S^2 C_0 + m_a^2 C_1 + m_b^2 C_2 \right) \right. \\ &\left. \left. + g^{LL} m_a m_F C_1 + g^{RR} m_b m_{F'} C_2 \right] \right\}, \\ \Delta_R^{SFF'} &= \Delta_L^{SFF'} \left[g^{LL} \rightarrow g^{RR}, g^{RR} \rightarrow g^{LL}, g^{RL} \rightarrow g^{LR}, \right. \\ &\left. g^{LR} \rightarrow g^{RL} \right],\end{aligned}\quad (B33)$$

where $g^{XY} = g_{SFF'}^{XY}$ and the arguments $(m_S^2, m_F^2, m_{F'}^2)$.

Form factors corresponding to sum of two diagram (9) and (10):

$$\begin{aligned}\Delta_L^{FS} &= \frac{g\delta_{hee}}{32\pi^2 m_W (m_a^2 - m_b^2)} \\ &\times \left[g^{LR} m_a m_b m_F \left(B_0^{(1)} - B_0^{(2)} \right) \right. \\ &\left. + g^{RL} m_F \left(m_b^2 B_0^{(1)} - m_a^2 B_0^{(2)} \right) \right. \\ &\left. + g^{LR} m_a m_b m_F \left(B_0^{(2)} - B_0^{(1)} \right) \right],\end{aligned}$$

$$\begin{aligned}
& -m_a m_b (g^{LL} m_b + g^{RR} m_a) (B_1^{(1)} - B_1^{(2)})], \\
\Delta_R^{FS} = & \Delta_L^{FS} [g^{LL} \rightarrow g^{RR}, g^{RR} \rightarrow g^{LL}, g^{RL} \rightarrow g^{LR}, \\
& g^{LR} \rightarrow g^{RL}], \quad (B34)
\end{aligned}$$

where $g^{XY} = g_{FS}^{XY}$ and $B_{0,1}^{(k)} = B_{0,1}(p_k^2; m_F^2, m_S^2)$ with $k = 1, 2$.

Appendix C: Details calculation to the LRIS model

1: C1: Higgs sector

The model consists of one singly charged Higgs boson, apart from two Goldstone bosons $G_{1,2}^\pm$ absorbed by the two respective gauge bosons $W^\pm$ and $W'^\pm$:

$$\begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \\ \chi_R^\pm \end{pmatrix} = C_{h^\pm}^T \begin{pmatrix} G_1^\pm \\ G_2^\pm \\ H^\pm \end{pmatrix}, \quad C_{h^\pm}^T = \begin{pmatrix} s_\beta s_\xi & c_\beta & c_\xi s_\beta \\ c_\beta s_\xi & -s_\beta & c_\beta c_\xi \\ c_\xi & 0 & -s_\xi \end{pmatrix}, \quad (C1)$$

where $t_\xi \equiv \frac{v c_{2\beta}}{v_R}$, and $C_{h^\pm}$ is the 3×3 unitary matrix satisfying the diagonal relations $C_{h^\pm} \mathcal{M}_+^2 C_{h^\pm}^T = \text{diag}(0, 0, m_{H^\pm}^2)$, and $m_{H^\pm}^2 = \alpha_{32} (c_{2\beta} v^2 + v_R^2 / (2c_{2\beta}))$ with $\alpha_{32} = \alpha_3 - \alpha_2$.

The CP-odd Higgs components generate two Goldstone bosons of Z and Z' , and a physical states A . A relation among these flavor and mass eigenstates is:

$$\begin{pmatrix} a_1 \\ a_2 \\ a_R \end{pmatrix} = C_a \begin{pmatrix} G_1^0 \\ G_2^0 \\ A^0 \end{pmatrix}, \quad C_a = \begin{pmatrix} 0 & -s_\beta & c_\beta \\ 0 & c_\beta & s_\beta \\ 1 & 0 & 0 \end{pmatrix}, \quad (C2)$$

where $m_A^2 = 2v^2(\lambda_3 - 2\lambda_2) + \frac{v_R^2(\alpha_3 - \alpha_2)}{2c_{2\beta}}$

We will summarize the physical spectrum of the Higgs, boson, and leptons based on previous works [52]. Some new conventions will be introduced for convenience.

Regarding to the CP-even Higgs sector. The mass matrix corresponding to the basis $r_H = (r_1, r_2, r_R)^T$, in which $\mathcal{L}_{mass}^H = -\frac{1}{2} r_H^T \mathcal{M}_H^2 r_H$ is:

$$\begin{aligned}
(\mathcal{M}_H^2)_{11} &= 2v^2 (c_\beta^2 \lambda_{23} + 2c_\beta \lambda_4 s_\beta + \lambda_1 s_\beta^2) + \frac{\alpha_{32} c_\beta^2 v_R^2}{2(c_\beta^2 - s_\beta^2)}, \\
(\mathcal{M}_H^2)_{22} &= 2v^2 (c_\beta^2 \lambda_1 + 2c_\beta \lambda_4 s_\beta + s_\beta^2 \lambda_{23}) + \frac{\alpha_{32} s_\beta^2 v_R^2}{2(c_\beta^2 - s_\beta^2)}, \\
(\mathcal{M}_H^2)_{33} &= 2\lambda_5 v_R^2, \\
(\mathcal{M}_H^2)_{12} &= (\mathcal{M}_H^2)_{21}
\end{aligned}$$

