

# $D^0 - \bar{D}^0$ MIXING

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The main challenge in the Standard Model calculation of the mass and width difference in  $D^0 - \bar{D}^0$  mixing is to estimate the size of  $SU(3)$  breaking. We prove that mixing occurs in the Standard Model only at second order in  $SU(3)$  violation. We calculate  $SU(3)$  breaking due to phase space effects, and find that it can naturally give rise to a width difference  $\Delta\Gamma/2\Gamma \sim 1\%$ , potentially reducing the sensitivity of  $D$  mixing to new physics.<sup>1,2</sup>

## 1 Introduction

It is a common assertion that the Standard Model (SM) prediction for mixing in the  $D^0 - \bar{D}^0$  system is very small, making this process a sensitive probe of new physics. Two physical parameters that characterize  $D^0 - \bar{D}^0$  mixing are

$$x \equiv \frac{\Delta M}{\Gamma}, \quad y \equiv \frac{\Delta\Gamma}{2\Gamma}, \quad (1)$$

where  $\Delta M$  and  $\Delta\Gamma$  are the mass and width differences of the neutral  $D$  meson mass eigenstates, and  $\Gamma$  is their average width. The  $D^0 - \bar{D}^0$  system is unique among the neutral mesons in that it is the only one whose mixing proceeds via intermediate states with down-type quarks, and is therefore very sensitive to certain classes of new physics models.<sup>3</sup> The mixing is very slow in the SM, because the third generation plays a negligible role due to the smallness of  $|V_{ub}V_{cb}|$  and  $m_b \ll m_W$ , and so the GIM cancellation is very effective.<sup>4,5,6,7</sup>

The current experimental upper bounds on  $x$  and  $y$  are of order  $10^{-2}$ , and are expected to improve significantly in the coming years. To regard a future discovery of nonzero  $x$  or  $y$  as a signal for new physics, we would need high confidence that the SM predictions lie significantly below the present limits. As we will show,  $x$  and  $y$  are generated only at second order in  $SU(3)$  breaking in the SM, so schematically

$$x, y \sim \sin^2 \theta_C \times [SU(3) \text{ breaking}]^2, \quad (2)$$

where  $\theta_C$  is the Cabibbo angle. Therefore, the SM values of  $x$  and  $y$  depend crucially on the size of  $SU(3)$  breaking. Although  $y$  is expected to be determined by SM processes, its value affects significantly the sensitivity of  $D$  mixing to new physics.<sup>2</sup>

At present, there are three types of experiments which measure  $x$  and  $y$ . Each is actually sensitive to a combination of  $x$  and  $y$ , rather than to either quantity directly. First, the  $D^0$  lifetime difference to  $CP$  even and  $CP$  odd final states<sup>8,9,10,11,12</sup> measures, to leading order,

$$y_{CP} = \frac{\Gamma(CP \text{ even}) - \Gamma(CP \text{ odd})}{\Gamma(CP \text{ even}) + \Gamma(CP \text{ odd})} \simeq \frac{\hat{\tau}(D \rightarrow \pi^+ K^-)}{\hat{\tau}(D \rightarrow K^+ K^-)} - 1 = y \cos \phi - x \sin \phi \frac{A_m}{2}, \quad (3)$$

where the  $D$  mass eigenstates are  $|D_{L,S}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle$ ,  $A_m = |q/p|^2 - 1$ , and  $\phi = \arg(q/p)$  is a possible  $CP$  violating phase of the mixing amplitude. The experimental results are

$$y_{CP} = \begin{cases} 0.8 \pm 3.1\% & \text{E791,}^8 \\ 3.4 \pm 1.6\% & \text{FOCUS,}^9 \\ -1.1 \pm 2.9\% & \text{CLEO,}^{10} \\ -0.5 \pm 1.3\% & \text{BELLE,}^{11} \\ 1.4 \pm 1.2\% & \text{BABAR,}^{12} \end{cases} \quad (4)$$

which yield a world average  $y_{CP} = 1.0 \pm 0.7\%$  at present. Second, the time dependence of doubly Cabibbo suppressed decays, such as  $D^0 \rightarrow K^+ \pi^-$ ,<sup>13</sup> is sensitive to the three quantities

$$(x \cos \delta + y \sin \delta) \cos \phi, \quad (y \cos \delta - x \sin \delta) \sin \phi, \quad x^2 + y^2, \quad (5)$$

where  $\delta$  is the strong phase between the Cabibbo allowed and doubly Cabibbo suppressed amplitudes. A similar study for  $D^0 \rightarrow K^- \pi^+ \pi^0$  would allow the strong phase difference to be extracted simultaneously from the Dalitz plot analysis.<sup>14</sup> Third, one can search for  $D$  mixing in semileptonic decays,<sup>15</sup> which is sensitive to  $x^2 + y^2$ .

