Continuous-Variable Quantum Communication with Quantum State Engineering

Author:
He, Mingjian

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Continuous-Variable Quantum Communication with Quantum State Engineering

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Mingjian HE

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Thesis Title and Abstract | **Declarations** | **Inclusion of Publications Statement** | **Corrected Thesis and Responses**

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Authors:	M He, R Malaney, J Green
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Abstract

Quantum information encoded in quantum states allows for various information processing tasks that cannot be realized by classical methods. However, the decoherence of states due to channel imperfection limits the transmission distance of such information. This thesis investigates quantum state engineering in various quantum communication protocols, aiming to find the engineered quantum states that provide the best loss tolerance under different conditions.

This thesis contains three parts. In the first part of this thesis, we quantify the non-Gaussian entanglement distributed between two locations. We consider the scenario where a satellite generates broadband pulses of twin beams. Each pulse contains a multitude of continuous-variable (CV) Gaussian entangled states in the orthogonal supermode basis. The entangled states are engineered by non-Gaussian operations and then partially sent to a ground station through an atmospheric channel. For comparison, we also consider the scenario where non-Gaussian operations are performed after the channel transmission. We evaluate the level of entanglement of the final non-Gaussian state distributed between the locations. We find that all the non-Gaussian operations we considered can improve entanglement over certain parameter regions. The regions depend on the locale of the operations, the squeezing of the initial entangled state, the channel loss, and the supermode structure of the system.

In the second part of this thesis, we move on to entanglement-based CV quantum key distribution (QKD) protocols. First, we consider a single-mode CV-QKD system. Focusing on photon subtraction and addition, we find that neither non-Gaussian operation can improve the key rates. We then extend the study to a multi-mode CV-QKD system and include photon catalysis. We find that all non-Gaussian operations can improve the key rates for the multi-mode system. However, the improvement is marginal. We later find that a specific arrangement of noiseless amplification and noiseless attenuation can significantly improve the key rates. Finally, we propose two possible implementations of noiseless amplifiers for multi-mode states.

In the third part of this thesis, we study non-Gaussian operations and non-Gaussian measurements in a teleportation protocol that uses CV entangled states. We investigate different states to be teleported, including DV qubits, CV qubits, and a more complex hybrid entangled state. We consider a realistic scenario where the sender and receiver do not share any resource state—a state that enables teleportation. The resource state is first prepared at a middle station and then distributed to the sender and receiver. The distributed resource state is then used for teleportation. We show that a modified non-Gaussian measurement improves the CV teleportation protocol. The modified protocol can be further improved by additional non-Gaussian operations.

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Publications during PhD candidature

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• Peer-Reviewed Journal Articles

[1] **M. He**, R. Malaney, and J. Green, "Global entanglement distribution with multi-mode non-Gaussian operations," *IEEE Journal on Selected Areas in Communications*, vol. 38, no. 3, 528–539, 2020.

[2] **M. He**, R. Malaney, and J. Green, "Multimode CV-QKD with non-Gaussian operations," *Quantum Engineering*, vol. 2, no. 2, e40, 2020.

[3] **M. He**, R. Malaney, and B. A. Burnett, "Noiseless linear amplifiers for multi-mode states," *Physical Review A*, vol. 103, no. 1, 012414, 2021.

[4] **M. He**, R. Malaney, and R. Aguinaldo, "Teleportation of discrete-variable qubits via continuous-variable lossy channels," *Physical Review A*, vol. 105, no. 6, 062407, 2022.

[5] **M. He** and R. Malaney, "Teleportation of hybrid entangled states with continuous-variable entanglement," *Scientific Reports*, vol 12, no. 17169, 2022.

• Peer-Reviewed Conference Articles

[6] **M. He**, R. Malaney, and J. Green, "Photonic engineering for CV-QKD over

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[7] **M. He**, R. Malaney, and B. A. Bumett, "Multi-mode CV-QKD with noiseless attenuation and amplification," in *2020 IEEE Globecom Workshops (GC Workshps)*, 1–7, IEEE, 2020.

Publications not included in this thesis

[8] E. Villaseñor, **M. He**, Z. Wang, R. Malaney, and M. Z. Win, "Enhanced uplink quantum communication with satellites via downlink channels," *IEEE Transactions on Quantum Engineering*, vol. 2, 1–18, 2021.

[9] R. Malaney, X. Ai, H. Do, **M. He**, E. Villaseñor, Z. Wang, and J. Green, "Quantum communications: From space to the nano," in *Proceedings of the Sixth Annual ACM International Conference on Nanoscale Computing and Communication*, 1–6, 2019.

[10] **M. He**, R. Malaney, and J. Green, "Quantum communications via satellite with photon subtraction," in *2018 IEEE Globecom Workshops (GC Wkshps)*, 1–6, IEEE, 2018. (Manuscript was submitted before candidature)

Inclusion of publications statement

My thesis has publications—either published or submitted for publication—incorporated into it in lieu of Chapters. I am the primary author of these publications and have contributed greater than 50% of the contents. As per UNSW thesis examination procedures, I have the approval to include the publications in the thesis in lieu of chapters from UNSW. The publications are not subject to any obligations or contractual agreements with a third party that would constrain their inclusion in the thesis. Following the Thesis Format Guide provided by UNSW, these publications are included in the thesis as they were submitted or published—no edits or modifications were made to the text. The figures in the publications are not listed in the List of figures. Details of these publications are provided below.

Chapter 2. Global entanglement distribution with multi-mode non-Gaussian operations.
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Contents

Acronyms

BSM	Bell State Measurement.
CF	Characteristic Function.
CM	Covariance Matrix.
CV	Continuous Variable.
CV-BSM	Continuous Variable Bell State Measurement.
CV-QKD	Continuous Variable Quantum Key Distribution.
DV	Discrete Variable.
EB-QKD	Entanglement Based Quantum Key Distribution.
EPR	Einstein–Podolsky–Rosen.
FWHM	Full Width at Half Maximum.
H-BSM	Hybrid Bell State Measurement.

LEO	Low-Earth-Orbit.
LO	Local Oscillator.
MSA	Multi-Mode Single-Photon Addition.
MSC	Multi-Mode Single-Photon Catalysis.
MSS	Multi-Mode Single-Photon Subtraction.
NLA	Noiseless Linear Amplification.
PA	Photon Addition.
PAS	Photon-Added State.
PC	Photon Catalysis.
PDC	Parametric Down-Conversion.
PDF	Probability Density Function.
PM-QKD	Prepare-And-Measure Quantum Key Distribution.
PNRD	Photon Number Resolving Detector.
PPT	Positive Partial Transpose.
PS	Photon Subtraction.
PSS	Photon-Subtracted State.
QKD	Quantum Key Distribution.
QS	Quantum Scissors.
TMSV	Two-Mode Squeezed Vacuum.

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CHAPTER 1

Introduction

1.1 Overview

As one of the most developed quantum communication protocols, quantum key distribution (QKD) has drawn much attention over the past decades for its unparalleled communication security. Indeed, new theoretical protocols have been designed regularly since the first QKD protocol was proposed in 1984 [1]. Numerous QKD protocols, or at least part of the protocols, have been implemented over optical fibers [2–8], terrestrial free-space channels [9–14], and even satellite-based channels [15–17]. Like other quantum communication protocols, the core of QKD is the distribution of quantum states. However, quantum states are highly sensitive to the decoherence during channel transmission, making communication distance a trade-off for information throughput. In quantum communication systems deployed over optical fiber channels, the channel loss increases exponentially as the distance grows, severely limiting the communication distance. Satellite-based quantum communication provides much longer communication distances relative to terrestrial-only links. Many breakthroughs have been made in the deployment of satellite-based quantum communication systems, e.g., [15–20]. However, the achievable information throughput for satellite-based quantum communication systems, e.g., ∼50 kbps for QKD [17], still cannot keep pace with its classical counterpart.

The multiplexing of quantum channels is one possible solution to the low-throughput issue. The ultra-fast pulses with high repetition rates carry the potential of rich channel capacity [21]. Such broadband pulses can be engineered to emit an array of entangled states [22], which are the critical resource for various quantum communication protocols. As an alternative, the photonic engineering of highly non-classical, non-Gaussian states can also achieve enhanced entanglement and other desirable properties. Indeed, non-Gaussian features are essential for various quantum information tasks, such as entanglement distillation [23–32], noiseless linear amplification [33–39], discrimination of noisy quantum states [40–43], and quantum computation [44–48]. Much effort has also been put into the study of non-Gaussian operations in quantum communication protocols including quantum teleportation [49–57] and QKD [58–68]. However, all these works only consider quantum communication systems under the single-mode narrowband setting. Whether non-Gaussian operations can be combined with the multiplexing of quantum channels to improve the information throughput is an interesting question. The study of non-Gaussian operations can never be exhausted due to the wide variety of such operations. Whether there are new combinations of non-Gaussian operations that can improve the throughput of a quantum communication system is another interesting question to ask.

In this thesis, we study the use of quantum states, engineered by various quantum operations, in different entanglement-based quantum communication protocols. Focusing on continuous-variable (CV) quantum states, we will consider three highly related protocols, namely entanglement distribution, QKD, and quantum teleportation. Our study will cover both the single-mode and the multi-mode settings of quantum systems. We will consider scenarios where the protocols are deployed over optical fiber channels or satellite-based channels. The results presented in this thesis should be of value in assessments of the different technology solutions under consideration for future quantum communication systems.

1.2 Thesis structure

We now present the structure of this thesis. Chapters 2-8 are self-contained papers presenting the results of this thesis. Chapter 9 summarizes the results and presents suggestions for possible future works. There is a strong narrative connecting the chapters, which is supported by the preambles of each chapter. The preambles are reproduced here for convenience:

Chapter 2. The distribution of entanglement is a prerequisite for entanglement-based quantum information tasks. In this chapter, we study photon subtraction, photon addition, and photon catalysis in the distribution of multi-mode entangled states over atmospheric channels. As the main contribution of this chapter, we propose a framework for multi-mode photon catalysis. Building on this framework, we consider scenarios where the non-Gaussian operations are performed on the entangled states before or after they pass through the atmospheric channels. We find that all three non-Gaussian operations considered can improve entanglement over certain parameter regions.

Chapter 3. We have shown in Chapter 2 that certain non-Gaussian operations can improve the entanglement distributed over atmospherics channels. In this chapter, we study the use of certain non-Gaussian operations in entanglement-based CV-QKD systems. In this chapter, we focus on the single-mode system only. We find that non-Gaussian operations can only improve the channel loss tolerance of a CV-QKD system when the initial squeezing of the entangled state is large. When the initial squeezing is adjusted to maximize the tolerable channel loss, non-Gaussian operations cannot improve the tolerance.

