Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B 926 (2018) 1–10

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

Turbulent mixing of a critical fluid: The non-perturbative renormalization

M. Hnatič [a,b,c], G. Kalagov [a,d,*], M. Nalimov [d]

[a] *Department of Theoretical Physics and Astrophysics, Faculty of Science, P.J. Safarik University, Park Angelinum 9, 041 54 Kosice, Slovak Republic*

[b] *Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Moscow Region, Russian Federation*

[c] *Peoples' Friendship University of Russia (RUDN University), 6 Miklukho-Maklaya St, 117198 Moscow, Russian Federation*

[d] *St. Petersburg State University, 7/9 Universitetskaya nab., 199034 St. Petersburg, Russian Federation*

Received 9 June 2017; received in revised form 13 September 2017; accepted 31 October 2017

Available online 3 November 2017

Editor: Hubert Saleur

Abstract

Non-perturbative Renormalization Group (NPRG) technique is applied to a stochastical model of a non-conserved scalar order parameter near its critical point, subject to turbulent advection. The compressible advecting flow is modeled by a random Gaussian velocity field with zero mean and correlation function $\langle \upsilon_j \, \upsilon_i \rangle \sim (P^{\perp}_{ji} + \alpha P^{\parallel}_{ji})/k^{d+\zeta}$. Depending on the relations between the parameters ζ, α and the space dimensionality d, the model reveals several types of scaling regimes. Some of them are well known (model A of equilibrium critical dynamics and linear passive scalar field advected by a random turbulent flow), but there is a new nonequilibrium regime (universality class) associated with new nontrivial fixed points of the renormalization group equations. We have obtained the phase diagram (d, ζ) of possible scaling regimes in the system. The physical point $d = 3$, $\zeta = 4/3$ corresponding to three-dimensional fully developed Kolmogorov's turbulence, where critical fluctuations are irrelevant, is stable for $\alpha \lesssim 2.26$. Otherwise, in the case of "strong compressibility" $\alpha \gtrsim 2.26$, the critical fluctuations of the order parameter become relevant for three-dimensional turbulence. Estimations of critical exponents for each scaling regime are presented.

* Corresponding author at: St. Petersburg State University, 7/9 Universitetskaya nab., 199034 St. Petersburg, Russian Federation.

E-mail addresses: hnatic@saske.sk (M. Hnatič), kalagov.g@gmail.com (G. Kalagov), mikhail.nalimov@pobox.spbu.ru (M. Nalimov).

https://doi.org/10.1016/j.nuclphysb.2017.10.024

0. Introduction

The application of the renormalization group approach to theoretical understanding of dynamical critical phenomena has a long history dating back to Kadanoff's and Wilson's ideas. The most significant technical advancements in this field were implemented within the quantum field renormalization group method, where the investigation of a system close to criticality is based on the study of the infrared behavior of a field model defined through an effective Lagrangian (the MSRJD formalism [1]) that inherits underlying properties of the system. One of the representatives of the effective models is the model A, according to [2], of the critical dynamics belonging to the Ising universality class, that consists of systems having short-range forces and a scalar order parameter. This class comprises the Ising model, liquid–gas critical point, binary fluid mixtures.

Interest in critical fluids is related to their specific physical, thermodynamic and transport properties, so the behavior of the medium near its critical point has long been a focus of research but later it became clear that hydrodynamic effects can not be ignored due to the strong sensitivity of such systems to external disturbances, for instance turbulence [3]. The classical models of critical dynamics, and in particular the model A, describe the relaxation processes and critical slowing down in out-of-equilibrium systems and allow one to determine the dispersion $\omega \sim k^z$ of order parameter fluctuations in the infrared limit $k \to 0$. These models are applicable only to "ideal" systems, which are not subject to any perturbations, e.g. stirring by the deterministic or random turbulent shear flow. It is therefore reasonable to extend the classical models [2] by including the coupling with a turbulent velocity field. Two accepted manners allow one to do this: the rapid-change Kraichnan model, where the velocity field $\upsilon_j(x,t)$ obeys a Gaussian distribution with zero mean and correlation function $\sim \delta(t-t')|x-x'|^{\zeta}$, and its modifications [4–8] and a way based on the stochastic Navier–Stokes equation [8,9]. The extensions of various critical dynamic models within both manners have been investigated in numerous papers by means of perturbation renormalization in $d = 4 - \varepsilon$ dimensions, see for example [10–17] and this field is actively being studied nowadays. It turns out that the coupling of the order parameter field to the stochastic velocity one in the infrared region gives rise to emergence of new scaling regimes. The corresponding critical exponents of power law correlations can be computed within regular expansions, e.g. (ε, ζ)-perturbation expansion. However, as in the case of the static field models, to establish the pattern of possible scaling regimes and obtain numerical values of critical exponents for a real physical system, where $\varepsilon \sim \zeta \gtrsim 1$, additional methods of the Borel summation of series are required. A difficulty with this approach is that multi-loop renormalization group analysis of complex models is not performed. For this reason, there have been many attempts in the literature to go beyond the perturbation schemes. The non-perturbation renormalization group (NPRG) is one of the methods that does not rely on any small parameters in the studied model.