In a large class of models, the best hope to discover new physics in  $D$  mixing is to observe the  $CP$  violating phase,  $\phi_{12} = \arg(M_{12}/\Gamma_{12})$ , which is very small in the Standard Model. However, if  $y \gg x$ , then the sensitivity of any physical observable to  $\phi_{12}$  is suppressed, since  $A_m \propto x/y$  and  $\phi \propto (x/y)^2$ , even if new physics makes a large contribution to  $M_{12}$  and  $\phi_{12}$ .<sup>2</sup> It is also clear from Eq. (5) that if  $y$  is significantly larger than  $x$ , then  $\delta$  must be known very precisely for experiments to be sensitive to new physics in the terms linear in  $x$  and  $y$ .

## 2 $SU(3)$ analysis of $D^0 - \bar{D}^0$ mixing

In this section we prove that  $D^0 - \bar{D}^0$  mixing arises only at second order in  $SU(3)$  breaking effects. The proof is valid when  $SU(3)$  violation enters perturbatively. The quantities  $M_{12}$  and  $\Gamma_{12}$ , which determine  $x$  and  $y$ , depend on matrix elements of the form  $\langle \bar{D}^0 | \mathcal{H}_w \mathcal{H}_w | D^0 \rangle$ , where  $\mathcal{H}_w$  is the  $\Delta C = -1$  part of the weak Hamiltonian. Denoting by  $D$  the field operator that creates a  $D^0$  meson and annihilates a  $\bar{D}^0$ , this matrix element may be written as

$$\langle 0 | D \mathcal{H}_w \mathcal{H}_w D | 0 \rangle. \quad (6)$$

Let us focus on the  $SU(3)$  flavor group theory properties of this expression.

Since the operator  $D$  is of the form  $\bar{c}u$ , it transforms in the fundamental representation of  $SU(3)$ , which we represent with a lower index,  $D_i$  (the index  $i = 1, 2, 3$  corresponds to  $u, d, s$ , respectively). The only nonzero element is  $D_1 = 1$ . The  $\Delta C = -1$  part of the weak Hamiltonian has the flavor structure  $(\bar{q}_i c)(\bar{q}_j q_k)$ , so its matrix representation is written with a fundamental index and two antifundamentals,  $H_k^{ij}$ . This operator is a sum of irreducible representations contained in the product  $3 \times \bar{3} \times \bar{3} = \bar{15} + 6 + \bar{3} + \bar{3}$ . In the limit in which the third generation

is neglected,  $H_k^{ij}$  is traceless, so only the  $\overline{15}$  (symmetric in  $i$  and  $j$ ) and 6 (antisymmetric in  $i$  and  $j$ ) appear. That is, the  $\Delta C = -1$  part of  $\mathcal{H}_w$  may be decomposed as  $\frac{1}{2}(\mathcal{O}_{\overline{15}} + \mathcal{O}_6)$ , where<sup>1</sup>

$$\begin{aligned}\mathcal{O}_{\overline{15}} &= (\bar{s}c)(\bar{u}d) + (\bar{u}c)(\bar{s}d) + s_1(\bar{d}c)(\bar{u}d) + s_1(\bar{u}c)(\bar{d}d) \\ &\quad - s_1(\bar{s}c)(\bar{u}s) - s_1(\bar{u}c)(\bar{s}s) - s_1^2(\bar{d}c)(\bar{u}s) - s_1^2(\bar{u}c)(\bar{d}s), \\ \mathcal{O}_6 &= (\bar{s}c)(\bar{u}d) - (\bar{u}c)(\bar{s}d) + s_1(\bar{d}c)(\bar{u}d) - s_1(\bar{u}c)(\bar{d}d) \\ &\quad - s_1(\bar{s}c)(\bar{u}s) + s_1(\bar{u}c)(\bar{s}s) - s_1^2(\bar{d}c)(\bar{u}s) + s_1^2(\bar{u}c)(\bar{d}s),\end{aligned}\quad (7)$$

and  $s_1 = \sin\theta_C \approx 0.22$ . This determines the nonzero elements of the matrix representations of  $H(\overline{15})_k^{ij}$  and  $H(6)_k^{ij}$ . We introduce  $SU(3)$  breaking through the quark mass operator  $\mathcal{M}$ , whose matrix representation is  $M_j^i = \text{diag}(m_u, m_d, m_s)$ . Although  $\mathcal{M}$  is a linear combination of the adjoint and singlet representations, only the 8 induces  $SU(3)$  violating effects. It is convenient to set  $m_u = m_d = 0$  and let  $m_s \neq 0$  be the only  $SU(3)$  violating parameter. All nonzero matrix elements built out of  $D_i$ ,  $H_k^{ij}$  and  $M_j^i$  must be  $SU(3)$  singlets.