Chapter 4. In this chapter, we extend the single-mode CV-QKD system in Chapter 3 to a multi-mode system. To better study the impact of non-Gaussian operations on a multi-mode CV-QKD system, in this chapter, we focus on the fiber channels with fixed loss only. Our results show that, considering the multi-mode

CV-QKD system only, non-Gaussian operations can slightly improve the secret key rates of the system. When comparing the multi-mode CV-QKD system with a single-mode system, the secret key rates for the multi-mode non-Gaussian operations can be orders of magnitude higher than the single-mode system.

Chapter 5. We have shown in Chapter 4 that certain non-Gaussian operations only provide minor improvement in a multi-mode CV-QKD system. In this chapter, we continue our search for quantum state engineering that can improve the secret key rates for a CV-QKD system. We show that noiseless attenuation, when combined with noiseless amplification, can significantly improve the secret key rates of both the single-mode and the multi-mode CV-QKD systems.

Chapter 6. We have shown in Chapter 5 that noiseless amplification can significantly improve the secret key rates of a CV-QKD system. However, no realization on noiseless amplification has thus far been proposed for multi-mode states. In this chapter, we propose two different schemes for the implementation of noiseless linear amplifiers for multi-mode states. We first generalize the exiting amplification scheme that uses quantum scissors to the multi-mode setting. We then propose a new amplification scheme that uses photon catalysis. The performances of the two schemes are compared in the contexts of coherent state amplification and entanglement distribution.

Chapter 7. We have shown in previous chapters that although non-Gaussian operations can improve the entanglement, they only provide a marginal improvement in CV-QKD. This contradiction motivates us to study non-Gaussian operations in other quantum communication protocols that also rely on entanglement. We have shown in 10.1109/TQE.2021.3091709 that non-Gaussian operation can improve the fidelity of teleportation of coherent states. In this chapter, we move on to the teleportation of DV qubits. We first show that the CV teleportation protocol can be improved by a non-Gaussian measurement, the DV Bell state measurement. We then show that the modified protocol can be further improved by introducing additional non-Gaussian operations.

Chapter 8. In this chapter, we extend the study in Chapter 7 to the teleportation of CV qubits and the hybrid entanglement between DV and CV qubits. We first compare the original CV-based teleportation protocol and our modified teleportation protocol proposed in Chapter 7 in teleporting CV qubits. We find that no protocol is always superior. Our modified protocol outperforms the original protocol only when the mean photon number of the CV qubit is below a certain threshold. We then compare the two protocols in teleporting a hybrid entangled state. Our modified protocol is again found to be better than the original protocol when the mean photon number of the qubit of the hybrid entangled state is below a certain threshold. Finally, we study the use of non-Gaussian operations in our modified protocol, finding that quantum scissors provide the most improvement.

In the remainder of this chapter, we present the basic concepts upon which the thesis is built, along with a consistent notation.

1.3 Background

This section reviews the fundamentals of continuous-variable quantum communications used in this thesis. We start with the definitions of quantum states of light and their phase-space representation. We then present non-Gaussian operations, which play a major role in various quantum information tasks and are the focus of this thesis. We later describe some widely studied quantum communication protocols including entanglement distribution, quantum teleportation, and quantum key distribution. Finally, we clarify some additional settings we assumed in the actual deployment of the quantum communication protocols.

1.3.1 Notation

The notation used in this section is fairly standard. The set of real (natural) numbers is denoted by $\mathbb{R}$ ($\mathbb{N}$). The imaginary unit is denoted by $i = \sqrt{-1}$. The real part (imaginary part) of a complex number α is denoted by $\mathrm{Re}\{\alpha\}$ ($\mathrm{Im}\{\alpha\}$). The conjugate of a complex number α is denoted by α^*. Elements of $\mathbb{R}^{2N}$ are real column vectors with $2N$ entries. A column vector is denoted by a lowercase letter in bold font, e.g., $\mathbf{x}$. The transpose of a vector is denoted by $\mathbf{x}^T$. A matrix is denoted by an uppercase letter in bold font, e.g., $\mathbf{V}$. The trace of a matrix is denoted by $\mathrm{tr}\{\mathbf{V}\}$. The symbol $\oplus$ denotes the direct sum of two matrices. The field operator of a quantum mode is denoted by a lowercase letter with the hat symbol, e.g., $\hat{a}$. The conjugate transpose of an operator is denoted by $\hat{a}^\dagger$. The symbol $\otimes$ denotes a tensor product between Hilbert spaces or an algebraic tensor product between operators. The symbol $[\hat{a}_1, \hat{a}_2] = \hat{a}_1\hat{a}_2 - \hat{a}_2\hat{a}_1$ denotes the commutation between two operators. The symbol $\langle\hat{a}\rangle$ denotes the mean value of an operator.

1.3.2 Continuous-variable quantum systems

A continuous-variable (CV) quantum system has an infinite-dimensional Hilbert space described by observables with a continuous eigenspectrum. The quantized mode of an electromagnetic field is an example of such a quantum system. The composite system of $N(N \geq 1)$ bosonic modes has a tensor-product Hilbert space $\otimes_{k=1}^N \mathcal{H}_k$ and N pairs of corresponding bosonic field operators, namely the annihilation operators $\{\hat{a}_k\}_{k=1}^N$ and the creation operators $\{\hat{a}_k^\dagger\}_{k=1}^N$. The index k is used here to identify different modes and will be dropped when the context is clear. These bosonic field operators satisfy the commutation relations:

$$[\hat{a}_k,\ \hat{a}_{k'}] = [\hat{a}_k^\dagger,\ \hat{a}_{k'}^\dagger] = 0,\ [\hat{a}_k,\ \hat{a}_{k'}^\dagger] = \delta_{kk'}, \tag{1.1}$$

where $\delta_{kk'} = 1$ for $k = k'$ and $\delta_{kk'} = 0$ otherwise.

Consider an arbitrary mode of the composite system. The bosonic field operators are not observables because they are not Hermitian. However, these operators can be combined linearly to form the so-called quadrature field operators ($\hbar = 2$ is assumed in this thesis),

$$\hat{q} = \hat{a} + \hat{a}^\dagger,\ \hat{p} = i(\hat{a}^\dagger - \hat{a}), \tag{1.2}$$

which are Hermitian and thus are observables. The two quadrature filed operators have eigenstates

$$\hat{q}\,|q\rangle = q\,|q\rangle\,,\ \hat{p}\,|p\rangle = p\,|p\rangle\,, \tag{1.3}$$

with continuous eigenvalues $q \in \mathbb{R}$ and $p \in \mathbb{R}$.

Consider the entire composite bosonic system of N modes. Let

$$\hat{\mathbf{x}} = [\hat{q}_1, \hat{p}_1, \ldots, \hat{q}_N, \hat{p}_N]^T, \tag{1.4}$$

be a vector of the quadrature field operators. The commutation relations of the quadrature field operators, which can be derived from Eq. (1.1), can then be written as

$$[\hat{x}_l,\ \hat{x}_j] = 2i\Omega_{lj}, \tag{1.5}$$

where the Ω_{lj}'s are entries of the matrix

$$\mathbf{\Omega} = \bigoplus_{k=1}^{N} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{1.6}$$

The bosonic field operators can also be combined as a product to form the so-called number operator, $\hat{n} = \hat{a}^\dagger \hat{a}$, which is also Hermitian and thus can be measured. The eigenstates of the number operator are the Fock states (or number states), which can be written in the bra-ket formalism in Dirac notation as $|n\rangle$, and satisfy

$$\hat{n}\,|n\rangle = n\,|n\rangle\,, \tag{1.7}$$

where $n \in \mathbb{N}$. Over these states, the actions of the bosonic operators are well-defined and are given by

$$\begin{aligned} \hat{a}\,|0\rangle &= 0, \\ \hat{a}\,|n\rangle &= \sqrt{n}\,|n-1\rangle\,,\ \text{for } n \geq 1, \\ \hat{a}^\dagger\,|n\rangle &= \sqrt{n+1}\,|n+1\rangle\,,\ \text{for } n \geq 0, \end{aligned} \tag{1.8}$$

where $|0\rangle$ is known as the vacuum state. The Fock states compose an infinite-dimensional orthonormal basis $\{|n\rangle\}_{n=0}^{\infty}$ called the Fock basis.

Quantum states and phase-space representation

Quantum state contains all the physical information of quantum systems. A quantum state can be defined by a state vector $|\phi\rangle$ if the state is pure. A more general definition of a quantum state is the density operator $\hat{\rho}$, which is a trace-one positive operator acting on the corresponding Hilbert space. The state is pure when $\hat{\rho} = |\phi\rangle \langle\phi|$ and $\hat{\rho}^2 = \hat{\rho}$.

Every quantum state has phase-space representations that are equivalent to the density operator. The earliest introduced phase-space representation is the Wigner function, which is a quasi-probability distribution because it can reach negative values for some states. The Wigner function for an N-mode state can be written as

$$W(\mathbf{x}) = \int_{\mathbb{R}^{2N}} \frac{d^{2N}\boldsymbol{\zeta}}{(2\pi)^{2N}} \exp(-i\mathbf{x}^T\boldsymbol{\Omega}\boldsymbol{\zeta})\chi(\boldsymbol{\zeta}), \tag{1.9}$$

where $\boldsymbol{\zeta} \in \mathbb{R}^{2N}$ is a column vector, $d^{2N}\boldsymbol{\zeta} = \Pi_{i=1}^{2N} d\zeta_i$, $\mathbf{x} = [q_1, p_1, \ldots, q_N, p_N]^T$ is a vector of the quadrature eigenvalues of the modes,

$$\chi(\boldsymbol{\zeta}) = \mathrm{tr}\{\hat{\rho} D_{\mathrm{w}}(\boldsymbol{\zeta})\}, \tag{1.10}$$

is the Wigner characteristic function, and

$$D_{\mathrm{w}}(\boldsymbol{\zeta}) = \exp(i\hat{\mathbf{x}}^T\boldsymbol{\Omega}\boldsymbol{\zeta}), \tag{1.11}$$

is the Weyl operator. In the scope of this thesis, we only consider the Wigner characteristic function and will drop the term “Wigner” for conciseness.

Gaussian states

As a specific class of states, Gaussian states are completely characterized by their mean value $\bar{\mathbf{x}}$ and covariance matrix (CM) $\mathbf{V}$ of the quadrature field operators, where

$$\bar{\mathbf{x}} = \mathrm{tr}(\hat{\mathbf{x}}\hat{\rho}) := \langle \hat{\mathbf{x}} \rangle \,, \tag{1.12}$$

and the entries of $\mathbf{V}$ are defined as

$$V_{ij} = \frac{1}{2}\langle \hat{x}_i\hat{x}_j + \hat{x}_j\hat{x}_i\rangle - \langle \hat{x}_i\rangle \langle \hat{x}_j\rangle . \tag{1.13}$$

The diagonal entries of $\mathbf{V}$ are variances of the quadrature field operators and the off-diagonal entries are covariances between the operators. To satisfy the uncertainty principle $\mathbf{V} + i\mathbf{\Omega} \geq 0$ the CM of a state must be positive definite.