$$\begin{aligned}
&= 2v^2 (c_\beta^2 \lambda_4 + c_\beta s_\beta (\lambda_1 + \lambda_{23}) + \lambda_4 s_\beta^2) \\
&\quad - \frac{\alpha_{32} c_\beta s_\beta v_R^2}{2(c_\beta^2 - s_\beta^2)}, \\
(\mathcal{M}_H^2)_{13} &= (\mathcal{M}_H^2)_{31} = v_R v [\alpha_4 c_\beta + s_\beta (\alpha_{12} + \alpha_{32})], \\
(\mathcal{M}_H^2)_{23} &= (\mathcal{M}_H^2)_{32} = v_R v (\alpha_{12} c_\beta + \alpha_4 s_\beta), \quad (C3)
\end{aligned}$$

where $\lambda_{23} = 2\lambda_2 + \lambda_3$. We see that $\text{Det} [\mathcal{M}_H^2]|_{v \rightarrow 0} = 0$, therefore the model consists of at least one neutral CP-even Higgs with mass $\propto v^2$, which can be identified with the SM-like Higgs found experimentally. In particular, in the limit $v = 0$, the transformation C_1 can be used to diagonal the squared mass matrix: $C_1 \mathcal{M}_H^2|_{v=0} C_1^T = \text{diag} \left(0, \frac{\alpha_{32} v_R^2}{2c_{2\beta}}, 2\lambda_5 v_R^2 \right)$, in which

$$\begin{aligned}
C_1 \mathcal{M}_H^2 C_1^T &= \mathcal{M}_{H,0}^2, \quad C_1 = \begin{pmatrix} s_\beta & c_\beta & 0 \\ c_\beta & -s_\beta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\
(\mathcal{M}_{H,0}^2)_{11} &= 2v^2 (\lambda_1 + \lambda_{23} s_{2\beta}^2 + 4\lambda_4 s_{2\beta}), \\
(\mathcal{M}_{H,0}^2)_{22} &= 2c_{2\beta}^2 \lambda_{23} v^2 + \frac{\alpha_{32} v_R^2}{2c_{2\beta}}, \\
(\mathcal{M}_{H,0}^2)_{33} &= 2\lambda_5 v_R^2, \\
(\mathcal{M}_{H,0}^2)_{12} &= (\mathcal{M}_{H,0}^2)_{21} = 2c_{2\beta} v^2 (\lambda_4 + \lambda_{23} s_{2\beta}), \\
(\mathcal{M}_{H,0}^2)_{13} &= (\mathcal{M}_{H,0}^2)_{31} = v_R v (\alpha_{12} + \alpha_4 s_{2\beta} + \alpha_{32} s_\beta^2), \\
(\mathcal{M}_{H,0}^2)_{23} &= (\mathcal{M}_{H,0}^2)_{32} = v_R v (\alpha_4 c_{2\beta} + \frac{\alpha_{32} s_{2\beta}}{2}). \quad (C4)
\end{aligned}$$

It can be seen that $(\mathcal{M}_{H,0}^2)_{11} \propto v^2 \ll v_R^2 \propto (\mathcal{M}_{H,0}^2)_{22}, (\mathcal{M}_{H,0}^2)_{33}$. In addition, non-diagonal entries of $\mathcal{M}_{H,0}^2 \propto v v_R \ll (\mathcal{M}_{H,0}^2)_{22}, (\mathcal{M}_{H,0}^2)_{33}$, therefore, the mixing matrix used to diagonalized $\mathcal{M}_{H,0}^2$ is close to identity. As a result, we will use C_1 as the relation between the flavor states and the physical states (h, h_1, h_2) , in which h is identified with the SM-like Higgs boson: $(r_1, r_2^T, r_R)^T = C_1^T (h, h_1, h_2)^T$. This corresponds to the strict relations of $(\mathcal{M}_{H,0}^2)_{12} = (\mathcal{M}_{H,0}^2)_{13} = (\mathcal{M}_{H,0}^2)_{23} = 0$, equivalently $\lambda_4 = -\lambda_{23} s_{2\beta}, \alpha_{12} = -\alpha_4 s_{2\beta} - \alpha_{32} s_\beta^2$, and $\alpha_4 = -\frac{\alpha_{32} t_{2\beta}}{2}$. As a result, $m_h^2 = 2v^2 (\lambda_1 + \lambda_{23} s_{2\beta}^2 + 4\lambda_4 s_{2\beta})$.

C2: One-loop contributions to decays LFVh and LFVZ

We list here the one-loop contributions from charge Higgs and gauge bosons $H^\pm, W^\pm$ and $W'^\pm$, which contain large contributions to Δa_{e_a} and LFV decay rates.