We are now ready to prove that  $D^0 - \overline{D}^0$  mixing arises only at second order in  $SU(3)$  violation, by which we mean second order in  $m_s$ . First, note that the pair of  $D$  operators is symmetric, and so  $D_i D_j$  transforms as a 6 under  $SU(3)$ . Second, the pair of  $\mathcal{H}_w$ 's is also symmetric, and the product  $H_k^{ij} H_n^{lm}$  is in one of the representations which appears in the product

$$\begin{aligned}[(\overline{15} + 6) \times (\overline{15} + 6)]_S &= (\overline{15} \times \overline{15})_S + (\overline{15} \times 6) + (6 \times 6)_S \\ &= (\overline{60} + \overline{24} + 15 + 15' + \overline{6}) + (42 + 24 + 15 + \overline{6} + 3) + (15' + \overline{6}).\end{aligned}\quad (8)$$

A straightforward computation shows that only the  $\overline{60}$ , 42, and  $15'$  representations appear in the decomposition of  $\mathcal{H}_w \mathcal{H}_w$ . So we have product operators of the form

$$DD = \mathcal{D}_6, \quad \mathcal{H}_w \mathcal{H}_w = \mathcal{O}_{\overline{60}} + \mathcal{O}_{42} + \mathcal{O}_{15'}, \quad (9)$$

where the subscripts denote the representation of  $SU(3)$ .

Since there is no  $\overline{6}$  in the decomposition of  $\mathcal{H}_w \mathcal{H}_w$ , there is no  $SU(3)$  singlet which can be made with  $\mathcal{D}_6$ , and so there is no  $SU(3)$  invariant matrix element of the form in Eq. (6). This is the well-known result that  $D^0 - \overline{D}^0$  mixing is prohibited by  $SU(3)$  symmetry.

Now consider a single insertion of the  $SU(3)$  violating spurion  $\mathcal{M}$ . The combination  $\mathcal{D}_6 \mathcal{M}$  transforms as  $6 \times 8 = 24 + \overline{15} + 6 + \overline{3}$ . There is still no invariant to be made with  $\mathcal{H}_w \mathcal{H}_w$ . This proves that  $D^0 - \overline{D}^0$  mixing is not induced at first order in  $SU(3)$  breaking.

With two insertions of  $\mathcal{M}$ , it becomes possible to make an  $SU(3)$  invariant. The decomposition of  $\mathcal{D} \mathcal{M} \mathcal{M}$  is

$$\begin{aligned}6 \times (8 \times 8)_S &= 6 \times (27 + 8 + 1) \\ &= (60 + \overline{42} + 24 + \overline{15} + \overline{15}' + 6) + (24 + \overline{15} + 6 + \overline{3}) + 6.\end{aligned}\quad (10)$$

There are three elements of the  $6 \times 27$  part which can give invariants with  $\mathcal{H}_w \mathcal{H}_w$ . Each invariant yields a contribution proportional to  $s_1^2 m_s^2$ . As promised,  $D^0 - \overline{D}^0$  mixing arises only at second order in the  $SU(3)$  violating parameter  $m_s$ .

### 3 Estimating the size of $SU(3)$ breaking

There is a vast literature on estimating  $x$  and  $y$  within and beyond the Standard Model, and the results span many orders of magnitudes.<sup>16</sup> Roughly speaking, there are two approaches, neither of which is very reliable, because  $m_c$  is in some sense intermediate between heavy and light.

**“Inclusive” approach** The inclusive approach is based on the operator product expansion (OPE). In the  $m_c \gg \Lambda$  limit, where  $\Lambda$  is a scale characteristic of the strong interactions, such as

Table 1: The enhancement of  $\Delta M$  and  $\Delta\Gamma$  relative to the box diagram (i.e., the 4-quark operator) contribution at higher orders in the OPE.  $\Lambda$  denotes a typical hadronic scale around 1 GeV, and  $\beta_0 = 11 - 2n_f/3 = 9$ .

ratio	4-quark	6-quark	8-quark
$\Delta M/\Delta M_{\text{box}}$	1	$\Lambda^2/m_s m_c$	$(\Lambda^2/m_s m_c)^2 (\alpha_s/4\pi)$
$\Delta\Gamma/\Delta M$	$m_s^2/m_c^2$	$\alpha_s/4\pi$	$\beta_0 \alpha_s/4\pi$

$m_\rho$  or  $4\pi f_\pi$ ,  $\Delta M$  and  $\Delta\Gamma$  can be expanded in terms of matrix elements of local operators.<sup>4,5,17</sup> The use of the OPE relies on local quark-hadron duality, and on  $\Lambda/m_c$  being small enough to allow a truncation of the series after the first few terms. However, the charm mass may not be large enough for these to be good approximations for nonleptonic  $D$  decays. While an observation of  $y$  of order  $10^{-2}$  could be ascribed to a breakdown of the OPE or of duality,<sup>17</sup> such a large value of  $y$  is not a generic prediction of OPE analysis.

