Hypernuclear density functional theory

H. Lenske, P. Konrad, U. Badarch, C. Keil, and A. Fedoseew

Institut für Theoretische Physik, U. Giessen, Heinrich-Buff-Ring 16, D-35392 Giessen, Germany

Abstract. Hypernuclei and infinite hypermatter are investigated by microscopic density functional theory using our Density Dependent Relativistic Hadron (DDRH) field theory. Based on a meson exchange picture for free space baryon-baryon interactions we derive in-medium interactions by Dirac-Brueckner theory. The results are cast into a field theory with density dependent meson-baryon vertex functionals. Applications to the equation of states of infinite hypermatter and Λ hypernuclei are discussed. Consequences of $\Lambda\Sigma^0$ mixing is explored.

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1 Introduction

A modern approach to nuclear many-body physics should go beyond the isospin sector and include beside protons and neutrons also the hyperons, thus covering all members the lowest baryon octet of hadron physics. Such a broadened view is a necessary prerequisite for understanding in-medium dynamics of the hadrons which appear are the asymptotic mass eigenstates of the fundamental u, d, s quark flavor multiplet. Moreover, in many instances hypernuclei will be the only source on reliable information on YN and YY interactions in cold nuclear matter close to equilibrium. In this respect, hypernuclear physics gives access to investigations of strangeness in an unique environment, not accessible in other reaction scenarios like high energy heavy ion collisions.

Theory is asked to extract from hypernuclear data information on the YY and YN interactions. The primary goal is to determine the strength of the genuine baryon-baryon (BB) interactions and eventually to conclude on the elementary BB-vertices as they would be measured in free space. Such a program requires a sound handling of many-body effects at all levels. Configuration mixing and other correlations in the nuclear wave function are an important issue. Given that those effects are under control, the medium dependence of the interactions themselves remain to be investigated. Over the last years, relativistic density functional theory has proven to be an extremely useful tool to explore the nuclear many-body problem. The main advantage are the built-in non-perturbative aspects because principally density functional theory is formulated in terms of the exact nuclear density, including all features of many-body dynamics. This is known since the ground-breaking work of Kohn and collaborators, culminating in the Hohenberg-Kohn [1] and Kohn-Sham [2] theorems which proof the existence of a density functional for quantum mechanical many-body systems. However, neither of these theorems do provide a method for construction of the appropriate density functional. Hence, we need to introduce models. In order to cover the whole variety of nuclear phenomena, from pure proton-neutron isospin states to the $S = -1$ single Λ hypernuclei and beyond, they should be formulated as general as possible, including the possibility to link with the fundamental BB-interactions.

Naturally, the direct derivation of BB interactions from QCD is the ultimate goal of theoretical hadron and nuclear physics. In fact, first attempts in this direction have been undertaken in the context of Lattice QCD [4, 5]. However, for the foreseeable future this will remain a program for research in progress. Independent of the outcome of LQCD, theories dealing with the effective degrees of freedom of low energy QCD, namely hadrons, are required to understand many-body dynamics in nuclear environments. This is also the area in which our approach is centered by describing BB interactions in a meson exchange picture with phenomenological coupling constants and vertex form factors.

Our approach to in-medium interactions and their manifestation in the DDRH density functional theory is summarized in sect. 2. Results for infinite hypermatter and finite hypernuclei are presented in sect. 3. The paper closes in sect. 4 with a short summary and outlook.

2 Density functional theory for hypernuclei

2.1 Concepts of relativistic density functional theory in nuclear systems

Leaving aside the purely empirical potential models, two classes of theoretical approaches are in use, both having their own advantages and disadvantages. The phenomenological models determine the parameters from direct fits to nuclear and hypernuclear data. This has been done non-relativistically on the basis of the rather successful Skyrme density functional by adding ΛN and $\Lambda\Lambda$ interaction densities [3]. Relativistically, Walecka-type models have been extended into the strangeness sector already some time ago, either using the original formulation [6–8] or including non-linear self-interactions of the scalar field [9]. By the way of derivation both the non-relativistic and the relativistic models describe the available nuclear and hypernuclear data with high precision but typically lead to unreliable or contradictory results when leaving the region covered by the data base.

A microscopic alternative is the Density Dependent Relativistic Hadron Field (DDRH) Theory, developed in recent years at Giessen [15]. The DDRH approach is a relativistic density functional theory incorporating the medium dependence of interactions by density dependent meson-baryon vertex functionals. The variation of the vertex functionals with density is obtained from free space BB interactions by means of Dirac-Brueckner theory. Applications to stable and unstable nuclei [13,14] have led to quite satisfying results for binding energies, charge radii, and separation energies. From a theoretical point of view an important advantage of DDRH theory is the well defined diagrammatic structure which allows us to improve the theory systematically by including additional classes of diagrams, e.g. extending the Brueckner ladder series to loops and ring diagrams. Obviously, phenomenological models do not allow such kind of extensions.