The NPRG originally applied to equilibrium models of statistical physics and field theory [18] affords a seamless application in out-of-equilibrium systems: the A [19] and C [20] models of critical dynamics, the Navier–Stokes stochastic equation [21] and the Kraichnan model [22], the reaction–diffusion process [23] and the Kardar–Parisi–Zhang model [24]. The central tool for the NPRG calculations is the functional equation for a scale dependent effective action that interpolates between "microscopic action" on small scales and free energy of a system on large scales.

In this paper, we are going to work in the NPRG formalism to reveal the effect of turbulent motion on the critical behavior systems belonging to the A model universality class. The systems can be considered to be opened, in the sense that we assume a pump which adds energy to the flow in order to maintain the fully developed turbulence. In Sec. 1, we define the model A of critical dynamics of a non-conserved order parameter coupled to Kraichnan's stochastic velocity field modeling a turbulent motion of a compressible fluid. We derive the NPRG flow equations of the scale dependent effective action in the leading order of the derivative and field expansion. The perturbation solutions near the upper critical dimension of the model are shown to be consistent with the one-loop results of (ε, ζ)-expansion obtained in [16]. In Sec. 2, we solve the NPRG equations numerically, find that they exhibit stable infrared behavior and investigate the effect of compressibility on possible types of the scaling regimes. A short summary closes this note.

1. Model and methods

Within the framework of the model A the dynamics of a non-conserved scalar order parameter $\phi = \phi(x,t)$ advected by a random velocity field $\upsilon_j = \upsilon_j(x,t)$ is described by the Langevin-type stochastic equation

$$\lambda\, \nabla_t \phi(x,t) = -\frac{\delta H_0[\phi]}{\delta \phi(x,t)} + \eta(x,t), \tag{1}$$

where by definition $\nabla_t = \partial_t + \upsilon_j \partial_j$ is the material derivative, $\lambda > 0$ is the reciprocal of the kinematic coefficient, the random Gaussian noise $\eta(x,t)$ with zero mean can be defined by specifying the correlation function $\langle \eta(x,t)\eta(x',t')\rangle = 2\lambda\, \delta(x-x')\delta(t-t')$, the noise simulates the background of molecular degrees of freedom. The functional $H_0[\phi]$ is a time-local "Hamiltonian" expressed through the field $\phi(x,t)$ and its space derivatives. Close to the critical point $H_0[\phi]$ takes the form

$$H_0[\phi] = \int \mathrm{d}^d x \left\{ \frac{1}{2}(\nabla\phi)^2 + \frac{\tau_0}{2}\phi^2 + \frac{g_0}{4!}\phi^4 \right\}. \tag{2}$$

The "bare mass" $\tau_0 \sim T - T_c$ is the deviation temperature from its mean-field critical value T_c, the coupling constant $g_0 > 0$.

In real systems, the velocity field satisfies the hydrodynamic equations, e.g. Navier–Stokes; however, we will assume the velocity to be a random variable distributed in accordance with the Gaussian statistics with the zero expectation value $\langle \upsilon_i(x,t)\rangle = 0$ and the covariance $\langle \upsilon_j(x,t)\upsilon_i(x',t')\rangle = D_{ji}$, where

$$D_{ji} = D_0\, \delta(t-t') \int\limits_{p>m} \frac{\mathrm{d}^d p}{(2\pi)^d} \frac{P^{\perp}_{ji} + \alpha P^{\parallel}_{ji}}{p^{d+\zeta}} \exp[ip(x-x')] \tag{3}$$

the tensors $P^{\parallel}_{ji} = p_j p_i / p^2$ and $P^{\perp}_{ji} = \delta_{ji} - P^{\parallel}_{ji}$ are longitudinal and transversal projectors, respectively, the amplitudes $D_0 > 0$ and $\alpha > 0$. The special case $\alpha = 0$ corresponds to an incompressible fluid flow. The scale m is an external macroscopic scale of turbulence, which provides infrared regularization. In the regime of fully developed turbulence in the inertial range the magnitude m is not contained in calculated values, e.g., in anomalous dimensions. Moreover, the m-regularization will not be needed due to the NPRG approach. The written down correlation function reproduces Kolmogorov's 5/3-spectrum for turbulent excitations in case ζ equals $\zeta_K = 4/3$. The $\delta(t-t')$-correlations of the velocity field are more naturally interpreted within the Stratonovich prescription, which corresponds to the finite correlation time. However, it is much shorter than dynamical timescales. At the computation stage this presumption is inherently regularization of the Heaviside step function at zero $\Theta(0) = 1/2$.