The leading contribution in the OPE comes from 4-quark operators in the  $|\Delta C| = 2$  effective Hamiltonian, corresponding to the short distance box diagram. The result is of the form

$$\Delta M_{\text{box}} \propto \frac{2X_D}{3\pi^2} \frac{m_s^4}{m_c^2}, \quad \Delta\Gamma_{\text{box}} \propto \frac{4X_D}{3\pi} \frac{m_s^6}{m_c^4}, \quad (11)$$

where  $X_D = |V_{cs}V_{cd}^*|^2 G_F^2 m_D f_D^2 B_D$ . Eq. (11) then yields the estimates

$$x_{\text{box}} \sim \text{few} \times 10^{-5}, \quad y_{\text{box}} \sim \text{few} \times 10^{-7}. \quad (12)$$

The  $m_s^6$  dependence of  $\Delta\Gamma_{\text{box}}$  comes from three sources: (i)  $m_s^2$  from an  $SU(3)$  violating mass insertion on each quark line in the box graph; (ii)  $m_s^2$  from an additional mass insertion on each line to compensate the chirality flip from the first insertion; (iii)  $m_s^2$  to lift the helicity suppression of the decay of a scalar meson into a massless fermion pair. The last factor of  $m_s^2$  is absent from  $\Delta M$ ; this is why at leading order in the OPE,  $y_{\text{box}} \ll x_{\text{box}}$ .

Higher order terms in the OPE are important, because the chiral suppressions can be lifted by quark condensates instead of mass insertions,<sup>4</sup> allowing  $\Delta M$  and  $\Delta\Gamma$  to be suppressed by  $m_s^2$  only. This is the minimal suppression required by  $SU(3)$  symmetry. The order of magnitudes of the resulting contributions are summarized in Table 1. The dominant contributions to  $x$  are from 6- and 8-quark operators, while the dominant contribution to  $y$  is from 8-quark operators. With some assumptions about the hadronic matrix elements, one finds

$$x \sim y \sim 10^{-3}. \quad (13)$$

A generic feature of OPE based analyses is that  $x \gtrsim y$ . We emphasize that at the present time these methods are useful for understanding the order of magnitude of  $x$  and  $y$ , but not for obtaining reliable results. Turning these estimates into a systematic computation of  $x$  and  $y$  would require the calculation of many nonperturbative matrix elements.

**“Exclusive” approach** A long distance analysis of  $D$  mixing is complementary to the OPE. Instead of assuming that the  $D$  meson is heavy enough for duality to hold between the partonic rate and the sum over hadronic final states, one explicitly examines certain exclusive decays. This is particularly interesting for studying  $\Delta\Gamma$ , which depends on real final states. However,  $D$  decays are not dominated by a small number of final states. Since there are cancellations between states within a given  $SU(3)$  multiplet, one needs to know the contribution of each state with high precision. In the absence of sufficiently precise data on many rates and on strong phases, one is forced to use some assumptions. While most studies find  $x, y \lesssim 10^{-3}$ , it has also been argued that  $SU(3)$  violation is of order unity and so  $x, y \sim 10^{-2}$  is possible.<sup>18,19,20</sup>

The importance of  $SU(3)$  cancellations in both the magnitudes and phases of matrix elements is nicely illustrated by two-body  $D$  decays to charged pseudoscalars ( $\pi^+\pi^-$ ,  $\pi^+K^-$ ,  $K^+\pi^-$ ,

$K^+K^-$ ). It is known that  $SU(3)$  breaking is sizable in certain decay rates, e.g.,  $\mathcal{B}(D^0 \rightarrow K^+K^-)/\mathcal{B}(D^0 \rightarrow \pi^+\pi^-) \simeq 2.8$ ,<sup>21</sup> whereas it should be unity in the  $SU(3)$  limit. Such effects were the basis for the claim that  $SU(3)$  is not applicable to  $D$  decays.<sup>18,a</sup> In contrast, we know very little about the strong phase  $\delta$  between the Cabibbo allowed and doubly Cabibbo suppressed amplitudes, which vanishes in the  $SU(3)$  limit. Some model calculation give  $\cos \delta \gtrsim 0.8$ ,<sup>23</sup> but it is possible to obtain much larger results for  $\delta$ .<sup>19</sup> The value of  $y$  corresponding to two-body decays to charged  $\pi$ 's and  $K$ 's is

$$y_{\pi K} = \mathcal{B}(D^0 \rightarrow \pi^+\pi^-) + \mathcal{B}(D^0 \rightarrow K^+K^-) - 2 \cos \delta \sqrt{\mathcal{B}(D^0 \rightarrow K^-\pi^+) \mathcal{B}(D^0 \rightarrow K^+\pi^-)}. \quad (14)$$