Because of the successful description of pure isospin nuclei it was tempting to extend DDRH theory also into the strangeness sector. There, however, the data base on free space YN and YY interactions is scarce, implying corresponding uncertainties for the BB interactions in nuclear matter. In the naïve flavor counting model one would assume that the strange quark in a hyperon would not couple to non-strange mesons, hence reducing the strength of e.g. the $\sigma\Lambda$ vertex to about $\frac{2}{3}$ of the σN vertex. Even if this might be true on the tree-level the relation has to be expected to change when summing up the full scattering series by means of the Bethe-Salpeter equation. A more detailed analysis of the ladder series has revealed that the in-medium meson-nucleon and the meson-Y vertices are related in leading order by the ratio of the free space vertices, modified next by $SU(3)_f$ symmetry breaking terms. The overall density dependence, however, is almost flavor-independent, only slightly modulated by effects of the order of the Fermi momenta of the various baryonic flavor species to the total baryon Fermi momentum [15,21]. This allows us to simplify hypernuclear calculation considerably, namely by introducing scaling factors $R_{\sigma,\omega\cdots}$ describing the reduction of the YN and YY interactions.

The fact that DDRH theory is based on sound theoretical foundations has initiated applications by several other groups, e.g. [23–26] using the approach on an empirical level. Without any special adjustment of parameters the DDRH results describe convincingly well nuclear data, thus confirming the approach.

2.2 Relativistic DDRH density functional theory

DDRH theory may be considered as the prototype for a relativistic density functional theory. In order to account for the dependence of BB interaction on the nuclear environment we formulate a field theory in terms of mesons and baryons but with interaction vertex functionals depending on the field operators:

$$\mathcal{L}_{BM} \sim \Gamma_{mBB'}(\Psi_B, \overline{\Psi}_B)\overline{\Psi}_B \Gamma_\alpha \Psi_{B'} \Phi^\alpha_M \quad . \tag{1}$$

The functional form of $\Gamma_{mBB'}(\Psi_B, \overline{\Psi}_B)$ is determined by the fundamental symmetries of the theory. For example, in order to conserve Lorentz-invariance of the Lagrangian the vertex functionals must be Lorentz scalars and conservation of isospin symmetry inhibits dependencies on isovector quantities. Other intrinsic symmetries will define further constraints [15]. As a relativistic generalization of the Kohn-Sham and Hohenberg-Kohn theory we follow [16,17] and express the functionals in terms of the square of the total baryon 4-current operator, $j_\mu = \sum_B \overline{\Psi}_B \gamma_\mu \Psi_B$, which in the rest system is given by the total baryon density ρ_B. Thus, the vertices have an intrinsic dynamical structure on their own. The quantum character of the system is retained and we obtain a covariant and thermodynamically consistent theory, as pointed out in refs. [12–15,21,22] .

We obtain an *ab initio* description of nuclear and hypernuclear interactions by combining DDRH theory with DBHF theory [11,13,15]. In the mean-field limit the functionals reduce to density dependent vertices $\Gamma_m(\rho)$, depending on the total baryon vector density $\rho = \rho_B$. Once we have obtained the density dependent DBHF vertices in infinite nuclear matter we construct a relativistic energy density functional from the time-like components of the DDRH energy-momentum tensor. Details are discussed in refs. [15,18]. The dynamical structure of the vertex functionals gives rise to rearrangement self-energies which in the context of relativistic DFT were studied the first time by DDRH theory in [12].

2.3 In-medium interactions

The question arises how to determine the DDRH vertex functionals. Since our aim is to investigate in-medium nuclear dynamics in an *ab initio* manner we choose a fully microscopic approach. For a general description, applicable equally well to free space and in-medium interactions, we use a Lagrangian

$$\mathcal{L} = \mathcal{L}_B + \mathcal{L}_M + \mathcal{L}_{BM} \quad . \tag{2}$$

Above, $\mathcal{L}_B$ and $\mathcal{L}_M$ are the Lagrangians of non-interacting baryons and mesons while their interactions are contained in

$$\mathcal{L}_{BM} \sim g\overline{\Psi}_B \Gamma_\alpha \Psi_B \Phi^\alpha_M \tag{3}$$

including a (set of) coupling constant(s) g Lorentz-invariant combinations of the baryon Ψ_B and meson field Φ_M operators including appropriate Dirac vertices Γ, see e.g. [11,15].

