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IMPARTING ARBITRARY CORRELATION ON LONGITUDINAL PHASE SPACE USING TRANSVERSE WIGGLERS AND DEFLECTING CAVITIES

Gwanghui Ha*, Northern Illinois University, DeKalb, IL, USA

Abstract

Imparting designed nonlinear correlation on the longitudinal phase space is nontrivial task. While RF cavities operating at different frequencies can generate arbitrary correlation in principle, it is hard to realize such system due to the lack of RF power sources and their costs. We present a new method that may overcome such practical limitation by adopting transverse wigglers and transverse deflecting cavities. Deflecting cavities introduce and eliminate linear correlation between longitudinal and transverse coordinates. We located transverse wigglers, which impart arbitrary correlation on the transverse phase space, where the longitudinal-to-transverse correlation is maximized. In principle, this system only requires deflecting cavities operating in the same frequency and several magnets such as transverse wigglers and quadrupoles.

INTRODUCTION

Recently, a new method has been introduced to impart arbitrary correlation on the transverse phase space [1]. This method utilizes the concept of approximating the desired correlation using a Fourier series or an arbitrary cosine sum. There are several equipment that can impart sinusoidal modulation on the phase space such as accelerating cavity or wakefield structure. However, accelerating structures have significant limitations in tunability. This limitation is a reason why the transverse wiggler is an attractive option for enabling arbitrary correlation [2]. While the transverse wiggler provides tunability for all three control factors (amplitude, period, and phase), it only imparts the modulation on the transverse phase space. Thus, it is necessary to adopt an emittance exchange beamline [3] to deliver the shaped correlation to the longitudinal phase space. However, it may not be preferred due to the exchange of the phase space.

We introduce a new method to impart arbitrary correlation on the longitudinal phase space. This method still utilizes the transverse wiggler but introduces the longitudinal modulation using transverse deflecting cavities (TDC) instead of the EEX beamline. This paper introduces the principle of the method and provides an example numerical test case. We also briefly discuss intrinsic limitations of the method. Although we developed the method to utilize the transverse wiggler for longitudinal control, any transverse modulator can be adopted instead of the transverse wiggler.

SHAPING PRINCIPLE AND NUMERICAL EXAMPLE

Imparting correlation on the longitudinal phase space occurs in several steps. This section describes the principle of the method and provides details of each step. Figure. 1 shows the beamline for imparting the correlation. For a numerical example, we generated a 4D Gaussian beam in x-z plane. Figure. 2 displays its initial state.

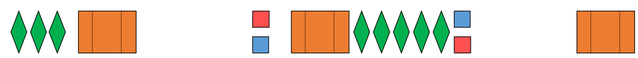

Figure 1: Beamline layout imparting arbitrary correlation. Green diamond, orange box, and red-blue pairs represent quadrupole magnet, TDC, and transverse wiggler, respectively.

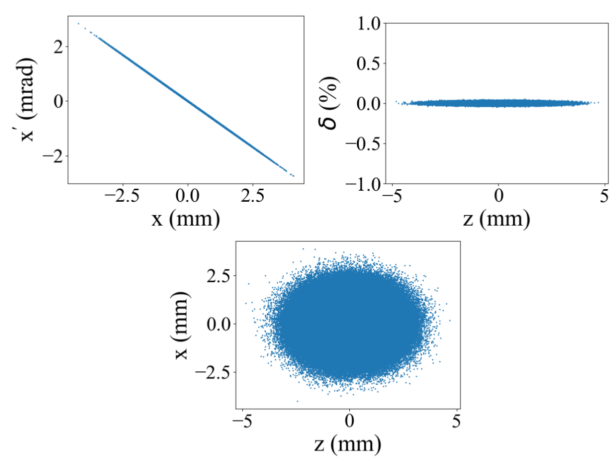

Figure 2: Initial beam images.