The experimental central values, allowing for  $D$  mixing in the doubly Cabibbo suppressed rates, yield  $y_{\pi K} \simeq (5.76 - 5.29 \cos \delta) \times 10^{-3}$ .<sup>2</sup> For small  $\delta$ , there is an almost perfect cancellation, even though the individual rates violate  $SU(3)$  significantly. In this “exclusive” approach,  $x$  is usually obtained from  $y$  using a dispersion relation, and one typically finds  $x \sim y$ .

At the present time, one cannot use the exclusive approach to reliably predict  $x$  or  $y$ , since the estimates depend very sensitively on  $SU(3)$  breaking in poorly known strong phases and doubly Cabibbo suppressed rates. While calculating these effects model independently is not tractable in general, one source of  $SU(3)$  breaking in  $y$ , from final state phase space, can be calculated with only minimal assumptions. We estimate these effects in the next section.

#### 4 $SU(3)$ breaking from phase space

There is a contribution to the  $D^0$  width difference from all final states common in  $D^0$  and  $\bar{D}^0$  decay. In the  $SU(3)$  limit these contributions cancel when one sums over complete  $SU(3)$  representations. The cancellations depend on  $SU(3)$  symmetry both in the matrix elements and in the final state phase space. Since model independent calculations of  $SU(3)$  violation in matrix elements are not available, we focus on  $SU(3)$  violation in the phase space. This depends only on the hadron masses in the final state, and can be computed with mild assumptions about the momentum dependence of the matrix elements. Below we estimate  $y$  solely from  $SU(3)$  violation in the phase space.<sup>b</sup> We find that this contribution to  $y$  is negligible for two-body pseudoscalar final states, but can be of the order of a percent for final states with mass near  $m_D$ .

Let us concentrate on final states  $F$  which transform in a single  $SU(3)$  representation  $R$ . Assuming  $CP$  symmetry in  $D$  decays, which in the Standard Model and in most new physics scenarios is an excellent approximation, relates  $\langle \bar{D}^0 | \mathcal{H}_w | n \rangle$  to  $\langle D^0 | \mathcal{H}_w | \bar{n} \rangle$ . Since  $|n\rangle$  and  $|\bar{n}\rangle$  are in the same  $SU(3)$  multiplet, these two matrix elements are determined by a single effective Hamiltonian. Hence the contribution of the states  $n \in F_R$  to  $y$  is

$$y_{F,R} = \frac{1}{\Gamma} \langle \bar{D}^0 | \mathcal{H}_w \left\{ \eta_{CP}(F_R) \sum_{n \in F_R} |n\rangle \rho_n \langle n| \right\} \mathcal{H}_w | D^0 \rangle = \frac{\sum_{n \in F_R} \langle \bar{D}^0 | \mathcal{H}_w | n \rangle \rho_n \langle n | \mathcal{H}_w | D^0 \rangle}{\sum_{n \in F_R} \Gamma(D^0 \rightarrow n)}, \quad (15)$$

where  $\rho_n$  is the phase space available to the state  $n$ , and  $\eta_{CP} = \pm 1$  is determined by the  $CP$  transformation of the final state,  $CP|n\rangle = \eta_{CP}|\bar{n}\rangle$ . In the  $SU(3)$  limit, the  $\rho_n$ 's are the same for  $n \in F_R$ . Since  $\rho_n$  depend only on the known masses of the particles in the state  $n$ , incorporating the true values of  $\rho_n$  in the sum is a calculable source of  $SU(3)$  breaking.

As the simplest example, consider  $D$  decays to states  $F = PP$  consisting of a pair of pseudoscalar mesons such as  $\pi$ ,  $K$ ,  $\eta$ . We neglect  $\eta - \eta'$  mixing, but checked that this has a

<sup>a</sup>The  $SU(3)$  breaking in the matrix elements may be modest, even though the ratio of the measured rates appears to be very far from the  $SU(3)$  limit.<sup>22</sup>

<sup>b</sup>The phase space difference alone can explain the large  $SU(3)$  breaking between the measured  $D \rightarrow K^* \ell \bar{\nu}$  and  $D \rightarrow \rho \ell \bar{\nu}$  rates, assuming no  $SU(3)$  breaking in the form factors.<sup>24</sup> Recently it was shown that the lifetime ratio of the  $D_s$  and  $D^0$  mesons may also be explained this way.<sup>25</sup>

Table 2: Values of  $y_{F,R}$  for two-body final states. This represents the value which  $y$  would take if elements of  $F_R$  were the only channel open for  $D^0$  decay.

