Physics Letters B 772 (2017) 16–20

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Hadronic structure functions in the $e^+e^- \to \bar{\Lambda}\Lambda$ reaction

Göran Fäldt *, Andrzej Kupsc

Division of Nuclear Physics, Department of Physics and Astronomy, Uppsala University, Box 516, 75120 Uppsala, Sweden

ARTICLE INFO

Article history:
Received 28 February 2017
Received in revised form 16 April 2017
Accepted 4 June 2017
Available online 8 June 2017
Editor: J.-P. Blaizot

Keywords:
Hadron production in e^-e^+ interactions
Hadronic decays

ABSTRACT

Cross-section distributions are calculated for the reaction $e^+e^- \to J/\psi \to \bar{\Lambda}(\to \bar{p}\pi^+)\Lambda(\to p\pi^-)$, and related annihilation reactions mediated by vector mesons. The hyperon-decay distributions depend on a number of structure functions that are bilinear in the, possibly complex, psionic form factors G_M^ψ and G_E^ψ of the Lambda hyperon. The relative size and relative phase of these form factors can be uniquely determined from the unpolarized joint-decay distributions of the Lambda and anti-Lambda hyperons. Also the decay-asymmetry parameters of Lambda and anti-Lambda hyperons can be determined.

1. Introduction

Two hadronic form factors, commonly called $G_M(s)$ and $G_E(s)$, are needed for the description of the annihilation process $e^-e^+ \to \Lambda\bar{\Lambda}$, Fig. 1a, and by varying the c.m. energy $\sqrt{s}$, their numerical values can in principle be determined for all s values above $\Lambda\bar{\Lambda}$ threshold. For the general case of annihilation via an intermediate photon, the joint $\Lambda(\to p\pi^-)\bar{\Lambda}(\to \bar{p}\pi^+)$ decay distributions were calculated and analyzed in Ref. [1], using methods developed in [2,3]. Recently, a first attempt to calculate the hyperon form factors $G_M(s)$ and $G_E(s)$ in the time-like region was reported in Ref. [4].

Previously, the interesting special case of annihilation through an intermediate J/ψ or $\psi(2S)$, Fig. 1b, has been investigated in several theoretical [5,6] and experimental papers [7–9]. This process has also been used for determination of the anti-Lambda decay-asymmetry parameter and for CP symmetry tests in the hyperon system. A precise knowledge of the Lambda decay-asymmetry parameter is needed for studies of spin polarization in Ω^-, Ξ^-, and Λ_c^+ decays.

Presently, a collected data sample of 1.31×10^9 J/ψ events [10] by the BESIII detector [11] permits high-precision studies of spin correlations.

In the experimental work referred to above, the joint-hyperon-decay distributions considered are not the most general ones possible, but seem to be curtailed. Incomplete distribution functions do not permit a reliable determination of the form factors and we therefore suggest to fit the experimental data to the general distribution described in [1], and further elaborated below.

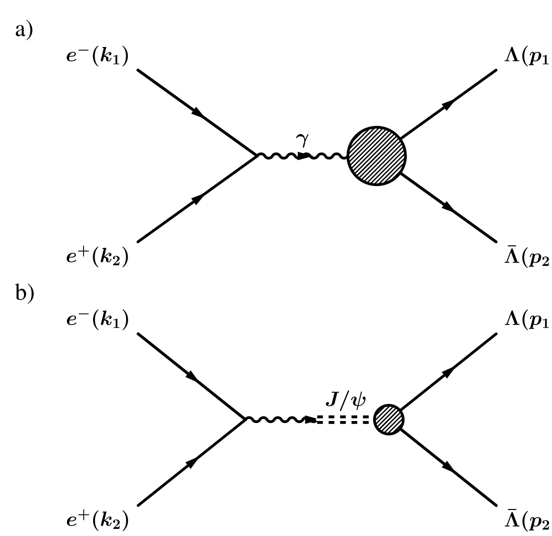

Fig. 1. Graph describing the reaction $e^+e^- \to \bar{\Lambda}\Lambda$; a) general case, and b) mediated by the J/ψ resonance.

Since the photon and the J/ψ are both vector particles, their corresponding annihilation processes will be similar. In fact, by a simple substitution, the cross-section distributions in Ref. [1], valid in the photon case, are transformed into distributions valid in the J/ψ case, but expressed in the corresponding psionic form factors G_M^ψ and G_E^ψ.

* Corresponding author.
E-mail addresses: goran.faldt@physics.uu.se (G. Fäldt), andrzej.kupsc@physics.uu.se (A. Kupsc).

http://dx.doi.org/10.1016/j.physletb.2017.06.011

In order to specify events and compare measured data with theoretical predictions, we need distribution functions expressed in some specific coordinate system. For this purpose we employ the coordinate system introduced in [1]. Many investigations employ different coordinate systems for the Lambda and anti-Lambda decays, a custom which in our opinion can lead to confusion.