According to the general formalism [1], stochastic problems (1) are tantamount to the MSRJD field model of the fields $\Theta = \{\phi, \phi', \upsilon_j\}$

$$S[\Theta] = \int \mathrm{d}^d x \mathrm{d}t \left\{ \lambda \phi' \nabla_t \phi + \phi' \frac{\delta H_0[\phi]}{\delta \phi} - \lambda \phi' \phi' + \frac{1}{2} \upsilon_i D_{ij}^{-1} \upsilon_j \right\}. \tag{4}$$

Formulation (4) means that statistical averages of random quantities in the original stochastic problem can be represented as functional averages with the weight $\exp(-S[\Theta])$.

The exact RG technique [25] is based on the consideration of the flow equation for the scale dependent effective average action functional $\Gamma_k[\Phi]$. By construction, this functional interpolates between the microscopic action $\Gamma_{k=\Lambda}[\Phi] = S[\Theta = \Phi]$ and the free energy $\Gamma[\Phi]$ or the generating functional of 1-particle irreducible Green functions, i.e. $\Gamma_{k=0}[\Phi] = \Gamma[\Phi]$, where Λ is an ultraviolet scale. The average action $\Gamma_k[\Phi]$ is the "free energy" of rapid modes $p > k$ that have been integrated out. Within this approach k plays the role of an infrared cutoff. Usually, to split collective modes of a system into fast and slow degrees of freedom the momentum-dependent mass term $\Delta S_k[\Theta]$ is added to the action $S[\Theta]$. The new term has the form

$$\Delta S_k[\Theta] = \frac{1}{2} \int \frac{\mathrm{d}\omega \, \mathrm{d}^d p}{(2\pi)^{d+1}} \Theta(p, \omega) \, R_k(p) \, \Theta(-p, -\omega), \tag{5}$$

and the scale-dependent partition function is given by

$$Z_k[J] = \int \mathcal{D}\Theta \exp\left(-S[\Theta] - \Delta S_k[\Theta] + J \, \Theta\right). \tag{6}$$

For the sake of simplicity we consider a frequency-independent cutoff function $R_k = R_k(p)$, which has to meet the conditions:

- when $k = 0$, $R_{k=0}(p) = 0$, $\forall p \lesssim \Lambda$ – all fluctuations are integrated out $\Gamma_{k=0}[\Phi] = \Gamma[\Phi]$;
- when $k = \Lambda$, $R_{k=\Lambda}(p) = \infty$ (or, at least, $\sim \Lambda^2$), $\forall p \lesssim \Lambda$ – all fluctuations are frozen $\Gamma_{k=\Lambda}[\Phi] = S[\Phi]$;
- for the slow modes $p \ll k$, $R_k(p) \sim k^2$ is *per se* an effective mass that provides the infrared cutoff;
- for the rapid modes $p \gg k$, $R_k(p) \simeq 0$ and they remain unaltered.

The average action $\Gamma_k[\Phi]$ is defined through the modified Legendre transformation

$$\Gamma_k[\Phi] = -\ln Z_k[J] + J \, \Phi - \Delta S_k(\Phi), \tag{7}$$

where the field Φ is, by definition, the average of Θ and is therefore $\Phi = \delta \ln Z_k[J]/\delta J$.

The exact Wetterich RG equation [25] on $\Gamma_k = \Gamma_k[\Phi]$ in the Fourier space is given by

$$\partial_s \Gamma_k = \frac{1}{2} \overline{\partial}_s \, \mathrm{Tr} \ln \left(\Gamma_k^{[2]} + R_k \right), \tag{8}$$

where the RG "time" $s = \ln(k/\Lambda)$, $\overline{\partial}_s$ acts only on R_k, i.e. $\overline{\partial}_s \equiv \partial_s R_k\, \partial/\partial R_k$, the symbol Tr designates a trace over matrix indices and integration over momentum and frequency, $(\Gamma_k^{[2]})_{ij} = \delta^2\Gamma_k/\delta\Phi_j\delta\Phi_i$ is the Hessian for the action $\Gamma_k[\Phi]$.

