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Sub-subleading soft gravitons: New symmetries of quantum gravity?

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ABSTRACT

Due to the seminal work of Weinberg, Cachazo and Strominger we know that tree level quantum gravity amplitudes satisfy three factorization constraints. Building on previous works which relate two of these constraints to symmetries of gravity at null infinity, we present strong evidence that the third constraint is also equivalent to a new set of symmetries. Our analysis suggests that the symmetry group of quantum gravity may be richer than the BMS group –or infinite dimensional extension thereof– previously considered.

There has been significant recent progress in our understanding of symmetries associated to quantum gravity in asymptotically flat spacetimes. We now understand that at least at perturbative level, these symmetries contain an infinite dimensional group which is one of two possible extensions of the Bondi–Metzner–Sachs (BMS) group [1] that has long been known to be a symmetry of classical general relativity.[1] The key evidence for these groups come from their relation with certain soft theorems in perturbative quantum gravity. In particular as shown in [3], the statement that supertranslations are symmetries of the gravitational S matrix is encoded in Weinberg's soft graviton theorem [4]. In [5] this idea was extended to the Cachazo–Strominger subleading soft theorem [6] where it was argued it implied a Virasoro symmetry of locally conformal Killing vector fields of the sphere at null infinity [7]. Based on these developments we showed [8] that the subleading soft theorem can alternatively be understood as the statement that the group of diffeomorphisms of the sphere at null infinity is a symmetry of the S matrix.

The soft theorems in themselves are rather fascinating statements. As argued in [6], when in a gravitational scattering process one of the gravitons becomes 'soft' (its energy goes to zero), the tree level scattering amplitude factorizes upto first order in the soft graviton energy E_q:

$$\mathcal{M}_{n+1}(k_1,\dots,k_n;q) = (E_q^{-1}S^{(0)} + S^{(1)} + E_q S^{(2)})\mathcal{M}_n(k_1,\dots,k_n) + O(E_q^2). \quad (1)$$

Since both $S^{(0)}$ and $S^{(1)}$ are associated to (one of the two possible) extensions of BMS, one may wonder if the final factorization term, namely $S^{(2)}$, is also associated to symmetries of quantum gravity. In this paper we present rather strong evidence that this is the case. As summarized below, our strategy involves looking at the problem from a slightly new perspective [9] that includes a dual or 'magnetic' version of the usual charges.

The expansion (1) yields the three equations:

$$\lim_{E_q\to 0} E_q\mathcal{M}_{n+1} = S^{(0)}\mathcal{M}_n \quad (2)$$

$$\lim_{E_q\to 0} \mathcal{M}_{n+1}|_{\text{fin}} = S^{(1)}\mathcal{M}_n \quad (3)$$

$$\lim_{E_q\to 0} E_q^{-1}\mathcal{M}_{n+1}|_{\text{fin}} = S^{(2)}\mathcal{M}_n \quad (4)$$

(in the last two lines one keeps the finite piece and discard terms proportional to E_q^{-1} and E_q^{-2}). Since the emitted soft graviton has two possible polarizations, each of these equations provides two independent identities (per point on the sphere of soft graviton directions). One would like to realize such identities as Ward identities associated to appropriate charges. In [3] it was shown that (2) corresponds to supertranslations Ward identities:

$$\langle \text{out}|[Q_f, S]|\text{in}\rangle = 0, \quad (5)$$

where Q_f is the charge associated to a supertranslation vector field $\xi^a \sim f\partial_u$ and $\langle \text{out}|S|\text{in}\rangle = \mathcal{M}_n(k_1,\dots,k_n)$. Now, since (5) is parametrized by functions on the sphere f, it counts as one identity per point on the sphere. Where is the second identity? In [3] this second identity is associated to certain Christodoulou–Klainerman (CK) condition imposed on the free data [10]. Now, it

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[1] See [2] for the fundamentals of BMS in quantum gravity.