Our calculation is performed in two steps. After some preliminaries we turn to the inclusive process of lepton annihilation into polarized hyperons. The results obtained are the starting point for the calculation of exclusive annihilation, i.e. the distribution for the hyperon-decay products. Our method of calculation consists in multiplying the hyperon-production distribution with the hyperon-decay distributions, averaging over intermediate hyperon-spin directions. The method is referred to as folding.

2. Basic necessities

Resolving the hyperon vertex in Fig. 1a uncovers a number of contributions. The one of interest to us is described by the diagram of Fig. 1b, whereby the photon interaction with the hyperons is mediated by the J/ψ vector meson, and the coupling of the initial-state leptons to the J/ψ related to the decay $J/\psi \to e^+e^-$.

For a J/ψ decay through an intermediate photon, tensor couplings can be ignored. Thus, the effective coupling of the J/ψ to the leptons is the same as that for the photon, provided we replace the electric charge e_{em} by a coupling strength e_ψ,

$$\Gamma^e_\mu(k_1,k_2) = -ie_\psi \gamma_\mu, \tag{2.1}$$

with e_ψ determined by the $J/\psi \to e^+e^-$ decay (see Appendix A).

At the J/ψ-hyperon vertex two form factors are possible and they are both considered. We follow Ref. [1] in writing the hyperon vertex as

$$\Gamma^\Lambda_\mu(p_1,p_2) = -ie_g\Big[G^\psi_M \gamma_\mu - \frac{2M}{Q^2}(G^\psi_M - G^\psi_E)Q_\mu\Big], \tag{2.2}$$

with $P = p_1 + p_2$, and $Q = p_1 - p_2$, and M the Lambda mass. The argument of the form factors equals $s = P^2$. The coupling strength e_g in Eq. (2.2) is determined by the hadronic-decay rate for $J/\psi \to \Lambda\bar{\Lambda}$ (see Appendix A).

In Ref. [1] polarizations and cross-section distributions were expressed in terms of structure functions, themselves functions of the form factors G^ψ_M and G^ψ_E. Here, we shall introduce combinations of form factors called D, α, and $\Delta\Phi$, which are employed by the experimental groups [7–9] as well.

The strength of form factors is measured by $D(s)$,

$$D(s) = s\left|G^\psi_M\right|^2 + 4M^2\left|G^\psi_E\right|^2, \tag{2.3}$$

a factor that multiplies all cross-section distributions. The ratio of form factors is measured by α,

$$\alpha = \frac{s\left|G^\psi_M\right|^2 - 4M^2\left|G^\psi_E\right|^2}{s\left|G^\psi_M\right|^2 + 4M^2\left|G^\psi_E\right|^2}, \tag{2.4}$$

with α satisfying $-1 \le \alpha \le 1$. The relative phase of form factors is measured by $\Delta\Phi$,

$$\frac{G^\psi_E}{G^\psi_M} = e^{i\Delta\Phi}\left|\frac{G^\psi_E}{G^\psi_M}\right|. \tag{2.5}$$

The diagram of Fig. 1 represents a J/ψ exchange of momentum P. J/ψ being a vector meson, its propagator takes the form

$$\frac{g_{\mu\nu} - P_\mu P_\nu/m^2_\psi}{s - m^2_\psi + im_\psi\Gamma(\psi)}, \tag{2.6}$$

where m_ψ is the J/ψ mass, and $\Gamma(\psi)$ the full width of the J/ψ. However, since the J/ψ couples to conserved lepton and hyperon currents, the contribution from the $P_\mu P_\nu$ term vanishes. In conclusion, the matrix element for e^+e^- annihilation through a photon will be structurally identical to that for annihilation through a J/ψ provided we make the replacement

$$\frac{e_\psi e_g}{s - m^2_\psi + im^2_\psi\Gamma(\psi)} \to \frac{e^2_{em}}{s}, \tag{2.7}$$

where e_{em} is the electric charge.