Final state representation		$y_{F,R}/s_1^2$	$y_{F,R}$ (%)
$PP$	8	-0.0038	-0.018
	27	-0.00071	-0.0034
$PV$	$8_S$	0.031	0.15
	$8_A$	0.032	0.15
	10	0.020	0.10
	$\overline{10}$	0.016	0.08
	27	0.040	0.19
$(VV)_{s\text{-wave}}$	8	-0.081	-0.39
	27	-0.061	-0.30
$(VV)_{p\text{-wave}}$	8	-0.10	-0.48
	27	-0.14	-0.70
$(VV)_{d\text{-wave}}$	8	0.51	2.5
	27	0.57	2.8

negligible effect on our numerical results. Since  $PP$  is symmetric in the two mesons, it must transform as an element of  $(8 \times 8)_S = 27 + 8 + 1$ . It is straightforward to construct the  $SU(3)$  invariants, and compute  $y_{PP,R}$  from them. For example, for the  $PP$  in an 8, there are invariants with  $\mathcal{H}_w$  in a  $\overline{15}$ ,  $A_8^{\overline{15}}(PP_8)_i^* H_k^{ij} D_j$ , and with  $\mathcal{H}_w$  in a 6,  $A_8^6(PP_8)_i^* H_k^{ij} D_j$ . In this particular case, the product  $H_k^{ij} D_j$  with  $(ij)$  symmetric (the  $\overline{15}$ ) is proportional to  $H_k^{ij} D_j$  with  $(ij)$  antisymmetric (the 6), and the linear combination  $A_8 \equiv A_8^{\overline{15}} - A_8^6$  is the only one which appears. We find

$$y_{PP,8} \propto s_1^2 \left[ \frac{1}{2} \Phi(\eta, \eta) + \frac{1}{2} \Phi(\pi^0, \pi^0) + \frac{1}{3} \Phi(\eta, \pi^0) + \Phi(\pi^+, \pi^-) + \Phi(K^+, K^-) - \frac{1}{6} \Phi(\eta, K^0) - \frac{1}{6} \Phi(\eta, \overline{K}^0) - \Phi(K^+, \pi^-) - \Phi(K^-, \pi^+) - \frac{1}{2} \Phi(K^0, \pi^0) - \frac{1}{2} \Phi(\overline{K}^0, \pi^0) \right], \quad (16)$$

where  $\Phi(P_1, P_2)$  is the phase space for  $D^0 \rightarrow P_1 P_2$  decay. In the  $SU(3)$  limit all  $\Phi$ 's are equal, and  $y_{PP,8}$  vanishes as  $m_\pi^2$ . In a two-body decay  $\Phi(P_1, P_2)$  is proportional to  $|\vec{p}|^{2\ell+1}$ , where  $\vec{p}$  and  $\ell$  are the spatial momentum and orbital angular momentum of the final state particles. For  $D^0 \rightarrow PP$ , the decay is into an  $s$ -wave, and it is straightforward to compute the phase space factors from the pseudoscalar masses. The results are not larger than one finds in the inclusive analysis (see Table 2), since as in the parton picture, the final states are far from threshold.

Next we turn to final states consisting of a pseudoscalar and a vector meson,  $F = PV$ . In this case there is no symmetry between the mesons, so all representations in the combination  $8 \times 8 = 27 + 10 + \overline{10} + 8_S + 8_A + 1$  can appear. For simplicity, we take the quark content of the  $\phi$  and  $\omega$  to be  $\bar{s}s$  and  $(\bar{u}u + \bar{d}d)/\sqrt{2}$  respectively, and consider only the combination which appears in the  $SU(3)$  octet. Reasonable variations of the  $\phi - \omega$  mixing angle have a negligible effect on our results. Both because the vector mesons are more massive, and because the decay is now into a  $p$ -wave, the phase space effects are larger than for the  $PP$  final state (see Table 2). Still, for all representation,  $y_{PV}$  are less than a percent. Note that three-body final states  $3P$  can resonate through  $PV$ , and so are partially included here.

For the  $VV$  final state, decays into  $s$ -,  $p$ - and  $d$ -waves are all possible. Bose symmetry and the restriction to zero total angular momentum together imply that only the symmetric  $SU(3)$  combinations appear. Because some  $VV$  final states, such as  $\phi K^*$ , lie near the  $D$  threshold, the vector meson widths are very important. We model the resonance line shapes with Lorentz invariant Breit-Wigner distributions,  $m^2 \Gamma_R^2 / [(m^2 - m_R^2)^2 + m^2 \Gamma_R^2]$ , where  $m_R$  and  $\Gamma_R$  are the

Table 3: Values of  $y_{F,R}$  for some three- and four-body final states.

