Various forms of BRST transformation and superspace formulation in gauge theories

THESIS SUBMITTED FOR THE DEGREE OF

Doctor of Philosophy

in

Physics

By

Manoj Kumar Dwivedi

Under the Supervision of

Head, Department of Physics

NUCLEAR AND PARTICLE PHYSICS SECTION
DEPARTMENT OF PHYSICS
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VARANASI - 221005
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ABSTRACT OF THE

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Three out of four fundamental interactions of nature (i.e., strong, weak, and electromagnetic) are very well explained by suitable gauge theories. These strong, weak and electromagnetic interactions are described by $SU(3) \times SU(2) \times U(1)$ gauge group on which the standard model of particle physics is based. Gauge theories are very well studied at the classical level but there is a problem in quantization. BRST formulation is one of the most intuitive theoretical approaches to quantize gauge theories of any kind. This formulation also helps in the proof of unitary and it is very handy in the context of renormalization.

The consequences of BRST invariance (formulated as Slavnov-Taylor identities) are central to the discussion of renormalizability, unitarity and the gauge independence of the non-Abelian gauge theory. The attempts that offer a reformulation and simplification of the understanding of BRST formulation have significance in the realm of particle physics.

The most important and useful generalization, known as the finite field-dependent BRST (FFBRST) transformation, is the one where the transformation parameter is finite and field dependent and it does not depend on the space-time explicitly. The speciality of such transformation is that these are also symmetry of the FP effective action but the path integral measure (in the generating functional) is not invariant under these and leads to non-trivial Jacobian. The non-trivial Jacobian of functional measure, under such transformation, gives rise to various interesting applications in gauge field theories. For example, these transformations connect theories in two different gauges by choosing a particular finite field dependent parameter

In this thesis, we would like to mention the various forms of BRST symmetry transformation and find new applications of BRST transformation in gauge field theories. We would also analyze the BRST symmetry generalizations and its applications to different gauge field theoretic models which are of interest in theoretical physics.

Detail contents of different chapters are as follows:

In the **first chapter**, we discuss the importance of gauge theories, BRST symmetry transformations and its generalizations with applications, basic techniques of field/anti-field formulation (BV formulation) for quantization of gauge theories, quantum gauge transfor-

mations and its importance with applications and the superspace description of topological field theories (particularly, CS theory and BF model)

The different mathematical techniques or Methodology of FFBRST transformation and BV (field/anti-field) formulation in brief have been discussed in the **second chapter**.

In **chapter three**, we show the application of FFBRST transformation to 3-form gauge theory. A 3-form gauge field theory is subject of interest as a 3-form field arises naturally in M-theory. The 3-form field automatically appears when multiple M2-branes are used to study M5-branes. We establish a connection between the noncovariant gauge and the covariant gauge in 3-form gauge theory using FFBRST transformation. In order to do so, we construct an infinitesimal but field dependent parameter and calculate the infinitesimal change in the Jacobian. This Jacobian adds some extra pieces to the effective action of the theory in covariant gauge which, in turn, reduces to the effective action in the noncovariant gauge.

In **chapter four**, we consider a super-renormalizable theory of massless QED in $(2+1)$ dimensions and discuss its BRST symmetry. We derive the Nielsen identities by extending the BRST transformation. Furthermore, we show that the Lowenstein-Zimmermann mass containing Lowenstein-Zimmermann parameter, which is important in the BPHZL renormalization (along with the external sources coupled to the nonlinear BRST variations) can appear naturally in the theory using the FFBRST transformations.

Chapter **five** is reserved for the study of quantum gauge transformations of Kalb and Ramond theory (reducible gauge theory). We observe that under the gauge transformations, the action of this reducible gauge theory remains form invariant but the gauge parameter is shifted. Furthermore, we study the BRST symmetric gaugeon formalism by introducing ghost fields and ghost of ghosts fields. After calculating BRST charge, we apply it on physical state which removes the both the unphysical modes i.e. gauge as well as gaugeon mode. This means that both the Gupta-Bleuler and the Kugo-Ojima type subsidiary conditions are converted into single Kugo-Ojima type condition. At the end of this **chapter**, we study the FFBRST symmetric transformations which introduce the gaugeon mode in reducible gauge theory through the Jacobian.

Four dimensional topological BF model in the Landau gauge and its BRST transformations are considered in **chapter six**. Further, we generalize the BSRT transformations by making the transformation parameter finite and field dependent. The infinitesimal change in the Jacobian corresponding to this field dependent parameter attributes a precious term in the generating functional which is useful to connect the Lorentz gauge action of BF model with the axial gauge action.

In **chapter seven**, we discuss the superspace description of CS theory in BV formulation. We begin by extending all the fields of the theory by introducing the shift fields. The main motivation behind this is that the anti-fields corresponding to each fields get identified. The extended action remains invariant under BRST symmetry together with the shift symmetry. Both these symmetries together are known as the extended BRST symmetry. This extended BRST invariant CS theory can be described in superspace using one extra Grassmannian coordinate, while using two extra Grassmannian coordinates one can describe both the extended BRST and extended anti-BRST invariant CS theory in superspace.

In **chapter eight**, we try to generalize the superspace formulation of BV action for BF model. Particularly, we first consider BRST invariant BF model in the Landau gauge and extend the BRST symmetry of the theory by including shift symmetry. By doing so, we find that the anti-ghosts of shift symmetry get identified as anti-fields of the standard BV formulation naturally. Further, we discuss a superspace formulation of extended BRST invariant BF model. Here we see that one additional Grassmannian coordinate is required if action admits only extended BRST symmetry. However, for both extended BRST and extended anti-BRST invariant BF model, two additional Grassmannian coordinates are required for their analysis.

In **chapter nine** the summary, results, conclusions and future perspectives have been discussed.

LIST OF PUBLICATIONS

1. *The noncovariant gauges in 3-form theories.*

   S. Upadhyay **M. K. Dwivedi** and B. P. Mandal, International Journal of Modern Physics A 10, 1350033 (2013).

2. *A superspace description of Chern-Simons theory in Batalin-Vilkovisky formulation.*
   S. Upadhyay **M. K. Dwivedi** and B. P. Mandal, **International Journal of Theoretical Physics** 54, 2076-2086 (2015).

3. *Emergence of Lowenstein Zimmermann mass terms for QED_3.*
   S. Upadhyay **M. K. Dwivedi** and B. P. Mandal, **International Journal of Modern Physics A** 30, 1550178 (2015).

4. *Quantum gauge symmetry for reducible gauge theory.*
   M. K. Dwivedi, **International Journal of Theoretical Physics** 56, 11 (2017).

5. *The quantum description of BF model in superspace.*
   M. K. Dwivedi, Advances in High Energy Physics 2018, 9291213, 11 (2018).

6. *Study of BF model in different gauges.*
   M. K. Dwivedi (Communicated)

7. *FFBRST in electroweak theory.*
   M. K. Dwivedi, (Work is in progress).

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SUBJECT APPROVED

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(Head of the Department)

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UNDERTAKING

I hereby declare that I have completed the research work for the full time period prescribed under the clause VIII.1 of the Ph. D. ordinances of the Banaras Hindu University, Varanasi and that the research work embodied in this thesis entitled **"Various forms of BRST transformation and superspcae formulation in gague theories"** is my own research work.

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Place: Varanasi

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CANDIDATE'S DECLARATION

I, **Manoj Kumar Dwivedi**, certify that the work embodied in this Ph. D. thesis is my own bonafide work carried out by me under the supervision of Head, Dept. of Physics for a period of ______________ to ______________ at Banaras Hindu University. The matter embodied in this Ph. D. thesis has not been submitted for the award of any other degree/diploma.

I declare that I have faithfully acknowledged, given credit to and referred to the research workers wherever their works have been cited in the text and the body of the thesis. I further certify that I have not willfully lifted up some other's work, para, text, data, results, etc. reported in the journals, books, magazines, reports, dissertations, theses, etc., or available at web-sites and included them in this Ph. D. thesis and cited as my own work.

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This is to certify that the above statements made by the candidate are correct to the best of my knowledge.

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(Head of the Department)

(Head and Supervisor)

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This is to certify that Mr. **Manoj Kumar Dwivedi**, a bonafide research scholar of this department, has successfully completed the pre-submission seminar requirement, which is a part of his Ph. D. programme, on his thesis entitled **"Various forms of BRST transformation and superspcae formulation in gague theories"**.

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Place: Varanasi

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Place: Varanasi

Prof. R. P. Malik
(Head of the Department)

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(Manoj Kumar Dwivedi)

Contents

Chapter 1

Introduction

There are four types of fundamental interactions (i.e. Gravitational, Strong, Weak, and Electromagnetic) that exist in nature. Out of these four fundamental interactions, only three fundamental interactions (i.e. Strong, Weak, and Electromagnetic) are very well explained by suitable gauge theories. These three fundamental interactions are well described by the $SU(3) \times SU(2) \times U(1)$ gauge group on which the standard model of particle physics is based. The basic features of these gauge theories are their invariant nature under some local transformations of gauge fields known as gauge transformations. The gauge transformations play a crucial role in determining the structure of these interactions. The associated group and algebra of gauge theories are called the Lie group and Lie algebra, respectively. The Lie algebra plays an important role in defining the representation of the gauge fields. If the generators of Lie group commute with one-another, the associated gauge theories are known as the Abelian gauge theories. However, if the generators do not commute among themselves then the associated gauge theories are known as non-Abelian gauge theories. For example the U(1) gauge theory is an Abelian gauge theory while the SU(N) gauge theory is a non-Abelian theory.

Gauge theories are very well studied at the classical level (but *not* at the quantum level). For instance, when we study the gauge theories at quantum level, there are redundancies in the gauge degree of freedom due to ill defined path integral. Also, Green's functions (an important ingredient in perturbative analysis of gauge theories) are not well-defined at quantum level. Therefore, it is required to remove the redundant degree of freedom from generating functional (path integral). In order to do so, we need some additional gauge variant piece (commonly known as gauge-fixing term) in the classical Lagrangian density such that the generating functional (and hence the Green function) becomes well-defined. The resultant Lagrangian density consists of gauge fields and arbitrary gauge parameter. In this way, we remove the problem of over counting in generating functional. But the cost we pay is that our physical theory turns into unphysical one which depends upon an arbitrary function of gauge field. In order to revert it back to the physical one, the Faddeev-Popov (FP) ghost and anti-ghost fields are introduced which do not participate in the physical Hilbert space of the quantum gauge theory. The nomenclatures (anti-)ghost fields are justified due to the violation of the spin and statistics relation as they are scalars satisfying the anti-commutation property [1]. These ghost and anti-ghost fields counterbalance the effect of arbitrary function present in the Lagrangian density and help in establishing the unitarity of the theory. However, after introduction of these unphysical fields (ghost and anti-ghost) in gauge-fixed Lagrangian density, the local gauge symmetry is broken which causes difficulties in renormalization. The resulting effective Lagrangian density is not invariant under a (local) gauge symmetry but is invariant under a new type of symmetry transformation invented by C. Becchi, A. Rouet, R. Stora and I. V. Tyutin independently [2, 3] which is known as BRST transformation. The so-called BRST transformation is characterized by the fact that it is (i) infinitesimal, (ii) global (i.e. does not depend on the space-time), and (iii) anti-commuting in nature (i.e. nilpotent of order 2) and, hence, fermionic in nature.

The FP effective action, consisting of gauge-fixing and ghost parts, remains invariant under BRST transformation. The key property of this BRST transformation is the nilpotency (of order 2). If the nilpotency of transformation is achieved by using equations of motion of any fields then this is known as the on-shell nilpotency. However, the off-shell nilpotency does not require any use of equations of motion. Remarkably, one can switch from the on-shell nilpotency to the off-shell nilpotency by introducing a Nakanishi-Lautrup type of auxiliary fields in the theory.

The BRST symmetry is prodigiously important in quantizing different gauge field theoretic models and greatly promotes the renormalization program [2-5]. However, the standard BRST quantization of gauge theories has found certain limitations. To improve this situation, I. A. Batalin and G. A. Vilkovisky gave a potent technique to quantize the wider class of gauge theories (including open or reducible gauge theories) known as BV formulation [4-8]. According to the BV formulation, one introduces anti-fields $\phi^\star$ (with opposite statistics) corresponding to each fields ϕ with ghost number $-gh(\phi)-1$, where

$gh(\phi)$ refers to the ghost number of the fields ϕ. This formulation is also known as the field/anti-field formulation and BV-BRST formulation. Due to anti-fields, the effective action gets extended and this extended action satisfies certain affluent mathematical formula known as quantum master equation which reflects the gauge symmetry in the zeroth-order of anti-fields and, in the first-order of anti-fields, it reflects the nilpotency of BRST transformation. The importance of this extended quantum action can be seen as the frame of gauge theories which are always endowed with first-class constraints in the language of Dirac's constraints analysis [9-12]. BV formulation is also useful in quantizing theories with the second-class constraints by converting them into the first-class in an extended phase space [2, 9, 11-13]. This was done *first* by I. A. Batalin, E. S. Fradkin and I. V. Tyutin [14, 15] which was further applied to various models [16-20]. Furthermore, a more general technique was introduced by I. A. Batalin, E. S. Fradkin and G. A. Vilkovisky which was completely different from earlier Dirac's method and known as the BFV quantization [7, 21, 22]. The main features of the BFV approach are the following: (i) it does not require closure off-shell of the gauge algebra and therefore does not need an auxiliary fields, (ii) this formalism relies on BRST transformation which is independent of gauge-fixing condition, and (iii) it is also applicable to the first-order Lagrangian density of the quantum gauge models.

To understand the BRST formalism geometrically, the well known superfield and superspace approaches provide a better grasp of the mathematical properties associated with the nilpotent BRST and anti-BRST symmetries and their corresponding generators (i.e. nilpotent conserved charges) [23-30]. In these formulations, BRST and anti-BRST symmetry transformations of gauge theories are realized on equal footing. Superspace is the extension of usual Minkowski space with extra scalar anticommuting coordinates. BRST and anti-BRST symmetry transformations are simply realized as translations along these anticommuting directions. Hence, the Ward-Takahasi (WT) identities are expressed in a very simple form (in the superspace formulation) leading to many applications in renormalization problems. A superspace description of the BV formalism has been studied in the context of many gauge theories [31-39]. For instance, these are studied in non-Abelian gauge theory [34], higher derivative theory [37], higher form gauge theory [38], topological Chern-Simons theory [39], perturbative gravity [40], FRW-model [41-43], ABJM theory [44] and in M-theory [45].