By the definitions given by Eqs. (1.9) and (1.10), the Wigner function of an N-mode Gaussian state can be written as

$$W(\mathbf{x}) = \frac{1}{(2\pi)^N\sqrt{\det \mathbf{V}}} \exp\left[-(1/2)(\mathbf{x}-\bar{\mathbf{x}})^T\mathbf{V}^{-1}(\mathbf{x}-\bar{\mathbf{x}})\right], \tag{1.14}$$

where $\det\cdot$ represents the determinant of matrices. The corresponding characteristic function can be written as

$$\chi(\boldsymbol{\zeta}) = \exp\left[-(1/2)\boldsymbol{\zeta}^T(\mathbf{\Omega}\mathbf{V}\mathbf{\Omega}^T)\boldsymbol{\zeta} - i(\mathbf{\Omega}\bar{\mathbf{x}})^T\boldsymbol{\zeta}\right]. \tag{1.15}$$

The Gaussian states used in this thesis are enumerated as follows.

Vacuum state: As given by Eq. (1.8), a vacuum state $|0\rangle$ is defined as the eigenstate of the annihilation operator with a zero eigenvalue. A vacuum state is a Fock state with zero mean photon number, i.e., $\bar{n} = \langle \hat{n}\rangle = 0$. It can also be viewed as a coherent state with zero amplitude. In terms of quadrature field operators, the mean value of $|0\rangle$ is zero (or more precisely, a vector of zeros) and the CM of $|0\rangle$ is the two-by-two identity matrix $\mathbf{I}_2$.

Thermal state: Thermal states are those about which we have a minimum of information, knowing only the mean value of the photon number. They are mixed states with non-zero mean photon number $\bar{n}$. In the Fock basis, the density operator for a thermal state can be written as

$$\hat{\rho}_{\text{thermal}} = \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(\bar{n}+1)^{n+1}} |n\rangle \langle n| . \tag{1.16}$$

In terms of quadrature field operators, the mean value of the above thermal state is zero and the CM is $(2\bar{n}+1)\mathbf{I}_2$.

Coherent state: Coherent states are the eigenstates of the annihilation operator with complex eigenvalue α, $\hat{a}\left|\alpha\right\rangle = \alpha\left|\alpha\right\rangle$. They are defined as the displaced vacuum state with a complex displacement α,

$$\left|\alpha\right\rangle := D(\alpha)\left|0\right\rangle = \exp(-\frac{1}{2}|\alpha|^2)\sum_{n=0}^{\infty}\frac{\alpha^2}{\sqrt{n!}}\left|n\right\rangle, \tag{1.17}$$

where

$$D(\alpha) = e^{\alpha\hat{a}^\dagger - \alpha^*\hat{a}} = e^{-\frac{1}{2}|\alpha|^2}e^{\alpha\hat{a}^\dagger}e^{-\alpha^*\hat{a}} = e^{\frac{1}{2}|\alpha|^2}e^{-\alpha^*\hat{a}}e^{\alpha\hat{a}^\dagger}, \tag{1.18}$$

is the displacement operator that is just the complex version of the Weyl operator. In terms of quadrature field operators, the mean value of a coherent state $\left|\alpha\right\rangle$ is $[\mathrm{Re}\{\alpha\},\ \mathrm{Im}\{\alpha\}]^T$ and the CM is $\mathbf{I}_2$.

Squeezed vacuum state: Squeezed states are characterized by the property that the variance of one of the quadrature field operators is less than the value associated with the vacuum state and the coherent states (1 for our case). A squeezed vacuum state is defined as [69]

$$\left|\mathrm{SV}\right\rangle = S(r,\phi)\left|0\right\rangle, \tag{1.19}$$

where

$$S(r,\phi) = \exp\left[-\frac{r\exp{(i\phi)}}{2}\hat{a}^{\dagger 2} + \frac{r\exp{(-i\phi)}}{2}\hat{a}^2\right], \tag{1.20}$$

is the single-mode squeezing operator, $r \in \mathbb{R}$ is the squeezing parameter that quantifies the squeezing. In terms of quadrature field operators, the squeezed vacuum states have zero mean and their CM is $\mathrm{diag}[\exp{(-2r)}, \exp{(2r)}]$.

Two-mode squeezed vacuum state: Two-mode squeezed vacuum (TMSV) states are an important class of entangled states. They can provide an arbitrary approximation to the ideal Einstein-Podolsky-Rosen (EPR) state. A TMSV state is defined as [69]

$$\left|\mathrm{TMSV}\right\rangle = S_{\mathrm{AB}}(r,\phi)\left|00\right\rangle_{\mathrm{AB}}, \tag{1.21}$$

where the subscript 'A' and 'B' indicate the two modes,

$$S_{\mathrm{AB}}(r,\phi) = \exp\left[-r\exp(i\phi)\hat{a}^{\dagger}\hat{b}^{\dagger} + r\exp(-i\phi)\hat{a}\hat{b}\right], \tag{1.22}$$

is the two-mode squeezing operator, $r \in \mathbb{R}$ again is the squeezing parameter that quantifies the squeezing, and $\hat{a}$ and $\hat{b}$ are the annihilation operators of the two modes. The form of operator $S_{\mathrm{AB}}(r,\phi)$ is similar to that of the single-mode operator $S(r,\phi)$ given in Eq. (1.20) but with $\hat{a}^{\dagger 2}/2$ replaced by $\hat{a}^{\dagger}\hat{b}^{\dagger}$. TMSV states can be created from mixing two squeezed vacuum states at a beam splitter. The squeezing in the unit of decibels is given by $r[\mathrm{dB}] = -10\log_{10}[\exp(-2r)]$.

In this thesis, we adopt $\phi = \pi$. In the Fock basis, the TMSV given by Eq. (1.21) can then be re-written as

$$|\mathrm{TMSV}\rangle = \sqrt{\lambda^2 - 1}\sum_{n=0}^{\infty}\lambda^n |nn\rangle_{\mathrm{AB}}, \tag{1.23}$$

where the parameter λ is related to the squeezing parameter by $\lambda = \tanh r$. In terms of quadrature field operators, the mean value of the TMSV state is zero and the CM is given by

$$\mathbf{V} = \begin{bmatrix} \cosh(2r)\mathbf{I}_2 & \sinh(2r)\mathbf{Z} \\ \sinh(2r)\mathbf{Z} & \cosh(2r)\mathbf{I}_2 \end{bmatrix}, \tag{1.24}$$

where $\mathbf{Z} = \mathrm{diag}[1,\ -1]$.

Gaussian measurement

A Gaussian measurement projects states into Gaussian states. Common Gaussian measurements include homodyne detection, which projects states into squeezed states, and heterodyne detection, which projects states into coherent states.

Homodyne detection is one method of extracting information encoded as the amplitude of the quadrature fields of optical light. Such detection can also be used to measure the $\hat{q}$ (or $\hat{p}$) quadratures of a mode. The measurement outcome q (or p) has a probability distribution, which can be written as the marginal integration of the Wigner function over the conjugate quadrature. In homodyne detection, a target mode (with annihilation operator $\hat{a}$) is mixed with a classical coherent light (with complex amplitude α_0) at a 50:50 beam splitter. The classical coherent light, which is called the local oscillator (LO), is usually assumed to be derived from the same source as the target mode. The annihilation operators of the two output modes of the beam splitter can then be written as $\hat{a}_1 = (\alpha_0 + \hat{a})/\sqrt{2}$ and $\hat{a}_2 = (\alpha_0 - \hat{a})/\sqrt{2}$. The intensities of the output modes, $I_1 = \langle \hat{a}_1^\dagger \hat{a}_1 \rangle$ and $I_2 = \langle \hat{a}_2^\dagger \hat{a}_2 \rangle$, are measured by two photo-detectors. The difference in these intensities is $I_1 - I_2 = \langle \alpha^* \hat{a} + \alpha \hat{a}^\dagger \rangle$. By setting the phase of the LO as 0 or $\pi/2$, which is just the argument of α_0, the $\hat{q}$ or $\hat{p}$ quadratures of the target mode can be measured.

Heterodyne detection allows the simultaneous measurements of both quadratures of a mode at the cost of additional measurement noise. The measurement outcome has a probability distribution named the Q, or Husimi, function. In heterodyne detection, the mode to be measured is first mixed with a vacuum ancillary mode at a 50:50 beam splitter. The $\hat{q}$ quadrature of one output mode of the beam splitter and the $\hat{p}$ quadrature of the other output mode are measured by homodyne detection.

Non-Gaussian states

As the opposite of Gaussian states, non-Gaussian states have non-zero high-order moments and cannot be simply characterized by their mean values and CMs. Therefore, the set of non-Gaussian states is much more vast and complicated than Gaussian states. Non-Gaussian states can be categorized into two sub-classes, namely the non-Gaussian mixture of Gaussian states and the quantum non-Gaussian states [70,71]. States in the first sub-class can be written as a classical mixture (with a non-Gaussian weighting function) of Gaussian states, e.g., $(|\alpha\rangle \langle\alpha| + |-\alpha\rangle \langle-\alpha|)/2$. States in the second sub-class cannot be represented by a mixture of Gaussian states. Within the scope of this thesis, it will be sufficient to consider only the quantum non-Gaussian states. Therefore, for brevity, we will drop the term 'quantum' when we refer to quantum non-Gaussian states.

The most common non-Gaussian states are the Fock states (except the vacuum state). They can be created by performing a photon-number resolving detection on one mode of a TMSV state. A Fock state $|n\rangle$ has the Wigner characteristic function (with complex variable ξ) [72]

$$\chi(\xi) = (-1)^n \exp(-|\xi|^2/2) L_n(|\xi|^2), \tag{1.25}$$

where $L_n(\xi) = \frac{\exp(\xi)}{n!}\frac{\partial^n}{\partial \xi^n}[\exp(-\xi)\xi^n]$ is the Laguerre polynomials. The Fock states are an important ingredient for various non-Gaussian operations, which will be discussed as follows.

1.3.3 Non-Gaussian operations

Non-Gaussian operations are defined as the operations that can transform Gaussian states into non-Gaussian states. Given an input state (i.e., the state to be non-Gaussified) $\hat{\rho}_{\text{in}}$, a non-Gaussian operation can be defined as a trace non-preserving operators $\hat{O}$, which implements the transformation

$$\hat{\rho}_{\text{out}} = \frac{1}{\mathcal{N}} \hat{O} \hat{\rho}_{\text{in}} \hat{O}^\dagger, \tag{1.26}$$

where $\mathcal{N}$ is a normalization constant, which is also the success probability of the operation.