Final state representation	$y_{F,R}/s_1^2$	$y_{F,R}$ (%)
$(3P)_{s\text{-wave}}$	8	-0.48
	27	-0.11
$(3P)_{p\text{-wave}}$	8	-1.13
	27	-0.07
$(3P)_{\text{form-factor}}$	8	-0.44
	27	-0.13
$4P$	8	3.3
	27	2.2
	27'	1.9

mass and width of the vector meson, and  $m^2$  is the square of its four-momentum in the decay. The results for  $s$ -,  $p$ -, and  $d$ -wave decays are shown in Table 2. With these heavier final states and with the higher partial waves, effects at the level of a percent are quite generic. If the vector meson widths were neglected, the results in the  $p$ - and  $d$ -wave channels would be larger by approximately a factor of three. The finite widths soften the  $SU(3)$  breaking which would otherwise be induced by a sharp phase space boundary. Again,  $4P$  and  $PPV$  final states can resonate through  $VV$ , so they are partially included here.

As we go to final states with more particles, the combinatoric possibilities begin to proliferate. We will only consider the final states  $3P$  and  $4P$ , and require that the pseudoscalars are in a totally symmetric 8 or 27 representation of  $SU(3)$ . This assumption is convenient, because the phase space integration is much simpler if it can be performed symmetrically. We have no reason to believe that this choice selects final state multiplets for which phase space effects are particularly enhanced or suppressed. The results for  $y_{3P}$  and  $y_{4P}$  are shown in Table 3.

In contrast to the two-body decays, for three-body final states the momentum dependence of the matrix elements is no longer fixed by angular momentum conservation. The simplest assumption is to take a momentum independent matrix element, with all three final state particles in an  $s$ -wave. We have also considered other matrix elements; for example, if one of the mesons has angular momentum  $\ell = 1$  in the  $D^0$  rest frame (balanced by the combination of the other two mesons). One could also imagine introducing a mild “form factor suppression,” with a weight such as  $\prod_{i \neq j} (1 - m_{ij}^2/Q^2)^{-1}$ , where  $m_{ij}^2 = (p_i + p_j)^2$ , and  $Q = 2 \text{ GeV}$  is a typical resonance mass. The resulting  $y_{F,R}$  values are shown in Table 3.

Finally, we studied  $4P$  final states with the mesons in fully symmetric 8 or 27. The results for momentum independent matrix elements are summarized in Table 3. (Note that the last two entries in the last column were mixed up in our paper.<sup>1</sup>) There are actually two symmetric 27's; we call 27 and 27' the representations of the form  $R_{kl}^{ij} = [M_m^i M_k^m M_n^j M_l^n + \text{symmetric} - \text{traces}]$  and  $R_{kl}^{ij} = [M_m^i M_n^m M_k^j M_l^n + \text{symmetric} - \text{traces}]$ , respectively. Here the partial contributions to  $y$  are very large, of the order of 10%. This is not surprising, since  $4P$  final states containing more than one strange particle are close to  $D$  threshold, and the ones with no pions are kinematically inaccessible. So there is no reason for  $SU(3)$  cancellations to persist effectively.

Formally, one can construct  $y$  from the individual  $y_{F,R}$  by weighting them by their  $D^0$  branching ratios,

$$y = \frac{1}{\Gamma} \sum_{F,R} y_{F,R} \left[ \sum_{n \in F_R} \Gamma(D^0 \rightarrow n) \right]. \quad (17)$$

However, the data on  $D$  decays are neither abundant nor precise enough to disentangle the decays to the various  $SU(3)$  multiplets, especially for the three- and four-body final states. Nor have we computed  $y_{F,R}$  for all or even most of the available representations. Thus, we

Table 4: Total  $D^0$  branching fractions to classes of final states, rounded to the nearest 5%.<sup>21</sup>

Final state	$PP$	$PV$	$(VV)_{s\text{-wave}}$	$(VV)_{d\text{-wave}}$	$3P$	$4P$
Branching fraction	5%	10%	5%	5%	5%	10%

can only estimate  $y$  by assuming that the representations for which we know  $y_{F,R}$  are typical for final states with a given multiplicity, and scale to the branching ratio of those final states. Table 4 summarizes the  $D^0$  branching ratios to two-, three- and four-body final states, rounded to the nearest 5%, to emphasize the uncertainties in the data.<sup>21</sup> Almost half of the  $D^0$  width is accounted for. Based on data in the  $\bar{K}^{0*}\rho^0$  channel, the  $VV$  final state is dominantly  $CP$  even, consistent with an equal distribution of  $s$ - and  $d$ -wave decays. Taking the product of the typical  $y_{F,R}$  with the approximate branching ratios in Table 4, we can estimate the contribution to  $y$  from a given type of final state. While in most cases the contributions are small, of the order of  $10^{-3}$  or less,  $D^0$  decays to nonresonant  $4P$  states naturally contribute to  $y$  at the percent level. The reason for so large  $SU(3)$  violating effects in  $y$  is that a sizable fraction of  $D^0$  decays are to final states for which the complete  $SU(3)$  multiplets are not kinematically accessible.