By standard techniques we express the mesonic degrees of freedom in terms of meson-exchange potentials $V_{BB'}$ acting among baryons BB' in the t- und u-channels. They are used to solve the Bethe-Salpeter equation which in momentum space is [15,19]

$$T(q',q|\sqrt{s},k_F) = V(q',q) + \int \frac{d^3k}{(2\pi)^3} V(q',k) g_{NN}(k|\sqrt{s},k_F) \cdot \cdot Q_F(k|\sqrt{s},k_F) T(k,q|\sqrt{s},k_F) \quad . \quad (4)$$

The solutions of eq. 4 include medium effects in the first place via the Pauli-projector Q_F excluding the Fermi-sphere $k \leq k_F$ from the intermediate two-particle states. Eq. 4 has to be solved at the total center of mass energy $\sqrt{s}$, the relative momenta q, q' and given Fermi-momentum k_F. The medium may lead to a mixing among the various baryons, e.g. for protons and neutrons in asymmetric nuclear matter [11] and $\Lambda\Sigma^0$ mixing in hypermatter [29].

Similar to the decomposition of the elementary BB Born-amplitudes we represent the full scattering amplitudes $T(q',q|\sqrt{s}k_F)$ in a basis of meson exchange potentials [15] by the *ansatz*

$$T(q,q'|\sqrt{s},k_F) = \sum_m z_m(\sqrt{s},k_F) V_m(q,q') \quad . \quad (5)$$

This allows us to express the correlations introduced by solving the Bethe-Salpeter equation by *density dependent* vertex renormalization factors $z_m(\sqrt{s},k_F) > 0$, with an additional dependence on the on-shell energy $\sqrt{s}$. For $\rho \to 0$ the $z_m(\sqrt{s},k_F)$ approach the free space vertex factors $\zeta_m(\sqrt{s})$, representing the full BB scattering matrix in terms of the elementary meson exchange amplitudes. In general $\zeta_m(\sqrt{s}) \neq 1$.

After averaging over the $\sqrt{s}$ dependence the above *ansatz* leads to the density dependent in-medium meson-nucleon vertices [11,15]

$$\Gamma_m(k_F) = z_m(k_F) g_m \sim g_m \left(1 - h_m(k_F^2) g_m^2 + \mathcal{O}(g_m^4)\right) \quad , \quad (6)$$

where $h_m(k_F^2)$ is directly related to the underlying Bethe-Salpeter equation.

The DBHF results for $\Gamma_m(k_F)$ are identified as the DDRH vertex functionals in the mean-field limit,

$$\langle 0|\Gamma_m(\overline{\Psi}_B, \Psi_B)|0\rangle \cong \Gamma_m(k_F) + \cdots \quad (7)$$

where we have neglected terms of the order $\langle 0|\overline{\Psi}_B\gamma_0\Psi_B - \rho_B|0\rangle$). The DBHF vertices are of a well defined diagrammatical structure, namely given solely by ladder diagrams [15].

The DBHF/DDRH vertices in the interaction channels contributing to the mean-field have been discussed before [13,15,27]. With the exception of the scalar-isovector channel ($\delta/a_0(980)$ meson) the vertices decrease with increasing density. The different behavior of the δ-meson vertex is caused by the highly non-linear behavior introduced because the scalar meson channels are coupled in a non-linear manner. Results of DDRH calculations in nuclear matter and finite, exotic nuclei, and neutron stars are found e.g. in [12–14,27].

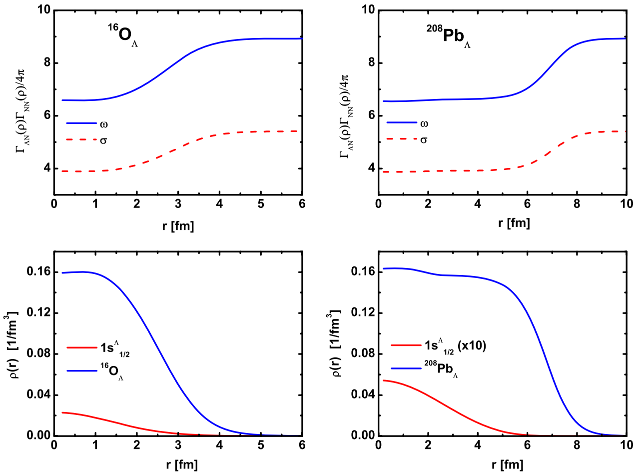

Fig. 1. DDRH vertices, ground state densities and the densities of Λ 1s-states in $^{16}_{\Lambda}O$ and $^{208}_{\Lambda}Pb$, respectively, are shown.