Photon number resolving detectors

The measurements that project states into non-Gaussian states are one typical class of non-Gaussian operations. Ideal photon number resolving detectors (PNRDs) implement measurements that project states into Fock states. A PNRD can be described by a complete set of projection operators $\{|n\rangle\langle n|\}_{n=0}^{\infty}$. In general, a post-selection on $|n\rangle\langle n|$ for a fixed n is performed on the detection result so that the detection is successful only if n photons are detected. The operator for the detection can then be simplified to $|n\rangle\langle n|$. A simplification of PNRDs is the on/off detectors, which can be described by the set $\{\hat{\Pi}_{\text{off}} = |0\rangle\langle 0|,\ \hat{\Pi}_{\text{on}} = \sum_{n=0}^{\infty} |n\rangle\langle n|\}$. The on/off detectors can detect the absence or presence of photons but cannot resolve the photon number. Such detectors can be used in a multiplexed fashion to approximate the ideal PNRD.

Photon subtraction, addition, and catalysis

One class of widely studied non-Gaussian operations applies a polynomial function of the annihilation operator $\hat{a}$ and the creation operator $\hat{a}^\dagger$ to a state. There are two basic types of such operations, namely photon subtraction and photon addition. Ideal photon subtraction and ideal photon addition apply $\hat{a}$ and $\hat{a}^\dagger$ to a state, respectively.

As shown in Fig. 1.1a, a procedure that can approximate both photon subtraction and photon addition contains beam splitters and PNRDs. In this procedure, the input mode is coupled with an ancillary Fock state $|n\rangle$ at a beam splitter with transmissivity T_{b}. The beam splitter can be represented by an operator

$$\hat{B} = \exp\left[\theta(\hat{a}\hat{b}^\dagger - \hat{a}^\dagger\hat{b})\right], \tag{1.27}$$

where $\sqrt{T_{\text{b}}} = \cos\theta$, and $\hat{a}$ and $\hat{b}$ are the annihilation operators of the input mode and the ancillary mode, respectively. The procedure has been successful if m photons are detected at the ancillary output of the beam splitter. The transformation that the procedure described above applies on an input mode can be represented by an operator

$$\hat{O}_{mn} = \frac{1}{\sqrt{m!n!}} \frac{\partial^{m+n}}{\partial\alpha^n\partial\beta^m} e^{-\hat{a}^\dagger\alpha\sqrt{1-T_{\text{b}}}} \sqrt{T_{\text{b}}}^{\hat{a}^\dagger\hat{a}} e^{\hat{a}\beta\sqrt{1-T_{\text{b}}}} \Big|_{\alpha=\beta=0}. \tag{1.28}$$

Setting $n = 0$ and $m = 1$ (Fig. 1.1b) leads to the approximation of ideal photon subtraction,

$$\hat{O}_{\text{PS}} = \sqrt{\frac{1-T_{\text{b}}}{T_{\text{b}}}} \hat{a} \sqrt{T_{\text{b}}}^{\hat{a}^\dagger\hat{a}}, \tag{1.29}$$

which approximates $\hat{a}$ for $T_{\text{b}} \to 1$ with a vanishing success probability. Setting $n = 1$

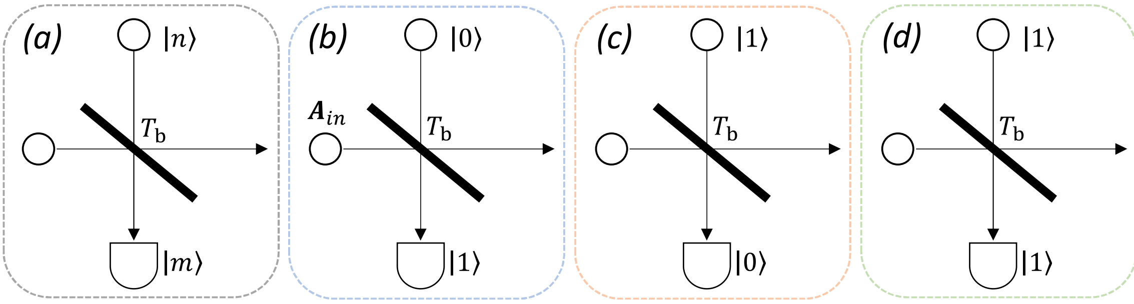

Figure 1.1: Non-Gaussian operations implemented by a beam splitter and a photon number resolving detector. a) A general non-Gaussian operation. b) Photon subtraction. c) Photon addition. d) Photon catalysis.

and $m = 0$ (Fig. 1.1c) leads to the approximation of ideal photon addition,

$$\hat{O}_{\mathrm{PA}} = -\sqrt{1 - T_{\mathrm{b}}}\hat{a}^\dagger \sqrt{T_{\mathrm{b}}}^{\hat{a}^\dagger \hat{a}}, \tag{1.30}$$

which approximates $\hat{a}^\dagger$ for $T_{\mathrm{b}} \to 1$ with a vanishing success probability. Higher-order photon subtraction $\hat{a}^i$ and photon addition $\hat{a}^{\dagger i}$ for $i > 1$ can be realized by a cascaded process of the procedures described above or by setting $m > 1$ or $n > 1$.

Setting $n = m = 1$ (Fig. 1.1d) leads to an operation that is different from both photon subtraction and photon addition. This operation is named single-photon photon catalysis since it might replace one photon from a state. The single-photon photon catalysis can be represented by an operator

$$\hat{O}_{\mathrm{PC}} = \sqrt{T_{\mathrm{b}}} \left(\frac{T_{\mathrm{b}} - 1}{T_{\mathrm{b}}} \hat{a}^\dagger \hat{a} + 1 \right) \sqrt{T_{\mathrm{b}}}^{\hat{a}^\dagger \hat{a}}, \tag{1.31}$$

where the two terms in the parentheses indicate that the output state after photon catalysis will be a superposition of the state with one photon replaced and the state with no photon replaced. The operation that performs the exact photon replacement can be implemented by a cascaded processing of photon subtraction and photon addition.

Higher order of photon catalysis can be implemented by setting $n = m = N_{\mathrm{c}}$. In this case, the output state after photon catalysis will be a superposition of the state with no photon replaced, the state with one photon replaced, and up to the state with N_{c} photons replaced (with different weightings).

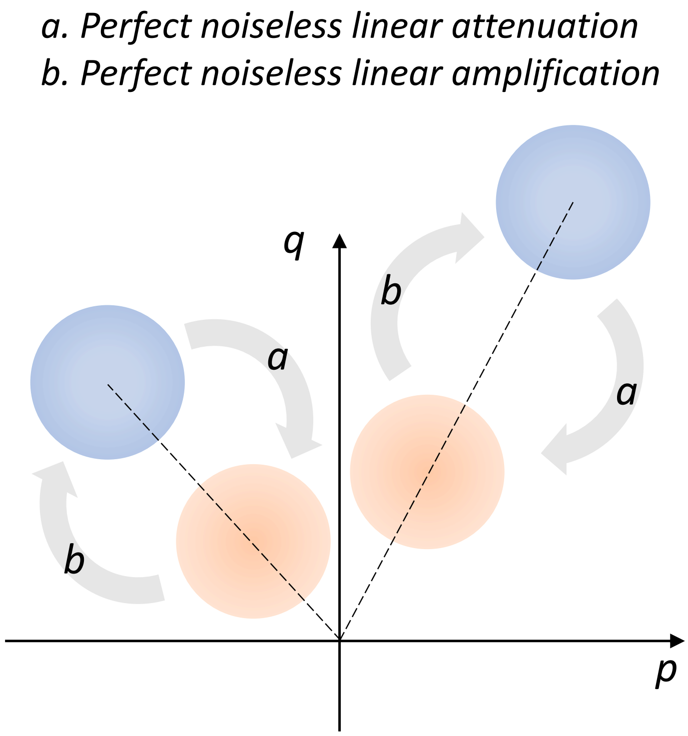

Figure 1.2: Phase-space representation of coherent states, before or after a) Perfect noiseless linear attenuation and b) Perfect noiseless linear amplification.

Setting $n = m = 0$ leads to the so-called zero-photon photon catalysis. Such an operation can be represented by the operator $\sqrt{T_\mathrm{b}}^{\hat{a}^\dagger \hat{a}}$, which is a trace-non-preserving Gaussian operation since no non-Gaussian ingredient is included in the operation. Zero-photon photon catalysis is one possible implementation to *perfect noiseless linear attenuation*. As shown in Fig. 1.2a, perfect noiseless linear attenuation can be used to scale down the amplitude of a coherent state without introducing extra noise:

$$\sqrt{T_\mathrm{b}}^{\hat{a}^\dagger \hat{a}} |\alpha\rangle = \frac{1}{\sqrt{\mathcal{N}}} |\sqrt{T_\mathrm{b}}\alpha\rangle \,, \tag{1.32}$$

where $\mathcal{N}$ is a normalization constant, which is also the success probability for the operation.

Quantum scissors

Similar to the operations discussed above, quantum scissors also consist of beam splitters, ancillary photons, and photon number resolving detectors. Quantum scissors was first proposed as a tool to truncate an input state into the subspace of

$\{|0\rangle\,,\ |1\rangle\}$. They can be represented by an operator [73]

$$\hat{O}_{\text{trun}} = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|), \tag{1.33}$$

which was later generalized to [33]

$$\hat{O}_{\text{QS}} = \sqrt{\frac{T_{\text{s}}}{2}}\,|0\rangle\langle 0| + \sqrt{\frac{1-T_{\text{s}}}{2}}\,|1\rangle\langle 1|\,, \tag{1.34}$$

where T_{s} is the transmissivity of one beam splitter in the quantum scissors. Besides truncation, the generalized quantum scissors can also change the weights of the vacuum component and the single-photon component of an input state. By preforming the generalized quantum scissors in a parallel way, higher order truncation can be implemented, albeit with a lower success probability than other methods, e.g. [39, 74].

Noiseless linear amplification

Noiseless linear amplification is an useful tool in combating the loss that occurs during the channel transmission of quantum states. As shown in Fig. 1.2b, perfect noiseless linear amplification (NLA) scales up the amplitude of a coherent state deterministically and without introducing extra noise: $|\alpha\rangle \to |g\alpha\rangle$, where $g > 1$ is a gain factor for the amplification. We refer to such amplification as the *perfect deterministic NLA*. Perfect deterministic NLA is impossible as it violates the non-cloning theorem, which states that an unknown state cannot be perfectly cloned. This can be seen by setting $g = \sqrt{2}$,

$$|\alpha\rangle|0\rangle \xrightarrow{\text{Perfect deterministic NLA}} |\sqrt{2}\alpha\rangle|0\rangle \xrightarrow{\text{beam splitter}} |\alpha\rangle|\alpha\rangle\,. \tag{1.35}$$

The non-cloning theorem can be satisfied by performing NLA in a probabilistic fashion. *Perfect probabilistic NLA* can be defined by an operator,

$$g^{\hat{a}^\dagger\hat{a}} = \sum_{n=0}^{\infty} g^n |n\rangle\langle n|\,, \tag{1.36}$$

where again g is the gain factor for the amplification. Similar to the noiseless linear attenuation defined in Eq. (1.32), perfect probabilistic NLA is a Gaussian operation and by definition it can be used to scale up the amplitude of a coherent state $|\alpha\rangle$. However, the operator $g^{\hat{a}^\dagger\hat{a}}$ is unbounded, meaning that $g^{\hat{a}^\dagger\hat{a}}|\alpha\rangle$ cannot be normalized by a constant less than one. Therefore, perfect probabilistic NLA is again impossible.