The BRST transformation has been generalized in many different ways in the past. For example, M. Lavelle and D. Mcmullan have found a generalized BRST symmetry for quantum electrodynamics (QED) which is non-local and non-covariant [46]. Their motivation was to refine the characterization of physical states given by the BRST charge. Further generalization of BRST transformation was done by Z. Tang and D. Finkelstein in which they found a non-local but covariant BRST symmetry [47]. The drawback of such a generalized BRST symmetry was that this was not nilpotent generally and required additional conditions (in auxiliary field formulation) to make them nilpotent. H. S. Yang and B. H. Lee had presented a local and non-covariant generalized BRST symmetry in the case of Abelian gauge theories [48]. The most important and useful generalization was developed by S. D. Joglekar and B. P. Mandal in 1995 [49], known as the finite field-dependent BRST (FFBRST) transformation, where the transformation parameter is finite and field dependent and it does not depend on the space-time explicitly. The speciality of such transformations is that these are also symmetry of the FP effective action but the path integral measure (in the generating functional) is not invariant under these and leads to the non-trivial Jacobian. The non-trivial Jacobian of functional measure under such transformation gives rise to various interesting applications to gauge field theories [50-59]. For instance, FFBRST transformations connect theories in two different gauges by choosing a particular finite field-dependent parameter [50-59]. These were useful to connect the FP effective action from Lorentz gauge to the (i) axial-gauge (ii) Coulomb gauge (iii) quadratic gauge. This transformation also connects two different gauge parameter of the same theory [56]. In fact, this approach connects the extended actions corresponding to different solutions of the quantum master equation (in field/anti-field formulation) [58-62]. For connecting two different gauges of the theory, the choice of finite field-dependent parameter is very important. Various applications of FFBRST transformation in the context of different kinds of gauge theories and gauge models are reported in Refs. [50-77].

Till now, we have discussed about the gauge theory from classical gauge symmetry perspective. In the standard quantization formalism of gauge theories, one does not consider the quantum gauge freedom and the quantum action is defined only after the fixing the gauge. In fact, the gauge-fixing term breaks the local (classical) gauge invariance. In order to improve this situation, Yokoyama introduced a wider framework to quantize the gauge theories, known as the gaugeon formalism, by developing the quantum gauge transformation [78-83]. The basic idea of gaugeon formalism is to introduce the extra (quantum) fields, so-called gaugeon fields, in the Lagrangian density describing the quantum gauge freedom. This formulation was originally proposed to settle the problem of renormalization of gauge parameter of quantum electrodynamics [84]. In this connection, the occurrence of the shift in gauge parameter, during the course of renormalization [84], was addressed naturally by connecting theories in two different gauges within the same family by a q-number gauge transformation [78]. Further, this formalism is applied to the case of Yang-Mills theory. The gaugeon modes contain negative-norm states that give rise to negative probability and one needs to remove this unphysical gaugeon modes. Yokoyama implemented a Gupta-Bleuler type subsidiary condition to remove this unnecessary modes. Due to certain limitations, this subsidiary condition can not be applied to the case where the interaction terms are present between the gaugeon fields. This limitation was improved substantially with the help of BRST charge which replaces the Yokoyama subsidiary condition to a more acceptable Kugo-Ojima type restriction [85-88]. The gaugeon formalism has been studied, so far, in various interesting contexts [68, 69, 76, 89-97]. Due to incredible importance of the gaugeon formalism, it was necessary to study this formalism in the case of Abelian rank-2 antisymmetric tensor gauge fields. There are various reasons to provide the full quantum description of the Abelian gauge rank-2 antisymmetric tensor fields.

Kalb and Ramond were first to discuss the idea of interaction of classical strings with Abelian rank-2 antisymmetric tensor fields [98]. Applications of this interaction can be seen in the Lorentz covariant description of the vortex motion in an irrotational incompressible fluid [99] and to the dual formulation of the Abelian Higgs model [100]. Importance of Abelian rank-2 antisymmetric tensor fields can also be found in the supergravity multiplets [101], excited states of quantized superstring theories and anomaly cancellation of certain superstring theory [102]. The Abelian rank-2 antisymmetric tensor field also generates the effective mass for an Abelian vector gauge field through a topological coupling between two fields [103]. Abelian rank-2 antisymmetric tensor field has also been studied for $U(1)$ gauge theory in loop space [104]. Covariant quantization of this field was first attempted by Townsend [105] and has been studied by many authors there after [106, 107]. This theory is also discussed in a superspace formulation where the Ward-Takahashi identities are derived [108].

On the other hand, the topological gauge field theories (TGFT) have some peculiar properties. Two distinct classes of TGFT are: topological Yang-Mills theory and Chern-Simons (CS) theory which are classified as the Witten-type and the Schwarz-type, respectively [109]. Besides these two-types, there are another Schwarz-type TGFT known as topological BF theory [110] which is an extension of Chern-Simons theory. The main difference between CS theory and BF model is that the action of CS theory exists only in odd-dimensions of space-time while BF model can be defined on the spacetime manifolds of any dimension. In string theory and non-linear sigma model, four dimensional BF model were introduced [111]. This model is interesting due to its topological nature [109] and their connection with lower dimensional quantum gravity. For example, three space-time dimensional Einstein-Hilbert action, with or without using cosmological constant term, can be naturally formulated in terms of the BF-models [109, 112, 113]. The coupling of an antisymmetric tensor field with the field strength tensor of Yang-Mills theory is described by the models in Refs. [114, 115]. Topological BF theory, in the Landau gauge, has a common feature with large class of topological models [116, 117]. However, the importance of the BF model in axial-gauge can never be suppressed as this trivializes the ghost sector and remains comfortable also in handling them in the Landau gauge [118]. Considering BF model in axial gauge is rather beneficial in the higher dimensional generalization of the model.

In this thesis, we would like to mention various forms of BRST symmetry transformations and would find new applications of BRST transformations in gauge field theories. We would also analyze the BRST symmetry generalizations and its applications to different gauge field theoretic models. FFBRST transformations have already been found many applications in 1-form gauge theories. However, little progress has been made so far in the context of 2-from gauge theories. But no one had shown any progress of the application of FFBRST in 3-form gauge theory previously, hence we have taken this opportunity to study the same. We show that the 3-form effective action, in the non-covariant gauge, can be obtained from that of the covariant gauge using FFBRST transformation. The 3-form gauge field theory is a subject of interest as the 3-form field arises naturally in M-theory. The theory of multiple M2-branes has been used to study M5-branes and a 3-form field naturally appears in the theory.

BRST symmetry shows a very rich structure in superspace formulation when certain shift symmetry is incorporated. It is always interesting and useful to write most general extended BRST and extended anti-BRST symmetric action in a compact way in superspace formulation. These extended formulations can be extended to BV formulation where natural identification for the antifields is possible through equation of motion. We generalize this idea to the BV actions of the CS theory and BF model in superspace. The reason for considering the CS theory is because of its topological nature which offers relevance in various areas of research, namely, supergravity theories, superstring theories and condensed matter theories, etc. On the other hand, the topological BF model describes the interaction between 1-form and 2-form gauge theory.

Further, we would like to show that we can generate the Lowenstein-Zimmermann mass terms for QED_3 using FFBRST transformation. Massless QED in $(2 + 1)$ dimensions (QED_3) has very interesting and crucial features. It is an ultraviolet finite, super-renormalizable and parity invariant theory leading to the subjects of interest in frontier areas of research. The massless QED_3 provides an ideal platform to tackle the infrared divergence present in the theory. The parity anomaly has been removed for such theories. The Lowenstein-Zimmerman scheme plays an important role in the algebraic proof of ultraviolet and infrared finiteness and in the removal of the parity anomaly. The QED_3 also plays a crucial role in the study of high temperature superconductivity. Moreover, the dynamical mass generation, using Hamiltonian lattice methods, is also studied which has been found in agreement with both the strong coupling expansion and with the Euclidean lattice simulations. The Lowenstein-Zimmermann subtraction scheme is adopted in the algebraic proof on the ultraviolet and infrared finiteness. It has also been used to show the absence of the parity and infrared anomalies, in the context of massless QED_3, which are based on the general theorems of perturbative quantum field theory.

We try to develop gaugeon formalism for the reducible gauge theories. Within the gaugeon formalism, the renormalized value of gauge parameter appears naturally. We also would like to investigate the quantum gauge symmetry for the case of reducible gauge theory. We also study the BRST symmetry which enables us to convert the Gupta-Bleuler type subsidiary condition to Kugo-Ojima type subsidiary condition.

This thesis is divided into following **nine Chapters**. The full contents of these

Chapters are given as follows:

In the **first Chapter**, we discuss the importance of gauge theories, BRST symmetry transformations and its generalizations with applications, basic techniques of field/anti-field formulation (BV formulation) for quantization of gauge theories, quantum gauge transformations and its importance with applications and the superspace description of topological field theories (particularly, the CS theory and BF model).

The different mathematical techniques or methodology of FFBRST transformation and BV (field/anti-field) formulation in brief have been discussed in the **second Chapter** of our present thesis.

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In **Chapter four**, we consider a super-renormalizable theory of the massless QED in $(2+1)$ dimensions and discuss its BRST symmetry. We derive the Nielsen identities by extending the BRST transformation. Furthermore, we show that the Lowenstein-Zimmermann mass, containing the Lowenstein-Zimmermann parameter (which is important in the BPHZL renormalization along with the external sources coupled to the nonlinear BRST variations) can appear naturally in the theory using the FFBRST transformations.

Chapter five is reserved for the study of quantum gauge transformations of Kalb and Ramond theory (reducible gauge theory). We observe that, under the gauge transformations, the action of this reducible gauge theory remains form invariant but the gauge parameter is shifted. Furthermore, we study the BRST symmetric gaugeon formalism by introducing the ghost fields and ghost of ghost fields. After calculating the BRST charge, we apply it on physical state which removes both the unphysical modes i.e. gauge as well as gaugeon modes. This means that both the Gupta-Bleuler and Kugo-Ojima type subsidiary conditions are converted into a single Kugo-Ojima type condition. At the end of this **Chapter**, we study the FFBRST symmetric transformations which introduce the gaugeon mode in reducible gauge theory through the Jacobian.

Four dimensional topological BF model in the Landau gauge and its BRST transformations are considered in **Chapter six**. Further, we generalize the BSRT transformations by making the transformation parameter finite and field dependent. The infinitesimal change in the Jacobian corresponding to this field dependent parameter contributes a precious term in the generating functional which is useful to connect the Lorentz gauge action of BF model with the axial gauge action.

In **Chapter seven**, we discuss the superspace description of the CS theory in BV formulation. We begin by extending all the fields of the theory by introducing the shift fields. The main motivation behind this is that the anti-fields, corresponding to every fields, get identified. The extended action remains invariant under BRST symmetry together with the shift symmetry. Both these symmetries together are known as the extended BRST symmetry. This extended BRST invariant CS theory can be described in superspace using one extra Grassmannian coordinate, while using two extra Grassmannian coordinates, one can describe *both* the extended BRST and extended anti-BRST invariant CS theory in superspace.

In **Chapter eight**, we try to generalize the superspace formulation of BV action for BF model. Particularly, we first consider the BRST invariant BF model in the Landau gauge and extend the BRST symmetry of the theory by including a shift symmetry. By doing so, we find that the anti-ghosts of the shift symmetry get identified to the anti-fields of the standard BV formulation naturally. Further, we discuss a superspace formulation of the extended BRST invariant BF model. Here, we see that one additional Grassmannian coordinate is required if action admits only the extended BRST symmetry. However, for both the extended BRST and extended anti-BRST invariant BF model, two additional Grassmannian coordinates are required for their analysis.

In **Chapter nine** the summary, results, conclusions and future perspective of our thesis have been discussed.

Chapter 2

Preliminaries

The aim of this **chapter** is to provide the basic background and mathematical tools to prepare the necessary theoretical inputs that are relevant to this thesis. In particular, we briefly outline the basic ideas of the FFBRST transformation and BV formulation in gauge theories. In the first section, we discuss the FFBRST transformation. Furthermore, in second section, we outline the BV formulation.

2.1 Finite field dependent BRST (FFBRST) transformation

We start with the basic FFBRST formulation of pure gauge theories. The usual BRST transformation for a generic field ϕ of an effective action is defined, compactly, as

$$\delta_b\phi\ (x,\kappa) = s_b\phi\ \delta\Lambda, \tag{2.1}$$

where $s_b\phi$ is the Slanove variation of the field ϕ with infinitesimal, anti-commuting and global parameter $\delta\Lambda$. This transformation (2.1) is nilpotent, i.e. $s_b^2 = 0$ and leaves the FP effective action (S_{eff}) invariant. It was observed by Joglekar and Mandal in Ref. [27] that $\delta\Lambda$ needs neither to be infinitesimal nor to be field-independent to maintain the symmetry of the FP effective action as long as $\delta\Lambda$ does not depend explicitly on space-time. The infinitesimal field dependent transformation is defined as [27]

$$\frac{d}{d\kappa}\ \phi(x,\kappa) = s_b\phi(x,\kappa)\ \Theta_b'[\phi(x,\kappa)], \tag{2.2}$$

where Θ'_b is an infinitesimal field dependent parameter. By integrating equation (2.2) from $\kappa = 0$ to $\kappa = 1$, the FFBRST transformation is obtained as [27]

$$\phi' \equiv \phi(x, \kappa = 1) = \phi(x, \kappa = 0) + s_b[\phi(x)]\Theta[\phi(x)], \tag{2.3}$$

where $\Theta[\phi(x)]$ is finite field dependent parameter. This transformation is also symmetry of the effective action but the functional measure $\mathcal{D}\phi$, defined in generating functional

$$Z = \int [\mathcal{D}\phi]\ e^{iS_{eff}}, \tag{2.4}$$

is not invariant under such FFBRST transformation and leads to a non-trivial Jacobian. Under the FFBRST transformation, the functional measure $\mathcal{D}\phi$ changes as

$$\mathcal{D}\phi \to J[\phi(\kappa)]\mathcal{D}\phi(\kappa). \tag{2.5}$$

It has been shown in [27] that, under certain condition, this non-trivial Jacobian $J[\phi]$ can be replaced (within the functional integral) as

$$J[\phi(\kappa)] \to e^{iS_1[\phi(\kappa)]}, \tag{2.6}$$

where $S_1[\phi(\kappa)]$ is some local functional of field $\phi(x)$. The condition for specifying S_1 is

$$\int [\mathcal{D}\phi] \left[\frac{1}{J}\frac{dJ}{d\kappa} - i\frac{dS_1}{d\kappa}\right] e^{i[S_{eff}+S_1]} = 0. \tag{2.7}$$

Thus, under FFBRST, the generating functional, changes as

$$Z \left(= \int [\mathcal{D}\phi]\ e^{iS_{eff}} \right) \xrightarrow{FFBRST} Z' \left(= \int [\mathcal{D}\phi]\ e^{i[S_{eff}(\phi)+S_1(\phi)]} \right), \tag{2.8}$$

where $S_1[\phi]$ depends on the finite field dependent parameter. From equation (2.8), one can see that the generating functional corresponding to the two different effective actions can be related through FFBRST transformation with an appropriate choice of finite field dependent parameter. The FFBRST transformation has also been used to solve many of the long-standing problems in quantum field theory [28-32, 45, 35-38]. For example, the gauge field propagators in non-covariant gauges contain singularities on the real momentum axis [30] and proper prescriptions for these singularities in gauge field propagators have been found by using FFBRST transformation [33].