The Wetterich functional equation rewritten as a set of equations for the Green functions is not closed; therefore, some approximations must be used to get around this problem and perform direct analysis. The most used truncation is the field and derivative expansion, and we use it here. From convergence studies it has been shown (for equilibrium model see [26]) that expanding the effective average action around its minimum configuration, in this model $\{\varphi' = v_j = 0, \varphi \neq 0\}$, improves the convergence properties when one is interested in the critical behavior. We consider ansatz for the average action

$$\Gamma_k = \int \mathrm{d}^d x \mathrm{d}t \left\{ X_k\, \varphi' \left\{\nabla_t + A_k\,(\partial_i v_i)\right\} \varphi + \varphi' \frac{\delta H_k[\varphi]}{\delta\varphi} - Y_k\, \varphi'\varphi' + \frac{1}{2} v_i D_{ij}^{-1} v_j \right\}, \tag{9}$$

and

$$H_k = \int \left\{ \frac{1}{2} Z_k\, (\nabla\varphi)^2 + U_k(\varphi) \right\} \mathrm{d}^d x.$$

Due to the Galilean symmetry the material derivative term is renormalized by the uniform coupling X_k; the term $v_i D_{ij}^{-1} v_j$ is not renormalized at all, see Appendix A; the dimensionless function A_k having the zero bare value $A_{k=\Lambda} = 0$ takes compressibility of the flow into analysis. The pure A model without velocity-terms possesses a specific symmetry under transformation $t \to -t$, $\varphi \to \varphi$, $\varphi' \to \varphi' - \partial_t\varphi$, which leads to $X_k = Y_k$. But, as a matter of fact, the inclusion of the velocity breaks this symmetry, so $X_k \neq Y_k$. Thus, the behavior of the system extends beyond the paradigm of the classical fluctuation–dissipation theorem. The renormalization couplings X_k, Y_k, Z_k, A_k depend on the field φ, thus proceeding along the general line of the derivative and the field expansions, we can use expansion $X_k = X_k(\rho_k) + X_k^1(\rho_k)(\rho - \rho_k) + \ldots$ and similarly for others; however, we confine ourselves to the lowest order of this expansion, assuming solely the k-dependence of the couplings. For the running potential $U_k(\varphi)$ we accept the approximation $U_k(\varphi) = \lambda_k(\rho - \rho_k)^2/2$, where $\rho = \varphi^2/2$ and ρ_k corresponds to the minimum of the running potential.

We choose the matrix R_k in the following block-diagonal form:

$$R_k = \begin{pmatrix} 0 & R_k^{\phi}(p) & 0 \\ R_k^{\phi}(p) & 0 & 0 \\ 0 & 0 & R_k^{v}(p) \end{pmatrix}. \tag{10}$$

A typical cutoff function, which is widely used since it allows for analytical results for the non-perturbative β-functions, is the theta-cutoff introduced by D.F. Litim [27]

$$R_k^{\phi} = (k^2 - p^2)\,\Theta(1 - p^2/k^2), \tag{11}$$

$$R_k^{v} = (k^{d+\zeta} - p^{d+\zeta})\,\Theta(1 - p^2/k^2). \tag{12}$$

It has been also shown in [27–29] that the Litim regulator optimizes truncated flow and provides the quickest convergence of the computed results within the leading order of the derivative expansion. For the higher order approximations the Litim regulator leads to difficulties due to its non-analytical structure; therefore, the more smooth cutoff functions that meet the necessary conditions of differentiability are employed.

At the criticality, the renormalization functions have a power law behavior $X_k \sim k^{-\gamma_*^X}$, $Y_k \sim k^{-\gamma_*^Y}$ and $Z_k \sim k^{-\eta_*}$, so let us define the running critical dimensions according to

$$\gamma_k^X = -\partial_s \ln X_k, \quad \gamma_k^Y = -\partial_s \ln Y_k, \quad \eta_k = -\partial_s \ln Z_k. \tag{13}$$

In doing so, one can express the dynamical critical exponent using the values of running dimensions at a fixed point $z = 2 - \eta_* + \gamma_*^X$. If the ansatz (7) is inserted into the flow equation (8), one obtains the flow equations for couplings. For instance, the function X_k can be defined by

$$X_k(\rho_k) = \lim_{\substack{\omega\to 0\\ q\to 0}} \partial_{i\omega} \left.\frac{\delta^2 \Gamma_k}{\delta\varphi(-q,-\omega)\delta\varphi'(q,\omega)}\right|_{\substack{\varphi'=\upsilon_j=0\\ \rho=\rho_k}}. \tag{14}$$

Taking the ∂_s-derivative of both sides of this equality and using the flow equation (8) yield

$$\partial_s X_k(\rho_k) = \frac{1}{2}\tilde{\partial}_s \lim_{\substack{\omega\to 0\\ \mathbf{q}\to \mathbf{0}}} \partial_{i\omega} \left.\frac{\delta^2 \,\mathrm{Tr}\ln\left(\Gamma_k^{[2]} + R_k\right)}{\delta\varphi(-q,-\omega)\delta\varphi'(q,\omega)}\right|_{\substack{\varphi'=\upsilon_j=0\\ \rho=\rho_k}}. \tag{15}$$