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turns out that this second condition may also be realized as Ward identities of 'dual' supertranslation charges [11]:

$$\langle \text{out}|[Q^*_f, S]|\text{in}\rangle = 0. \tag{6}$$

Here Q^*_f is the 'magnetic' version of Q_f that is obtained by dualizing the Weyl tensor [11,12]. Thus, the two identities contained in (2) are equivalent to the two identities (5) and (6).

In [8] we showed that (3) is equivalent to certain Diff(S^2) Ward identities,

$$\langle \text{out}|[Q_V, S]|\text{in}\rangle = 0, \tag{7}$$

associated to 'generalized BMS' vector fields $\xi^a \sim V^A\partial_A$. In this case the charges are parametrized by arbitrary sphere vector fields V^A and so (7) counts as two identities per sphere point. This was a key point in showing the equivalence with (3) without further CK-type conditions. What about the magnetic version of (7)? It turns out [11] that in this case $Q^*_{V^A} = Q_{\epsilon^A_B V^B}$ and hence no further charges arise (in consistency with the number of independent identities). We finally come to the results presented in this paper. We will show that (4) is equivalent to two identities,

$$\langle \text{out}|[Q_{rX}, S]|\text{in}\rangle = 0, \qquad \langle \text{out}|[\tilde{Q}_{rX}, S]|\text{in}\rangle = 0, \tag{8}$$

associated to vector fields $\xi^a \sim rX^A\partial_A$. The charges are now parametrized by *divergence-free* sphere vector fields X^A and so *each equation corresponds to one identity* (per point on the sphere). We will show that Q_{rX} can be computed by phase space methods in the same way as done for Q_V and Q_f. We currently lack a first principles derivation of $\tilde{Q}_{rX}$. From the structure of the leading and subleading cases in gravity and electromagnetism we expect that $\tilde{Q}_{rX}$ is the magnetic version of Q_{rX}. We will later comment further on this point, whose final clarification is left for future investigations.

We motivate our search for the new symmetry by looking at how soft theorem → Ward identities is accomplished in the known cases. To simplify the analysis we restrict attention to the case where the external particles are massless scalars. Let us for concreteness look at the leading soft theorem (2). Using the relation between the graviton Fock operator and the Fourier transform of the radiative free data [2]:

$$a_-(\omega,\hat{q}) = \sqrt{\gamma}\, 2\pi i\, C^{zz}(\omega,\hat{q}), \tag{9}$$

($\sqrt{\gamma}$ is the area element on the sphere of soft graviton directions $\hat{q}$ parametrized by stereographic coordinates $(z,\bar{z})$) one can rewrite Eq. (2) (for an outgoing negative helicity soft graviton) as:

$$\sqrt{\gamma} 2\pi i \lim_{\omega\to 0} \omega \langle \text{out}|C^{zz}(\omega,\hat{q})S|\text{in}\rangle = S^{(0)-}\langle \text{out}|S|\text{in}\rangle, \tag{10}$$

where $S^{(0)-} = \omega\sum_{i=1}^n \frac{k_i^\mu \epsilon^-_{\mu\nu} k_i^\nu}{k_i\cdot q}$ is a function of the soft graviton direction $\hat{q}$ and the external momenta k_i. On the other hand the 'soft' (linear in C^{zz}) part of the supertranslation charge can be written as [3]:

$$Q^{\text{soft}}_f = i\lim_{\omega\to 0}\,\omega\int d^2z\sqrt{\gamma}\, f D_z^2 C^{zz}(\omega,\hat{q}) + c.c. \tag{11}$$

This motivates one to perform the operation $\int d^2z f D_z^2$ on both sides of (10). The identity

$$\frac{1}{2\pi} D_z^2 S^{(0)-} = -\sum_{i=1}^n E_i \delta^{(2)}(z,z_i) \tag{12}$$

(z_i parametrizes the direction of the i-th external particle with energy E_i) allows one to write the right hand side term as a local function of the external particle momenta and subsequently identify it with the action of the 'hard' (quadratic) part of the supertranslation charge Q^{hard}_f. Thus, by smearing both sides of the soft theorem (10) with $f D_z^2$ one arrives at (5) with $Q_f = Q^{\text{soft}}_f + Q^{\text{hard}}_f$.