3. Cross section for $e^+e^- \to \Lambda(s_1)\bar{\Lambda}(s_2)$

Our first task is to calculate the cross-section distribution for e^+e^- annihilation into polarized hyperons. From the squared matrix element $|\mathcal{M}|^2$ for this process we remove a factor $\mathcal{K}_\psi$, to get

$$d\sigma = \frac{1}{2s}\mathcal{K}_\psi\,|\mathcal{M}_{red}|^2\ \mathrm{dLips}(k_1+k_2;p_1,p_2), \tag{3.8}$$

with dLips the phase-space factor, with $s = P^2$, and with

$$\mathcal{K}_\psi = \frac{e^2_\psi e^2_g}{(s - m^2_\psi)^2 + m^2_\psi\Gamma^2(m_\psi)}. \tag{3.9}$$

The square of the reduced matrix element can be factorized as

$$\left|\mathcal{M}_{red}(e^+e^- \to \Lambda(s_1)\overline{\Lambda}(s_2))\right|^2 = L\cdot K(s_1,s_2), \tag{3.10}$$

with $L(k_1,k_2)$ and $K(p_1,p_2;s_1,s_2)$ lepton and hadron tensors, and s_1 and s_2 hyperon spin four-vectors.

Lepton tensor including averages over lepton spins;

$$\begin{aligned} L_{\nu\mu}(k_1,k_2) &= \tfrac{1}{4}\mathrm{Tr}\big[\gamma_\nu \not{k}_1 \gamma_\mu \not{k}_2\big] \\ &= k_{1\nu}k_{2\mu} + k_{2\nu}k_{1\mu} - \tfrac{1}{2}sg_{\nu\mu}. \end{aligned} \tag{3.11}$$

Hadron tensor for polarized hyperons;

$$\begin{aligned} K_{\nu\mu}(s_1,s_2) = \mathrm{Tr}\Big[&\overline{\Gamma}^\Lambda_\nu(\not{p}_1 + M)\tfrac{1}{2}(1+\gamma_5\not{s}_1) \\ &\times \Gamma^\Lambda_\mu(\not{p}_2 - M)\tfrac{1}{2}(1+\gamma_5\not{s}_2)\Big]/e^2_g, \end{aligned} \tag{3.12}$$

with p_1 and s_1 momentum and spin for the Lambda hyperon and p_2 and s_2 correspondingly for the anti-Lambda hyperon. The trace itself is symmetric in the two hyperon variables.

The spin four-vector $s(\mathbf{p},\mathbf{n})$ of a hyperon of mass M, three-momentum $\mathbf{p}$, and spin direction $\mathbf{n}$ in its rest system, is

$$s(\mathbf{p},\mathbf{n}) = \frac{n_\parallel}{M}(|\mathbf{p}|;E\hat{\mathbf{p}}) + (0;\mathbf{n}_\perp). \tag{3.13}$$

Here, longitudinal and transverse designations refer to the $\hat{\mathbf{p}}$ direction; $n_\parallel = \mathbf{n}\cdot\hat{\mathbf{p}}$ and $\mathbf{n}_\perp = \mathbf{n} - \hat{\mathbf{p}}(\mathbf{n}\cdot\hat{\mathbf{p}})$ are parallel and transverse components of the spin vector $\mathbf{n}$. Also, observe that the four-vectors p and s are orthogonal, i.e. $p\cdot s(p) = 0$.

For the evaluation of the matrix element we turn to the global c.m. system where kinematics simplifies. Here, three-momenta $\mathbf{p}$ and $\mathbf{k}$ are defined such that

$$\mathbf{p}_1 = -\mathbf{p}_2 = \mathbf{p}, \tag{3.14}$$

$$\mathbf{k}_1 = -\mathbf{k}_2 = \mathbf{k}, \tag{3.15}$$

and scattering angle by,

$$\cos\theta = \hat{\mathbf{p}}\cdot\hat{\mathbf{k}}. \tag{3.16}$$

The phase-space factor becomes

$$\mathrm{dLips}(k_1 + k_2; p_1, p_2) = \frac{p}{32\pi^2 k}\, \mathrm{d}\Omega, \tag{3.17}$$

with $p = |\mathbf{p}|$ and $k = |\mathbf{k}|$.

The matrix element in Eq. (3.10) can be written as a sum of four terms that depend on the hyperon spin directions in their respective rest systems, $\mathbf{n}_1$ and $\mathbf{n}_2$,

$$\left|\mathcal{M}_{red}(e^+e^- \to \Lambda(s_1)\overline{\Lambda}(s_2))\right|^2 = sD(s)\Big[H^{00}(0,0) + H^{05}(\mathbf{n}_1, 0) + H^{50}(0, \mathbf{n}_2) + H^{55}(\mathbf{n}_1, \mathbf{n}_2)\Big]. \tag{3.18}$$

The polarization distributions H^{ab} are each expressed in terms of structure functions that depend on the scattering angle θ, the ratio function $\alpha(s)$, and the phase function $\Delta\Phi(s)$. There are six such structure functions,

$$\mathcal{R} = 1 + \alpha\cos^2\theta, \tag{3.19}$$

$$\mathcal{S} = \sqrt{1-\alpha^2}\sin\theta\cos\theta\sin(\Delta\Phi), \tag{3.20}$$

$$\mathcal{T}_1 = \alpha + \cos^2\theta, \tag{3.21}$$

$$\mathcal{T}_2 = -\alpha\sin^2\theta, \tag{3.22}$$

$$\mathcal{T}_3 = 1 + \alpha, \tag{3.23}$$

$$\mathcal{T}_4 = \sqrt{1-\alpha^2}\cos\theta\cos(\Delta\Phi). \tag{3.24}$$

The definitions and notations are slightly different from those of Ref. [1]. In particular, a factor $sD(s)$ has been pulled out from the structure functions, and placed in front of the sum of the polarization distributions of Eq. (3.18).