2.1.1 Evaluation of Jacobian

Due to the field dependent nature of FFBRST transformation, the Jacobian of functional measure is not unity and hence leads to non-trivial contribution in an effective action. In this subsection, we present the general method to evaluate the non-trivial Jacobian of path integral measure corresponding to FFBRST transformation. Here, we utilize the fact that FFBRST transformation can be written as a succession of infinitesimal transformation given in Eq. (2.2). Now, one can write the path integral measure as follows:

$$\mathcal{D}\phi = J(\kappa)\mathcal{D}\phi(\kappa) = J(\kappa + d\kappa)\mathcal{D}\phi(\kappa + d\kappa). \tag{2.9}$$

Since the transformation $\phi(\kappa)$ to $\phi(\kappa + d\kappa)$ is an infinitesimal one, the above equation leads to

$$\frac{J(\kappa)}{J(\kappa + d\kappa)} = \int d^4x \sum_\phi \pm \frac{\delta\phi(x, \kappa + d\kappa)}{\delta\phi(x, \kappa)}, \tag{2.10}$$

where Σ_ϕ sums over all fields in the measure. Here this plus (+) sign will be used for the bosonic field and the negative (−) sign for fermionic field. By using the Taylor expansion in the above equation, the infinitesimal change in Jacobian is obtained as follows:

$$\frac{1}{J(\kappa)}\frac{dJ(\kappa)}{d\kappa} = -\int d^4x \sum_\phi \left[\pm s_b\phi \frac{\delta\Theta_b'[\phi(x, \kappa)]}{\delta\phi(x, \kappa)}\right]. \tag{2.11}$$

In the next section, we will describe the BV formalism.

2.2 Batalin-Vilkovisky (BV) formalism

The BV formulation (also known as field/anti-field formulation) is a powerful technique in the Lagrangian framework to deal with more general gauge theories. This method is applicable to gauge theories with both reducible (open) as well as irreducible (close) algebras. The basic trick of BV formulation is to introduce anti-field ($\phi^\star$) for each field (ϕ) of a given gauge theory.

With the introduction of anti-fields ($\phi^\star$), the effective action (S_{eff}) gets extended as

$$S_{eff}[\phi, \phi^\star] = I[\phi] + (s_b\phi)\phi^\star, \tag{2.12}$$

where $I[\phi]$ is the gauge invariant action. The anti-fields ($\phi^\star$) can be obtained from the gauge-fixing fermion Ψ as

$$\phi^\star = \frac{\delta\Psi}{\delta\phi}. \tag{2.13}$$

The extended effective action can also be written in terms of Ψ as

$$S_{eff}[\phi] = I[\phi] + s_b\Psi. \tag{2.14}$$

The effective action (S_{eff}) satisfies certain mathematical relation which is known as the quantum master equation. The latter is defined as

$$(S_{eff}, S_{eff}) - 2i\Delta S_{eff} = 0, \tag{2.15}$$

where the anti-bracket of effective action, (S_{eff}, S_{eff}), is defined by

$$(S_{eff}, S_{eff}) = \frac{\delta_r S_{eff}}{\delta\phi}\frac{\delta_l S_{eff}}{\delta\phi^\star} - \frac{\delta_r S_{eff}}{\delta\phi^\star}\frac{\delta_l S_{eff}}{\delta\phi}, \tag{2.16}$$

and Δ is the Laplacian, defined with left and right differentials (δ_l and δ_r respectively), as

$$\Delta = \frac{\delta_r}{\delta\phi^\star}\frac{\delta_l}{\delta\phi}. \tag{2.17}$$

Usually, it is easy to construct an action which satisfies the classical master equation as

$$(S_{eff}, S_{eff}) = 0. \tag{2.18}$$

The generating functional, given in Eq. (2.4), can also be written in a compact form as given below

$$Z = \int [\mathcal{D}\phi] \exp [iW_\Psi(\phi, \phi^\star)], \tag{2.19}$$

where $W_\Psi(\phi, \phi^\star)$ is an extended action satisfying the following quantum master equation :

$$\Delta e^{iW_\Psi[\phi,\phi^\star]} = 0. \tag{2.20}$$

The quantum master equation, in the zeroth-order of the anti-fields, gives the condition of the gauge invariance. On the other hand, it reflects the nilpotency of BRST transformation in the first order of anti-fields. We will be using this formulation within the framework of FFBRST formulation (applied to various branches of gauge theories) in upcoming **Chapters**.

Chapter 3

The noncovariant gauges in 3-form theories

In this **Chapter**, we show that how can generating functional of the non-covariant gauge of Abelian 3-form gauge theory be obtained by the generating functional in the covariant gauge using FFBRST symmetric transformation. Further, we show the same connection with the help of BV formulation.

3.1 Abelian 3-form gauge theory

We consider the action of Abelian 3-form gauge theory in $(1+5)$ dimensions as follows,

$$S_0 = \frac{1}{24}\int d^6x \; H_{\mu\nu\eta\chi}H^{\mu\nu\eta\chi}, \tag{3.1}$$

where $H_{\mu\nu\eta\chi}$ is the field strength (curvature) tensor, described in terms of the totally antisymmetric tensor gauge field $B_{\mu\nu\eta}$, defined as

$$H_{\mu\nu\eta\chi} = \partial_\mu B_{\nu\eta\chi} - \partial_\nu B_{\eta\chi\mu} + \partial_\eta B_{\chi\mu\nu} - \partial_\xi B_{\mu\nu\eta}. \tag{3.2}$$

The given action (3.1) is invariant under the following gauge transformation

$$\delta B_{\mu\nu\eta} = \partial_\mu \lambda_{\nu\eta} + \partial_\nu \lambda_{\eta\mu} + \partial_\eta \lambda_{\mu\nu}, \tag{3.3}$$

where $\lambda_{\mu\nu}$ is an arbitrary antisymmetric transformation parameter. To quantize this theory with BRST technique, we extend the action (3.1) by introducing the following (covariant) gauge-fixing and ghost terms:

$$\begin{aligned}
S_{gf+gh} &= \int d^6x \left[\partial_\mu B^{\mu\nu\eta} B_{\nu\eta} + \frac{1}{2} B_{\mu\nu} \tilde{B}^{\mu\nu} \right. \\
&+ \left(\partial_\mu \tilde{C}_{\nu\eta} + \partial_\nu \tilde{C}_{\eta\mu} + \partial_\eta \tilde{C}_{\mu\nu} \right) \partial^\mu C^{\nu\eta} \\
&- \left(\partial_\mu \tilde{\beta}_\nu - \partial_\nu \tilde{\beta}_\mu \right) \partial^\mu \beta^\nu - B B_2 \\
&- \frac{1}{2} B_1^2 + \left(\partial_\mu \tilde{C}^{\mu\nu} \right) f_\nu - \left(\partial_\mu C^{\mu\nu} \right) \tilde{F}_\nu \\
&+ \partial_\mu \tilde{C}_2 \partial^\mu C_2 + \tilde{f}_\mu f^\mu - \tilde{F}_\mu F^\mu \\
&+ \left. \partial_\mu \beta^\mu B_2 + \partial_\mu \phi^\mu B_1 - \partial_\mu \tilde{\beta}^\mu B \right], \qquad (3.4)
\end{aligned}$$

where $C_{\mu\nu}$ and $\tilde{C}_{\mu\nu}$ are the ghost and anti-ghost fields, respectively, which are fermionic in nature. The vector field (ϕ_μ), antisymmetric auxiliary fields $(B_{\mu\nu}, \tilde{B}_{\mu\nu})$ and auxiliary fields (B, B_1, B_2) are bosonic in nature and the fields $(f_\mu, \tilde{f}_\mu, F_\mu, \tilde{F}_\mu)$ are auxiliary fermionic fields. The complete effective action for the Abelian 3-form gauge theory, is then written as

$$S_{eff} = S_0 + S_{gf+gh}. \qquad (3.5)$$

The effective action given in (3.5) is invariant under the following BRST symmetric transformations:

$$\begin{aligned}
\delta_b B_{\mu\nu\eta} &= -\left(\partial_\mu C_{\nu\eta} + \partial_\nu C_{\eta\mu} + \partial_\eta C_{\mu\nu} \right) \delta\Lambda, \\
\delta_b C_{\mu\nu} &= \left(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu \right) \delta\Lambda, \ \delta_b \tilde{C}_{\mu\nu} = B_{\mu\nu}\ \delta\Lambda, \\
\delta_b \tilde{B}_{\mu\nu} &= -\left(\partial_\mu f_\nu - \partial_\nu f_\mu \right) \delta\Lambda, \quad \delta_b \tilde{\beta}_\mu = -\tilde{F}_\mu\ \delta\Lambda, \\
\delta_b \beta_\mu &= -\partial_\mu C_2\ \delta\Lambda, \quad \delta_b F_\mu = -\partial_\mu B\ \delta\Lambda, \\
\delta_b \tilde{C}_2 &= B_2\ \delta\Lambda, \quad \delta_b \tilde{f}_\mu = \partial_\mu B_1\ \delta\Lambda, \\
\delta_b C_1 &= -B\ \delta\Lambda, \quad \delta_b \phi_\mu = -f_\mu\ \delta\Lambda, \\
\delta_b \tilde{C}_1 &= B_1\ \delta\Lambda, \quad \delta_b \mathcal{M} = 0, \\
\mathcal{M} &\equiv \{ C_2, f_\mu, \tilde{F}_\mu, B, B_1, B_2, B_{\mu\nu} \}, \qquad (3.6)
\end{aligned}$$

where the nature of transformation parameter $\delta\Lambda$ is infinitesimal, anticommuting and global (i.e. it does not depend on space and time). It is easy to check that the given gauge-fixing and ghost part of the effective action given in (3.4), is BRST exact and hence can be written in terms of BRST variation of the gauge-fixed fermion (Ψ_L) as

$$\begin{aligned} S_{gf+gh} &= s_b\Psi_L = s_b \int d^6x\ [-\partial_\mu \tilde{C}_{\nu\eta} B^{\mu\nu\eta} - \frac{1}{2}\tilde{C}_2 B \\ &+ \frac{1}{2} C_1 B_2 - \frac{1}{2}\tilde{C}_1 B_1 - C^{\mu\nu}\partial_\mu \tilde{\beta}_\nu - \partial_\mu \tilde{C}_2 \beta^\mu \\ &+ \frac{1}{2}\tilde{C}_{\mu\nu}\tilde{B}^{\mu\nu} - F^\mu \tilde{\beta}_\mu - \tilde{f}^\mu \phi_\mu], \end{aligned} \tag{3.7}$$

where the expression of Ψ_L is given as

$$\begin{aligned} \Psi_L &= \int d^6x\ \psi_L = \int d^6x[-\partial_\mu \tilde{C}_{\nu\eta} B^{\mu\nu\eta} - \frac{1}{2}\tilde{C}_2 B \\ &+ \frac{1}{2} C_1 B_2 - \frac{1}{2}\tilde{C}_1 B_1 - C^{\mu\nu}\partial_\mu \tilde{\beta}_\nu - \partial_\mu \tilde{C}_2 \beta^\mu \\ &+ \frac{1}{2}\tilde{C}_{\mu\nu}\tilde{B}^{\mu\nu} - F^\mu \tilde{\beta}_\mu - \tilde{f}^\mu \phi_\mu]. \end{aligned} \tag{3.8}$$

In the path integral formulation, the generating functional for the 3-form gauge theory in covariant gauge is defined as,

$$Z_{eff} = \int \mathcal{D}\phi\ e^{iS_{eff}}. \tag{3.9}$$

Here, the expression of S_{eff} is given by

$$\begin{aligned} S_{eff} &= \frac{1}{24}\int d^6x \Big[H_{\mu\nu\eta\chi}H^{\mu\nu\eta\chi} + \partial_\mu B^{\mu\nu\eta} B_{\nu\eta} + \frac{1}{2} B_{\mu\nu}\tilde{B}^{\mu\nu} \\ &+ (\partial_\mu \tilde{C}_{\nu\eta} + \partial_\nu \tilde{C}_{\eta\mu} + \partial_\eta \tilde{C}_{\mu\nu})\partial^\mu C^{\nu\eta} \\ &- (\partial_\mu \tilde{\beta}_\nu - \partial_\nu \tilde{\beta}_\mu)\partial^\mu \beta^\nu - BB_2 \\ &- \frac{1}{2}B_1^2 + (\partial_\mu \tilde{C}^{\mu\nu}) f_\nu - (\partial_\mu C^{\mu\nu})\tilde{F}_\nu \\ &+ \partial_\mu \tilde{C}_2 \partial^\mu C_2 + \tilde{f}_\mu f^\mu - \tilde{F}_\mu F^\mu \\ &+ \partial_\mu \beta^\mu B_2 + \partial_\mu \phi^\mu B_1 - \partial_\mu \tilde{\beta}^\mu B \Big]. \end{aligned} \tag{3.10}$$

where $\mathcal{D}\phi$ indicates the path integral measure which includes all the fields ϕ, generically. Next, following the technique discussed in **Chapter two**, we will generalize the BRST transformation given in (3.6).

