EPJ Web of Conferences **271**, 01005 (2022)
HYP2022
https://doi.org/10.1051/epjconf/202227101005

Baryonic effective field theory for light hypernuclei

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Abstract. Light hypernuclei containing one or two Λ baryons are the subject of an ongoing experimental campaign aiming to study the spectrum of these systems, as well as the 2 and 3-body interaction between Λ hyperons and nucleons. Here we shortly review the theoretical study of these systems within the framework of baryonic effective field theory.

1 Introduction

Baryonic effective field theory (BEFT) describing the dynamics of hypernuclei composed of protons (p), neutrons (n), and Λ-particles as the only degrees of freedom, has proven itself over the last few years as an useful tool to study bound and resonance states in light single- and double-Λ, hypernuclear systems. Constrained to reproduce the available low energy data, BEFT solves the longstanding overbinding problem of the $^{5}_{\Lambda}$He hypernucleus [1, 2], and predicts the existence of bound double-Λ hypernuclei still under debate. Its application to study the continuum spectrum of hypernuclear trios reveals the existence of a virtual state in the Λ-neutron-proton (Λnp) $J^\pi = \frac{3}{2}^+$ channel, leading to cross-section enhancement near the Λ-deuteron threshold. For the Λnn $J^\pi = \frac{1}{2}^+$ channel it predicts a resonance state, depending, however, on the value of the Λ-nucleon scattering length. Recently, BEFT was also applied to study the $^{4}_{\Lambda}$H-$^{4}_{\Lambda}$He charge symmetry breaking, yielding an estimate for the $\Lambda - \Sigma^0$ admixture amplitude $\mathcal{A}_{I=1} \approx 1.5\%$ in agreement with the value deduced by Dalitz and von-Hippel from the baryon octet mass [3].

In this article we shortly review the foundation of BEFT, the determination of its low energy constants (LECs) using the available experimental data, and its application to the study of light single- and double-Λ hypernuclear bound and resonance states, as presented in [1–6].

2 Baryonic EFT

BEFT is a nuclear effective field theory (EFT), see *e.g.* [7], having baryons as the only degrees of freedom. As such, BEFT provides a low energy realization of QCD, the fundamental theory of the strong interactions. The details of the QCD dynamics are encoded in the BEFT interaction strengths, the LECs. Being a low energy theory, the physical observables

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in BEFT are calculated as expansions in Q/M_h, with Q being the typical momentum scale of the physics we want to describe, and M_h the scale where the EFT breaks down.

Here we shall limit our attention to the three lightest baryons - namely the neutron, the proton and the Λ-particle - and discuss the appropriate BEFT. Denoting by $N = (p, n)$ the nucleon field and by Λ the Λ-particle field, the BEFT Lagrangian takes the form

$$\mathcal{L} = N^\dagger(i\partial_0 + \frac{\nabla^2}{2M_N})N + \Lambda^\dagger(i\partial_0 + \frac{\nabla^2}{2M_\Lambda})\Lambda + \mathcal{L}_{2B} + \mathcal{L}_{3B} + \ldots, \quad (1)$$

with M_N the nucleon mass, M_Λ the Λ mass, and $\mathcal{L}_{2B}$, $\mathcal{L}_{3B}$ the two and three-baryon interaction Lagrangians. Naively, the various terms appearing in the Lagrangian are ordered by powers of (Q/M_h). However this naive ordering is modified to ensure that the EFT is renormalizable in the sense that at each order in the expansion the sensitivity to unaccounted short-range physics is small, of the order $O(Q/M_h)$.

Ignoring spin and flavor, the 2-body Lagrangian $\mathcal{L}_{2B}$ is nothing but a low energy expansion of the 2-body interaction, equivalent to the effective range expansion [7]. In momentum space, the resulting s-wave $2B$ potential can be written as

$$\begin{aligned} V_{\mathrm{N}^n\mathrm{LO}} =& C_0 + C_2\left(p^2 + p'^2\right) \\ &+ C_4\left(p^4 + p'^4\right) + C_{22}p^2p'^2 + \cdots , \end{aligned} \quad (2)$$

where $\boldsymbol{p}$ is the incoming relative momentum and $\boldsymbol{p}'$ the outgoing momentum. To avoid the UV divergences embedded in these contact terms the interaction must be regularized.