The polarization distributions H^{ab} are,

$$H^{00} = \mathcal{R} \tag{3.25}$$

$$H^{05} = \mathcal{S}\left[\frac{1}{\sin\theta}(\hat{\mathbf{p}} \times \hat{\mathbf{k}}) \cdot \mathbf{n}_1\right] \tag{3.26}$$

$$H^{50} = \mathcal{S}\left[\frac{1}{\sin\theta}(\hat{\mathbf{p}} \times \hat{\mathbf{k}}) \cdot \mathbf{n}_2\right] \tag{3.27}$$

$$H^{55} = \Big\{\mathcal{T}_1 \mathbf{n}_1 \cdot \hat{\mathbf{p}}\mathbf{n}_2 \cdot \hat{\mathbf{p}} + \mathcal{T}_2 \mathbf{n}_{1\perp} \cdot \mathbf{n}_{2\perp} + \mathcal{T}_3 \mathbf{n}_{1\perp} \cdot \hat{\mathbf{k}}\mathbf{n}_{2\perp} \cdot \hat{\mathbf{k}} + \mathcal{T}_4\Big(\mathbf{n}_1 \cdot \hat{\mathbf{p}}\mathbf{n}_{2\perp} \cdot \hat{\mathbf{k}} + \mathbf{n}_2 \cdot \hat{\mathbf{p}}\mathbf{n}_{1\perp} \cdot \hat{\mathbf{k}}\Big)\Big\} \tag{3.28}$$

Transverse components $\mathbf{n}_{1\perp}$ and $\mathbf{n}_{2\perp}$ are orthogonal to the Lambda hyperon momentum $\mathbf{p}$ in the global c.m. system. Also, transverse $\mathbf{n}_\perp$ and longitudinal $n_\parallel = \hat{\mathbf{p}} \cdot \mathbf{n}$ polarization components enter differently, since they transform differently under Lorentz transformations.

All polarization observables, single and double, can be directly read off Eqs. (3.25)–(3.28), and there are no other possibilities. The set of scalar products involving $\mathbf{n}_1$ and $\mathbf{n}_2$ is complete. As an example, the Lambda-hyperon polarization is obtained from Eq. (3.26) which shows that the polarization is directed along the normal to the scattering plane, $\hat{\mathbf{p}} \times \hat{\mathbf{k}}$, and that the value of the polarization is

$$P_\Lambda(\theta) = \frac{\mathcal{S}}{\mathcal{R}} = \frac{\sqrt{1-\alpha^2}\cos\theta\sin\theta}{1+\alpha\cos^2\theta}\sin(\Delta\Phi) \tag{3.29}$$

From Eq. (3.27) we conclude that the polarization of the anti-Lambda is exactly the same, but then one should remember that $\mathbf{p}$ is the momentum of the Lambda hyperon but $-\mathbf{p}$ that of the anti-Lambda.

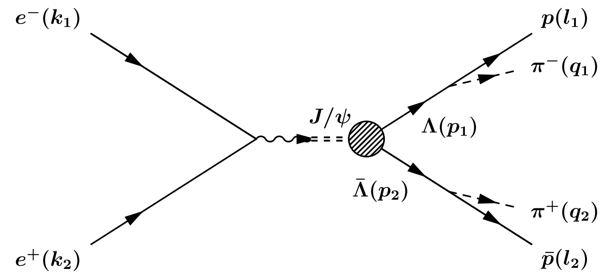

Fig. 2. Graph describing the reaction $e^+e^- \to \Lambda(\to p\pi^-)\bar{\Lambda}(\to \bar{p}\pi^+)$.

4. Folding of distributions

Our next task is to calculate the cross-section distribution for e^+e^- annihilation into hyperon pairs, followed by the hyperon decays into nucleon–pion pairs. This reaction is described by the connected diagram of Fig. 2.