3.2 Generalized BRST formulation of Abelian 3-form gauge theory

We first, generalize the BRST transformation by making the transformation parameter finite and field dependent as following:

$$\begin{aligned}
\delta_b B_{\mu\nu\eta} &= -(\partial_\mu C_{\nu\eta} + \partial_\nu C_{\eta\mu} + \partial_\eta C_{\mu\nu})\ \Theta[\phi],\\
\delta_b C_{\mu\nu} &= (\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)\ \Theta[\phi],\\
\delta_b \tilde{C}_{\mu\nu} &= B_{\mu\nu}\ \Theta[\phi],\\
\delta_b \tilde{\beta}_\mu &= -\tilde{F}_\mu\ \Theta[\phi],\\
\delta_b \tilde{B}_{\mu\nu} &= -(\partial_\mu f_\nu - \partial_\nu f_\mu)\ \Theta[\phi],\\
\delta_b \beta_\mu &= -\partial_\mu C_2\ \Theta[\phi],\\
\delta_b F_\mu &= -\partial_\mu B\ \Theta[\phi],\\
\delta_b \tilde{C}_2 &= B_2\ \Theta[\phi],\\
\delta_b \tilde{f}_\mu &= \partial_\mu B_1\ \Theta[\phi],\\
\delta_b C_1 &= -B\ \Theta[\phi],\\
\delta_b \phi_\mu &= -f_\mu\ \Theta[\phi],\\
\delta_b \tilde{C}_1 &= B_1\ \Theta[\phi],\\
\delta_b \varpi &= 0,\\
\varpi &\equiv [C_2, f_\mu, \tilde{F}_\mu, B, B_1, B_2, B_{\mu\nu}].
\end{aligned} \tag{3.11}$$

This BRST transformation with finite field-dependent parameter $\Theta[\phi]$ is also symmetry of the effective action (3.5). However, the path integral measure defined in Eq. (3.9), is not invariant under such transformation as the BRST parameter is finite and field dependent in nature. Now, we will compute the Jacobian of the path integral measure under the BRST transformation (3.11), which helps in obtaining the gauge-fixed action in non-covariant gauge.

3.3 3-form gauge theory in non-covariant gauge

In order to obtain the generating functional for 3-form gauge theory in non-covariant gauge, we choose the infinitesimal field-dependent parameter as follows

$$\begin{aligned}
\Theta' &= i\gamma\int d^6y[-\tilde{C}_{\nu\eta}\partial_\mu B^{\mu\nu\eta}+\tilde{C}_{\nu\eta}\eta_\mu B^{\mu\nu\eta}\\
&+ C_{\mu\nu}\partial^\mu\tilde{\beta}^\nu - C_{\mu\nu}\eta^\mu\tilde{\beta}^\nu-\tilde{C}_2\partial_\mu\beta^\mu\\
&+ \tilde{C}_2\eta_\mu\beta^\mu],
\end{aligned}\tag{3.12}$$

where γ is an arbitrary constant parameter. The infinitesimal change in the Jacobian of functional integral is calculated as

$$\begin{aligned}
\frac{1}{J}\frac{dJ}{d\kappa} &= -i\gamma\int d^6y\,[B_{\nu\eta}\partial_\mu B^{\mu\nu\eta}-B_{\nu\eta}\eta_\mu B^{\mu\nu\eta}\\
&+ \partial^\mu(\partial_\mu C_{\nu\eta}+\partial_\nu C_{\eta\mu}+\partial_\eta C_{\mu\nu})\tilde{C}^{\nu\eta}\\
&- \eta^\mu(\partial_\mu C_{\nu\eta}+\partial_\nu C_{\eta\mu}+\partial_\eta C_{\mu\nu})\tilde{C}^{\nu\eta}\\
&- (\partial_\mu\tilde{\beta}_\nu-\partial_\nu\tilde{\beta}_\mu)\partial^\mu\beta^\nu+C_{\mu\nu}\partial^\mu\tilde{F}^\nu\\
&+ (\eta_\mu\tilde{\beta}_\nu-\eta_\nu\tilde{\beta}_\mu)\partial^\mu\beta^\nu-C_{\mu\nu}\eta^\mu\tilde{F}^\nu\\
&+ \left.B_2\partial_\mu\beta^\mu-B_2\eta_\mu\beta^\mu-\tilde{C}_2\partial_\mu\partial^\mu C_2+\tilde{C}_2\eta_\mu\partial^\mu C_2\right].
\end{aligned}\tag{3.13}$$

One can replace the Jacobian J by e^{iS_1}, if S_1 satisfies condition (2.7). Now, we make the following ansatz for functional S_1:

$$\begin{aligned}
S_1 &= \int d^6x\;[\xi_1(\kappa)\;B_{\nu\eta}\partial_\mu B^{\mu\nu\eta}+\xi_2(\kappa)\;B_{\nu\eta}\eta_\mu B^{\mu\nu\eta}\\
&+ \xi_3(\kappa)\;\partial^\mu(\partial_\mu C_{\nu\eta}+\partial_\nu C_{\eta\mu}+\partial_\eta C_{\mu\nu})\;\tilde{C}^{\nu\eta}\\
&+ \xi_4(\kappa)\;\eta^\mu(\partial_\mu C_{\nu\eta}+\partial_\nu C_{\eta\mu}+\partial_\eta C_{\mu\nu})\;\tilde{C}^{\nu\eta}\\
&+ \xi_5(\kappa)\;(\partial_\mu\tilde{\beta}_\nu-\partial_\nu\tilde{\beta}_\mu)\;\partial^\mu\beta^\nu\\
&+ \xi_6(\kappa)\;(\eta_\mu\tilde{\beta}_\nu-\eta_\nu\tilde{\beta}_\mu)\;\partial^\mu\beta^\nu\\
&+ \xi_7(\kappa)\;C_{\mu\nu}\partial^\mu\tilde{F}^\nu+\xi_8(\kappa)\;C_{\mu\nu}\eta^\mu\tilde{F}^\nu\\
&+ \xi_9(\kappa)\;B_2\;\partial_\mu\beta^\mu+\xi_{10}(\kappa)\;B_2\;\eta_\mu\beta^\mu\\
&+ \xi_{11}(\kappa)\;\tilde{C}_2\;\partial_\mu\partial^\mu\;C_2+\xi_{12}(\kappa)\;\tilde{C}_2\;\eta_\mu\partial^\mu\;C_2],
\end{aligned}\tag{3.14}$$

where $\xi_1, \xi_2, ...\xi_{12}$ are arbitrary κ dependent constants. This functional S_1 together with the expressions (3.13) and (2.7) leads to

$$\begin{aligned}
&\int \mathcal{D}\phi(x)\,[(\xi_1' + \gamma)B_{\nu\eta}\partial_\mu B^{\mu\nu\eta} \\
+ \;& (\xi_2' - \gamma)B_{\nu\eta}\eta_\mu B^{\mu\nu\eta} \\
+ \;& (\xi_3' + \gamma)\partial^\mu(\partial_\mu C_{\nu\eta} + \partial_\nu C_{\eta\mu} + \partial_\eta C_{\mu\nu})\tilde{C}^{\nu\eta} \\
+ \;& (\xi_4' - \gamma)\eta^\mu(\partial_\mu C_{\nu\eta} + \partial_\nu C_{\eta\mu} + \partial_\eta C_{\mu\nu})\tilde{C}^{\nu\eta} \\
+ \;& (\xi_5' - \gamma)(\partial_\mu\tilde{\beta}_\nu - \partial_\nu\tilde{\beta}_\mu)\partial^\mu\beta^\nu \\
+ \;& (\xi_6' + \gamma)(\eta_\mu\tilde{\beta}_\nu - \eta_\nu\tilde{\beta}_\mu)\partial^\mu\beta^\nu \\
+ \;& (\xi_7' + \gamma)C_{\mu\nu}\partial^\mu\tilde{F}^\nu + (\xi_8' - \gamma)C_{\mu\nu}\eta^\mu\tilde{F}^\nu \\
+ \;& (\xi_9' + \gamma)B_2\partial_\mu\beta^\mu + (\xi_{10}' - \gamma)B_2\eta_\mu\beta^\mu \\
+ \;& (\xi_{11}' - \gamma)\tilde{C}_2\partial_\mu\partial^\mu C_2 + (\xi_{12}' + \gamma)\tilde{C}_2\eta_\mu\partial^\mu C_2 \\
- \;& \beta_{\nu\eta}\partial_\mu(\partial^\mu C^{\nu\eta} + \partial^\nu C^{\eta\mu} + \partial^\eta C^{\mu\nu})\Theta'(\xi_1 - \xi_3) \\
- \;& \beta_{\nu\eta}\eta_\mu(\partial^\mu C^{\nu\eta} + \partial^\nu C^{\eta\mu} + \partial^\eta C^{\mu\nu})\Theta'(\xi_2 - \xi_4) \\
- \;& (\partial_\mu\tilde{F}_\nu - \partial_\nu\tilde{F}_\mu)\partial^\mu\beta^\nu\Theta'(\xi_5 + \xi_7) \\
- \;& (\eta_\mu\tilde{F}_\nu - \eta_\nu\tilde{F}_\mu)\partial^\mu\beta^\nu\Theta'(\xi_6 + \xi_8) \\
- \;& B_2\partial_\mu\partial^\mu C_2\Theta'(\xi_9 + \xi_{11}) \\
- \;& B_2\eta_\mu\partial^\mu C_2\Theta'(\xi_{10} + \xi_{12})] = 0. \qquad (3.15)
\end{aligned}$$

Here, all the nonlocal (Θ' dependent) terms will vanish, if the following conditions are satisfied:

$$\begin{aligned}
&\xi_1 - \xi_3 = \xi_2 - \xi_4 = \xi_5 + \xi_7 = 0, \\
&\xi_6 + \xi_8 = \xi_9 + \xi_{11} = \xi_{10} + \xi_{12} = 0. \qquad (3.16)
\end{aligned}$$

Equating the L.H.S. and R.H.S. of the coefficients of local terms of the Eq. (3.15), we get the following differential equations:

$$\begin{aligned}
\xi_1' + \gamma &= 0, \quad \xi_2' - \gamma = 0, \\
\xi_3' + \gamma &= 0, \quad \xi_4' - \gamma = 0, \\
\xi_5' - \gamma &= 0, \quad \xi_6' + \gamma = 0, \\
\xi_7' + \gamma &= 0, \quad \xi_8' - \gamma = 0, \\
\xi_9' + \gamma &= 0, \quad \xi_{10}' - \gamma = 0, \\
\xi_{11}' - \gamma &= 0, \quad \xi_{12}' + \gamma = 0.
\end{aligned} \tag{3.17}$$

The solution of the above differential equations, satisfying the initial conditions [$\xi_i(\kappa = 0) = 0$ $(i = 1, 2, ..., 12)$], are

$$\begin{aligned}
\xi_1 = -\gamma\kappa, \quad \xi_2 = \gamma\kappa, \quad \xi_3 = -\gamma\kappa, \\
\xi_4 = \gamma\kappa, \quad \xi_5 = \gamma\kappa, \quad \xi_6 = -\gamma\kappa, \\
\xi_7 = -\gamma\kappa, \quad \xi_8 = \gamma\kappa, \quad \xi_9 = -\gamma\kappa, \\
\xi_{10} = \gamma\kappa, \quad \xi_{11} = \gamma\kappa, \quad \xi_{12} = -\gamma\kappa.
\end{aligned} \tag{3.18}$$

Putting these values of ξ's in Eq. (3.14), the specific expression of S_1 becomes

$$\begin{aligned}
S_1 &= \int d^6x \, [-\gamma \; \kappa \; B_{\nu\eta}\partial_\mu B^{\mu\nu\eta} + \gamma \; \kappa \; B_{\nu\eta}\eta_\mu B^{\mu\nu\eta} \\
&- \gamma \; \kappa \; \partial^\mu(\partial_\mu C_{\nu\eta} + \partial_\nu C_{\eta\mu} + \partial_\eta C_{\mu\nu}) \; \tilde{C}^{\nu\eta} \\
&+ \gamma \; \kappa \; \eta^\mu(\partial_\mu C_{\nu\eta} + \partial_\nu C_{\eta\mu} + \partial_\eta C_{\mu\nu}) \; \tilde{C}^{\nu\eta} \\
&+ \gamma \; \kappa \; (\partial_\mu \tilde{\beta}_\nu - \partial_\nu \tilde{\beta}_\mu) \; \partial^\mu \beta^\nu \\
&- \gamma \; \kappa \; (\eta_\mu \tilde{\beta}_\nu - \eta_\nu \tilde{\beta}_\mu) \; \partial^\mu \beta^\nu \\
&- \gamma \; \kappa \; C_{\mu\nu}\partial^\mu \tilde{F}^\nu + \gamma \; \kappa \; C_{\mu\nu}\eta^\mu \tilde{F}^\nu \\
&- \gamma \; \kappa \; B_2 \; \partial_\mu \beta^\mu + \gamma \; \kappa \; B_2 \; \eta_\mu \beta^\mu \\
&+ \gamma \; \kappa \; \tilde{C}_2 \; \partial_\mu \partial^\mu \; C_2 - \gamma \; \kappa \; \tilde{C}_2 \; \eta_\mu \partial^\mu \; C_2].
\end{aligned} \tag{3.19}$$

At $\kappa = 1$, the above expression becomes

$$
\begin{aligned}
S_1 &= \int d^6x \, [-\gamma \, B_{\nu\eta}\partial_\mu B^{\mu\nu\eta} + \gamma \, B_{\nu\eta}\eta_\mu B^{\mu\nu\eta} \\
&- \gamma \, \partial^\mu(\partial_\mu C_{\nu\eta} + \partial_\nu C_{\eta\mu} + \partial_\eta C_{\mu\nu}) \, \tilde{C}^{\nu\eta} \\
&+ \gamma \, \eta^\mu(\partial_\mu C_{\nu\eta} + \partial_\nu C_{\eta\mu} + \partial_\eta C_{\mu\nu}) \, \tilde{c}^{\nu\eta} \\
&+ \gamma \, (\partial_\mu \tilde{\beta}_\nu - \partial_\nu \tilde{\beta}_\mu) \, \partial^\mu \beta^\nu \\
&- \gamma \, (\eta_\mu \tilde{\beta}_\nu - \eta_\nu \tilde{\beta}_\mu) \, \partial^\mu \beta^\nu \\
&- \gamma \, C_{\mu\nu}\partial^\mu \tilde{F}^\nu + \gamma \, C_{\mu\nu}\eta^\mu \tilde{F}^\nu \\
&- \gamma \, B_2 \, \partial_\mu \beta^\mu + \gamma \, B_2 \, \eta_\mu \beta^\mu \\
&+ \gamma \, \tilde{C}_2 \, \partial_\mu \partial^\mu \, C_2 - \gamma \, \tilde{C}_2 \, \eta_\mu \partial^\mu \, C_2].
\end{aligned}
\tag{3.20}
$$