The first TDC introduces a coupling between the transverse and longitudinal phase spaces. Operating at the zero-crossing phase, TDC kicks particles transversely based on their arrival time and accelerates/decelerates particles based on their transverse position. This behavior can be found in the transfer matrix of TDC [4],

$$\begin{bmatrix} 1 & L_c & \frac{L_c}{2}\kappa & 0 \\ 0 & 1 & \kappa & 0 \\ 0 & 0 & 1 & 0 \\ \kappa & \frac{L_c}{2}\kappa & \frac{L_c}{4}\kappa^2 & 1 \end{bmatrix}. \tag{1}$$

The $x' - z$ coupling that the first TDC introduced becomes $x - z$ via the following drift section. At this point, the particles' horizontal position will be written as,

$$x_1 = x_0 + (L_c + d_1)x'_0 + (L_c + d_1)\kappa z_0. \tag{2}$$

* gha@niu.edu

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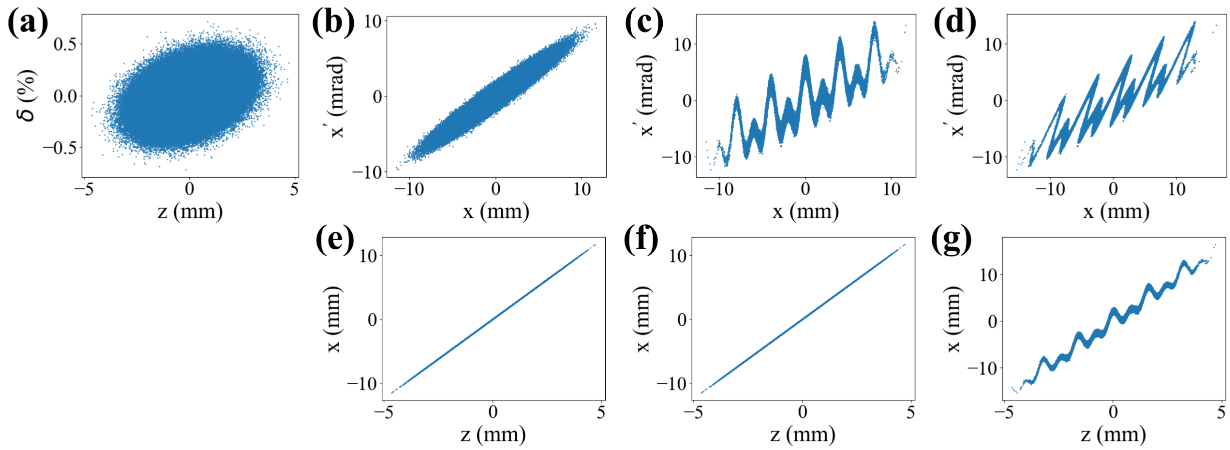

Figure 3: Simulated phase space images. While the first column shows the longitudinal phase space between the first and the second TDC, the remaining columns correspond to the following locations: before the first wiggler, after the wiggler, and before the second TDC.