Similar strategy applies to the subleading case where the appropriate smearing is $\int d^2z V^z D_z^3$. Whence the way to deduce the asymptotic charge from the soft theorem hinges on smearing both sides of the soft theorems with appropriate tensors. We use the same logic to find asymptotic charges from the sub-subleading soft theorem. We will then show that these charges are associated to certain symmetries of (perturbative) gravity.

In the notation of Eq. (10) the sub-subleading relation (4) for a negative helicity soft graviton takes the form

$$\sqrt{\gamma} 2\pi i \lim_{\omega\to 0} \omega^{-1}\langle \text{out}|C^{zz}(\omega,\hat{q})S|\text{in}\rangle|_{\text{fin}} = S^{(2)-}\langle \text{out}|S|\text{in}\rangle, \tag{13}$$

where $S^{(2)-} = \omega^{-1}\sum_{i=1}^n (2k_i\cdot q)^{-1}(\epsilon^\mu_- q^\nu J^i_{\mu\nu})^2$ is a function of $\hat{q}$ and a differential operator on the external particles. Looking at the smearing employed in the leading and subleading cases, it is natural to attempt a smearing of the form $\int d^2z Y^{zz} D_z^4$. One then finds an identity

$$\frac{1}{2\pi} D_z^4 S^{(2)-} = -3\sum_{i=1}^n E_i^{-1}\delta^{(2)}(z,z_i)\partial^2_{z_i} + \dots \tag{14}$$

in which all terms are proportional to (derivatives) of delta functions. Hence upon smearing with $Y^{zz}D_z^4$ the right hand side of (13) becomes a differential operator that is local in the external momenta. Furthermore, each term may be realized as the action of a hard charge Q^{hard}_Y. Thus, just as in the case of the previous soft theorems by smearing both sides of the sub-subleading theorem with $\int d^2z Y^{zz} D_z^4$ we arrive at a relation of the form

$$\langle \text{out}|[\mathcal{Q}_Y, S]|\text{in}\rangle = 0 \tag{15}$$

where $\mathcal{Q}_Y = \mathcal{Q}^{\text{soft}}_Y + \mathcal{Q}^{\text{hard}}_Y$ with [11]:

$$\mathcal{Q}^{\text{soft}}_Y = \int_{-\infty}^{\infty} du \int_{-\infty}^{u} du' \int d^2z\sqrt{\gamma}\, Y^{zz} D_z^4 C^{zz}(u',\hat{q}) + c.c. \tag{16}$$

$$\mathcal{Q}^{\text{hard}}_Y = -\int_{-\infty}^{\infty} du \int d^2z\sqrt{\gamma}\,\big(3Y^{zz}\partial_z\phi\partial_z\phi - D_z^2 Y^{zz}\phi^2 + 2u\, D_z Y^{zz}\partial_z\phi\,\partial_u\phi + \frac{u^2}{2}D_z^2 Y^{zz}(\partial_u\phi)^2\big) + c.c. \tag{17}$$

The double integral in (16) comes from the ω^{-1} factor in (13). The field ϕ in (17) is the radiative data of the external massless particles.[2] As in the leading and subleading cases, one can also go in the reverse direction by an appropriate choice of Y^{zz} and recover (4) from (15). Note that we have only explicitly shown negative helicity contributions. The positive helicity terms appear in the complex conjugated (c.c.) piece.