Without any loss of generality, we set the arbitrary constant $\gamma = 1$ in the above expression and by adding the expression (3.20) at $\gamma = 1$ to the effective action for 3-form gauge theory in covariant-gauge, we get

$$
\begin{aligned}
S_{eff} &+ S_1(\kappa = 1) = \int d^6x \, \left[\frac{1}{24} H_{\mu\nu\eta\chi} H^{\mu\nu\eta\chi}\right. \\
&+ \eta_\mu B^{\mu\nu\eta} B_{\nu\eta} + \frac{1}{2} B_{\mu\nu} \tilde{B}^{\mu\nu} \\
&+ (\partial_\mu \tilde{C}_{\nu\eta} + \partial_\nu \tilde{C}_{\eta\mu} + \partial_\eta \tilde{C}_{\mu\nu}) \eta^\mu C^{\nu\eta} \\
&- (\eta_\mu \tilde{\beta}_\nu - \eta_\nu \tilde{\beta}_\mu) \partial^\mu \beta^\nu - B B_2 \\
&- \frac{1}{2} B_1^2 + (\partial_\mu \tilde{C}^{\mu\nu}) f_\nu + C_{\mu\nu} \eta^\mu \tilde{F}^\nu \\
&- \tilde{C}_2 \eta_\mu \partial^\mu C_2 + \tilde{f}_\mu f^\mu - \tilde{F}_\mu F^\mu \\
&+ \left. \eta_\mu \beta^\mu B_2 + \partial_\mu \phi^\mu B_1 - \partial_\mu \tilde{\beta}^\mu B \right],
\end{aligned}
\tag{3.21}
$$

which is nothing but the 3-form effective action in a non-covariant gauge. We end this section by stating that the non-covariant gauge formulation of the 3-form gauge theory is obtained from the covariant gauge formulation through a FFBRST transformation with an appropriate finite field dependent parameter. Even though, we have shown the connection between Lorentz-(covariant)-gauge and axial-(non-covariant)-gauge, our formulation is valid for the connection between any covariant and non-covariant gauges.

3.4 Mapping of covariant and non-covariant gauges in the 3-form gauge theory: BV formulation

We consider the BV formulation for the Abelian 3-form gauge theory to re-establish the results of the previous section. For this purpose, we express the generating functional (3.9) in field/anti-field formulation by introducing the anti-field $\phi^\star$ corresponding to each generic field ϕ with opposite statistics, as follows

$$
\begin{aligned}
Z_{eff} &= \int \mathcal{D}\phi \exp\left[i\int d^6x \left\{\frac{1}{24}\, H_{\mu\nu\eta\chi}H^{\mu\nu\eta\chi}\right.\right. \\
&- B^\star_{\mu\nu\eta}(\partial^\mu C^{\nu\eta} + \partial^\nu c^{\eta\mu} + \partial^\eta C^{\mu\nu}) \\
&+ C^\star_{\mu\nu}(\partial^\mu \beta^\nu - \partial^\nu \beta^\mu) + \tilde{C}^\star_{\mu\nu} B^{\mu\nu} \\
&- \tilde{B}^\star_{\mu\nu}(\partial^\mu f^\nu - \partial^\nu f^\mu) - \beta^\star_\mu \partial^\mu\, C_2 - \tilde{\beta}^\star_\mu \tilde{F}^\mu \\
&- F^\star_\mu \partial^\mu\, B + \tilde{f}^\star_\mu \partial^\mu\, B_1 + \tilde{C}^\star_2\, B_2 + \tilde{C}^\star_1\, B_1 \\
&\left.\left. - C^\star_1\, B - \phi^\star_\mu f^\mu \right\}\right]. \qquad (3.22)
\end{aligned}
$$

The above generating functional can further be written in compact form as

$$
Z_{eff} = \int \mathcal{D}\phi\; e^{iW_{\Psi_L}(\phi,\phi^\star)}, \qquad (3.23)
$$

where $W_{\Psi_L}(\phi,\phi^\star)$ is an extended action for the Abelian 3-form gauge theory in the covariant gauge corresponding to the gauge-fixed fermion Ψ_L given in (Eq. (3.8)) with Grassmann parity 1 and ghost number -1. The generating functional Z_{eff} does not depend on the choice of gauge-fixed fermion. This extended quantum action, $W_{\Psi_L}(\phi,\phi^\star)$, satisfies rich mathematical relation, which is so-called quantum master equation, given by

$$
\Delta e^{iW_{\Psi_L}[\phi,\phi^\star]} = 0, \quad \Delta \equiv \frac{\partial_r}{\partial\phi}\frac{\partial_r}{\partial\phi^\star}(-1)^{\epsilon+1}. \qquad (3.24)
$$

The anti-field $\phi^\star(=\frac{d\psi_L}{d\phi})$ corresponding to the each generic field ϕ for this particular theory is obtained from the gauge-fixed fermion (3.8) as

$$
\begin{aligned}
&B^\star_{\mu\nu\eta} = -\partial_\mu \tilde{C}_{\nu\eta}, \quad C^\star_{\mu\nu} = -\partial_\mu \tilde{\beta}_\nu, \quad \tilde{B}^\star_{\mu\nu} = \frac{1}{2}\tilde{C}_{\mu\nu}, \\
&\tilde{C}^\star_{\mu\nu} = \frac{1}{2}\tilde{B}_{\mu\nu} + \partial^\eta B_{\mu\nu\eta}, \quad \beta^\star_\mu = -\partial_\mu \tilde{C}_2, \\
&\tilde{\beta}^\star_\mu = -F_\mu + \partial^\nu C_{\nu\mu}, \quad F_{\mu\star} = -\tilde{\beta}_\mu, \quad \tilde{f}^\star_\mu = -\phi_\mu, \\
&\tilde{C}^\star_2 = -\frac{1}{2}B + \partial_\mu \beta^\mu, \quad C^\star_1 = \frac{1}{2}B_2, \quad \tilde{C}^\star_1 = -\frac{1}{2}B_1, \\
&\phi^\star_\mu = -\tilde{f}_\mu, \quad B^\star = -\frac{1}{2}\tilde{C}_2, \quad B^\star_1 = -\frac{1}{2}\tilde{C}_1, \\
&B^\star_2 = \frac{1}{2}C_1, \quad \{B^\star_{\mu\nu}, \tilde{F}^\star_\mu, f^\star_\mu, C^\star_2\} = 0.
\end{aligned}
\tag{3.25}
$$

Now, we construct the infinitesimal field-dependent parameter written in field/anti-field formulation as

$$
\begin{aligned}
\Theta' &= i\gamma \int d^6y \Big[- B^\star_{\mu\nu\eta} B^{\mu\nu\eta} + \bar{B}^\star_{\mu\nu\eta} B^{\mu\nu\eta} \\
&- C^\star_{\mu\nu} C^{\mu\nu} + \bar{C}^\star_{\mu\nu} C^{\mu\nu} - \beta^\star_\mu \beta^\mu + \bar{\beta}^\star_\mu \beta^\mu \Big],
\end{aligned}
\tag{3.26}
$$

where fields with star ($\star$) are the anti-fields corresponding to the fields satisfying axial (non-covariant) gauge-fixing condition. Under the FFBRST transformation, corresponding to (3.26), the path integral measure is not invariant and gives rise to a factor e^{iS_1}, in the functional integral. The expression for functional S_1 is calculated with the help of eqs. (2.7), (3.13) and (3.26) as

$$
\begin{aligned}
S_1 &= \int d^6x\ [\kappa\ B^\star_{\mu\nu\eta}(\partial^\mu C^{\nu\eta} + \partial^\nu C^{\eta\mu} + \partial^\eta C^{\mu\nu}) \\
&- \kappa\ \bar{B}^\star_{\mu\nu\eta}(\partial^\mu C^{\nu\eta} + \partial^\nu C^{\eta\mu} + \partial^\eta C^{\mu\nu}) \\
&- \kappa\ C^\star_{\mu\nu}(\partial^\mu \beta^\nu - \partial^\nu \beta^\mu) + \kappa\ \bar{C}^\star_{\mu\nu}(\partial^\mu \beta^\nu \\
&- \partial^\nu \beta^\mu) - \kappa\ \tilde{C}^\star_{\mu\nu} B^{\mu\nu} + \kappa \bar{\tilde{C}}^\star_{\mu\nu} B^{\mu\nu} \\
&+ \kappa\ \tilde{B}^\star_{\mu\nu}(\partial^\mu f^\nu - \partial^\nu f^\mu) - \kappa\ \bar{\tilde{B}}^\star_{\mu\nu}(\partial^\mu f^\nu \\
&- \partial^\nu f^\mu) + \kappa\ \beta^\star_\mu \partial^\mu\ C_2 - \kappa\ \bar{\beta}^\star_\mu \partial^\mu\ C_2 + \kappa\ \tilde{\beta}^\star_\mu \tilde{F}^\mu \\
&- \kappa\ \bar{\tilde{\beta}}^\star_\mu \tilde{F}^\mu + \kappa\ F^\star_\mu \partial^\mu B - \kappa\ \bar{F}^\star_\mu \partial^\mu B \\
&- \kappa\ \tilde{f}^\star_\mu \partial^\mu B_1 + \kappa\ \bar{\tilde{f}}^\star_\mu \partial^\mu B_1 - \kappa\ \tilde{C}^\star_2\ B_2 \\
&+ \kappa\ \bar{\tilde{C}}^\star_2 B_2 - \kappa\ \tilde{C}^\star_1 B_1 + \kappa\ \bar{\tilde{C}}^\star_1 B_1 + \kappa\ C^\star_1 B \\
&- \kappa\ \bar{C}^\star_1 B + \kappa\ \phi^\star_\mu f^\mu - \kappa\ \bar{\phi}^\star_\mu f^\mu]. \qquad (3.27)
\end{aligned}
$$

At $\kappa = 1$, the above equation reduces to

$$
\begin{aligned}
S_1 &= \int d^6x\ [B^\star_{\mu\nu\eta}(\partial^\mu C^{\nu\eta} + \partial^\nu C^{\eta\mu} + \partial^\eta C^{\mu\nu}) \\
&- \bar{B}^\star_{\mu\nu\eta}(\partial^\mu C^{\nu\eta} + \partial^\nu C^{\eta\mu} + \partial^\eta C^{\mu\nu}) \\
&- C^\star_{\mu\nu}(\partial^\mu \beta^\nu - \partial^\nu \beta^\mu) + \bar{C}^\star_{\mu\nu}(\partial^\mu \beta^\nu \\
&- \partial^\nu \beta^\mu) - \tilde{C}^\star_{\mu\nu} B^{\mu\nu} + \bar{\tilde{C}}^\star_{\mu\nu} B^{\mu\nu} \\
&+ \tilde{B}^\star_{\mu\nu}(\partial^\mu f^\nu - \partial^\nu f^\mu) - \bar{\tilde{B}}^\star_{\mu\nu}(\partial^\mu f^\nu \\
&- \partial^\nu f^\mu) + \beta^\star_\mu \partial^\mu\ C_2 - \bar{\beta}^\star_\mu \partial^\mu\ C_2 + \tilde{\beta}^\star_\mu \tilde{F}^\mu \\
&- \bar{\tilde{\beta}}^\star_\mu \tilde{F}^\mu + F^\star_\mu \partial^\mu\ B - \bar{F}^\star_\mu \partial^\mu\ B \\
&- \tilde{f}^\star_\mu \partial^\mu\ B_1 + \bar{\tilde{f}}^\star_\mu \partial^\mu\ B_1 - \tilde{C}^\star_2\ B_2 \\
&+ \bar{\tilde{C}}^\star_2\ B_2 - \tilde{C}^\star_1\ B_1 + \bar{\tilde{C}}^\star_1\ B_1 + C^\star_1\ B \\
&- \bar{C}^\star_1\ B + \phi^\star_\mu f^\mu - \bar{\phi}^\star_\mu f^\mu]. \qquad (3.28)
\end{aligned}
$$

The resulting generating functional in the BV formulation is given by

$$
\begin{aligned}
Z'_{eff} &= \int \mathcal{D}\phi \; e^{i\{W_{\Psi_L}+S_1(\kappa=1)\}}, \\
&\equiv \int \mathcal{D}\phi \; e^{iW_{\Psi_A}},
\end{aligned} \tag{3.29}
$$

where Z'_{eff} is the generating functional for the Abelian 3-form gauge theory in the axial-gauge with gauge-fixing fermion

$$
\begin{aligned}
\Psi_A &= \int d^6x \; \Big[-\eta_\mu \tilde{C}_{\nu\eta} B^{\mu\nu\eta} - \frac{1}{2}\tilde{C}_2 B + \frac{1}{2} C_1 B_2 \\
&- \frac{1}{2}\tilde{C}_1 B_1 - C^{\mu\nu}\eta_\mu \tilde{\beta}_\nu - \eta_\mu \tilde{C}_2 \beta^\mu + \frac{1}{2}\tilde{C}_{\mu\nu}\tilde{B}^{\mu\nu} \\
&- F^\mu \tilde{\beta}_\mu - \tilde{f}^\mu \phi_\mu\Big].
\end{aligned} \tag{3.30}
$$

The extended action W_{Ψ_A} for the 3-form gauge theory in axial-gauge also satisfies the quantum master equation (3.24). Such connection of generating functionals can be established for any covariant and non-covariant gauges in BV formulation.

3.5 Conclusions

The 3-form gauge theory in non-covariant gauge has been developed with the help of the finite field dependent BRST transformation. Usual BRST transformations have been generalized for the Abelian 3-form gauge theory in the covariant gauge. The generating functional in covariant gauge is connected to that of the non-covariant (axial) gauge through FFBRST transformation. We established this connection by constructing an appropriate finite field dependent BRST parameter. However, various non-covariant gauges like the Coulomb gauge, the light-cone gauge, the planer gauge and the temporal gauge can *also* be connected with such formulation. Thus, our formulation enables one to study the 3-form gauge theory in the non-covariant gauges. We, further, extend our study in the context of BV formulation.

Chapter 4

Emergence of Lowenstein-Zimmermann mass terms for QED$_3$

In this **Chapter**, we consider the super-renormalizable theory of massless QED in 2+1 dimensions and also discuss its BRST symmetry. Further, we show that the Lowenstein-Zimmermann mass containing the Lowenstein-Zimmermann parameter (which is important in the BPHZL renormalization along with the external sources coupled to the non-linear BRST variations) appear naturally in the theory using FFBRST transformations.