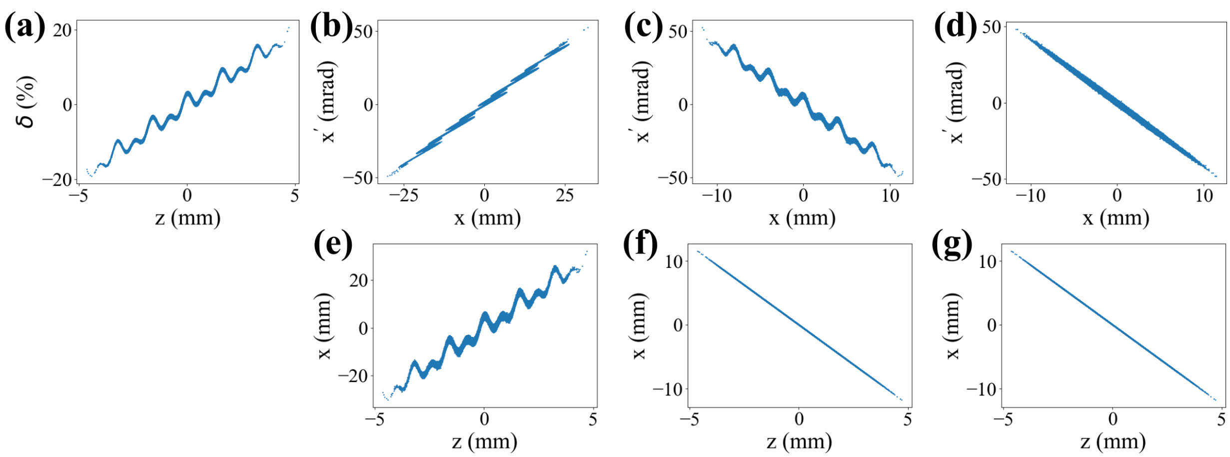

Figure 4: Simulated phase space images. While the first column shows the longitudinal phase space between the second and the third TDC, the remaining columns correspond to the following locations: after the second TDC, after the reverse drift, and after the compensating wiggler.

If we introduce quadrupoles in front of the first TDC as shown in Fig. 1, it is possible to focus the beam, which makes the cancellation of the first two transverse terms. In this way, the initial longitudinal position of the particles determines the horizontal position at this location; see the second column of Fig. 3. Then, the transverse wiggler, providing arbitrary correlation, will impart the correlation on (x_1, x_1'), which is actually (z_0, x_1'). Thus, while the beam in the z-x plane right after the wiggler does not show any changes (see the third column of Fig. 3), we observe the correlation in the z-x plane after the following drift (the fourth column).

So far, we have generated $z - x$ and $z - x'$ correlations. Now, we need to convert these to $z - \delta$ correlation. The second TDC enables this conversion because it introduces acceleration/deceleration based on the particles' horizontal positions. The particles' δ coordinates will be updated based on their horizontal positions. Therefore, $z - x$ correlation we generated earlier also introduces $z - \delta$ correlation after the second TDC as shown in Fig. 4. At this point, the process of imparting arbitrary correlation on the longitudinal phase space is completed. However, there are still remaining $z - x$, $z - x'$, and $x - x'$ correlations and couplings that are unwanted. The rest of the beamline is designed to eliminate these remaining correlations and couplings.

To eliminate the remaining correlations, two elements are placed after the second TDC. Firstly, the beam passes through a quadrupole channel that forms a reverse drift. This quadrupole section provides a transfer matrix for transverse coordinates as,

$$\begin{bmatrix} 1 & -d_r \\ 0 & 1 \end{bmatrix}, \tag{3}$$

which rewinds the beam transport. This process allows the particles' horizontal coordinates at the first wiggler location to be regained at the second wiggler location. The horizontal phase space at the entrance of the second wiggler (the third

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column of Fig. 4) shows the same correlation as the one introduced by the first wiggler, except for the slope and the amplitude. Note that the size can also be altered depending on the optics. Therefore, the second transverse wiggler can remove the correlation by appropriately adjusting its strength and width. Finally, all correlations are removed except for the one in the longitudinal phase space. As demonstrated in TDC-shaping [4], three-TDC configuration with appropriately balanced kick strengths can introduce and remove the transverse-to-longitudinal couplings. Thus, the remaining $z-x$ and $z-x'$ couplings can be eliminated by the third TDC. The final result is displayed in Fig. 5.