Again, we extract a prefactor, $\mathcal{K} = \mathcal{K}_\psi\mathcal{K}_1\mathcal{K}_2$, from the squared matrix element, writing

$$|\mathcal{M}|^2 = \mathcal{K}|\mathcal{M}_{red}|^2. \tag{4.30}$$

The prefactor originates, as before with the propagator denominators. Due to the smallness of the hyperon widths each of the hyperon propagators can, after squaring, be approximated as,

$$\mathcal{K}_i = \frac{1}{(p_i^2 - M^2)^2 + M^2\Gamma^2(M)} = \frac{2\pi}{2M\Gamma(M)}\delta(p_i^2 - M^2). \tag{4.31}$$

Effectively, this approximation puts the hyperons on their mass shells.

Hyperon-decay distributions are obtained by a folding calculation, whereby hyperon-production and -decay distributions are multiplied together and averaged over the intermediate hyperon-spin directions. It was proved in Ref. [2] that the folding prescription gives the same result as the evaluation of the connected-Feynman-diagram expression. Hence, summing over final hadron spins,

$$|\mathcal{M}|^2 = \sum_{\pm s_1, \pm s_2}\Big\langle\left|\mathcal{M}(e^+e^- \to \Lambda(s_1)\bar{\Lambda}(s_2))\right|^2 \times \left|\mathcal{M}(\Lambda(s_1) \to p\pi^-)\right|^2\left|\mathcal{M}(\bar{\Lambda}(s_2) \to \bar{p}\pi^+)\right|^2\Big\rangle_{\mathbf{n}_1\mathbf{n}_2}. \tag{4.32}$$

Production and decay distributions are,

$$\left|\mathcal{M}(e^+e^- \to \Lambda(s_1)\bar{\Lambda}(s_2))\right|^2 = L \cdot K(s_1, s_2), \tag{4.33}$$

$$\left|\mathcal{M}(\Lambda(s_1) \to p\pi^-)\right|^2 = R_\Lambda\left[1 - \alpha_1 l_1 \cdot s_1/l_\Lambda\right], \tag{4.34}$$

$$\left|\mathcal{M}(\bar{\Lambda}(s_2) \to \bar{p}\pi^+)\right|^2 = R_\Lambda\left[1 - \alpha_2 l_2 \cdot s_2/l_\Lambda\right], \tag{4.35}$$

with l_Λ the decay momentum in the Lambda rest system. R_Λ is determined by the Lambda decay rate.

The notation in Eq. (4.34) is the following; s_1 denotes the Lambda four-spin vector, l_1 the four-momentum of the decay proton, and α_1 the decay-asymmetry parameter. Similarly for the anti-Lambda hyperon parameters of Eq. (4.35).

We evaluate the hyperon-decay distributions in the hyperon-rest systems, where

$$\left|\mathcal{M}(\Lambda(s_1) \to p\pi^-)\right|^2 = R_\Lambda\left[1 + \alpha_1\hat{\mathbf{l}}_1 \cdot \mathbf{n}_1\right], \tag{4.36}$$

$$\left|\mathcal{M}(\bar{\Lambda}(s_2) \to \bar{p}\pi^+)\right|^2 = R_\Lambda\left[1 + \alpha_2\hat{\mathbf{l}}_2 \cdot \mathbf{n}_2\right], \tag{4.37}$$

where $\hat{\mathbf{l}}_1 = \mathbf{l}_1/l_\Lambda$ is the unit vector in the direction of the proton momentum in the Lambda-rest system, and correspondingly for the anti-Lambda case.

Angular averages in Eq. (4.32) are calculated according to the prescription

$$\langle(\mathbf{n}\cdot\mathbf{l})\mathbf{n}\rangle_{\mathbf{n}} = \mathbf{l}. \tag{4.38}$$

The folding of the production distributions, Eqs. (3.25)–(3.28), with the decay distributions, Eqs. (4.36, 4.37), yields

$$|\mathcal{M}_{red}|^2 = sD(s)R_\Lambda^2\left[G^{00}+G^{05}+G^{50}+G^{55}\right], \tag{4.39}$$

with the G^{ab} functions defined as

$$G^{00} = \mathcal{R}, \tag{4.40}$$

$$G^{05} = \alpha_1\mathcal{S}\left[\frac{1}{\sin\theta}(\hat{\mathbf{p}}\times\hat{\mathbf{k}})\cdot\hat{\mathbf{l}}_1\right], \tag{4.41}$$

$$G^{50} = \alpha_2\mathcal{S}\left[\frac{1}{\sin\theta}(\hat{\mathbf{p}}\times\hat{\mathbf{k}})\cdot\hat{\mathbf{l}}_2\right], \tag{4.42}$$