Our goal now is to show that such charges are associated to large spacetime diffeomorphisms. At first this may seem impossible as the charges are parametrized by Y^{zz} or equivalently by symmetric, trace-free sphere tensors Y^{AB}. However every such tensor can be written as (symmetric, trace free part of) $D^A X^B$ for some sphere vector field X^A. We will show that for *divergence-free* X^A the charge $\mathcal{Q}_{Y^{AB}=D^A X^B}$ is associated to a spacetime vector field with a leading $O(r)$ component $\xi^a \sim rX^A\partial_A$. This however captures

[2] Throughout the paper we assume $C_{AB} = O(u^{-2-\epsilon})$ and $\phi = O(u^{-1/2-\epsilon})$ at $u \to \pm\infty$ to ensure convergence of u integrals.

only 'half' of the $\mathcal{Q}_Y$ charges, the remaining half being labelled by $Y^{AB} = \epsilon^B_C D^A X'^C$ with X'^A divergence-free. This is the second charge alluded to in Eq. (8), namely

$$\tilde{Q}_{rX'} := \mathcal{Q}_{\epsilon^B_C D^A X'^C}. \tag{18}$$

In short, using the splitting $Y^{AB} = D^A X^B + \epsilon^B_C D^A X'^C$ (with X^A, X'^A divergence-free), the charges $\mathcal{Q}_Y$ are reinterpreted as a pair of charges Q_{rX} and $\tilde{Q}_{rX'}$.[3]

As generalized BMS symmetries are known to be equivalent to leading and subleading soft graviton theorems we know that we need a genuine extension of this group. Looking for such an extension is subtle in Bondi gauge as generalized BMS appear to exhaust all such symmetries as far as smooth diffeomorphisms are concerned [8]. Whence we look for such an extension in de Donder gauge. That is, we look for vector fields on flat spacetime which satisfy the wave equation

$$\Box \xi^a = 0. \tag{19}$$

The computation of asymptotic charges associated to symmetries in de Donder gauge also brings a nice structural coherence to the entire program. As the soft theorems are usually formulated in de Donder gauge as opposed to Bondi gauge, our analysis has a nice corollary which shows that the "Ward identities ≡ soft theorem" can be formulated in de Donder gauge for all generators of the generalized BMS group [11]. In the present case, taking a cue from the large gauge transformations in QED which give rise to the subleading theorem [9] we look for large diffeomorphism generators such that the $O(r^0)$ component of ξ^A is linear in u.

It turns out that a self-consistent asymptotic solution of (19) compatible with the prescribed boundary behavior is given by

$$\begin{aligned} \xi^A &= rX^A + \frac{u}{4}(\Delta + 5)X^A + O(r^{-1}) \\ \xi^u &= O(r^{-1}), \quad \xi^r = O(r^{-1}), \end{aligned} \tag{20}$$

with X^A a divergence-free, u-independent sphere vector field that plays the role of 'independent data' in terms of which the remaining components are determined. The form of the solution (20) ensure the asymptotic charges satisfy certain regularity conditions detailed below. The vector field shares with generalized BMS generators the property of being asymptotically divergence free, $\nabla_a \xi^a \to 0$ [8].

It is important to note that at this stage we do not understand in what sense these large gauge transformations are symmetries of asymptotically flat spacetimes. Due to their diverging behavior at infinity, they naively do not seem to preserve asymptotic flatness. However as the Ward identities associated to their charges capture the sub-subleading soft theorem, we believe there should be a characterization of these large gauge transformations as symmetries of the theory. We leave this important question for future investigation.

We now proceed to compute the associated charges and show that they precisely yield the charge obtained from the sub-subleading theorem. The computation of charges is best done via covariant phase space techniques [14]. Instead of considering pure gravity (for which the sub-subleading theorem is originally derived) we consider gravity coupled to massless scalar field as it simplifies the analysis.

[3] The situation is analogous to the subleading case in QED where the charges are parametrized by vector fields Y^A [13]. For $Y^A = D^A\mu$ the charge is associated to $O(r)$ large gauge transformations with leading piece $r\mu$. The magnetic dual of such charge is associated to $Y^A = \epsilon^{AB} D_B \mu$ [9].