Behind its apparent simplicity BEFT exhibits some non-trivial features, such as (a) The Thomas collapse, compelling the promotion of a 3-body contact term to leading-order (LO) [8], (b) The appearance of a 4-body force at next-to-leading-order (NLO) [9], and (c) The Wigner bound which limits the possible values of the effective range and forces a perturbative treatment of all but the LO terms [10, 11]. These features have led to the power counting scheme depicted in Fig. 1 [7]. The accuracy of the BEFT expansion depends on the ratio between the physical scale Q and the breakup scale M_h. For bound and resonance states we can take Q to be the binding momentum. For non-strange nuclei the breakup scale of BEFT is the pion mass m_π. Thus, considering the binding energy of light nuclei

$$\left(\frac{Q}{M_h}\right) = \frac{\sqrt{2B_N M_N}}{m_\pi} \approx 0.3 - 0.8$$

when the large range reflects the strong variation in the single nucleun binding energy B_N between the deutron and ^{4}He. For ΛN interaction the one-pion exchange diagram is forbidden, and therefore we may assume the breaking scale to be $M_h \approx 2m_\pi$. This leads to the expansion parameter

$$\left(\frac{Q}{M_h}\right) = \frac{\sqrt{2B_\Lambda M_\Lambda}}{2m_\pi} \approx 0.05 - 0.3 \, .$$

In view of these expansion parameters, we expect that already al leading order (LO) BEFT would give a very reasonable predictions for the Λ particle binding in the nucleus. It should be noted however, that in practice non-strange BEFT seems to work better than indicated by the large value of the nuclear expansion parameter. It predicts the binding energy of ^{4}He with an accuracy of $\approx 10\%$.

3 The low energy constants

The BEFT of single- and double-Λ hypernuclei contains, at leading order, 10 LECs, see Fig. 2. In the non-strange sector (strangeness = 0) there are 3 LECs - 2 two-body LECs

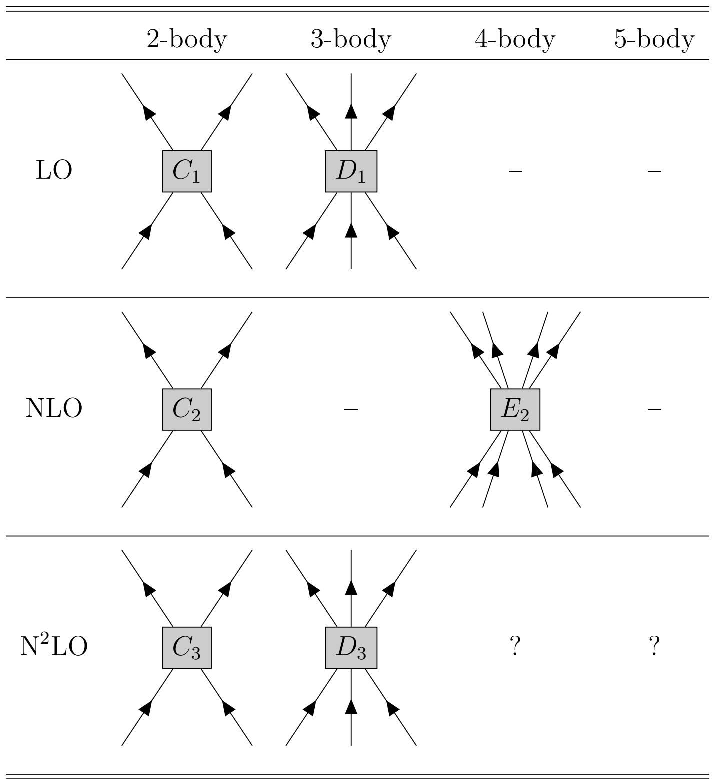

Figure 1. Diagrammatic representation of the multi baryon interaction in BEFT. The order n of each term is indicated by N^{n}LO. The letters in each diagram stand for the corresponding LECs.

which can be fitted either to the NN scattering lengths or to the np isospin triplet $I = 1$ scattering length and to the deutron binding energy, and 1 three-body term which can be fitted to the triton binding energy.