For LFVh decays, one-loop contributions are collected as follows. Diagram (1) of Fig. 3 give:

$$\Delta_{L,R}^{(1+2+7+8)} = \sum_{i=1}^9 \left[\Delta_{L,R}^{iWW} + \Delta_{L,R}^{iW'W'} + \Delta_{L,R}^{iWW'} + \Delta_{L,R}^{iW'W} + \Delta_{L,R}^{FW} + \Delta_{L,R}^{FW'} \right] + \sum_{i,j=1}^9 \left[\Delta_{L,R}^{Wij} + \Delta_{L,R}^{W'ij} \right], \quad (C5)$$

where $g_{hVV'} = g_{hWW}, g_{hW'W'}, g_{hWW'}$, and $g_{hWW'}$ were given in Table 4; and $\delta_{hee} = m_W/(gv/2)$ derived from Eq. (36). Diagram from singly charged Higgs boson $H^\pm$:

$$\Delta_{L,R}^{(5+6+9+10)} = \sum_{i=1}^9 \left[\Delta_{L,R}^{iH^+H^+} + \Delta_{L,R}^{iH^+} \right] + \sum_{i,j=1}^9 \Delta_{L,R}^{H^+ij}, \quad (C6)$$

Diagram from both exchanges of singly charged Higgs boson $H^\pm$ and gauge boson:

$$\Delta_{L,R}^{(3+4)} = \sum_{i=1}^9 \left(\Delta_{L,R}^{iH^+W} + \Delta_{L,R}^{iWH^+} + \Delta_{L,R}^{iH^+W'} + \Delta_{L,R}^{iW'H^+} \right), \quad (C7)$$

where $g_{VSh} = g_{W^+H^-h}, g_{W'^+H^-h}$ are given in Table 4.

The one-loop contributions to LFVZ decay rates presented in Fig. 2 are collected as follows. The total form factors are sum of all particular contributions as follows

$$\begin{aligned} \bar{a}_{L,R} &= \bar{a}_{L,R}^{(1+2+7+8)} + \bar{a}_{L,R}^{(5+6+9+10)} + \bar{a}_{L,R}^{(3+4)}, \\ \bar{b}_{L,R} &= \bar{b}_{L,R}^{(1+2+7+8)} + \bar{b}_{L,R}^{(5+6+9+10)} + \bar{b}_{L,R}^{(3+4)}, \\ \bar{a}_{L,R}^{(1+2+7+8)} &= \sum_{i=1}^9 \left[\bar{a}_{L,R}^{iWW} + \bar{a}_{L,R}^{iW'W'} + \bar{a}_{L,R}^{iWW'} + \bar{a}_{L,R}^{iW'W} + \bar{a}_{L,R}^{iW} + \bar{a}_{L,R}^{iW'} \right] \\ &\quad + \sum_{i,j=1}^9 \left[\bar{a}_{L,R}^{Wij} + \bar{a}_{L,R}^{W'ij} \right], \\ \bar{b}_{L,R}^{(1+2+7+8)} &= \sum_{i=1}^9 \left[\bar{b}_{L,R}^{iWW} + \bar{b}_{L,R}^{iW'W'} + \bar{b}_{L,R}^{iWW'} + \bar{b}_{L,R}^{iW'W} + \bar{b}_{L,R}^{iW} + \bar{b}_{L,R}^{iW'} \right] \\ &\quad + \sum_{i,j=1}^9 \left[\bar{b}_{L,R}^{Wij} + \bar{b}_{L,R}^{W'ij} \right], \end{aligned} \quad (C8)$$

From singly charged Higgs boson:

$$\begin{aligned} \bar{a}_{L,R}^{(5+6+9+10)} &= \sum_{i=1}^9 \left[\bar{a}_{L,R}^{iH^+H^-} + \bar{a}_{L,R}^{iH^+} \right] + \sum_{i,j=1}^9 \bar{a}_{L,R}^{H^+ij}, \\ \bar{b}_{L,R}^{(5+6+9+10)} &= \sum_{i=1}^9 \left[\bar{b}_{L,R}^{iH^+H^+} + \bar{b}_{L,R}^{iH^+} \right] + \sum_{i,j=1}^9 \bar{b}_{L,R}^{H^+ij}, \end{aligned} \quad (C9)$$

From both Higgs and gauge boson exchanges in diagram (3+4)

$$\begin{aligned} \bar{a}_{L,R}^{(3+4)} &= \sum_{i=1}^9 \left(\bar{a}_{L,R}^{iH^+W} + \bar{a}_{L,R}^{iWH^+} + \bar{a}_{L,R}^{iH^+W'} + \bar{a}_{L,R}^{iW'H^+} \right), \\ \bar{b}_{L,R}^{(3+4)} &= \sum_{i=1}^9 \left(\bar{b}_{L,R}^{iH^+W} + \bar{b}_{L,R}^{iWH^+} + \bar{b}_{L,R}^{iH^+W'} + \bar{b}_{L,R}^{iW'H^+} \right), \end{aligned} \quad (C10)$$

where $g_{SVZ} = g_{H^-W^+Z}, g_{H^-W'^+Z}$ are given in Table 5.

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