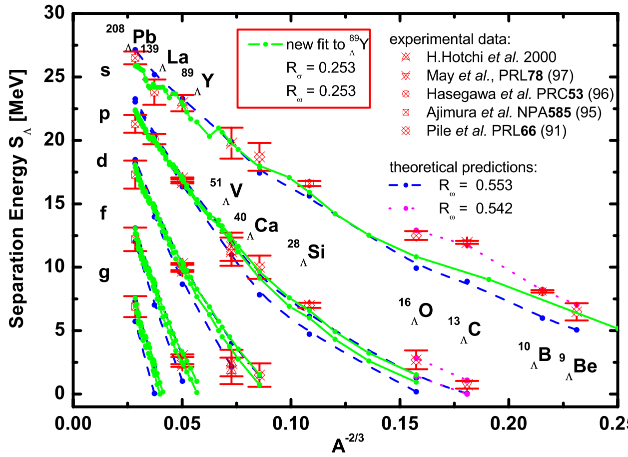

Fig. 2. DDRH results for separation energies of single-Λ hypernuclei.

The density dependence of the DDRH ΛN vertices, varying over the volume of a finite nucleus, is illustrated in Fig. 1 for $^{16}_{\Lambda}O$ and $^{208}_{\Lambda}Pb$, respectively. There, also the total DDRH densities and the density of a Λ in the $1s$ ground state Dirac-orbit are displayed.

3 Applications to hypermatter and hypernuclei

In [15,21,22] the DDRH approach has been extended into the strangeness sector by considering infinite hyper matter and single Λ hypernuclei. Using meson-hyperon vertices obtained in a semi-microscopic approach the available Λ hypernuclear data are well described. Our predictions for the equation of state of infinite hypermatter, a mixture of symmetric nuclear matter with a hyperon fraction $Y = \rho_Y/\rho_B$ are found e.g. in [27]. Compared to pure nuclear matter, the binding energy per particle increases to $B = -18$ MeV and the minimum is shifted to a larger density, $\tilde{\rho}_0 = 0.21 fm^{-3}$ when adding 10% Λ hyperons.

DDRH results of Λ single particle separation energies are compared to the world data base on Λ hypernuclei in Fig. 2. The fit to the older data leads to scaling factors $R_{\sigma,\omega} \sim 0.5$ [21]. However, the newer measurements of Hotchi *et al.* [30] changed that picture considerably. As seen in Fig.2 the more recent KEK data are compatible with scaling factors as low as $R_\sigma = R_\omega = 0.25$. Obviously, the presently available data cannot explain the reason for this strong suppression. However, these small values seem to be in rough agreement with the (non-relativistic) results of Lanskoy *et al.* [3]. A considerable uncertainty and complication is introduced by the high core spin in the nuclei studied in [30].

In hypernuclei with a charge-asymmetric core the isovector mean-fields introduces a mixing of Λ and Σ_0 states via the rho- and delta-meson self-energy parts, $U_{\Lambda\Sigma} = U_\rho - U_\delta$. We describe this phenomenon by coupled equations leading to new in-medium mass eigenstates $|\tilde{\Lambda}\rangle$ and $|\tilde{\Sigma}_0\rangle$ which are superpositions of the free space flavor eigenstates. The mixing results in energy shifts of the Λ levels and a reduced occupation probability of the orbits. In lowest (i.e. second) order of the (reduced) isovector mixing potential the spectroscopic factor is

$$S_\Lambda(\omega) \sim \left[1 + \frac{N-Z}{A} \frac{U^2_{\Lambda\Sigma}}{(e_\Lambda - \omega - \delta M)^2}\right]^{-1} \quad , \qquad (8)$$

where e_Λ denotes the unperturbed mean-field eigenenergy of the unmixed Λ hyperon. Hence, measuring spectroscopic factors in hypernuclei might reveal the mixing probability and give access to the underlying dynamics. Relativistically, $U_{\Lambda\Sigma}$ is given by the scalar (δ) and vector (ρ) iso-vector interactions which implies a genuine dependence on density.

4 Summary and conclusions

DDRH theory has been used to describe nuclei and hypernuclei in terms of relativistic density functional theory. The special feature of DDRH theory is the derivation of the in-medium interactions from DBHF theory, resulting in an *ab initio* description of nuclei and hypernuclei. The separation of the known single-Λ hypernuclei are well described. However, the existing data are not precise enough for a save determination of the ΛN interaction parameters. While here we have discussed subjects mainly related to mean-field phenomena the theory is, in fact, presently extended beyond that level into the region of dynamical correlations, describing the distribution of single particle of baryons in a nuclear medium due to short correlations and the respective spectral functions [28,29]. In [18] special attention is given to the proper definition and derivation of residual interactions using relativistic Fermi Liquid Theory.

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