In the single-Λ sector (strangeness =-1) there are 5 LECs - 2 two-body LECs which can be associated with the spin-singlet $S = 0$ and spin-triplet $S = 1$ ΛN scatterering lengths, and 3 three-body LECs. The problem here is the lack of experimental data. As there is no available low energy ΛN scattering data, the ΛN LECs can be fitted to theoreticaly extracted ΛN scattering lengths rather than directly to experimental data, see *e.g.* [2]. In the three-body sector the situation is more severe as there is only one ΛNN bound state, no scattering data, and we have 3 LECs. The solution suggested in [2] was to fit 2 LECs to the 2 available ${}^4_\Lambda$H states.

In the double-Λ sector (strangeness =-2) there are 2 LECs - 1 two-body LEC which can be associated with the spin-singlet $S = 0$ $\Lambda\Lambda$ scatterering length, and one three-body $\Lambda\Lambda N$ LEC. Here the two-body LEC can be fitted again to theoretical estimates of the $\Lambda\Lambda$ scattering length. We note that the avaialable theoretical value is consistent with the values extracted from heavy ion femtoscopy experiments and from LQCD calculations [12]. The three-body $\Lambda\Lambda N$ LEC can be fitted to the ${}^{\ 6}_{\Lambda\Lambda}$He Nagara event [5].

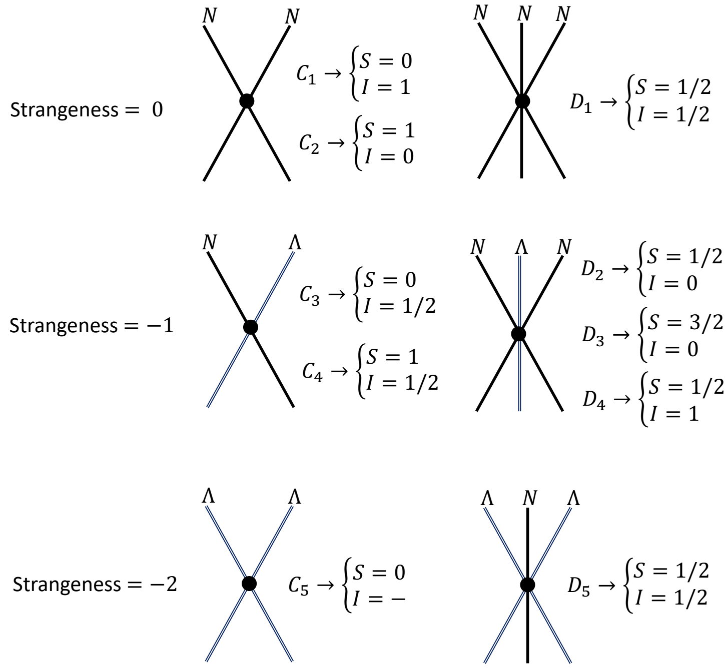

Figure 2. Diagrammatic representation of 2,3-baryon intearactions apearing at LO in single- and double-Λ BEFT. Here C_i stand for the two-body LECs, and D_j for the three-body LECs. S is the spin, and I the isospin. Figure taken from [5].

Inspecting Fig. 1 and Fig. 2 we can conclude that, at NLO, BEFT of single and double Λ hypernuclei contains 5 two-body LECs, each corresponding to the same quantum numbers as the LO two-body terms, no new three-body terms, and 5 four-body LECs. The first four-body LEC is associated with the nuclear - zero strangeness - sector, the next 2 four-body LECs are associated with the $S = 0, 1$ $^{4}_{\Lambda}$H states, and the last 2 are associated with the $S = 0, 1$ $^{4}_{\Lambda\Lambda}$H states. The lack of the needed experimental 3,4-body data hinders the development of BEFT at NLO.