However, there are procedures, mostly including non-Gaussian operations, that approximate perfect probabilistic NLA. The core of such procedures is to only amplify a limited number $N_{\rm lim}$ of components of states, e.g., $N_{\rm lim} = 2$ for amplifying only the $|0\rangle$ and the $|1\rangle$ components. Photon replacement (i.e., cascaded processing of photon subtraction and photon addition) can be used to realize the probabilistic NLA for $N_{\rm lim} = 2$ with a fixed gain $g = 2$ [75]. The quantum scissors defined by Eq. (1.34) can also be used for the probabilistic NLA with $N_{\rm lim} = 2$ and an adjustable gain. The photon catalysis defined by Eq. (1.31) also allows the probabilistic NLA for coherent states with small amplitude. Probabilistic NLA with $N_{\rm lim} > 2$ is possible with a paralleled processing of quantum scissors or photon catalysis.

1.3.4 Quantum communication systems

In this section, we first discuss the quantum communication protocols considered in this thesis. We then present some additional settings we will consider in the actual deployment of the protocols. Before presenting the protocols, we first discuss quantum entanglement, which is an important resource for the protocols.

Quantum entanglement

Entanglement is one of the most important properties of quantum mechanics (for review, see [76,77]). It is also one critical resource for various quantum information tasks. Consider a bipartite system $\hat{\rho}$ with subsystems labeled as A and B and each subsystem can have more than one mode. The system is said to be entangled if it cannot be written as

$$\hat{\rho} = \sum_i p_i \hat{\rho}_i{}^A \otimes \hat{\rho}_i{}^B, \tag{1.37}$$

where $\sum_i p_i = 1$ and the $\hat{\rho}_i{}^A$'s and $\hat{\rho}_i{}^B$'s are density operators on the Hilbert spaces of the subsystems. For an arbitrary bipartite state $\hat{\rho}$ with dimension 2×2 or 2×3, the positive partial transpose (PPT) criterion can be used to determine if the state is entangled [78, 79]. Let $\hat{\rho}^{T_\mathrm{B}}$ be the partial transpose of $\hat{\rho}$ with respect to subsystem B. The system is entangled if $\hat{\rho}^{T_\mathrm{B}}$ has a negative eigenvalue. The state is said to be separable otherwise. The PPT criterion was shown later to be able to detect entanglement for two-mode [80] and $1 \times N$ mode [81] Gaussian states for CV systems. For DV states with higher dimension or CV non-Gaussian states, higher-order criteria, e.g., [82, 83], is needed to detect entanglement.

There is no standard measure of the amount of entanglement. In this thesis, we adopt the logarithm negativity, E_LN, as the entanglement measure. The logarithm negativity is easily computable and sets an upper bound on the distillable entanglement. It is defined as

$$E_\mathrm{LN}(\hat{\rho}) = \log_2[1 + 2N(\hat{\rho})], \tag{1.38}$$

where $N(\hat{\rho})$ is the negativity that is defined as the absolute value of the sum of the negative eigenvalues of the partially transposed $\hat{\rho}$.

The logarithm negativity has the following properties: (1) $E_\mathrm{LN}(\hat{\rho}) \geq 0$. (2) $E_\mathrm{LN}(\hat{\rho}) = 0$ when $\hat{\rho}$ is separable. (3) $E_\mathrm{LN}(\hat{\rho})$ is additive on tensor product: $E_\mathrm{LN}(\hat{\rho}_1 \otimes \hat{\rho}_2) = E_\mathrm{LN}(\hat{\rho}_1) + E_\mathrm{LN}(\hat{\rho}_2)$. (4) $E_\mathrm{LN}(\hat{\rho})$ does not increase under local operations and classical communications.

Entanglement distribution

The distribution of entanglement between users in different locations is a prerequisite for various quantum communication protocols. Focusing on bipartite entanglement only, entanglement distribution can be realized in three different schemes discussed as follows. Before introducing the schemes, we first define two types of

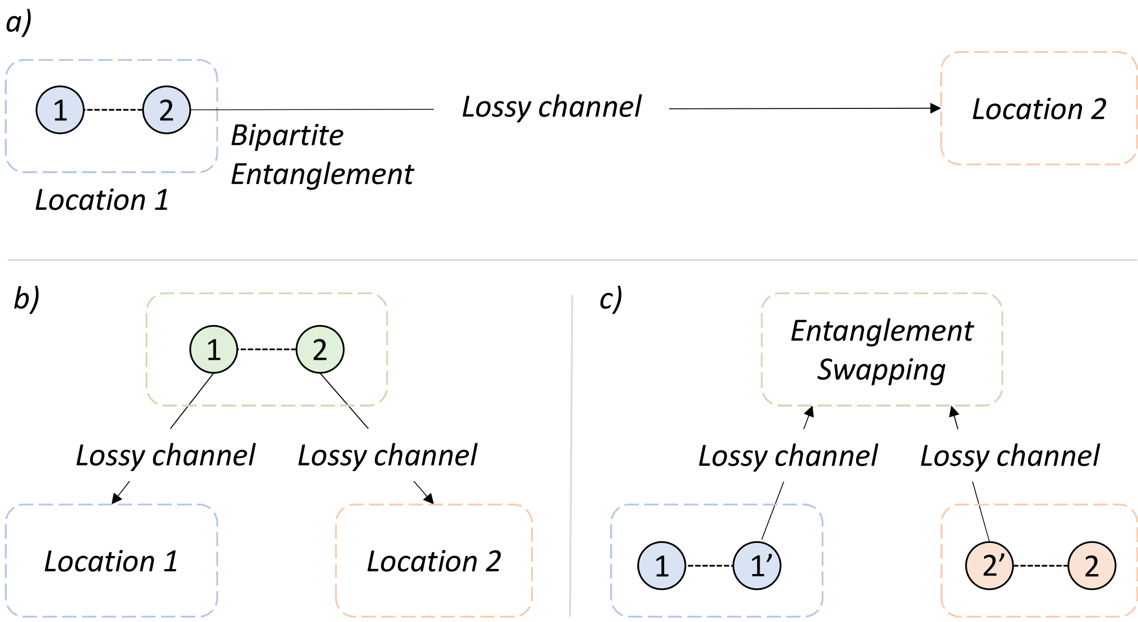

Figure 1.3: Three schemes for entanglement distribution. a) Direct transmission. b) Entanglement in the middle. c) Entanglement swapping.

channels that will be used in this thesis. The first type is named *quantum channel*, which is the channel through which quantum states are sent. Quantum channels are characterized by their transmissivity T and noise ε. Such channels can be modeled by a beam splitter with transmissivity T and a thermal state (or vacuum state) with variance ε. In this model, the state sent through the channel is mixed with the thermal state (or vacuum state) at the beam splitter. The subsystem at the reflection path of the beam splitter is discarded. By *lossy channel* we refer to the quantum channel with loss. The second type is named *classical channel*, which is the channel through which classical signals are sent. We will always assume the classical channel is perfect and no loss occurs during the transmission of classical signals. We will drop the terms "quantum" or "classical" when the context is clear. We now briefly discuss the three schemes for entanglement distribution. The goal is to establish entanglement between two users at two locations.

Direct transmission: As shown in Fig. 1.3a, the bipartite entangled state is prepared at one location by one user and one part of the state is directly sent to a remote location through a channel.

Entanglement in the middle: As shown in Fig. 1.3b, the bipartite entangled state is prepared at a middle station and one part of the state is sent to one location through one channel. Another part of the state is sent to another location through another channel.

Entanglement swapping: As shown in Fig. 1.3c, both locations prepare their own bipartite entangled states and send one part of their states to a middle station through two independent channels. An entanglement swapping is performed at the middle station. In this way, one part of the state kept by one location is entangled with one part of another state kept by another location.

For all schemes the logarithm negativity of the entangled state after the distribution will be used as the performance metric.

Quantum key distribution

Quantum key distribution (QKD) provides unparalleled information security by the lays of quantum mechanics. The goal of QKD is to establish a secure bit string between two legitimate users labeled Alice and Bob. The bit string, which is also named the secure key, is known only by the two users and will then be used as the one-time pad for encrypted communications.

The first and perhaps the most famous QKD protocol is the BB84 protocol proposed in 1984 [84]. As shown in Fig. 1.4, in the BB84 protocol, Alice prepares photons, of which the polarizations are determined by two pre-generated random bit strings. Four possible states, chosen from two mutually unbiased bases[1], are used in her preparation. She then sends the modulated photons to Bob through a quantum channel controlled by an Eavesdropper, Eve. Bob measures the polarizations of the incoming photons and informs Alice on the measurement bases he used for the measurement. To ensure security Bob's measurement bases are determined by one randomly generated bit string. By classical communications, Alice and Bob agree on which bits to keep. Two correlated bit strings are then established between the two users. Finally, Alice and Bob perform channel estimation, error correction and

[1]Mutually unbiased bases: If a system is prepared in a state belonging to one of the bases, then all outcomes of the measurement using other bases are predicted to occur with equal probability.