$$\begin{aligned} G^{55} = \alpha_1\alpha_2\Big\{ & \mathcal{T}_1\hat{\mathbf{l}}_1\cdot\hat{\mathbf{p}}\hat{\mathbf{l}}_2\cdot\hat{\mathbf{p}} + \mathcal{T}_2\hat{\mathbf{l}}_{1\perp}\cdot\hat{\mathbf{l}}_{2\perp} \\ & + \mathcal{T}_3\hat{\mathbf{l}}_{1\perp}\cdot\hat{\mathbf{k}}\hat{\mathbf{l}}_{2\perp}\cdot\hat{\mathbf{k}} \\ & + \mathcal{T}_4\left(\hat{\mathbf{l}}_1\cdot\hat{\mathbf{p}}\hat{\mathbf{l}}_{2\perp}\cdot\hat{\mathbf{k}} + \hat{\mathbf{l}}_2\cdot\hat{\mathbf{p}}\hat{\mathbf{l}}_{1\perp}\cdot\hat{\mathbf{k}}\right)\Big\}. \end{aligned} \tag{4.43}$$

Thus, we conclude the connection between joint-hadron production and joint-hadron decay distributions simply to be,

$$G^{ab}(\hat{\mathbf{l}}_1;\hat{\mathbf{l}}_2) = H^{ab}(\mathbf{n}_1\to\alpha_1\hat{\mathbf{l}}_1;\mathbf{n}_2\to\alpha_2\hat{\mathbf{l}}_2). \tag{4.44}$$

We repeat the notation; $\mathbf{p}$ and $\mathbf{k}$ are momenta of Lambda and electron in the global c.m. system; $\mathbf{l}_1$ and $\mathbf{l}_2$ are momenta of proton and anti-proton in Lambda and anti-Lambda rest systems; orthogonal means orthogonal to $\mathbf{p}$; and structure functions $\mathcal{R}$, $\mathcal{S}$, and $\mathcal{T}$ are functions of θ, α, and $\Delta\Phi$. The angular functions multiplying the structure functions form a set of seven mutually orthogonal functions, when integrated over the proton and anti-proton decay angles.

5. Cross section for $e^+e^-\to\Lambda(\to p\pi^-)\bar{\Lambda}(\to\bar{p}\pi^+)$

Our last task is to find the properly normalized cross-section distribution. We start from the general expression,

$$d\sigma = \frac{1}{2s}\mathcal{K}|\mathcal{M}_{red}|^2\,\mathrm{dLips}(k_1+k_2;q_1,l_1,q_2,l_2), \tag{5.45}$$

with dLips the phase-space density for four final-state particles. The prefactor $\mathcal{K}$ contains on the mass shell delta functions for the two hyperons. This effectively separates the phase space into production and decay parts. Repeating the manipulations of Ref. [2] we get

$$\begin{aligned} d\sigma = & \frac{1}{64\pi^2}\frac{p}{k}\frac{\alpha_g\alpha_\psi}{(s-m_\psi^2)^2+m_\psi^2\Gamma^2(\psi)}\frac{\Gamma_\Lambda\Gamma_{\bar{\Lambda}}}{\Gamma^2(M)}\cdot \\ & \cdot\left(D(s)\sum_{a,b}G^{ab}\right)d\Omega_\Lambda d\Omega_1 d\Omega_2, \end{aligned} \tag{5.46}$$

with k and p the initial- and final-state momenta; Ω_Λ the hyperon scattering angle in the global c.m. system; Ω_1 and Ω_2 decay angles measured in the rest systems of Λ and $\bar{\Lambda}$; Γ_Λ and $\Gamma_{\bar{\Lambda}}$ channel widths; and $\Gamma(M)$ and $\Gamma(\psi)$ total widths.

Integration over the angles Ω_1 makes the contributions from the functions G^{05} and G^{55} disappear [2], and correspondingly for the angles Ω_2. Integration over both angular variables results in the cross-section distribution for the reaction $e^+e^-\to J/\psi\to\Lambda\bar{\Lambda}$.

Suppose we integrate over the angles Ω_2. Then, the predicted hyperon-decay distribution becomes proportional to the sum

$$G^{00}+G^{05} = \mathcal{R}\left(1+\alpha_1\mathbf{P}_\Lambda\cdot\hat{\mathbf{l}}_1\right), \tag{5.47}$$

$$P_\Lambda = \frac{\mathcal{S}}{\mathcal{R}}, \tag{5.48}$$

with the polarization P_Λ as in Eq. (3.29), and the polarization vector $\mathbf{P}_\Lambda$ directed along the normal to the scattering plane