4 Light single- and double-Λ hypernuclear systems

Table 1 summarizes the available low energy data used to fit the Λ LECs used for BEFT calculations in [1, 2, 5, 6]. The table also presents the main predictions obtained in these works regarding the properties of the light single- and double-Λ hypernuclei bound and resonance states.

It is worth noting that constrained to reproduce the available low energy data, BEFT reproduce within error bars the $^{5}_{\Lambda}$He hypernucleus [1, 2], and predicts the existence of a bound $^{5}_{\Lambda\Lambda}$H system. In the application of BEFT to study the continuum spectrum of hypernuclear trios, it was found that the excited $J^\pi = \frac{3}{2}^+$ hypertriton state $^{3}_{\Lambda}\text{H}^{J=3/2}$ is a virtual state. For the

Table 1. Compilation of the experimental data reproduced or predicted by the BEFT. Here, a_J is the scattering length with J being the channel's total angular momentum, E is the state's energy, B_λ the Λ particle separation energy and $B_{\Lambda\Lambda}$ the double Λ separation energy. All energies are given in MeV, scattering lengths are given in fm. Observables used to fit the LECs are denoted by (LEC).

System	observable	BEFT	Exp./Theo. extraction
ΛN	a_0 (LEC)	-2.35 ± 0.55	-2.35 ± 0.55 [13–16]
	a_1 (LEC)	-1.60 ± 0.1	-1.60 ± 0.1 [13–16]
$\Lambda\Lambda$	a_0 (LEC)	-1.2 ± 0.7	-1.2 ± 0.7 [17–19]
${}^{3}_{\Lambda}$H	B_Λ (LEC)	0.13	0.13$\pm$ 0.05 [20]
Λnn	Re(E)	0.0$\pm$0.2 [6]	?
	Im(E)	-0.75$\pm$ 0.15 [6]	?
${}^{3}_{\Lambda}$H$^{J=3/2}$	E (virtual)	-0.012 ± 0.08 [6]	?
Λ-deuteron	$a_{3/2}$	-16 ± 9 [6]	?
${}^{4}_{\Lambda}$H$_0$	B_Λ (LEC)	2.16	2.16$\pm$ 0.08 [21]
${}^{4}_{\Lambda}$H$_1$	B_Λ (LEC)	1.07	1.07$\pm$ 0.08 [22]
${}^{5}_{\Lambda}$He	B_Λ	3.01$\pm$ 0.10 [1]	3.12$\pm$ 0.02 [20]
${}^{5}_{\Lambda\Lambda}$H	B_Λ	$1.14^{+0.44}_{-0.26}$ [5]	?
${}^{6}_{\Lambda\Lambda}$He	$B_{\Lambda\Lambda}$ (LEC)	6.91	6.91 ± 0.17 [23]

Λnn $J^\pi = \frac{1}{2}^+$ state it was found that the pole is very close to the real axis, and that its exact position depends on the value of the ΛN scattering lengths. Therefore at this point we cannot conclude weather it is a physical resonance state. We note that while at the moment there is no available experimental data regarding these continuum states, both systems are currently a subject of an ongoing experimental study.

Taking into account the Coulomb corrections the charge symmetry breaking (CSB) between the mirror nuclei ^{3}H and ^{3}He is about 70 keV, or about 1% of its binding energy. In comparison the CSB between the $S = 0$ and $S = 1$ states in the Λ hypernuclei ${}^{4}_{\Lambda}$H-${}^{4}_{\Lambda}$He is about 300 keV, i.e. $\Delta E_{CSB}/\Delta E_{01} \approx 22\%$. The most plausible source for this large CSB is the $\Lambda - \Sigma^0$ admixture as suggested by Gal [24], generalizing the work of Dalitz and von-Hippel [25]. Building upon this hypothesis, BEFT was applied to study the ${}^{4}_{\Lambda}$H- ${}^{4}_{\Lambda}$He CSB in [3]. Utilizing SU$_3$ flavor symmetry BEFT was used to estimate the $\Lambda - \Sigma^0$ admixture amplitude $\mathcal{A}_{I=1} \approx 1.5\%$. The obtained result was found to be in agreement with the value deduced by Dalitz and von-Hippel from the baryon octet mass [25].