In this way one can deduce the flow equations for all running couplings in ansatz (7). The next step of the RG program is to go first to the dimensionless variables to find a fixed point

$$\begin{aligned} &g_1 = X_k^{-1} Y_k Z_k^{-2} k^{d-4} \lambda_k, \quad g_2 = X_k Z_k^{-1} k^{-\zeta} D_0,\\ &g_3 = X_k Y_k^{-1} Z_k k^{2-d} \rho_k, \quad g_4 = A_k, \end{aligned} \tag{16}$$

the variables λ_k, ρ_k can be express through the potential $\lambda_k = U_k''(\rho_k)$ and $U_k'(\rho_k) = 0$, where $U_k' = \partial U_k/\partial\rho$. Let us redefine new variables according to $g_1 \to g_1/c(d)$, $g_2 \to g_2/c(d)$, $g_3 \to c(d)\, g_3$ and $c(d) = \left(2^{d+1}\pi^{d/2}\Gamma(d/2)\right)^{-1}$, then anomalous dimensions and the flow equations for running couplings are given

$$\gamma_k^X = -\tilde{\partial}_s \left\{ \frac{9}{2} g_1^2 g_3\, Q(3,0,1,0) - \frac{3}{2} g_1 g_2 g_3 g_4^2 \alpha\, Q(2,1,0,1|0,0) \right\}, \tag{17}$$

$$\begin{aligned} \gamma_k^Y &= -\tilde{\partial}_s \Big\{ \frac{9}{2} g_1^2 g_3\, Q(3,0,2,0|0,0) + 3 g_1 g_2 g_3 g_4 (1-g_4)\alpha\, Q(2,1,1,1|0,0)\\ &\quad + \frac{1}{2} g_2 (1-g_4)^2 \alpha\, Q(1,1,1,1|0,0) \Big\},\\ \eta_k &= \tilde{\partial}_s \Big\{ \frac{9}{2d} g_1^2 g_3 [2\,Q(4,0,1,1|1,2) - 2\,Q(3,0,1,1|2,1) - d\,Q(3,0,1,0|1,1)]\\ &\quad + \frac{3\alpha}{2d} g_1 g_2 g_3 g_4 [2\,Q(2,1,0,1|1,1) - 2 g_4\, Q(3,1,0,2|1,2)\\ &\quad + 2 g_4 Q(2,1,0,2|2,1) + d\, g_4\, Q(2,1,0,1|1,1)]\\ &\quad - g_2 \frac{d-1+\alpha}{2d} Q(0,1,0,0|0,0) \Big\}, \end{aligned}$$

$$\begin{aligned} \partial_s g_1 &= (d - 4 + \gamma_k^X - \gamma_k^Y + 2\eta_k) g_1\\ &\quad - \tilde{\partial}_s \left\{ \frac{9}{2} g_1^2 Q(2,0,1,0|0,0) - \frac{3}{2} g_1 g_2 g_4^2 \alpha Q(1,1,0,1|0,0) \right\},\\ \partial_s g_2 &= -\zeta\, g_2 + (\eta_k - \gamma_k^X)\, g_2, \end{aligned} \tag{18}$$

$$
\begin{aligned}
\partial_s g_3 &= -(d-2+\gamma_k^X-\gamma_k^Y+\eta_k)\, g_3 \\
&\quad -\widetilde{\partial}_s \left\{ \frac{3}{2} Q(1,0,1,0|0,0) + \frac{\alpha g_2\, g_4(1-g_4)}{2\, g_1} Q(0,1,0,1|0,0) \right\}, \\
\partial_s g_4 &= g_4\, \gamma_k^X + \widetilde{\partial}_s \Bigg\{ 18\, g_1^2\, g_3 \left(g_4 Q(3,0,1,0|0,0) - \frac{3}{2d} Q(4,0,1,1|1,1) \right) - \\
&\quad - 6\alpha g_1\, g_2\, g_3\, g_4^2 \left(\frac{g_4}{2} Q(2,1,0,1|0,0) - Q(3,1,0,2|1,1) \right) \\
&\quad - 3\, g_1 \left(\frac{g_4}{2} Q(2,0,1,0|0,0) - \frac{1}{2d} Q(3,0,1,1,|1,1) \right) \Bigg\}.
\end{aligned}
$$