6. Differential distributions

We first define our coordinate system. The scattering plane with the vectors $\mathbf{p}$ and $\mathbf{k}$ make up the xz-plane, with the y-axis along the normal to the scattering plane. We choose a right-handed coordinate system with basis vectors

$$\mathbf{e}_z = \hat{\mathbf{p}}, \tag{6.49}$$

$$\mathbf{e}_y = \frac{1}{\sin\theta}(\hat{\mathbf{p}}\times\hat{\mathbf{k}}), \tag{6.50}$$

$$\mathbf{e}_x = \frac{1}{\sin\theta}(\hat{\mathbf{p}}\times\hat{\mathbf{k}})\times\hat{\mathbf{p}}. \tag{6.51}$$

Expressed in terms of them the initial-state momentum

$$\hat{\mathbf{k}} = \sin\theta\,\mathbf{e}_x + \cos\theta\,\mathbf{e}_z. \tag{6.52}$$

This coordinate system is used for fixing the directional angles of the decay proton in the Lambda rest system, and the decay anti-proton in the anti-Lambda rest system. The spherical angles for the proton are θ_1 and ϕ_1, and the components of the unit vector in direction of the decay-proton momentum are,

$$\hat{\mathbf{l}}_1 = (\cos\phi_1\sin\theta_1, \sin\phi_1\sin\theta_1, \cos\theta_1), \tag{6.53}$$

so that

$$\hat{\mathbf{l}}_{1\perp} = (\cos\phi_1\sin\theta_1, \sin\phi_1\sin\theta_1, 0). \tag{6.54}$$

The momentum of the decay proton is by definition $\mathbf{l}_1 = l_\Lambda\hat{\mathbf{l}}_1$. This same coordinate system is used for defining the directional angles of the decay anti-proton in the anti-Lambda rest system, with spherical angles θ_2 and ϕ_2.

Now, we have all ingredients needed for the calculation of the G functions of Eqs. (4.40)–(4.43), the functions that in the end determine the cross-section distributions.

An event of the reaction $e^+e^-\to\Lambda(\to p\pi^-)\bar{\Lambda}(\to\bar{p}\pi^+)$ is specified by the five dimensional vector $\boldsymbol{\xi} = (\theta,\Omega_1,\Omega_2)$, and the differential-cross-section distribution as summarized by Eq. (4.39) reads,

$$d\sigma \propto \mathcal{W}(\boldsymbol{\xi})\,d\cos\theta\,d\Omega_1 d\Omega_2.$$

At the moment, we are not interested in the absolute normalization of the differential distribution. The differential-distribution function $\mathcal{W}(\boldsymbol{\xi})$ is obtained from Eqs. (4.40)–(4.43) and can be expressed as,

$$\begin{aligned} \mathcal{W}(\boldsymbol{\xi}) = & \mathcal{F}_0(\boldsymbol{\xi}) + \alpha\mathcal{F}_5(\boldsymbol{\xi}) \\ & + \alpha_1\alpha_2\left(\mathcal{F}_1(\boldsymbol{\xi}) + \sqrt{1-\alpha^2}\cos(\Delta\Phi)\mathcal{F}_2(\boldsymbol{\xi}) + \alpha\mathcal{F}_6(\boldsymbol{\xi})\right) \\ & + \sqrt{1-\alpha^2}\sin(\Delta\Phi)\left(\alpha_1\mathcal{F}_3(\boldsymbol{\xi}) + \alpha_2\mathcal{F}_4(\boldsymbol{\xi})\right), \end{aligned} \tag{6.55}$$

using a set of seven angular functions $\mathcal{F}_k(\boldsymbol{\xi})$ defined as:

$$\mathcal{F}_0(\boldsymbol{\xi}) = 1$$
$$\mathcal{F}_1(\boldsymbol{\xi}) = \sin^2\theta \sin\theta_1 \sin\theta_2 \cos\phi_1 \cos\phi_2 + \cos^2\theta \cos\theta_1 \cos\theta_2$$
$$\mathcal{F}_2(\boldsymbol{\xi}) = \sin\theta \cos\theta \,(\sin\theta_1 \cos\theta_2 \cos\phi_1 + \cos\theta_1 \sin\theta_2 \cos\phi_2)$$
$$\mathcal{F}_3(\boldsymbol{\xi}) = \sin\theta \cos\theta \sin\theta_1 \sin\phi_1$$
$$\mathcal{F}_4(\boldsymbol{\xi}) = \sin\theta \cos\theta \sin\theta_2 \sin\phi_2$$
$$\mathcal{F}_5(\boldsymbol{\xi}) = \cos^2\theta$$
$$\mathcal{F}_6(\boldsymbol{\xi}) = \cos\theta_1 \cos\theta_2 - \sin^2\theta \sin\theta_1 \sin\theta_2 \sin\phi_1 \sin\phi_2. \tag{6.56}$$

The differential distribution of Eq. (6.55) involves two parameters related to the $e^+e^- \to \Lambda\bar{\Lambda}$ process that can be determined by data: the ratio of form factors α, and the relative phase of form factors $\Delta\Phi$. In addition, the distribution function $\mathcal{W}(\boldsymbol{\xi})$ can be used to extract separately Λ and $\bar{\Lambda}$ decay-asymmetry parameters: α_1 and α_2, and hence allowing a direct test of CP conservation in the hyperon decays.