In the context of tree-level amplitudes we are interested, it suffices to consider the phase space of linearized gravity coupled to the massless scalar field. Given a symmetry generator ξ^a, its associated charge has two contributions. One contribution comes from the matter phase space and is given by

$$Q^{\text{matter}}[\xi] = -\lim_{t\to\infty} \int_{\Sigma_t} d^3V\, T^t_{\ b} \xi^b, \tag{21}$$

where Σ_t is a $t =$ constant hypersurface approaching null infinity and $T^a_{\ b}$ the stress tensor of the scalar field. The other contribution comes from the gravitational phase space and is given by

$$\delta Q^{\text{grav}}[\xi] = \lim_{t\to\infty} \delta_\xi \theta_t(\delta) - \delta\theta_t(\delta_\xi) \tag{22}$$

where $\theta_t(\delta) = \frac{1}{2}\int_{\Sigma_t} d^3V\, \Gamma^t_{bc}\delta h^{bc}$ is the symplectic potential in linearized gravity.

As shown in [11], the computation of such a charge requires determining the linearized metric which is sourced by the matter stress tensor. As we are working in de Donder gauge, we need to analyze solutions to linearized Einstein's equations

$$\Box \overline{h}_{ab} = -2T_{ab} \tag{23}$$

where $\overline{h}_{ab} = h_{ab} - \frac{1}{2}\eta^{mn}h_{mn}h_{ab}$.

A solution to this equation can be written as $\overline{h}_{ab} = \overline{h_1}_{ab} + \overline{h_2}_{ab}$ where h_1, h_2 satisfy

$$\begin{aligned} \Box \overline{h_1}_{ab} &= 0 \\ \Box \overline{h_2}_{ab} &= -2T_{ab}. \end{aligned} \tag{24}$$

Here h_1 is the linearized metric which is determined by the radiative data C_{AB} at null infinity and h_2 is the linearized metric which is sourced by the matter and is independent of the radiative gravitational data.

The gravitational contribution to the charge is hence given by

$$Q^{\text{grav}}[\xi] = Q^{\text{grav}}_{\text{soft}}[\xi] + Q^{\text{grav}}_{\text{hard}}[\xi] \tag{25}$$

where

$$Q^{\text{grav}}_{\text{soft}}[\xi] = \frac{1}{2}\lim_{t\to\infty} \int d^3V \left(\Gamma^t_{ab}[h_1]\delta_\xi h^{ab} - \delta_\xi \Gamma^t_{ab} h_1^{ab}\right) \tag{26}$$

$$Q^{\text{grav}}_{\text{hard}}[\xi] = \frac{1}{2}\lim_{t\to\infty} \int d^3V \left(\Gamma^t_{ab}[h_2]\delta_\xi h^{ab} - \delta_\xi \Gamma^t_{ab} h_2^{ab}\right). \tag{27}$$

The 'soft' piece is linear in the radiative gravitational mode C_{AB}. The 'hard' piece is linear in the matter stress tensor and adds to the contribution coming from the matter phase space (21).

Collecting all terms one finds the resulting charge is divergent. However the nature of the divergent terms points to a natural prescription for obtaining the finite charge. More in detail, one finds

$$Q[\xi] = \lim_{t\to\infty} \left(t\, Q^{(1)}[\xi] + Q^{(0)}[\xi]\right), \tag{28}$$

with $Q^{(1)}[\xi]$ the charge associated to generalized BMS sphere vector fields. Thus, extracting the finite piece in (28) amounts to discarding the contributions from the subleading soft gravitons (see [9] for similar prescription in QED). It is at this stage that the form (20) of ξ^a is crucial: Other vector fields satisfying (19) yield divergent contributions that cannot be associated to generalized BMS charges.