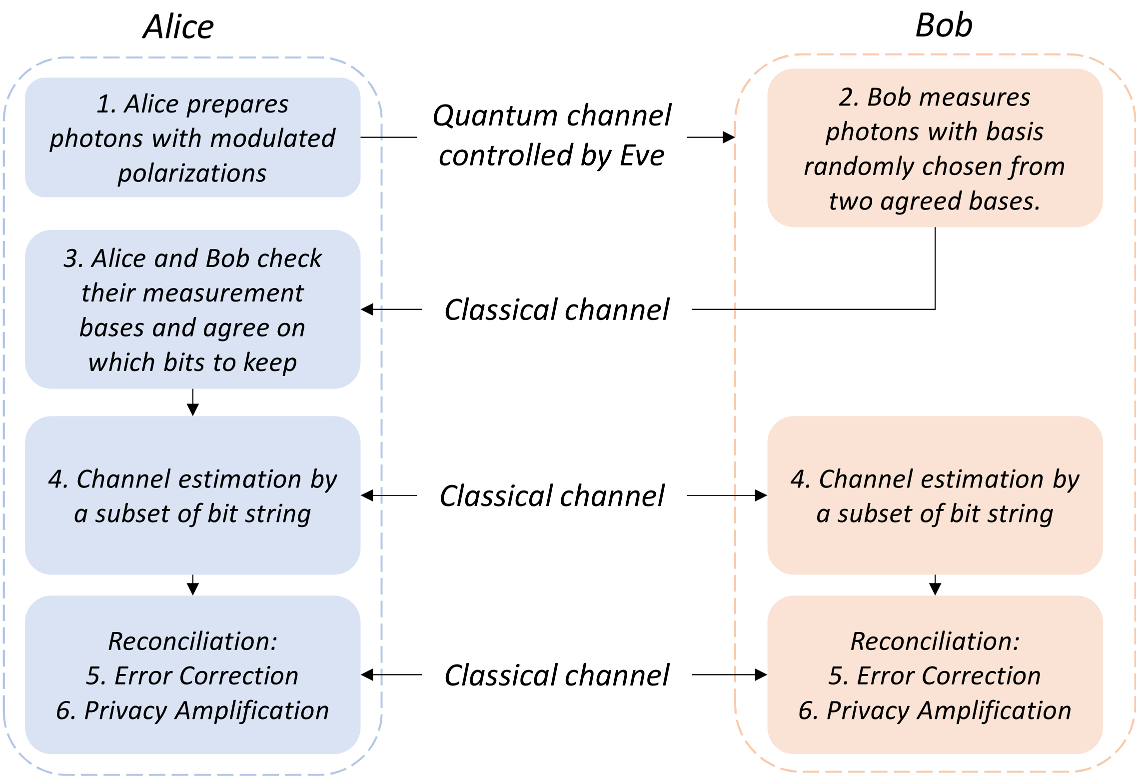

Figure 1.4: The BB84 protocol.

privacy amplification, all through a classical channel, to ensure the identity between their strings and to sift out the information that has leaked to Eve.

The BB84 protocol is referred to as a discrete-variable (DV) QKD protocol because it encodes information into the DV states of light. As a variant of DV-QKD protocols, in CV-QKD protocols CV states of light are used as the information carrier. In CV-QKD protocols, Alice generates a string of random variables from a Gaussian distribution. She then prepares pulses of light beam, of which the states of the quadrature fields are determined by the string. Typical states include coherent states and squeezed states. The encoded beam is then sent to Bob through a quantum channel, again assumed to be controlled by Eve. After receiving the beam, Bob performs homodyne detection on the quadrature fields of the beam and obtains data correlated to Alice's string. The measurement bases (on $\hat{p}$ or $\hat{q}$ quadratures) are randomly chosen to ensure security. Finally, Alice and Bob perform a reconciliation process on their data to extract bit strings from the data and sift out Eve's information.

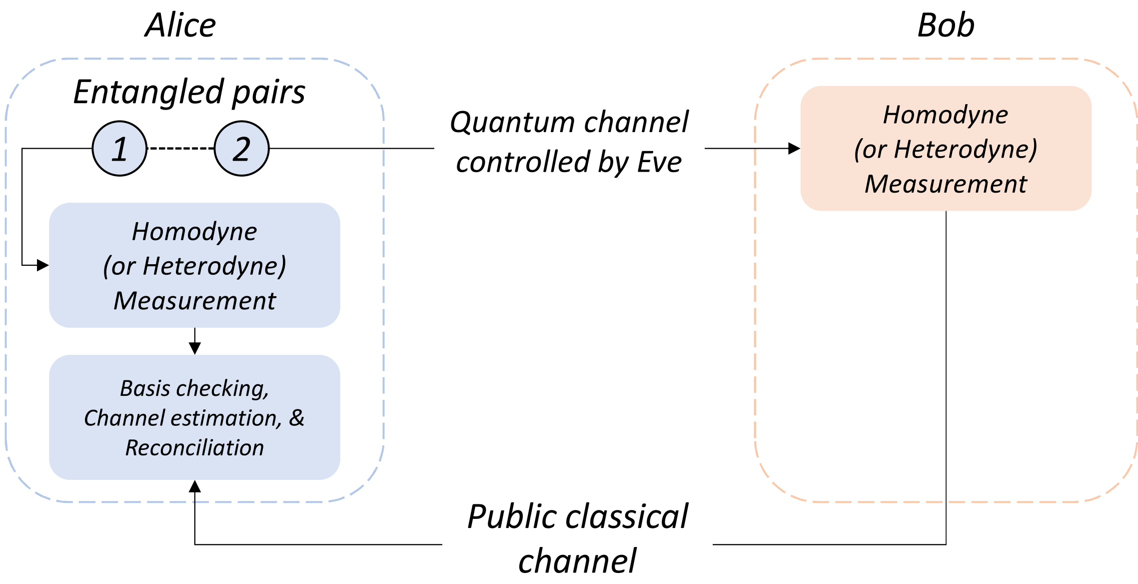

Figure 1.5: An entanglement-based CV-QKD protocol.

The aforementioned protocols, either DV or CV, are referred to as prepare-and-measure (PM) protocols. Every PM-QKD protocol has an equivalent entanglement-based (EB) form. As shown in Fig. 1.5, in the EB version of a CV-QKD protocol, Alice first prepares pulses of entangled twin beams. She then sends one beam to Bob and performs a homodyne measurement (or heterodyne) on the quadrature fields of the remaining beam. The measurement bases are randomly chosen if a homodyne measurement is adopted. After receiving the beam, Bob also performs a homodyne measurement (or heterodyne) on the quadrature fields of the beam. Similar to Alice, Bob's measurement bases are randomly chosen if a homodyne measurement is adopted. Bob's measurement results are correlated to Alice's measurement results due to the entanglement between the beams. The rest procedures of the EB form of the QKD protocol are the same as the original PM-QKD protocol.

In this thesis, we will always analyze the security of the CV-QKD protocols by their EB forms. We will adopt the usual assumption that the number of the pulses (i.e., states) exchanged between Alice and Bob is infinite (i.e., the asymptotic limit). We will use the secure key rates at given transmission distances (or equivalently, channel losses) as the performance measure of the QKD protocol. In the asymptotic limit, a lower bound for the key rates (in the unit of bit/pulse) can be written as

$$R \geq \eta_{\mathrm{r}} I(A : B) - \chi(E), \tag{1.39}$$

where $\eta_{\mathrm{r}} < 1$ is the reconciliation efficiency, $I(A : B)$ is the mutual information between Alice and Bob, and $\chi(E)$ is the information that Eve has obtained from the quantum channel and the classical channel used in the reconciliation process.

Although proposed much later than DV-QKD, some CV-QKD technologies now provide similar performance and security levels as DV-QKD technologies. In the practical implementation of a QKD system, CV-QKD offers the advantage of highly efficient homodyne detectors. Furthermore, CV-QKD systems are compatible with current telecommunication technologies. CV-QKD systems can be fully built using "off-the-shelf" coherent optical hardware. However, CV-QKD systems require much more signals to achieve the same key rate as DV-QKD [85], e.g., to achieve a key rate of 0.1 bits per pulse a CV-QKD protocol studied in [86] requires $\sim 10^9$ signals. To achieve the same key rate the DV-QKD protocol studied in [87] only requires $\sim 10^4$ signals. The cost of more complex data processing is a trade-off to be considered in the deployment of CV-QKD systems.

Quantum teleportation

As shown in Fig. 1.6, quantum teleportation enables the remote transmission of quantum states, either DV or CV, from one user to another by using pre-distributed entangled states. In this thesis, we name the entangled states used for teleportation as teleportation resource states (or just resource states). The first teleportation protocol uses a DV entangled state, the Bell state, as its resource state [88]. The resource state is distributed between two users named Alice and Bob. We define the qubit to be teleported as the *input qubit*, and the qubit after teleportation as the *output qubit*. Alice performs a Bell state measurement on the input qubit and her part of the resource state. She then informs Bob about her measurement result through a classical channel. Depending on the results, Bob performs a unitary operation on his part of the resource state, which is the output qubit. In the limit

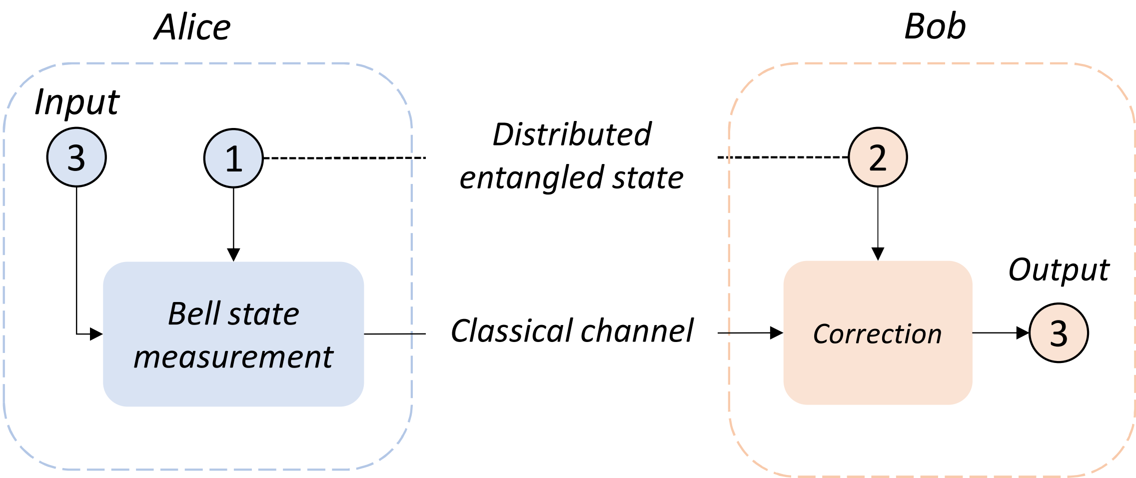

Figure 1.6: A teleportation protocol.

of perfect entanglement shared between Alice and Bob, The output qubit after the unitary operation approaches the input qubit. No information on the input qubit is obtained through Alice's measurement. In this way, the input qubit is recovered by Bob. Let $\hat{\rho}_{\mathrm{in}}$ and $\hat{\rho}_{\mathrm{out}}$ be the density operators for the states of the input qubit and the output qubit. The teleportation fidelity, which measures the closeness between the states, can then be written as [89]

$$\mathcal{F} = \left(\mathrm{tr} \left\{ \sqrt{\sqrt{\hat{\rho}_{\mathrm{out}}}\hat{\rho}_{\mathrm{in}}\sqrt{\hat{\rho}_{\mathrm{out}}}} \right\} \right)^2 , \tag{1.40}$$

which reduces to $\mathrm{tr}\{\hat{\rho}_{\mathrm{in}}\hat{\rho}_{\mathrm{out}}\}$ when either of the states is pure.