The term proportional to $\sin(\Delta\Phi)$ in Eq. (6.55) originates with Eqs. (4.41) and (4.42), and can be rewritten as,

$$\mathcal{S}(\theta)\,(\alpha_1 \sin\theta_1 \sin\phi_1 + \alpha_2 \sin\theta_2 \sin\phi_2)\,,$$

with the structure function $\mathcal{S}$ defined by Eq. (3.20). The relation between the structure functions and the polarization $P_\Lambda(\theta)$ was discussed in Sect. 3, where it was shown that the polarization, $P_\Lambda(\theta)$ of Eq. (3.29), and the polarization vector, $\mathbf{e}_y$, are the same for Lambda and anti-Lambda hyperons. This information tells us that Λ is polarized along the normal to the production plane, and that the polarization vanishes at $\theta = 0°$, $90°$ and $180°$. The maximum value of the polarization is for $\cos\theta = \pm 1/(2+\alpha)$, and $|P_\Lambda(\theta)| < \frac{2}{3}\sin(\Delta\Phi)$.

It should be stressed that the simplified distributions used in previous analyses, such as Ref. [9], assume the hyperons to be unpolarized and therefore terms containing $P_\Lambda(\theta)$ are missing. In fact, such decay distributions, only permit the determination of two parameters: the ratio of form factors α, and the product of hyperon-asymmetry parameters $\alpha_1\alpha_2$.

In our opinion, the formulas presented in this letter should be employed for the exclusive analysis of the new BESIII data [10]. Due to huge and clean data samples: (440675 ± 670) $J/\psi \to \Lambda\bar{\Lambda}$ and (31119 ± 187) $\psi(3686) \to \Lambda\bar{\Lambda}$, precision values for the decay-hadronic-form factors could be extracted as well as precision values for Λ and $\bar{\Lambda}$ decay-asymmetry parameters. The formulas presented could easily be generalized to neutron decays of the Λ and to production of other $J = 1/2$ hyperons with analogous decay modes.

Acknowledgements

We are grateful to Tord Johansson who provided the inspiration for this work.

Appendix A

The coupling of the initial-state leptons to the J/ψ vector meson is determined by the decay $J/\psi \to e^+e^-$. Assuming the decay to go via an intermediate photon, Fig. 1b, we can safely ignore any tensor coupling. The vector coupling of the J/ψ to leptons is therefore the same as for the photon, if replacing the electric charge e_{em} by a coupling strength e_ψ. From the decay $J/\psi \to e^+e^-$ one derives

$$\alpha_\psi = e_\psi^2/4\pi = 3\Gamma(J/\psi \to e^+e^-)/m_\psi. \tag{A.57}$$

In a similar fashion we relate the strength e_g of J/ψ coupling to the hyperons to the decay $J/\psi \to \Lambda\bar{\Lambda}$. In analogy with Eq. (A.57) we get

$$\alpha_g = e_g^2/4\pi = 3\left((1 + 2M^2/m_\psi^2)\sqrt{1 - 4M^2/m_\psi^2}\right)^{-1} \times \Gamma(J/\psi \to \Lambda\bar{\Lambda})/m_\psi. \tag{A.58}$$

When the Λ mass M is replaced by the lepton mass $m_l = 0$ we recover Eq. (A.57).

References

[1] G. Fäldt, Eur. Phys. J. A 52 (2016) 141.
[2] G. Fäldt, Eur. Phys. J. A 51 (2015) 74.
[3] H. Czyż, A. Grzelińska, J.H. Kühn, Phys. Rev. D 75 (2007) 074026.
[4] J. Haidenbauer, U.-G. Meißner, Phys. Lett. B 761 (2016) 456.
[5] Hong Chen, Rong-Gang Ping, Phys. Rev. D 76 (2007) 036005.
[6] Bin Zhong, Guangrui Liao, Acta Phys. Pol. 46 (2015) 2459.
[7] D. Pallin, et al., Nucl. Phys. B 292 (1987) 653.
[8] M.H. Tixier, et al., Phys. Lett. B 212 (1988) 523.
[9] M. Ablikim, et al., Phys. Rev. D 81 (2010) 012003.
[10] M. Ablikim, et al., BESIII Collaboration, Phys. Rev. D 95 (2017) 052003.
[11] M. Ablikim, et al., BESIII Collaboration, Nucl. Instrum. Methods A 614 (2010) 345.