We have not considered all possible final states which may give large contributions to  $y$ . For example,  $\mathcal{B}(D^0 \rightarrow K^- a_1^+) = (7.3 \pm 1.1)\%$ ,<sup>21</sup> although the available phase space is very small. Unfortunately, the identities of the  $SU(3)$  partners of the  $a_1(1260)$ , which has  $J^{PC} = 1^{++}$ , are not well established. While it is natural to identify the  $K_1(1400)$  and  $f_1(1285)$  as the analogues of the  $K^*$  and  $\omega$ , respectively, there is no natural candidate for the  $\bar{s}s$  analogue of the  $\phi$ . The value of  $y_{PV^*}$  is also sensitive to the poorly known width of the  $a_1$ . If we take the  $\bar{s}s$  state to be the  $f_1(1420)$ , and  $\Gamma(a_1) = 400$  MeV, we find  $y_{PV^*,g_s} = 1.8\%$ . If instead we take the  $f_1(1510)$ , we find  $y_{PV^*,g_s} = 1.7\%$ . With  $\Gamma(a_1) = 250$  MeV, these numbers become 2.5% and 2.4%, respectively. Although percent level contributions to  $y$  are clearly possible from this channel, the data are still too poor to draw firm conclusions.

From our analysis, in particular as applied to the  $4P$  final state, we conclude that  $y$  of the order of a percent appears completely natural. An order of magnitude smaller result would require significant cancellations which would only be expected if they were enforced by the OPE. The hypothesis underlying the present analysis is that this is not the case.

## 5 Conclusions

The motivation most often cited in searches for  $D^0 - \bar{D}^0$  mixing is the possibility of observing a signal from new physics which may dominate over the Standard Model contribution. But to look for new physics in this way, one must be confident that the Standard Model prediction does not already saturate the experimental bound. Previous analyses based on short distance expansions have consistently found  $x, y \sim 10^{-3}$ , while naive estimates based on known  $SU(3)$  breaking in charm decays allow an effect an order of magnitude larger. Since current experimental sensitivity is at the level of a few percent, the difference is quite important.

We proved that if  $SU(3)$  violation can be treated perturbatively, then  $D^0 - \bar{D}^0$  mixing in the Standard Model is generated only at second order in  $SU(3)$  breaking effects. Within the exclusive approach, we identified an  $SU(3)$  breaking effect,  $SU(3)$  violation in final state phase space, whose contribution to  $y$  can be calculated with small model dependence. We found that phase space effects in  $D$  decays to final states near threshold can induce  $y \sim 10^{-2}$ .

The implication of our results for the Standard Model prediction for  $x$  is less apparent. While analyses based on the “inclusive” approach generally yield  $x \gtrsim y$ , it is not clear what the “exclusive” approach predicts. If  $x > y$  is found experimentally, it may still be an indication of a new physics contribution to  $x$ , even if  $y$  is also large. On the other hand, if  $y > x$  then



it will be hard to find signals of new physics, even if such contributions dominate  $\Delta M$ . The linear sensitivity to new physics in the analysis of the time dependence of  $D^0 \rightarrow K^+\pi^-$  is from  $x' = x \cos \delta + y \sin \delta$  and  $y' = y \cos \delta - x \sin \delta$  instead of  $x$  and  $y$ . If  $y > x$ , then  $\delta$  would have to be known precisely for these terms to be sensitive to new physics in  $x$ .

There remain large uncertainties in the Standard Model predictions of  $x$  and  $y$ , and values near the current experimental bounds cannot be ruled out. Therefore, it will be difficult to find a clear indication of physics beyond the Standard Model in  $D^0 - \bar{D}^0$  mixing measurements. We believe that at this stage the only robust potential signal of new physics in  $D^0 - \bar{D}^0$  mixing is  $CP$  violation, for which the Standard Model prediction is very small. Unfortunately, if  $y > x$ , then the observable  $CP$  violation in  $D^0 - \bar{D}^0$  mixing is necessarily small, even if new physics dominates  $x$ . Thus, to disentangle new physics from Standard Model contributions, it will be crucial to (i) improve the measurements of both  $x$  and  $y$ ; (ii) extract the relevant strong phase in the time dependence of doubly Cabibbo suppressed decays; and (iii) look for  $CP$  violation, which remains a potentially robust signal of new physics.

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