The dimensionless functions of the charges

$$
Q(n_1,n_2,n_3,n_4|m_1,m_2) \equiv \int_0^\infty \frac{(1+f(y))^{n_3}}{h_1(y)^{n_1}\, h_2(y)^{n_2}} \left[h_1^{(m_1)}\right]^{m_2} y^{(d/2-1+n_4)} \mathrm{d}y,
$$

$$
\widetilde{\partial}_s = \int_{-\infty}^{\infty} \mathrm{d}y \left\{ s_1(y) \frac{\delta}{\delta r_1(y)} + s_2(y) \frac{\delta}{\delta r_2(y)} \right\}, \tag{19}
$$

reflect the non-perturbative structure of the RG equations. Here $h_1(y) = y + r_1(y) + 2\, g_1\, g_3$, $h_2(y) = y + r_2(y)$, $s_1(y) = (2-\eta_k)\, r_1(y) - 2\, y\, r'(y)$, $s_2(y) = (d+\zeta)\, r_2(y) - 2\, y\, r_2'(y)$, $h_1^{(m_1)}(y) = \partial^{m_1} h_1(y)/\partial y^{m_1}$, $r_1(y) = (1-y)\Theta(1-y)$, $r_2(y) = (1-y^{(d+\zeta)/2})\Theta(1-y)$, $f(y) = g_2\, g_3\, g_4^2 \alpha y/h_2(y)$. In the limit of weak coupling, where $\varepsilon = 4-d \to 0$ and $\zeta \to 0$, these equations obtained by means of the NPRG approach recover the results of one-loop perturbation renormalization [16] in $4-\varepsilon$ space dimensions.

2. Results

The system (18) depends on three parameters α, d, ζ, and we will consider the pattern of possible scaling regimes in the (d,ζ) space at given α. These long-wave asymptotic regimes are determined by the infrared attractive fixed points of the corresponding RG equations. The type of fixed point is defined by the stability matrix $\Omega_{ji} = \partial\beta_j/\partial g_i$ evaluated at the fixed point. Let us assume that Ω is diagonalized, that is $\Omega = \mathrm{diag}\{-w_1, w_2, w_3, w_4\}$, where the eigenvalue w_1 characterizes the relevant direction of the RG flow. For the infrared attractive fixed point all parameters w_j are positive, and the critical exponent ν is given by the relation $\nu = 1/w_1$. The results of numerical computation are shown in Fig. 1.

Four infrared stable regimes may occur in the considered model:

- I. The Gaussian fixed point, where $g_{1*} = g_{2*} = 0, \forall g_{4*}$. The critical and velocity fluctuations are infrared irrelevant, and $z = 2$.
- II. The "pure" A model, where $g_{1*} \neq 0$, $g_{2*} = 0$, $\forall g_4$. The scaling regime is completely determined by the order parameter fluctuations; the velocity fluctuations, whose correlation function is defined for $\zeta < 0$, are inessential in the long-wave limit. The numerical estimation yields $z \approx 2.046$ in the $d = 3$ space and $z \approx 2.151$ in two dimensions. The A model has been investigated in [19] within the UZA approximation that assumes dependence of the renormalization functions X_k, Z_k on fields.

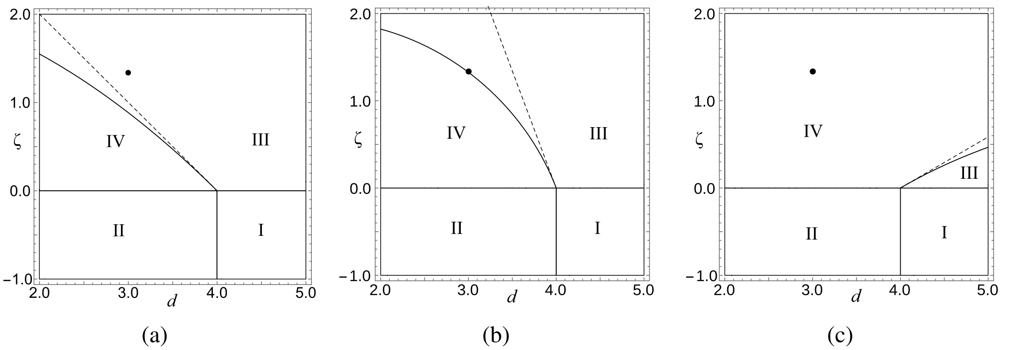

Fig. 1. Sets of infrared stable fixed points in the model A. The dashed line is the boundary between areas *III* and *IV* obtained within the one-loop approximation [16]. The dot $(3, 4/3)$ is the physical point. Dependence of the phase diagram on α: (a) and (b) "weak compressibility" cases, $\alpha = 1$ and $\alpha = 2.2$, respectively; (c) "strong compressibility" case $\alpha = 10$.