The total, finite charge associated to ξ^a is finally given by $Q_\xi := Q^{(0)}[\xi] = Q^{\text{hard}}_\xi + Q^{\text{soft}}_\xi$ with [11]:

$$Q_{\xi}^{\text{soft}} = \frac{1}{16}\int du\, d^2V\, s^{AB}(\hat{x}) \int_{-\infty}^{u} C_{AB}(u', \hat{x})du', \quad (29)$$

$$Q_{\xi}^{\text{hard}} = \frac{1}{4}\int du\, d^2V \left(3D^A X^B T_{AB}^{(-2)} + u(\Delta X^A + X^A)T_{uA}^{(-2)}\right), \quad (30)$$

where:

$$s^{AB} = \Delta^2 D^A X^B - 6\Delta D^A X^B + 8D^A X^B, \quad (31)$$

and $T_{AB}^{(-2)} = (\partial_A \phi\, \partial_B \phi)^{\text{TF}}$, $T_{uA}^{(-2)} = \partial_u \phi\, \partial_A \phi$ are leading terms of the stress tensor. Upon the identification $Y^{AB} = -\frac{1}{4}D^A X^B$ and using $D_A X^A = 0$ one finds that (29) and (30) exactly match the respective charges (16) and (17) that were obtained from the sub-subleading theorem.

As the large gauge transformations are parametrized by divergence free-vector fields on the sphere it corresponds to one factorization theorem for each direction of soft graviton as opposed to the two factorization theorems given in Eq. (4). Whence we are missing "half" of the Ward identities which would correspond to the remaining half of the sub-subleading theorem. It is here that we take a motivation from [9] where it is shown that a single large gauge transformation gives rise to both sub-leading relations (for two photon helicities) in massless QED. This is due to the fact that the magnetic and electric charge for a large gauge transformation are unequal and their Ward identities are equivalent to the subleading soft photon theorem. Whence a couple of questions naturally arise: (i) Is the charge Q_{ξ} we have computed analogous to the electric charge in the case of QED? and if it is (ii) What is the corresponding magnetic charge? There are strong reasons to believe that the answer to the first question is in the affirmative due to a reinterpretation of generalized BMS charges presented in [11]. As shown there, these charges can be obtained from the "electric" part of the Weyl tensor. Electric and magnetic part of the Weyl tensor whose leading piece contains information about radiative mode can be defined as[4]

$$\mathcal{E}^a_b := r\, C^{at}{}_{br}, \qquad \mathcal{B}^a_b := r * C^{at}{}_{br}. \quad (32)$$

Using these tensors, the generalized BMS charges can be obtained as [11]:

$$Q_{\mathcal{E}}[\xi] = \lim_{t\to\infty} \int_{\Sigma_t} d^3V\, \partial_a(\mathcal{E}^a_b \xi^b). \quad (33)$$

The 'magnetic' dual charges are then defined by replacing $\mathcal{E}^a_b$ with $\mathcal{B}^a_b$ in (33). Thus for a supertranslation vector field $\xi^a_f \sim f\partial_u$, $Q_{\mathcal{E}}[\xi_f]$ reproduces the usual supertranslation charge. The corresponding magnetic charge turns out to be[5]:

$$Q_{\mathcal{B}}[\xi_f] = \int du \int d^2V f \epsilon^{AB} D_A D^M \partial_u C_{MB}. \quad (34)$$

The charge is linear in the graviton and putting it equal to zero precisely gives the so-called CK condition used in [3]. For the remaining generators of generalized BMS, $\xi^a_V \sim V^A\partial_A$, it can also be shown that the electric charges match with those obtained in [8]. Whence we expect that the charge associated to $\xi^a \sim rX^A\partial_A$ computed above can be obtained as an electric charge whose magnetic counterpart provides the 'remaining' information of the $\mathcal{Q}_Y$ charges, Eq. (18). However a detailed proof of this statement remains outside the scope of this work.