The fidelity averaged over the outcomes of the Bell state measurement and the distribution of the input qubit, $\bar{\mathcal{F}}$, is a performance metric of the teleportation protocol. An important threshold is the 2/3 classical limit, which is the achievable average fidelity with local operations and classical communications only (i.e., no entanglement is shared between the users). A teleportation protocol is meaningful only when $\bar{\mathcal{F}} > 2/3$ can be achieved.

Similar to QKD, the DV teleportation protocol was later extended to the CV domain [90,91]. In a CV-based teleportation protocol, a CV two-mode entangled state (e.g., a TMSV state) is used as the resource state. Similar to the DV teleportation, in CV teleportation we define the mode to be teleported as the *input mode* and the teleported mode as the *output mode*. After the distribution of the entangled state, Alice couples the input mode with her part of the resource state by a 50:50 beam splitter. She then measure the $\hat{p}$ quadrature of one coupled mode and the $\hat{q}$ quadrature of another coupled mode, both through homodyne measurement. Depending on the results received from Alice, Bob performs a displacement operation on his part of the resource state to recover the input mode. The characteristic formalism provides a concise description to the CV teleportation protocol [92]. Let $\chi_{\text{in}}(\xi)$, $\chi_{\text{out}}(\xi)$, and $\chi_{\text{res}}(\xi_{\text{a}}, \xi_{\text{b}})$ be the characteristic functions (all with complex variables) of the input mode, the output mode (averaged over the probability distribution of the measurement outcomes), and the resource state, respectively. The input-output relation of the CV teleportation protocol can then be described by [93]

$$\chi_{\text{out}}(\xi) = \chi_{\text{in}}(g\xi)\chi_{\text{res}}(g\xi^*, \xi), \tag{1.41}$$

where g is a gain factor in the displacement operation. For a CV teleportation protocol with pure input mode, the teleportation fidelity given by Eq. (1.40) can be re-written as [94]

$$\bar{\mathcal{F}} = \frac{1}{\pi} \int d^2\xi \chi_{\text{in}}(\xi)\chi_{\text{out}}(-\xi), \tag{1.42}$$

where we have used the averaged symbol because the fidelity can also be viewed as the fidelity averaged over the outcome of the homodyne measurements.

The teleportation fidelity of coherent states can be used as a performance metric for a CV teleportation protocol. For the teleportation of coherent states, independent of the amplitude of the states, a fidelity of 1/2 can always be achieved with local operations and classical communications only. In teleporting coherent states, a CV teleportation is meaningful only when $\bar{\mathcal{F}} > 1/2$ can be achieved.

Multi-mode quantum systems

In this thesis, a multimode is simply a generic collection of single modes, each with a certain center frequency and a bandwidth. In a multimode system, supermodes are mutually orthogonal broadband modes that are linear combinations of the same single modes.

The parametric down-conversion (PDC) process is commonly used to create TMSV states. In this process, a pump is fed into a nonlinear crystal, creating pulses of two entangled beams. However, instead of producing single TMSV states, the PDC process creates a multitude of finitely squeezed TMSV states in the supermodes. Each output pulse consists of a multitude of TMSV states written as [95]

$$|\mathrm{PDC}\rangle = \bigotimes_{k=1}^{K} \exp\left[r_k \left(\hat{A}_k^\dagger \hat{B}_k^\dagger - \hat{A}_k \hat{B}_k\right)\right] |0\rangle\,, \tag{1.43}$$

where the $\hat{A}_k$'s and $\hat{B}_k$'s represent annihilation operators of the supermodes in the two beams, and the r_k's are the squeezing parameters of the TMSV state. The upper limit K is determined by the resolution of the detectors applied to the multimode state, which is the total number of supermodes in the system and also the total number of single modes. The annihilation operators satisfy the commutation relations

$$[\hat{A}_k,\ \hat{B}_{k'}] = [\hat{A}_k^\dagger,\ \hat{B}_{k'}^\dagger] = 0,\ [\hat{A}_k,\ \hat{A}_{k'}^\dagger] = [\hat{B}_k,\ \hat{B}_{k'}^\dagger] = \delta_{kk'}. \tag{1.44}$$

For common PDC sources, the squeezing parameters r_k form an exponentially decaying distribution, which can also be engineered from emitting a single EPR state to creating an array of EPR states.

Satellite-based quantum channels

For optical signals, the loss of the quantum channels can be characterized by the channel transmissivity, which is given by the ratio of the power captured by the receiver. For satellite-based optical communications, the sources of loss are diffraction, scattering, absorption, and atmospheric turbulence. Diffraction is a natural wave phenomenon of all light beams. It causes broadening of the light beam as it propagates and reduces the amount of energy within any given spot size inside the beam diameter. Diffraction only depends on the parameters of the light beam sent and is independent of the atmosphere channel. Scattering and absorption by certain gases and particulates in the atmosphere are highly frequency dependent and cause attenuation of a light beam. Atmospheric turbulence is a consequence of the

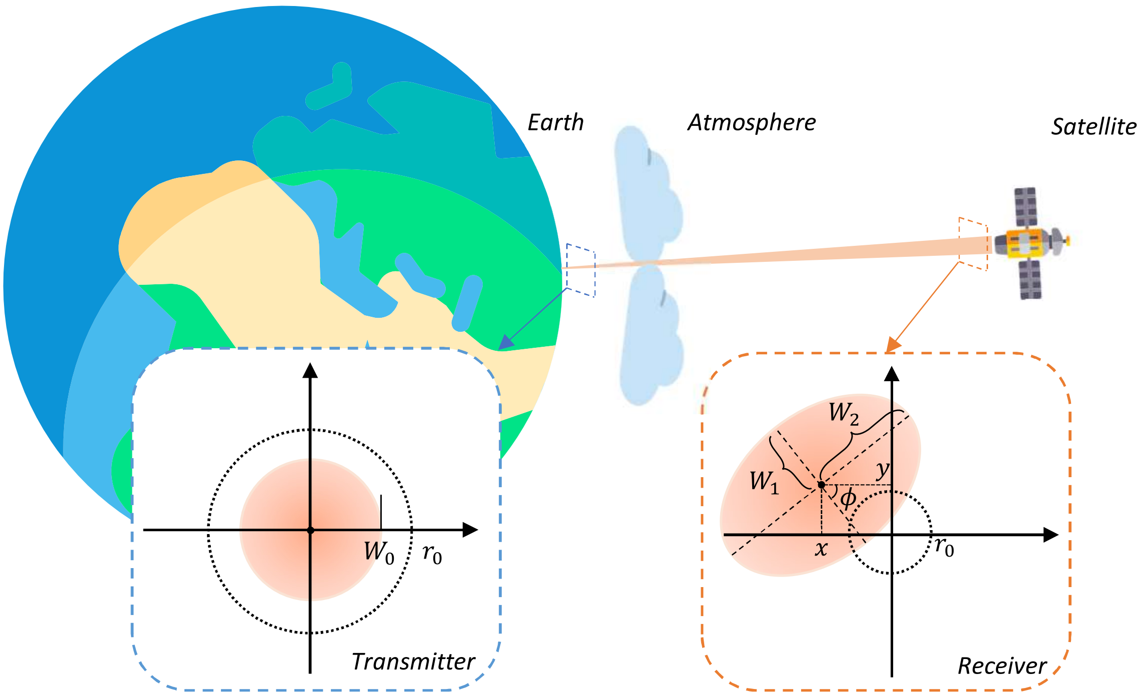

Figure 1.7: Evolution of the beam profile over an Earth-satellite channel. An up-link channel (from a ground station to a satellite) is illustrated in the figure. In such a channel, beam-wandering causes the deviation of the beam-centroid, beam-broadening expands the beam profile, and beam-deformation alters the shape of the beam profile. The orange circle represents the beam profile at the transmitter, the dotted circle with radius r_0 illustrates the detector aperture, and the orange eclipse represents the beam profile at the receiver. We have used different scales for the transmitter and receiver for better illustration.

temperature gradient in the atmosphere. The temperature gradient causes eddies, which result in a non-isotropic spreading (beam-broadening and beam-deformation) and a random wandering to the light beam.

Diffraction, scattering, and absorption are all deterministic effects and can be compensated with well-engineered transceivers. However, the atmospheric turbulence can be highly stochastic. Over the atmospheric channel, the channel transmissivity varies with time and is usually unpredictable. This type of channel is referred to as a fading channel, of which the transmissivity can be described by a probability density function. Albeit with certain technologies (e.g. beam tracking) the beam wandering effect can be alleviated to some extent, the random spreading effect is still a problem yet to be solved.

There have been many experimental advances demonstrating successful quantum light transmission over horizontal atmospheric channels, e.g. [9,96,97]. A useful model for the atmospheric channel is the log-normal model [98,99]. This model describes the transmissivity over various turbulence conditions with reasonable accuracy. Over the horizontal atmospheric channel, a so-called beam-wandering model is proposed to describe the beam wandering effect [100]. As illustrated in Figure 1.7, the beam-wandering model is later improved to consider other channel effects including the beam-broadening and the beam-deformation effects [101]. We refer to this model as the elliptical model. Among the three models, the elliptical model yields the best agreement with the recent experimental results of [97]. In this thesis, we will use the elliptical model when we consider the deployment of quantum communication protocols over satellite-based channels.

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CHAPTER 2

Global entanglement distribution with multi-mode non-Gaussian operations

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Preamble

The distribution of entanglement is a prerequisite for entanglement-based quantum information tasks. In this chapter, we study photon subtraction, photon addition, and photon catalysis in the distribution of multi-mode entangled states over atmospheric channels. As the main contribution of this chapter, we propose a framework for multi-mode photon catalysis. Building on this framework, we consider scenarios where the non-Gaussian operations are performed on the entangled states before or after they pass through the atmospheric channels. We find that all three non-Gaussian operations considered can improve entanglement over certain parameter regions.

CHAPTER 3

Photonic engineering for CV-QKD over Earth-satellite channels

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Preamble

We have shown in Chapter 2 that certain non-Gaussian operations can improve the entanglement distributed over atmospherics channels. In this chapter, we study the use of certain non-Gaussian operations in entanglement-based CV-QKD systems. In this chapter, we focus on the single-mode system only. We find that non-Gaussian operations can only improve the channel loss tolerance of a CV-QKD system when the initial squeezing of the entangled state is large. When the initial squeezing is adjusted so as to maximize the tolerable channel loss, non-Gaussian operations cannot improve the tolerance.