- III. Mixing of passive scalar field, where $g_{1*} = 0$, $g_{2*} \neq 0$. In this case the order parameter field corresponds to a passive admixture stirred by turbulent flows. The exponents have the exact form $\nu^{-1} = 2 - \zeta$, $\eta = \zeta$, $z = 2 - \zeta$, and for Kolmogorov's turbulence ($\zeta = \zeta_K$) they reproduce well-known Richardson's law [30]. The regime is stable for $\alpha < \alpha_c \approx 2.26$.
- IV. New regime, where $g_{1*} \neq 0$, $g_{2*} \neq 0$, $g_{4*} \neq 0$. Complex interaction of the order parameter and velocity fluctuations causes the establishing of this regime that is stable for $\alpha > \alpha_c \approx 2.26$. The exact expression $z = 2 - \zeta$ takes place in the regime *IV* as well. The critical exponents ν^{-1}, η depend on α; therefore, the system manifests a non-universal critical behavior. The numerical estimations for the critical exponent amplitudes are shown in Fig. 2. Note, one-loop perturbation RG analysis [16] predicts the existence of the regime IV and the threshold of "compressibility" $\alpha_c^{\text{1-loop}} = 15/7 \approx 2.14$ at which regime IV in real systems $d = 3$ and $\zeta = \zeta_K$ becomes stable; nonetheless, it is impossible to estimate amplitudes of the critical exponent correctly using only one-loop outcomes. In Fig. 1(b) dotted line is the boundary between areas III and IV obtained in [16] ($\alpha = 2.2$ as an example). One can see the alteration of the inter-regimes boundary by higher order corrections taken into analysis within the NPRG. The physical point corresponding to turbulently moving critical fluids is in the regime III up to $\alpha = \alpha_c$, although the difference between α_c and $\alpha^{\text{1-loop}}$ is not very substantial. The NPRG analysis shows that the region IV increases with the parameter α, see Fig. 1(c).

3. Conclusion

In this work we have studied the scaling phenomena in a turbulently moving fluid whose dynamics close to criticality is described by the model A of the non-conserved scalar order parameter coupled to the Gaussian random velocity field. The scaling regimes of the model are associated with infrared stable fixed points of the corresponding RG equations, which have been derived here within the NPRG approach. To obtain a solution of the functional Wetterich equation, we used the lowest order of the derivative and field expansion keeping only the scale dependence of renormalization functions. In the weak limit $\varepsilon = 4 - d \to 0$, $\zeta \to 0$ the obtained outcomes reproduce one-loop results found by means the perturbation renormalization

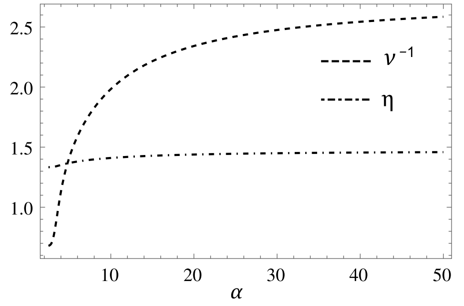

Fig. 2. The critical exponents of the scaling regime *IV* for $\alpha \gtrsim 2.26$.

group [16], where a new stable scaling regime has been found. We showed that this regime, in which both thermal and turbulent fluctuations are relevant, continues to be stable and higher order contributions grouped in the NPRG equations do not alter the quantitative picture. Within the applied Kraichnan model comprising the phenomenological parameter of compressibility α, we computed α-dependent critical exponents ν^{-1}, η, see Fig. 2; the dynamical critical exponent takes the exact value $z = 2 - \zeta$ and does not depend on α. In the "weak" compressible system $0 < \alpha < 2.26$ only turbulent fluctuations determine the scaling behavior, where critical exponents are exactly computed $z = 2 - \zeta$, $\eta = \zeta$ and $\nu^{-1} = 2 - \zeta$. In the complementary case $\alpha \gtrsim 2.26$ the system is in the new regime *IV*, where for "infinite" compressible flow the values of critical exponents Fig. 2 can be extrapolated at $\alpha = \infty$, as a result $\eta \approx 1.47$ and $\nu^{-1} \approx 2.75$.

Acknowledgements

This research was supported by the Ministry of Education and Science of the Russian Federation (Agreement No. 02.a03.21.0008), the Ministry of Education, Science, Research, and Sport of the Slovak Republic (VEGA Grant No. 1/0345/17), and VVGS grant 2016-72639 of PF UPJS.

Appendix A

This appendix is dedicated to the question of the free velocity action renormalization in (4). In the first place, we will show how Galilean invariance is connected to the time delta-correlations in the velocity propagator (3). To this end, let us consider the Galilean transformation $x - x' \to x - x' + V(t - t')$, then the velocity correlator $\langle \upsilon_j(x,t)\upsilon_i(x',t')\rangle = D_{ji}(x - x', t - t')$ changes in accordance with the formula $D_{ji}(x - x', t - t') \to D_{ji}(x - x' + V(t - t'), t - t')$. Thus, in order to provide the invariance of the velocity term we should employ the velocity correlator in the form $D_{ji}(x - x', t - t') = \delta(t - t')\bar{D}_{ji}(x - x')$, where $\bar{D}_{ji}(x - x')$ is a function containing solely space variables; therefore, the last therm in Γ_k (9) is time-local. If it were dependent on the coarse-grained scale k, it would satisfy the following RG equation

$$\partial_s D^{-1}_{ij} = \frac{1}{2}\,\bar{\partial}_s \lim_{\omega\to 0} \left.\frac{\delta^2 \operatorname{Tr}\ln\left(\Gamma_k^{[2]} + R_k\right)}{\delta\upsilon_i(-q,-\omega)\delta\upsilon_j(q,\omega)}\right|_{\substack{\varphi'=\upsilon_j=0\\ \rho=\rho_k}} . \tag{20}$$