[4] This is not the standard definition of $\mathcal{E}$ and $\mathcal{B}$ but it contains complete information of the Weyl tensor and at null infinity has trivial projection in the outgoing null direction.

[5] This charge was first obtained in [12] by conformal methods.

We thus believe to have provided enough evidence that the extensions of the BMS algebra previously considered in the literature are not the end of the story. The sub-subleading theorem of tree level quantum gravity amplitudes suggests the existence of a further extension of such algebras to a potentially larger symmetry. However in what sense this extension is a symmetry of asymptotically flat spacetimes and hence whether sub-subleading soft gravitons can also be understood as Goldstone modes of a spontaneously broken symmetry remains to be seen.

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References

[1] H. Bondi, M.G.J. van der Burg, A.W.K. Metzner, Gravitational waves in general relativity. 7. Waves from axisymmetric isolated systems, Proc. R. Soc. Lond. A 269 (1962) 21;
R.K. Sachs, Gravitational waves in general relativity. 8. Waves in asymptotically flat space–times, Proc. R. Soc. Lond. A 270 (1962) 103.
[2] A. Ashtekar, Asymptotic quantization of the gravitational field, Phys. Rev. Lett. 46 (1981) 573;
A. Ashtekar, M. Streubel, Symplectic geometry of radiative modes and conserved quantities at null infinity, Proc. R. Soc. Lond. A 376 (1981) 585;
A. Ashtekar, Radiative degrees of freedom of the gravitational field in exact general relativity, J. Math. Phys. 22 (1981) 2885;
A. Ashtekar, Asymptotic Quantization, Bibliopolis, Naples, Italy, 1987.
[3] T. He, V. Lysov, P. Mitra, A. Strominger, BMS supertranslations and Weinberg's soft graviton theorem, J. High Energy Phys. 1505 (2015) 151.
[4] S. Weinberg, Infrared photons and gravitons, Phys. Rev. 140 (1965) B516.
[5] D. Kapec, V. Lysov, S. Pasterski, A. Strominger, Semiclassical Virasoro symmetry of the quantum gravity S-matrix, J. High Energy Phys. 1408 (2014) 058.
[6] F. Cachazo, A. Strominger, Evidence for a new soft graviton theorem, arXiv:1404.4091 [hep-th].
[7] G. Barnich, C. Troessaert, Symmetries of asymptotically flat 4 dimensional spacetimes at null infinity revisited, Phys. Rev. Lett. 105 (2010) 111103, arXiv:0909.2617 [gr-qc];
G. Barnich, C. Troessaert, Aspects of the BMS/CFT correspondence, J. High Energy Phys. 1005 (2010) 062, arXiv:1001.1541 [hep-th].
[8] M. Campiglia, A. Laddha, Asymptotic symmetries and subleading soft graviton theorem, Phys. Rev. D 90 (12) (2014) 124028;
M. Campiglia, A. Laddha, New symmetries for the gravitational S-matrix, J. High Energy Phys. 1504 (2015) 076.
[9] M. Campiglia, A. Laddha, Subleading soft photons and large gauge transformations, J. High Energy Phys. 1611 (2016) 12.
[10] A. Strominger, On BMS invariance of gravitational scattering, J. High Energy Phys. 1407 (2014) 152.
[11] M. Campiglia, A. Laddha, Sub-subleading soft gravitons and large diffeomorphisms, arXiv:1608.00685.
[12] A. Ashtekar, A. Sen, NUT 4-momenta are forever, J. Math. Phys. 23 (1982) 2168.
[13] V. Lysov, S. Pasterski, A. Strominger, Low's subleading soft theorem as a symmetry of QED, Phys. Rev. Lett. 113 (11) (2014) 111601.
[14] A. Ashtekar, L. Bombelli, O. Reula, The covariant phase space of asymptotically flat gravitational fields, in: M. Francaviglia (Ed.), Analysis, Geometry and Mechanics: 200 Years After Lagrange, North-Holland, 1991.