CHAPTER 4

Multi-mode CV-QKD with non-Gaussian operations

DOI: 10.1002/que2.40, arXiv: 1911.10310

Preamble

In this chapter, we extend the single-mode CV-QKD system in Chapter 3 to a multi-mode system. To better study the impact of non-Gaussian operations on a multi-mode CV-QKD system, in this chapter, we focus on the fiber channels with fixed loss only. Our results show that, considering the multi-mode CV-QKD system only, non-Gaussian operations can slightly improve the secret key rate of the system. When comparing the multi-mode CV-QKD system with a single-mode system, the secret key rate for the multi-mode non-Gaussian operations can be orders of magnitude higher than the single-mode system.

CHAPTER 5

Multi-mode CV-QKD with noiseless attenuation and amplification

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Preamble

We have shown in Chapter 4 that certain non-Gaussian operations only provide minor improvement in a multi-mode CV-QKD system. In this chapter, we continue our search for quantum state engineering that can improve the secret key rate for a CV-QKD system. We show that noiseless attenuation, when combined with noiseless amplification, can significantly improve the secret key rate of both the single-mode and the multi-mode CV-QKD systems.

CHAPTER 6

Noiseless linear amplifiers for multi-mode states

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Preamble

We have shown in Chapter 5 that noiseless amplification can significantly improve the secret key rate of a CV-QKD system. However, no realization on noiseless amplification has thus far been proposed for multi-mode states. In this chapter, we propose two different schemes for the implementation of noiseless linear amplifiers for multi-mode states. We first generalize the exiting amplification scheme that uses quantum scissors to the multi-mode setting. We then propose a new amplification scheme that uses photon catalysis. The performances of the two schemes are compared in the contexts of coherent state amplification and entanglement distribution.

CHAPTER 7

Teleportation of discrete-variable qubits via continuous-variable lossy channels

DOI: 10.1103/PhysRevA.105.062407, arXiv: 2202.00841

Preamble

We have shown in previous chapters that although non-Gaussian operations can improve the entanglement, they only provide a marginal improvement in CV-QKD. This contradiction motivates us to study non-Gaussian operations in other quantum communication protocols that also rely on entanglement. We have shown in 10.1109/TQE.2021.3091709 that non-Gaussian operation can improve the fidelity of teleportation of coherent states. In this chapter, we move on to the teleportation of DV qubits. We first show that the CV teleportation protocol can be improved by a non-Gaussian measurement, the DV Bell state measurement. We then show that the modified protocol can be further improved by introducing additional non-Gaussian operations.

CHAPTER 8

Teleportation of hybrid entangled states with continuous-variable entanglement

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Preamble

In this chapter, we extend the study in Chapter 7 to the teleportation of CV qubits and the hybrid entanglement between DV and CV qubits. We first compare the original CV-based teleportation protocol and our modified teleportation protocol proposed in Chapter 7 in teleporting CV qubits. We find that no protocol is always superior. Our modified protocol outperforms the original protocol only when the mean photon number of the CV qubit is below a certain threshold. We then compare the two protocols in teleporting a hybrid entangled state. Our modified protocol is again found to be better than the original protocol when the mean photon number of the qubit of the hybrid entangled state is below a certain threshold. Finally, we study the use of non-Gaussian operations in our modified protocol, finding that quantum scissors provide the most improvement.

CHAPTER 9

Conclusions and future works

In this chapter, we conclude this thesis by summarizing the contributions and briefly discussing some potential future works.

9.1 Conclusions

In this thesis, we investigated quantum state engineering in three quantum communication protocols, aiming to find the optimal quantum states under different conditions. We divide this thesis into three parts based on the protocol studied. In the first part, we investigated the application of non-Gaussian operations in a satellite-based entanglement distribution system. This part includes Chapter 2. Our main contributions for this part are:

- We developed a general framework for photon catalysis in the multi-mode setting. We show that single-mode photon catalysis can be engineered to implement photon catalysis on broadband modes.
- We used the framework to investigate improvements that can be obtained by

photon catalysis in a satellite-based entanglement distribution system.

- We compared photon catalysis with photon subtraction and photon addition in the multi-mode setting. We determined the optimal non-Gaussian operations for different parameter regions.

In the second part, we investigated the application of non-Gaussian operations in different entanglement-based CV-QKD systems. This part includes Chapters 3-6. Our main contributions for this part are:

- For a single-mode CV-QKD system, photon subtraction, photon addition, and photon catalysis can only improve the key rates with a large initial squeezing of the entangled state. When the squeezing is set to optimize the key rates, no operations can improve the optimized key rates.

- For a multi-mode CV-QKD system, all three types of non-Gaussian operations can improve the key rates. However, the improvement is marginal. We show that noiseless attenuation can be combined with noiseless amplification to provide significant improvement in the key rates.

- We generalized an existing procedure for noiseless amplification from the single-mode setting to the multi-mode setting. We proposed a new procedure for noiseless amplification in the multi-mode setting.

An important result for the second part of the thesis is as follows. The non-Gaussianity, offered by non-Gaussian operations, does not have a direct connection with the optimized key rate (optimization performed independently over the squeezing of the initial TMSV state) of a CV-QKD system. None of the non-Gaussian operations considered in this thesis can improve the optimized key rate when applied before the channel transmission of the initial TMSV state. Recent work shows that photon catalysis, when performed after the channel transmission of the TMSV state, can improve the optimized key rate [1]. However, such an improvement is due to the noise-cancelling effect of photon catalysis, i.e., an approximation to noiseless linear amplification [2].

In the third part, we investigated the application of non-Gaussian operations in different teleportation systems. This part includes Chapters 7-8. Our main contributions for this part are:

- We proposed a hybrid teleportation protocol that combines a DV form of Bell state measurement (BSM) and a CV entangled resource state. We also proposed a possible implementation of the DV-BSM.
- We compared our protocol with an existing CV-BSM teleportation protocol in teleporting DV qubits, CV qubits, and hybrid entangled states. We show that our protocol outperforms the CV-BSM protocol when the mean photon number of the target state is small.
- We investigated the use of non-Gaussian operations in our teleportation protocol, showing that certain operations offer a significant improvement.

In this thesis, we have considered various noise sources that occur during the transmission or detection of entangled states, including the channel input excess noise, the vacuum or thermal noise during channel transmission, and the noise due to imperfect state detection. However, we have assumed that the entangled states before channel transmission is noiseless. We have not considered the noise sources that might occur in preparing the entangled states. For TMSV states with non-Gaussian operations applied before channel transmission, such noise sources can be categorized into two classes:

1. The noise sources that occur before non-Gaussian operations. An example is the thermal noise in the preparation of TMSV states. The two-mode squeezed thermal state is a more generalized two-mode entangled state comparing to a TMSV state. With a fixed initial squeezing, the level of entanglement of the state decreases as the noise level increases. For two-mode squeezed thermal states engineered by non-Gaussian operations, the level of entanglement of the resultant state also decreases as the thermal noise increases [3, 4]. For the entanglement-based quantum communication protocols considered in this thesis, when the thermal noise in the initial entangled state is considered, the performance of the protocols (e.g., the log-

negativity for entanglement distribution or fidelity for teleportation) will decrease with increasing noise levels. However, our conclusion that non-Gaussian operations improve the protocol over certain parameter regions will not change.

2. The noise sources that occur during non-Gaussian operations. An example is the noise in preparing the ancillary single-photon states needed for the operations. Considering the technical challenge associate with photon-number resolving detectors, the heralded single-photon states can be used as the ancillary states for the operations [5]. Such states can be created by performing on-off detection on one mode of a TMSV state, which approximate single-photon states for small squeezing. Reducing the squeezing can alleviate the noise from the heralded single-photon states. However, the success probability for the on-off detection and the overall success probability for the following non-Gaussian operations will also decrease with decreasing squeezing.

9.2 Future works

We finish this thesis by suggesting possible future directions. This thesis has focused on the simulation and theoretical aspects of the CV multi-mode quantum communication systems. We believe future works should focus on their experimental realization. The following challenges, which are exclusive to multi-mode quantum systems, might occur in the real-world implementation of such systems.

Firstly, an efficient method for detecting the quadrature fields of the supermodes in the multi-mode system is yet to be found. With a properly matched local oscillator (LO), a single homodyne detection can extract information from one supermode. Multiple homodyne defections with different LOs should be able to extract the information from all the supermodes. However, the supermodes cannot be easily separated [6]. Even though techniques, such as quantum pulses gates [7–10], have been invented to distinguish the supermodes with some efficiency, for the detection of one supermode the contributions from other supermodes cannot be perfectly eliminated. Such contributions lead to a cross-talk between the supermodes and the orthogonality between the supermodes is lost.

The cross-talk effect might also occur during the channel transmission of both the classical LO signal and the quantum signal [11], leading to the second challenge. An efficient method for obtaining the channel information, especially for a fading channel (e.g., Earth-satellite channels), and correcting the signals based on the information obtained, without the cost of a considerable increase in system complexity, is yet to be designed [12]. The attenuation on the classical signal can be compensated by the frequency-dependent amplification given perfect channel information. However, it is impossible to amplify the quantum signal without introducing extra noise due to the non-cloning theorem. A method for alleviating the cross-talk between the supermodes caused by frequency-dependent channels is also yet to be found.

Thirdly, the implementation of the multi-mode non-Gaussian operations considered in this thesis requires an array of photon number resolving detectors (PNRDs). However, there has been no single-photon detector thus far that can excel in all attributes, including counting rate, system detection efficiency, and photon number resolving functionality [13]. Besides the low detection success probability caused by low counting rates or low detection efficiency, the realization of the PNRD array is also of particular challenge in space-limited situations such as onboard a satellite. Future work could thus focus on the implementation of PNRDs with a higher counting rate, higher detection efficiency, and lower complexity.

Besides the quantum communication protocols considered in this thesis, the use of multi-mode quantum states, engineered by non-Gaussian operations, in other protocols, including quantum secure direct communications [14] and quantum streams piggybacking [15], is also to be explored. Beyond the quantum communication systems based on bipartite entanglement considered in this thesis, future work could also study the experimental realization of the distribution of the multi-partite entangled states created from ultra-fast pulses. Such states are the enabler for various quantum information tasks, such as measurement-based quantum computation [16], distributed quantum sensing [17], and quantum secret sharing [18]. The distribution of the multi-partite entangled states is an important prerequisite for these tasks.

While there have been various experiments demonstrating the feasibility of producing large-scale multi-partite entangled states (e.g., [19–23]), no experiments have been performed in the long-distance distribution of such states between multiple nodes. Realistic issues such as queuing delay [24] and distribution rate maximization [25] in such an entanglement distribution task are also to be solved. Whether the multi-mode non-Gaussian operations we have considered in this thesis can be used to improve the loss tolerance of the multi-partite entangled states or to distill the entanglement after the distribution of such states is also an interesting question. We believe that the real-world demonstration of a multi-partite entanglement distribution system, aided by non-Gaussian operations, will lead to a significant breakthrough in the development of large-scale quantum networks.

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