5 Summary

In this article we presented a short review of BEFT and its application to the study of light Λ hypernuclei. At leading order BEFT seems to be working rather well, reproducing the binding energy of ${}^{5}_{\Lambda}$He, and the charge symmetry breaking in the mirror hypernuclei ${}^{4}_{\Lambda}$H-${}^{4}_{\Lambda}$He. Going beyond LO seems to be a difficult step in view of the appearance of 4-body terms in Lagrangian and the shortage of experimental data to fit the LECs.

Acknowledgment The work of MS and NB was supported by the Pazy Foundation and by the Israel Science Foundation grant 1086/21. Furthermore, the work of AG and NB is part of a project funded by the European Union's Horizon 2020 research and innovation programme under grant agreement No. 824093.

References

[1] L. Contessi, N. Barnea, A. Gal, Phys. Rev. Lett. **121**, 102502 (2018)
[2] L. Contessi, N. Barnea, A. Gal, AIP Conf. Proc. **2130**, 040012 (2019)
[3] M. Schäfer, N. Barnea, A. Gal Phys. Rev. C **106**, L031001 (2022)
[4] M. Schäfer, B. Bazak, N. Barnea, A. Gal, J. Mareš, Phys.Rev.C **105**, 015202 (2022)
[5] L. Contessi, M. Schäfer, N. Barnea, A. Gal, J. Mareš, Phys. Lett. B **797**, 134893 (2019)
[6] M. Schäfer, B. Bazak, N. Barnea, J. Mareš, Phys. Lett. B **808** 135614 (2020)
[7] H.-W. Hammer, S. König, U. van Kolck, Rev. Mod. Phys. **92**, 025004 (2020)
[8] P. F. Bedaque, H.-W. Hammer, U. van Kolck, Phys. Rev. Lett. **82**, 463 (1999)
[9] B. Bazak, J. Kirscher, S. König, M. P. Valderrama, N. Barnea, and U. van Kolck, Phys. Rev. Lett. **122**, 143001 (2019)
[10] D. R. Phillips and T. D. Cohen, Phys. Lett. B **390**, 7 (1997)
[11] D. R. Phillips, S. R. Beane, and T. D. Cohen, Ann. Phys. **263**, 255 (1998)
[12] Y. Kamiya et al., Phys. Rev. C **105**, 014915 (2022).
[13] G. Alexander, U. Karshon, A. Shapira, et al. Phys. Rev. **173**, 1452 (1968)
[14] T.A. Rijken, V.J.G Stoks, Y. Yamamoto, Phys. Rev. C **59**, 21 (1999)
[15] H. Polinder, J. Haidenbauer, U.-G. Meißner, Nucl. Phys. A **779**, 244 (2006)
[16] J. Haidenbauer, S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga, W. Weise, Nucl. Phys. A **915**, 24 (2013)
[17] K. Morita, T. Furumoto, A. Ohnishi, Phys. Rev. C 91 (2015) 024916; A. Ohnishi, K. Morita, K. Miyahara, T. Hyodo, Nucl. Phys. A **954**, 294 (2016)
[18] L. Adamczyk, et al. (STAR Collaboration), Phys. Rev. Lett. **114**, 022301 (2015)
[19] S. Acharya, et al. (ATLAS Collaboration), Phys. Rev. C **99**, 024001 (2019)
[20] D. H. Davis, Nucl. Phys. A **754**, 3c (2005)
[21] A. Esser et al. (MAMI A1 Collaboration), Phys. Rev. Lett. **114** 232501 (2015) ; F. Schulz et al. (MAMI A1 Collaboration), Nucl. Phys. A **954**, 149 (2016)
[22] T.O. Yamamoto et al. (J-PARC E13 Collaboration), Phys. Rev. Lett. **115**, 222501 (2015)
[23] H. Takahashi, et al., Phys. Rev. Lett. **87**, 212502 (2001)
[24] A. Gal, Phys. Lett. B **744**, 352 (2015)
[25] R.H. Dalitz and F. von Hippel, Phys. Lett **10**, 153 (1964)