4.1 The massless QED$_3$

In this section, we reiterate the massless QED$_3$ *with* and *without* Lowenstein-Zimmermann mass terms. The expression of the massless action for QED$_3$, which is gauge invariant, is given by

$$S_0 = \int d^3x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi}\ \gamma^\mu D_\mu\ \psi \right], \tag{4.1}$$

where D_μ refers to the covariant derivative defined by $D_\mu = \partial_\mu + i\ e\ A_\mu$ and e is a coupling constant. For quantization of the above gauge invariant QED$_3$, we introduce the following gauge-fixing and corresponding ghost terms:

$$S_{gf+gh} = \int d^3x \left[B\ \partial^\mu A_\mu + \frac{\xi}{2} B^2 + \bar{C}\ \partial_\mu \partial^\mu\ C \right], \tag{4.2}$$

where B is a Nakanishi-Lautrup auxiliary field, C and $\bar{C}$ are the ghost and anti-ghost fields, respectively. Now, the total effective action is given by

$$S_{eff} = S_0 + S_{gf+gh}. \tag{4.3}$$

The effective action (4.3) remains invariant under the following BRST transformations:

$$\begin{aligned} \delta_b\ A_\mu &= \frac{1}{e}\ \partial_\mu C\ \delta\Lambda, \\ \delta_b\ \psi &= iC\ \psi\ \delta\Lambda, \\ \delta_b\ \bar{\psi} &= -iC\ \bar{\psi}\ \delta\Lambda, \\ \delta_b\ C &= 0, \\ \delta_b\ \bar{C} &= \frac{1}{e} B\ \delta\Lambda, \\ \delta_b\ B &= 0, \end{aligned} \tag{4.4}$$

where $\delta\Lambda$ is an infinitesimal and anticommuting parameter.

Now, we define the gauge invariant Lowenstein-Zimmermann mass term for massless QED_3 as follows,

$$S_m = \int d^3x\ \left[\frac{\mu}{2}(s-1)\ \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + m(s-1)\ \bar{\psi}\psi \right]. \tag{4.5}$$

Here, $0 \leq s \leq 1$ is the Lowenstein-Zimmermann parameter which plays an important role in BPHZL renormalization program. Now, including this Lowenstein-Zimmermann mass term, the effective action for QED_3 becomes

$$S^m_{eff} = S_{eff} + S_m. \tag{4.6}$$

The external source terms for the theory is written by

$$S_{ext} = \int d^3x \left[\bar{\Omega}\ s_b\psi - s_b\bar{\psi}\ \Omega \right], \tag{4.7}$$

where Ω and $\bar{\Omega}$ are the sources. The importance of this source term can be seen in demonstrating the Slavnov-Taylor identity which guarantees the renormalizability of the theory. The complete action including source term is

$$S_T = S_{eff} + S_m + S_{ext}. \tag{4.8}$$

This complete action (S_T) is invariant under the same set of BRST transformation as given in (4.4)

4.2 Emergence of Lowenstein-Zimmermann mass terms using FFBRST transformation

In this section, we explicitly show how does the Lowenstein-Zimmermann mass term for the massless QED$_3$ theory emerges using FFBRST formulation. For doing so, we construct the FFBRST transformation by making the transformation parameter finite and field-dependent as following:

$$\begin{aligned}
\delta_b\ A_\mu &= \frac{1}{e}\ \partial_\mu C\ \Theta[\phi], \\
\delta_b\ \psi &= iC\ \psi\ \Theta[\phi], \\
\delta_b\ \bar{\psi} &= -iC\ \bar{\psi}\ \Theta[\phi], \\
\delta_b\ C &= 0, \\
\delta_b\ \bar{C} &= \frac{1}{e}\ B\ \Theta[\phi], \\
\delta_b\ B &= 0,
\end{aligned} \tag{4.9}$$

where $\Theta[\phi]$ is an arbitrary field-dependent transformation parameter. We choose a particular infinitesimal field-dependent parameter $\Theta'[\phi]$ given as

$$\Theta' = i \int d^3x \; \frac{\bar{C}B}{B^2} \left[\frac{\mu}{2}(s-1)\; \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + m(s-1)\; \bar{\psi}\psi \right]. \tag{4.10}$$

With the help of expression (2.11), we calculate the change in Jacobian under FFBRST transformation with above parameter as follows,

$$\frac{d}{d\kappa} \ln J(\kappa) = \int d^3x \; \left[\frac{\mu}{2}(s-1)\; \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + m(s-1)\; \bar{\psi}\psi \right]. \tag{4.11}$$

In order to replace the Jacobian $J \rightarrow e^{iS_1}$, we make an ansatz for the local functional S_1 as following:

$$S_1[\phi(x,\kappa),\kappa] = \int d^3x \; \left[\xi_1(\kappa)\; \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + \xi_2(\kappa)\; \bar{\psi}\psi \right], \tag{4.12}$$

where $\xi_1(\kappa)$ and $\xi_2(\kappa)$ are explicit κ-dependent constant parameters. However, all the fields depend on κ implicitly. Making use of relation (2.2), we calculate the change in S_1 with respect to κ as follows

$$\frac{dS_1[\phi(x,\kappa)]}{d\kappa} = \int d^3x \; \left[\xi_1'\; \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + \xi_2'\; \bar{\psi}\psi \right], \tag{4.13}$$

where prime denotes the differentiation with respect to κ. The existence of the functional (4.12) is valid when it satisfies the essential requirement given in (2.7) along with (4.11). This leads to the following condition:

$$\int d^3x \; \left[\left(\xi_1' - \frac{\mu}{2}(s-1) \right) \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + \left(\xi_2' - m(s-1) \right) \right] \bar{\psi}\psi = 0. \tag{4.14}$$

Equating each local terms to zero, we get the following differential equations

$$\begin{aligned} \xi_1' - \frac{\mu}{2}(s-1) &= 0, \\ \xi_2' - m(s-1) &= 0. \end{aligned} \tag{4.15}$$

The solutions of the above equations satisfying the initial condition $\xi_1(\kappa = 0) = \xi_2(\kappa = 0) = 0$ are

$$\xi_1 = \frac{\mu}{2}(s-1)\kappa, \qquad \xi_2 = m(s-1)\kappa. \tag{4.16}$$

Plugging in these values of ξ's back into (4.12), we get the following identification of S_1

$$S_1 \left[\phi(x,\kappa),\kappa\right] = \kappa \int d^3x \left[\frac{\mu}{2}(s-1)\ \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + m(s-1)\ \bar{\psi}\psi\right]. \tag{4.17}$$

The exact expression of S_1 under the FFBRST transformation (i.e. $S_1|_{\kappa=1}$) is

$$S_1 \left[\phi(x,\kappa=1),\kappa=1\right] = \int d^3x \left[\frac{\mu}{2}(s-1)\ \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + e(s-1)\ \bar{\psi}\psi\right]. \tag{4.18}$$

This S_1 extends the effective action of the theory as follows:

$$\begin{aligned} S_{eff} + S_1(\kappa=1) &= \int d^3x \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}D\psi + B\ \partial^\mu A_\mu + \frac{\xi}{2}B^2 + \bar{C}\ \partial_\mu\partial^\mu\ C \right. \\ &+ \left. \frac{\mu}{2}(s-1)\ \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + m(s-1)\ \bar{\psi}\psi\right], \end{aligned} \tag{4.19}$$

which exactly coincides with the effective action given in (4.8). Hence, this justified our claim of emergence of the Lowenstein-Zimmermann mass term naturally under celebrated FFBRST technique.

We would like to show how can FFBRST formulation be useful in the renormalization of such theory. To show that, we further apply FFBRST transformation on the generating functional by choosing the following infinitesimal field-dependent parameter

$$\Theta' \, [\phi] = i \int d^3x \; [\psi \; \bar{\Omega} - \bar{\psi} \; \Omega], \tag{4.20}$$

where $\bar{\Omega}$ and Ω are the external sources corresponding to the non-linear BRST transformation. Following standard FFBRST technique mentioned earlier, we calculate the infinitesimal change in the Jacobian as follows

$$\frac{d}{d\kappa} \ln J(\kappa) = - \int d^3x \; [-s_b\psi \; \bar{\Omega} + s_b\bar{\psi} \; \Omega]. \tag{4.21}$$

An ansatz for S_1 to write the Jacobian as an exponent is given by

$$S_1[\phi(x,\kappa),\kappa] = \int d^3x \; [\xi_3(\kappa) \; \bar{\Omega} \; s_b\psi + \xi_4(\kappa) \; s_b\bar{\psi} \; \Omega], \tag{4.22}$$

where ξ_3 and ξ_4 are arbitrary κ dependent constant. The essential condition given in (2.7) together with (4.21) and (4.22) yields the following linear differential equations :

$$\xi_3' - 1 = 0, \qquad \xi_4' + 1 = 0. \tag{4.23}$$

The solutions of above equations satisfying initial boundary conditions are

$$\xi_3 = \kappa, \qquad \xi_4 = -\kappa. \tag{4.24}$$

With these identification of ξ's, the expression of S_1 at $\kappa = 1$ is given by

$$S_1[\phi(x,1),1] = \int d^3x \; [\bar{\Omega} \; s_b\psi - s_b\bar{\psi} \; \Omega] = S_{ext}. \tag{4.25}$$

It means that under the FFBRST transformation with parameter (4.20), the effective action (within functional integral) changes to

$$S_{eff} + S_m + S_1[\phi(x,1),1] = S_T. \tag{4.26}$$

Hence, the whole mechanism is, precisely, given by

$$\int \mathcal{D}\phi\ e^{iS_{eff}} \xrightarrow{FFBRST} \int \mathcal{D}\phi\ e^{i\left(S_{eff}+S_m\right)} \xrightarrow{FFBRST} \int \mathcal{D}\phi\ e^{i\left(S_{eff}+S_m+S_{ext}\right)} \tag{4.27}$$

where the generic path integral measure ($\mathcal{D}\phi$) is explicitly given by $\mathcal{D}\phi = \mathcal{D}\mathcal{A}_\mu \mathcal{D}B\mathcal{D}C\mathcal{D}\bar{C}\mathcal{D}\psi\mathcal{D}\bar{\psi}$. Therefore, the Lowenstein-Zimmermann mass term and the external source term are generated by calculating the Jacobian of the path integral measure under the generalized BRST transformations with appropriate transformation parameter.

4.3 Conclusions

In this **Chapter**, we have studied the BRST symmetry for the ultraviolet finite, superrenormalizable theory of massless QED_3. Further, we have generalized the BRST symmetry of the given theory by making the transformation parameter finite and field dependent. The fascinating feature of FFBRST transformation is that, under change of variables, it leads to a non-trivial Jacobian for the path integral measure of the generating functional. This Jacobian, written as e_1^{iS} for some local functional of fields S_1, depends on the choice of finite field-dependent parameter. We have computed the Jacobian for FFBRST transformation with appropriate finite field-dependent parameter. Remarkably, we have found that the Lowenstein-Zimmermann mass term together with the external sources for massless QED_3 emerge naturally within the functional integral through the Jacobian of a single FFBRST transformation. The remarkable feature of FFBRST symmetry is that, any gauge invariant (BRST exact) quantity, can be generated through the FFBRST symmetry. Although these Lowenstein-Zimmermann terms are mass terms but are gauge invariant also. Thus, we have seen that the extra physical degrees of freedom emerges due to the nonlinear BRST transformations where the parameter exhibits the extra physical degrees of freedom due to the mass terms. Though, we have illustrated our results for QED_3 only, but certainly these are not limited to a particular theory. It is a general result and can be applied to any gauge theory to get gauge invariant mass terms incorporating their dynamics.

Chapter 5

Quantum gauge symmetry of reducible gauge theory

In this **Chapter**, we study the quantum gauge symmetry of the reducible (2-form) gauge theory. Using generalized BRST (FFBRST) symmetry, we show the appearance of the gaugeon fields in the context of a reducible gauge theory.

5.1 Abelian rank-2 tensor field theory: A reducible gauge theory

We start with the kinetic part of the Lagrangian density of the Abelian gauge theory for rank-2 antisymmetric tensor field $B_{\mu\nu}$, defined by

$$\mathcal{L}_0 = \frac{1}{12} F_{\mu\nu\rho} F^{\mu\nu\rho}, \tag{5.1}$$

where the field strength tensor is defined as $F_{\mu\nu\rho} \equiv \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$. This Lagrangian density is invariant under the gauge transformation $\delta B_{\mu\nu} = \partial_\mu \zeta_\nu - \partial_\nu \zeta_\mu$. where $\zeta_\mu(x)$ is a vector gauge parameter.

In order to quantize this theory using the BRST transformation, it is necessary to introduce the following ghost and auxiliary fields: anticommuting vector fields ρ_μ and $\tilde{\rho}_\mu$, a commuting vector field β_μ, anticommuting scalar fields χ and $\tilde{\chi}$, and commuting scalar fields σ, φ, and $\tilde{\sigma}$. The BRST transformation for $B_{\mu\nu}$ is defined by replacing ζ_μ of the gauge transformation by the ghost field ρ_μ.