Computing these functional derivatives, we can see that the right-hand side of (20) is proportional to the integral

$$\tilde{\partial}_s \int \frac{d\omega d^d q}{(2\pi)^d} \frac{1}{(iX_k\omega + Z_k q^2 + R_k^\phi(q) + 2\lambda_k \rho_k)^2}, \tag{21}$$

which is obviously equal to zero. Hence one obtains the zero RG flow $\partial_s D_{ij}^{-1} = 0$.

References

[1] A.N. Vasiliev, Quantum-Field Renormalization Group in the Critical Behavior Theory and in Stochastic Dynamics, St. Petersburg Institute for Nuclear Physics, St. Petersburg, 1998.
[2] P.C. Hohenberg, B.I. Halperin, Rev. Mod. Phys. 49 (1977) 435.
[3] D.Yu. Ivanov, Critical Behaviour of Non-Ideal Systems, Wiley-VCH, Weinheim, Germany, 2008.
[4] R.H. Kraichnan, Phys. Rev. Lett. 72 (1994) 1016;
R.H. Kraichnan, Phys. Fluids 11 (1968) 945.
[5] M. Chertkov, I. Kolokolov, M. Vergassola, Phys. Rev. E 56 (1997) 5483;
M. Chertkov, I. Kolokolov, M. Vergassola, Phys. Rev. Lett. 80 (1998) 512.
[6] L.Ts. Adzhemyan, N.V. Antonov, Phys. Rev. E 58 (1998) 7381.
[7] N.V. Antonov, Phys. Rev. E 60 (1999) 6691.
[8] G. Falkovich, K. Gawedzki, M. Vergassola, Rev. Mod. Phys. 73 (2001) 913.
[9] L.Ts. Adzhemyan, N.V. Antonov, A.N. Vasiliev, The Field Theoretic Renormalization Group in Fully Developed Turbulence, Gordon and Breach, Amsterdam, 1999.
[10] N.V. Antonov, A.S. Kapustin, A.V. Malyshev, Theor. Math. Phys. 169 (1) (2011) 124.
[11] N.V. Antonov, P.I. Kakin, Theor. Math. Phys. 185 (1) (2015) 1391.
[12] N.V. Antonov, M. Hnatic, A.S. Kapustin, T. Lucivjansky, L. Mizisin, Phys. Rev. E 93 (1) (2016) 012151.
[13] N.V. Antonov, A.S. Kapustin, J. Phys. A 45 (50) (2012) 505001.
[14] N.V. Antonov, V.I. Iglovikov, A.S. Kapustin, J. Phys. A 45 (25) (2012) 255004.
[15] N.V. Antonov, M. Hnatich, J. Honkonen, J. Phys. A 39 (25) (2006) 7867.
[16] N.V. Antonov, A.S. Kapustin, J. Phys. A 43 (40) (2010) 405001.
[17] G. Satten, D. Ronis, Phys. Rev. Lett. 55 (1985) 91.
[18] J. Berges, N. Tetradis, C. Wetterich, Phys. Rep. 363 (2002) 223.
[19] L. Canet, H. Chate, J. Phys. A 40 (2007) 1937.
[20] D. Mesterházy, J.H. Stockemer, L.F. Palhares, J. Berges, Phys. Rev. B 88 (2013) 174301.
[21] L. Canet, B. Delamotte, N. Wschebor, Phys. Rev. E 93 (2016) 063101.
[22] C. Pagani, Phys. Rev. E 92 (2015) 033016.
[23] L. Canet, B. Delamotte, O. Deloubriére, N. Wschebor, Phys. Rev. Lett. 92 (2004) 195703.
[24] L. Canet, H. Chaté, B. Delamotte, N. Wschebor, Phys. Rev. Lett. 104 (2010) 150601.
[25] C. Wetterich, Phys. Lett. B 301 (1993) 90.
[26] K. Aoki, K. Morikawa, W. Souma, J. Sumi, H. Terao, Prog. Theor. Phys. 9 (1998) 451.
[27] D.F. Litim, Phys. Rev. D 64 (2001) 105007.
[28] D.F. Litim, J. High Energy Phys. 0111 (2001) 059.
[29] J.M. Pawlowski, Ann. Phys. 769 (2007) 105.
[30] A.S. Monin, A.M. Yaglom, Statistical Fluid Mechanics, vol. 2, MIT Press, Cambridge, MA, 1975.