These cancellations require certain relationships between beamline parameters, which are summarized below.

$$\kappa_1 + \kappa_2 + \kappa_3 = 0, \tag{4}$$

$$(L_c + d_1 + d_2)\kappa_1 = (L_c + d_3 + d_r)\kappa_3, \tag{5}$$

$$d_r + d_2 + L_c = 0. \tag{6}$$

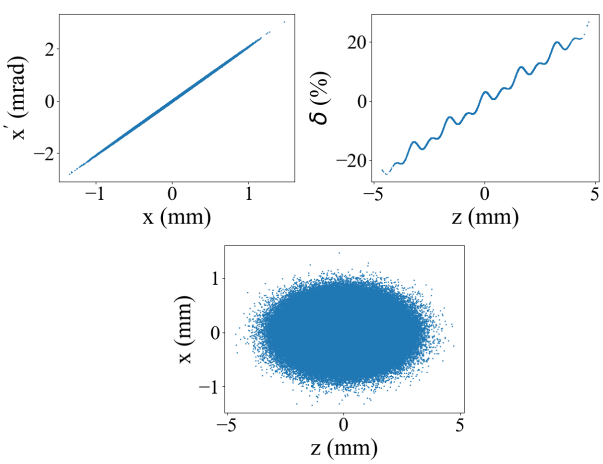

Figure 5: Final beam images.

LIMITATION OF METHOD

While the previously introduced process works perfectly for a zero-emittance beam, non-zero emittance prohibits the exact cancellation of remaining correlations. At the first wiggler location, the particle's horizontal position can be written as,

$$x_{TW1} = \left[x_0 + (L_c + d_1)x_0'\right] + \left(\frac{L_c}{2} + d_1\right)\kappa_1 z_0. \tag{7}$$

On the other hand, the horizontal position at the second wiggler location is,

$$x_{TW2} = \left[x_0 + (L_c + d_1)x_0'\right] + K z_0, \tag{8}$$

where $K = \left(\frac{L_c}{2} + d_1\right)\kappa_1 - \left(\frac{L_c}{2} + d_2\right)\kappa_2$. The coefficients for z_0 are not the same while the transverse terms are identical. Due to this discrepancy, particles will not be located at exactly the same spot. Thus, the correlation can be eliminated when the transverse terms are negligible. Since the beam will be focused transversely, particles will be randomly spread when the transverse terms are not negligible. The magnitude of the spread will depend on the emittance. We have tested a few different initial horizontal emittances for this example. As shown in Fig. 6, emittances larger than 0.5 µm result in clear spreads from the remaining correlation. Note that the emittance response will depend on the beamline design. The spread also depends on what correlation is imparted.

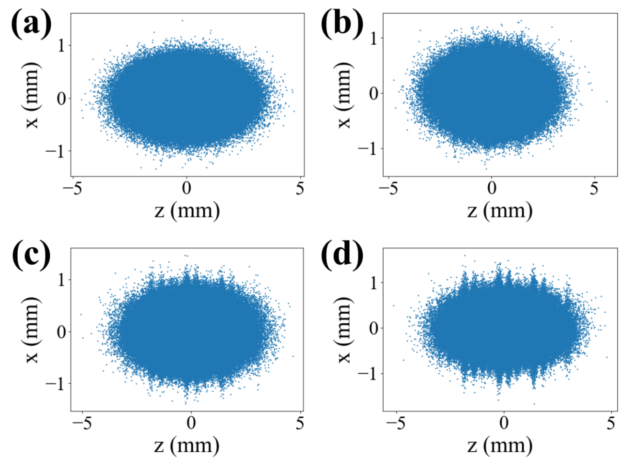

Figure 6: Final $x-x'$ phase spaces with different horizontal emittances.

SUMMARY

We have introduced a new method to impart arbitrary correlation on the longitudinal phase space using transverse wigglers and TDCs. The method introduced transverse-to-longitudinal coupling using TDCs and imparted the desired correlation using transverse wigglers. The electric field of the second TDC converts the $z-x$ correlation imparted by the wiggler into a longitudinal correlation. Any remaining unwanted correlations are then removed by the second transverse wiggler. Reverse drift was adopted to provide a cancellation condition for the second wiggler. The remaining transverse-to-longitudinal coupling is eliminated by the third TDC. While the method enables the generation of arbitrary correlation on the longitudinal phase space, its performance is limited by the emittance. The method leaves unwanted spread in $x-x'$ phase space when the initial transverse emittance is not small enough to make transverse component's contribution ignorable.

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