The complete effective Lagrangian density for this theory, in the covariant gauge, using the Faddeev-Popov method, is given by

$$\mathcal{L}^L_{eff} = \mathcal{L}_0 + \mathcal{L}_{gf} + \mathcal{L}_{gh}, \tag{5.2}$$

where the gauge-fixing and ghost terms are as follows

$$\begin{aligned} \mathcal{L}_{gf} + \mathcal{L}_{gh} &= -i\partial_\mu\tilde{\rho}_\nu(\partial^\mu\rho^\nu - \partial^\nu\rho^\mu) + \partial_\mu\tilde{\sigma}\partial^\mu\sigma + \beta_\nu(\partial_\mu B^{\mu\nu} + \lambda_1\beta^\nu - \partial^\nu\varphi) \\ &- i\tilde{\chi}\partial_\mu\rho^\mu - i\chi(\partial_\mu\tilde{\rho}^\mu - \lambda_2\tilde{\chi}). \end{aligned} \tag{5.3}$$

Here λ_1 and λ_2 are the Lagrangian multiplier fields. The effective Lagrangian density (5.2) is invariant under the following BRST and anti-BRST transformations (i.e. s_b and s_{ab}).

$$\begin{aligned} s_b B_{\mu\nu} &= (\partial_\mu\rho_\nu - \partial_\nu\rho_\mu), \\ s_b\rho_\mu &= -i\partial_\mu\sigma, \qquad s_b\sigma = 0, \\ s_b\tilde{\rho}_\mu &= i\beta_\mu, \qquad s_b\beta_\mu = 0, \\ s_b\tilde{\sigma} &= \tilde{\chi}, \qquad s_b\tilde{\chi} = 0, \\ s_b\varphi &= \chi, \qquad s_b\chi = 0, \end{aligned} \tag{5.4}$$

$$\begin{aligned} s_{ab} B_{\mu\nu} &= (\partial_\mu\tilde{\rho}_\nu - \partial_\nu\tilde{\rho}_\mu), \\ s_{ab}\tilde{\rho}_\mu &= -i\partial_\mu\tilde{\sigma}, \qquad s_{ab}\tilde{\sigma} = 0, \\ s_{ab}\rho_\mu &= -i\beta_\mu, \qquad s_{ab}\beta_\mu = 0, \\ s_{ab}\sigma &= \chi, \qquad s_{ab}\chi = 0, \\ s_{ab}\varphi &= -\tilde{\chi}, \qquad s_{ab}\tilde{\chi} = 0. \end{aligned} \tag{5.5}$$

Here we note that the anti-BRST transformations (s_{ab}) are similar to the BRST transformations (s_b), where the role of ghost and anti-ghost fields are interchanged with some modification in the coefficients.

5.2 Abelian rank-2 tensor field theory in gaugeon formalism

In this section, we study the Yokoyama gaugeon formalism to analyse the quantum gauge freedom for the Abelian rank-2 tensor field theory. To analyse the gaugeon formalism for the Abelian rank-2 tensor field theory, let us start by writting the effective Lagrangian density for the (3+1) dimensional theory in Landau-gauge

$$\begin{aligned} \mathcal{L}_Y &= \frac{1}{12}F_{\mu\nu\rho}F^{\mu\nu\rho} - i\partial_\mu\tilde{\rho}_\nu(\partial^\mu\rho^\nu - \partial^\nu\rho^\mu) + \partial_\mu\tilde{\sigma}\partial^\mu\sigma + \beta_\nu(\partial_\mu B^{\mu\nu} - \partial^\nu\varphi) \\ &+ \epsilon(Y^\star_\nu + \alpha\beta_\nu)^2 - (\partial_\mu Y^\star_\nu - \partial_\nu Y^\star_\mu)\partial^\mu Y^\nu - i\tilde{\chi}\partial_\mu\rho^\mu - i\chi(\partial_\mu\tilde{\rho}^\mu - \lambda_2\tilde{\chi}), \end{aligned} \tag{5.6}$$

where Y_ν and $Y^\star_\nu$ are the gaugeon fields, respectively. The above Lagrangian density (5.6) is invariant under the following BRST transformations:

$$\begin{aligned} \delta_b B_{\mu\nu} &= (\partial_\mu\rho_\nu - \partial_\nu\rho_\mu)\ \delta\lambda, \\ \delta_b\rho_\mu &= -i\partial_\mu\sigma\ \delta\lambda, \\ \delta_b\sigma &= 0, \\ \delta_b\tilde{\rho}_\mu &= i\beta_\mu\ \delta\lambda, \\ \delta_b\beta_\mu &= 0, \\ \delta_b\tilde{\sigma} &= \tilde{\chi}\ \delta\lambda, \\ \delta_b\tilde{\chi} &= 0, \\ \delta_b\varphi &= \chi\ \delta\lambda, \\ \delta_b\chi &= 0, \\ \delta_b Y_\mu &= 0, \\ \delta_b Y^\star_\mu &= 0. \end{aligned} \tag{5.7}$$

Now, we construct the following quantum gauge transformation under which the given Lagrangian density (5.6) remains form-invariant :

$$\begin{aligned}
&B_{\mu\nu} \longrightarrow \hat{B}_{\mu\nu} = B_{\mu\nu} + \tau(\partial_\mu Y_\nu - \partial_\nu Y_\mu), \\
&\rho_\mu \longrightarrow \hat{\rho}_\mu = \rho_\mu, \\
&\sigma \longrightarrow \hat{\sigma} = \sigma, \\
&\tilde{\rho}_\mu \longrightarrow \hat{\tilde{\rho}}_\mu = \tilde{\rho}_\mu, \\
&\beta_\mu \longrightarrow \hat{\beta}_\mu = \beta_\mu, \\
&\tilde{\sigma} \longrightarrow \hat{\tilde{\sigma}} = \tilde{\sigma}, \\
&\tilde{\chi} \longrightarrow \hat{\tilde{\chi}} = \tilde{\chi}, \\
&\varphi \longrightarrow \hat{\varphi} = \varphi, \\
&\chi \longrightarrow \hat{\chi} = \chi, \\
&Y_\nu \longrightarrow \hat{Y}_\nu = Y_\nu, \\
&Y^\star_\nu \longrightarrow \hat{Y}^\star_\nu = Y^\star_\nu - \tau\beta_\nu,
\end{aligned} \tag{5.8}$$

where τ is an infinitesimal commuting transformation parameter. The form-invariance of the Lagrangian density (5.6), under the quantum gauge transformation (5.8), reflects the following shift in parameter:

$$\alpha \longrightarrow \hat{\alpha} = \alpha + \tau\alpha. \tag{5.9}$$

Furthermore, following the Yokoyama approach to remove the unphysical gauge and gaugeon modes of the theory and to define physical states, we impose two subsidiary conditions (the Kugo-Ojima type and Gupta-Bleuler type) as follows

$$\begin{aligned}
Q_b|\text{phys}\rangle &= 0, \\
(Y^\star_\nu + \alpha B_\nu)^{(+)}|\text{phys}\rangle &= 0,
\end{aligned} \tag{5.10}$$

where Q_b is the BRST charge. The expression for the BRST charge using Noether's theorem is calculated as

$$\begin{aligned} Q_b &= \int d^3x \left[-2F^{0\nu\rho}(\partial_0\rho_\nu - \partial_\nu\rho_0) + \beta_\nu(\partial^0\rho^\nu - \partial^\nu\rho^0) - \partial_\nu\sigma(\partial^0\tilde{\rho}^\nu - \partial^\nu\tilde{\rho}^0)\right. \\ &+ \left.\tilde{\chi}\partial^0\sigma - \chi B^0\right]. \end{aligned} \tag{5.11}$$

The Kugo-Ojima type subsidiary condition is essential to remove the unphysical modes corresponding to gauge field from the total Fock space. However, the Gupta-Bleuler type condition is used to remove the unphysical gaugeon modes from the physical states. The second subsidiary condition is valid when the combination $(Y_\nu^\star + \alpha B)$ satisfies the following free equation:

$$\partial_\mu\partial^\mu(Y_\nu^\star) = 0, \tag{5.12}$$

which we have derived using equations of motion. The free equation (5.12) guarantees the decomposition of $(Y_\nu^\star + \alpha B)$ in positive and negative frequency parts. Consequently, the subsidiary conditions (5.10) warrant the positivity of the semi-definite metric of our physical state-vector space

$$\langle \text{phys}|\text{phys}\rangle \geq 0, \tag{5.13}$$

and, hence, we have a desirable physical subspace on which our unitary physical S-matrix exists in the quantum Hilbert space.

5.3 BRST symmetric gaugeon formalism

In this section, we discuss the BRST symmetric gaugeon formalism for the Abelian 2-form gauge theory. For this purpose, we first define the Lagrangian density of such a model as follows:

$$\begin{aligned} \mathcal{L}_Y &= \frac{1}{12}F_{\mu\nu\rho}F^{\mu\nu\rho} - i\partial_\mu\tilde{\rho}_\nu(\partial^\mu\rho^\nu - \partial^\nu\rho^\mu) + \partial_\mu\tilde{\sigma}\partial^\mu\sigma + \beta_\nu(\partial_\mu B^{\mu\nu} - \partial^\nu\varphi) \\ &+ \epsilon(Y_\nu^\star + \alpha\beta_\nu)^2 - (\partial_\mu Y_\nu^\star - \partial_\nu Y_\mu^\star)\partial^\mu Y^\nu - i\tilde{\chi}\partial_\mu\rho^\mu - i\chi(\partial_\mu\tilde{\rho}^\mu - \lambda_2\tilde{\chi}) \\ &- i\partial_\mu K_\nu^\star(\partial^\mu K^\nu - \partial^\nu K^\mu) + \partial_\mu Z^\star\partial^\mu Z, \end{aligned} \tag{5.14}$$

where $K_\nu, K_\nu^\star$ and $Z, Z^\star$ are the ghost fields and ghost of ghost fields corresponding to the gaugeon fields, respectively. The gaugeon fields and respective ghost and ghost of ghost fields transform, under the BRST transformation, as:

$$\begin{aligned} \delta_b Y_\nu &= K_\nu\, \delta\Lambda, \\ \delta_b K_\nu &= 0, \\ \delta_b K_\nu^\star &= i Y_\nu^\star\, \delta\Lambda, \\ \delta_b Y_\nu^\star &= 0, \\ \delta_b Z^\star &= 0, \\ \delta_b Z &= 0. \end{aligned} \tag{5.15}$$

Therefore, the gaugeon Lagrangian density (5.14) remains intact under the application of the combined BRST transformations (5.4) and (5.15). The BRST charge, corresponding to the symmetry transformations (5.4) and (5.15), is given by

$$\begin{aligned} Q_b &= \int d^3x \left[-2F^{0\nu\rho}(\partial_0\rho_\nu - \partial_\nu\rho_0) + \beta_\nu(\partial^0\rho^\nu - \partial^\nu\rho^0) - \partial_\nu\sigma(\partial^0\tilde{\rho}^\nu - \partial^\nu\tilde{\rho}^0) \right. \\ &+ \left. \tilde{\chi}\partial^0\sigma - \chi B^0 - K_\nu(\partial^0 Y^{\star\nu} - \partial^\nu Y^{\star 0}) + Y_\nu^\star(\partial^0 K^\nu - \partial^\nu K^0) \right]. \end{aligned} \tag{5.16}$$

This BRST charge Q_b annihilates the physical subspace of the $\mathcal{V}_{phys}$ of the total quantum Hilbert space as given below

$$Q_b|\text{phys}\rangle = 0. \tag{5.17}$$

This single subsidiary condition of Kugo-Ojima type removes both the unphysical gauge modes as well as the unphysical gaugeon modes.

The gaugeon Lagrangian density (5.14) also admits the following quantum gauge transformations:

$$
\begin{aligned}
&B_{\mu\nu} \longrightarrow \hat{B}_{\mu\nu} = B_{\mu\nu} + \tau(\partial_\mu Y_\nu - \partial_\nu Y_\mu),\\
&\rho_\mu \longrightarrow \hat{\rho}_\mu = \rho_\mu + \tau K_\mu,\\
&\sigma \longrightarrow \hat{\sigma} = \sigma,\\
&\tilde{\rho}_\mu \longrightarrow \hat{\tilde{\rho}}_\mu = \tilde{\rho}_\mu,\\
&\beta_\mu \longrightarrow \hat{\beta}_\mu = \beta_\mu,\\
&\tilde{\sigma} \longrightarrow \hat{\tilde{\sigma}} = \tilde{\sigma},\\
&\tilde{\chi} \longrightarrow \hat{\tilde{\chi}} = \tilde{\chi},\\
&\varphi \longrightarrow \hat{\varphi} = \varphi,\\
&\chi \longrightarrow \hat{\chi} = \chi,\\
&Y_\nu \longrightarrow \hat{Y}_\nu = Y_\nu,\\
&Y_\nu^\star \longrightarrow \hat{Y}_\nu^\star = Y_\nu^\star - \tau\beta_\nu,\\
&K_\nu^\star \longrightarrow \hat{K}_\nu^\star = K_\nu^\star - \tau\tilde{\rho}_\mu,\\
&K_\nu \longrightarrow \hat{K}_\nu = K_\nu,\\
&Z \longrightarrow \hat{Z} = Z,\\
&Z^\star \longrightarrow \hat{Z}^\star = Z.
\end{aligned}
\tag{5.18}
$$

Under the quantum gauge transformation (5.18), the Lagrangian density (5.14) is form-invariant. In other words, we have the following

$$
\mathcal{L}(\phi^A, \alpha) = \mathcal{L}(\hat{\phi}^A, \hat{\alpha}), \tag{5.19}
$$

where

$$
\hat{\alpha} = \alpha + \tau\alpha. \tag{5.20}
$$

Here we observe that the quantum gauge transformations (5.18) commute with the BRST transformations mentioned in (5.15). It confirms that the quantum Hilbert space, spanned by the physical states (that are annihilated by BRST charge), is also invariant under the quantum gauge transformations, i.e.,

$$\hat{Q}_b = Q_b \tag{5.21}$$

Hence, the physical subspace $\hat{\mathcal{V}}_{phys}$ is also *invariant* under the quantum gauge transformation.

5.4 FFBRST symmetric Gaugeon formalism

In order to obtain the gaugeon formalism for Abelian 2-form gauge theory, we construct FFBRST transformations, corresponding to the BRST transformations (5.18) and (5.15) using the technique, discussed in **Chapter two**. The infinitesimal field dependent transformation parameter is chosen by

$$\begin{aligned}\Theta' &= \int d^3x[-\bar{\rho}_\mu\frac{\beta_\mu}{\beta_\mu^2}(Y_\nu^\star)^2 - \epsilon\alpha^2\bar{\rho}_\mu\frac{\beta_\mu}{\beta_\mu^2}(\beta_\nu)^2 - \epsilon\bar{\rho}_\mu\frac{\beta_\mu}{\beta_\mu^2}(Y_\mu^\star\beta^\mu)\\ &- \bar{\rho}_\mu\frac{\beta_\mu}{\beta_\mu^2}(\partial_\mu Y_\nu^\star - \partial_\nu Y_\mu^\star)\partial^\mu Y^\nu].\end{aligned} \tag{5.22}$$

where $\Theta'[\phi]$ is infinitesimal field-dependent parameter.

We calculate the infinitesimal change in the Jacobian of functional integral as

$$\frac{1}{J}\frac{dJ}{d\kappa} = i[(Y_\nu^\star)^2 + \epsilon\alpha^2(\beta_\nu)^2 + 2\epsilon\alpha(Y_\mu^*\beta^\mu) - (\partial_\mu Y_\nu^* - \partial_\nu Y_\mu^*)\partial^\mu Y^\nu]. \tag{5.23}$$

We make the following ansatz for functional S_1 in this case:

$$\begin{aligned}S_1[\phi(x,\kappa),\kappa] &= [\xi_1(\kappa)Y_\nu^{*2} + \xi_2(\kappa)\beta_\nu^2 + \xi_3(\kappa)Y_\mu^*\beta^\mu + \xi_4(\kappa)\partial_\mu Y_\nu^*\partial^\mu Y^\nu\\ &+ \xi_5(\kappa)\partial_\nu Y_\mu^*\partial^\mu Y^\nu].\end{aligned} \tag{5.24}$$

The Jacobian J can be written as e^{iS_1} when the condition $(i\frac{dS_1}{d\kappa} - \frac{1}{J}\frac{dJ}{d\kappa}) = 0$ is satisfied. Here

$$i\frac{dS_1}{d\kappa}-\frac{1}{J}\frac{dJ}{d\kappa} = [\xi_1'(\kappa)-1]Y_\nu^2+[\xi_2'(\kappa)-\epsilon\alpha^2]\beta_\mu^2+[\xi_3'(\kappa)-2\epsilon\alpha]Y_\mu^*\beta^\mu + [\xi_4'(\kappa)+1]\partial_\mu Y_\nu^*\partial^\mu Y^\nu+[\xi_5'(\kappa)-1]\partial_\nu Y_\mu^\star\partial^\mu Y^\nu=0. \quad (5.25)$$

Equating the coefficient of each local term to zero, we obtain

$$\begin{aligned}\xi_1'(\kappa)-1 &= 0,\\ \xi_2'(\kappa)-\epsilon\alpha^2 &= 0,\\ \xi_3'(\kappa)-2\epsilon\alpha &= 0,\\ \xi_4'(\kappa)+1 &= 0,\\ \xi_5'(\kappa)-1 &= 0.\end{aligned} \quad (5.26)$$

The solutions of the above differential equations are

$$\begin{aligned}\xi_1(\kappa) &= \kappa,\\ \xi_2(\kappa) &= \epsilon\alpha^2\kappa,\\ \xi_3(\kappa) &= 2\epsilon\alpha\kappa,\\ \xi_4(\kappa) &= -\kappa,\\ \xi_5(\kappa) &= \kappa.\end{aligned} \quad (5.27)$$

After plugging in the values of $\xi_1(\kappa),\xi_2(\kappa),\xi_3(\kappa),\xi_4(\kappa)$ and $\xi_5(\kappa)$ into equation (5.24), we obtain the following

$$S_1[\phi(x,\kappa),\kappa]=Y_\nu^{*2}\kappa+\epsilon\alpha^2(\beta_\nu)^2\kappa+2\epsilon\alpha(Y_\mu^*\beta^\mu)\kappa-(\partial_\mu Y_\nu^*-\partial_\nu Y_\mu^*)\partial^\mu Y^\nu\kappa. \quad (5.28)$$

At $\kappa=1$, it reduces to the following

$$S_1[\phi(x,1),1] = Y_\nu^{*2} + \epsilon\alpha^2(\beta_\nu)^2 + 2\epsilon\alpha(Y_\mu^*\beta^\mu) - (\partial_\mu Y_\nu^* - \partial_\nu Y_\mu^*)\partial^\mu Y^\nu. \quad (5.29)$$

After adding this S_1 (at $\kappa = 1$) in the effective action of the Abelian 2-form gauge theory, we obtain

$$S_1[\phi(x,1),1] + S_{eff} = S_{eff}^Y, \quad (5.30)$$

where S_{eff}^Y is the effective action of gaugeon formalism for the Abelian 2-form gauge theory.

5.5 Conclusions

Starting from the most general gauge-fixing Lagrangian density (including the gaugeon fields), we have presented a general form of the BRST symmetric gaugeon formalism for the reducible gauge theory. This most general gauge-fixing Lagrangian density possesses the quantum gauge symmetry under which the Lagrangian remains form-invariant. The theory contains two gauge parameters from which one gets shifted by the quantum gauge transformation. By introducing the FP ghosts and ghosts of ghosts (corresponding to the gaugeon fields), we have provided a BRST symmetric gaugeon formalism for the Abelian 2-form gauge theory. The BRST symmetry enables us to improve the Yokoyama subsidiary conditions by replacing these to a single Kugo-Ojima type subsidiary condition. We have found that the quantum gauge transformation commutes with the BRST transformation. As a result, the BRST charge is invariant, and thus, the physical subspace is also gauge invariant. Finally, using FFBRST transformation, we have shown that we can generate gaugeon fields in the reducible gauge theory.

Chapter 6

BF models in different gauges

In this **Chapter**, we consider the BRST symmetric four dimensional BF theory, a topological theory, containing antisymmetric tensor fields. Further, we generalize the BRST transformation into finite field-depend transformation by making the transformation parameter finite and field-dependent. Applying the resulting generalized BRST transformation on the generating functional, we get an extra piece which changes the BRST exact part of the action due to the Jacobian of functional measure. For an specific choice of finite field- dependent parameter, the generalized BRST transformation connect the Landau-gauge to axial-gauge for BF model. In the last section of this **Chapter**, we show that the gaugeon fields can be incorporated into the Lagrangian density of the BF model using FFBRST formulation.

6.1 BRST symmetric four dimensional BF theory

We consider the BF model described by the classical action

$$S_{inv} = -\frac{1}{4}\int d^4x\ \epsilon^{\mu\nu\rho\sigma}\ F^a_{\mu\nu}\ B^a_{\rho\sigma}. \tag{6.1}$$

This action describes the interaction between a two form field $B^a_{\mu\nu}$ and a gauge field A^a_μ. The field strength tensor is defined as

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^a_\mu A^a_\nu. \tag{6.2}$$

For quantization of the BF theory, one needs a gauge-fixing term and induced ghost terms in the Landau-gauge as follows

$$
\begin{aligned}
S_{gf+gh} &= \int d^4x \Big[(B^a\partial^\mu A^a_\mu + \bar{C}^a\partial^\mu(D_\mu C)^a + h^{a\nu}(\partial^\mu B^a_{\mu\nu}) + w^a\partial^\mu\xi^a_\mu + h^a_\mu(\partial^\mu e^a) \\
&+ w^a\lambda^a + (\partial^\mu\bar{\xi}^a_\mu)\lambda^a - (\partial^\mu\bar{\phi}^a)[(D_\mu\phi)^a + f^{abc}C^b\xi^c_\mu] \\
&- (\partial^\mu\bar{\xi}^{a\nu})[(D_\mu\xi_\nu)^a - (D_\nu\xi_\mu)^a + f^{abc}B^b_{\mu\nu}C^c] \\
&+ \frac{1}{2}f^{abc}\epsilon^{\mu\nu\rho\sigma}(\partial_\mu\bar{\xi}^a_\nu)(\partial_\rho\bar{\xi}^b_\sigma)\phi^c)\Big]. \qquad (6.3)
\end{aligned}
$$

The complete effective action for the BF theory in the Landau-(covariant)-gauge, is given by

$$
\begin{aligned}
S_{eff} &= \int d^4x \Big[-\frac{1}{4}(\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}B^a_{\rho\sigma}) + (B^a\partial^\mu A^a_\mu) \\
&+ \bar{C}^a\partial^\mu(D_\mu C)^a + h^{a\nu}(\partial^\mu B^a_{\mu\nu}) + w^a\partial^\mu\xi^a_\mu + h^a_\mu(\partial^\mu e^a) \\
&+ w^a\lambda^a + (\partial^\mu\bar{\xi}^a_\mu)\lambda^a - (\partial^\mu\bar{\phi}^a)[(D_\mu\phi)^a + f^{abc}C^b\xi^c_\mu] \\
&- (\partial^\mu\bar{\xi}^{a\nu})[(D_\mu\xi_\nu)^a - (D_\nu\xi_\mu)^a + f^{abc}B^b_{\mu\nu}C^c] \\
&+ \frac{1}{2}f^{abc}\epsilon^{\mu\nu\rho\sigma}(\partial_\mu\bar{\xi}^a_\nu)(\partial_\rho\bar{\xi}^b_\sigma)\phi^c)\Big]. \qquad (6.4)
\end{aligned}
$$

The above effective action (6.4) is invariant under the following BRST symmetric transformations

$$
\begin{aligned}
&\delta_b A^a_\mu = -(D_\mu C)^a\ \delta\Lambda, \\
&\delta_b C^a = \frac{1}{2}f^{abc}C^b c^c\ \delta\Lambda, \\
&\delta_b \xi^a_\mu = (D_\mu\phi)^a + f^{abc}C^b\xi^c_\mu\ \delta\Lambda, \\
&\delta_b \phi^a = f^{abc}C^b\phi^c\ \delta\Lambda, \\
&\delta_b B^a_{\mu\nu} = -(D_\mu\xi_\nu - D_\nu\xi_\mu)^a - f^{abc}B^b_{\mu\nu}C^c + f^{abc}\epsilon_{\mu\nu\rho\sigma}(\partial^\rho\bar{\xi}^{b\sigma})\phi^c\ \delta\Lambda, \\
&\delta_b \bar{\xi}^a_\mu = h^a_\mu\ \delta\Lambda, \\
&\delta_b \bar{C}^a = B^a\ \delta\Lambda,
\end{aligned}
$$

$$\begin{aligned}
\delta_b \bar{\phi}^a &= \omega^a \ \delta\Lambda, \\
\delta_b e^a &= \lambda^a \ \delta\Lambda, \\
\delta_b h^a_\mu &= 0, \\
\delta_b B^a &= 0, \\
\delta_b \omega^a &= 0, \\
\delta_b \lambda^a &= 0,
\end{aligned} \tag{6.5}$$

where the nature of transformation parameter $\delta\Lambda$ is infinitesimal, anti-commuting and global (i.e. it does not depend on space and time). In the path integral formulation, the generating functional for the effective action (6.4) of the the BF model in the Landau gauge is defined as

$$Z^L_{eff} = \int \mathcal{D}\phi \ e^{iS_{eff}}, \tag{6.6}$$

where $\mathcal{D}\phi$ indicates the path integral measure which includes all the fields ϕ, generically. Next, we will generalize the BRST transformation (6.5) by following the technique discussed in **Chapter two**.

6.2 FFBRST formulation of BF model

In this section, we generalize the BRST transformations (6.5) by making the transformation parameter finite and field-dependent. Hence, the finite field-dependent BRST transformations for the BF model is constructed as

$$\begin{aligned}
\delta_b A_\mu^a &= -(D_\mu C)^a\ \Theta[\phi], \\
\delta_b C^a &= \frac{1}{2} f^{abc} C^b c^c\ \Theta[\phi], \\
\delta_b \xi_\mu^a &= (D_\mu \phi)^a + f^{abc} C^b \xi_\mu^c\ \Theta[\phi], \\
\delta_b \phi^a &= f^{abc} C^b \phi^c\ \Theta[\phi], \\
\delta_b B_{\mu\nu}^a &= -(D_\mu \xi_\nu - D_\nu \xi_\mu)^a - f^{abc} B_{\mu\nu}^b C^c + f^{abc} \epsilon_{\mu\nu\rho\sigma} (\partial^\rho \bar{\xi}^{b\sigma}) \phi^c\ \Theta[\phi], \\
\delta_b \bar{\xi}_\mu^a &= h_\mu^a\ \Theta[\phi], \\
\delta_b \bar{C}^a &= B^a\ \Theta[\phi], \\
\delta_b \bar{\phi}^a &= \omega^a\ \Theta[\phi], \\
\delta_b e^a &= \lambda^a\ \Theta[\phi], \\
\delta_b h_\mu^a &= 0, \\
\delta_b B^a &= 0, \\
\delta_b \omega^a &= 0, \\
\delta_b \lambda^a &= 0. \qquad (6.7)
\end{aligned}$$

This BRST transformation with finite field-dependent parameter $\Theta[\phi]$ is also symmetry of the effective action (6.4). However, the path integral measure, defined in Eq. (6.6), is *not* invariant under such transformation as the BRST parameter is finite and field-dependent in nature. Now, we will compute the Jacobian of the path integral measure under the generalized BRST transformation (6.7) which helps to connect the effective action of the BF model from Landau-gauge to axial-gauge.

6.3 An application of FFBRST transformation to connect from Landau-gauge to axial-gauge

In order to obtain the generating functional for the BF model in axial-gauge, we choose the following infinitesimal field-dependent parameter

$$\Theta[\phi] = \int_0^1 d\kappa' \; \Theta'[\phi(\kappa')] \;=\; -i\int_0^1 d\kappa \int d^4x \; [\bar{C}^a(\partial^\mu A^a_\mu - \eta^\mu A^a_\mu) - \bar{\phi}^a(\partial^\mu \xi^a_\mu - \eta^\mu \xi^a_\mu)$$
$$+ \; \bar{\xi}^{a\nu}(\partial^\mu B^a_{\mu\nu} - \eta^\mu B^a_{\mu\nu}) + \bar{\xi}^{a\mu}(\partial_\mu e^a - \eta_\mu e^a) + \bar{\phi}^a\lambda^a]. \quad (6.8)$$

We calculate the Jacobian $J[\phi]$ for the above Θ' exploiting following the relation as given in Ref. [73]

$$\begin{aligned} J &= \exp i\int d^4x \; \sum_\phi \pm s_b\phi(x)\frac{\delta}{\delta\phi(x)}[\Theta'[\phi]] \\ &= \exp i\int d^4x\Big[[(-D_\mu C^a)\partial^\mu\bar{C}^a - \eta^\mu\bar{C}^a) - B^a(\partial^\mu\bar{C}^a - \eta^\mu\bar{C}^a - w^a(\partial^\mu\xi^a_\mu - \eta^\mu\xi^a_\mu)] \\ &- [(D_\mu\phi)^a + f^{abc}C^b\xi^c_\mu][\partial^\mu\bar{\phi}^a + \eta^\mu\bar{\phi}^a]) - h^{\mu a}(\partial^\nu B^a_{\mu\nu} - \eta^\nu B^a_{\mu\nu}) \\ &+ [-(D_\mu\xi_\nu - D_\nu\xi_\mu)^a - f^{abc}B^b_{\mu\nu}C^c + f^{abc}\epsilon_{\mu\nu\rho\sigma}\partial^\rho_\mu\bar{\xi}^{\sigma b\mu}\phi^c](\partial^\mu\bar{\xi}^{\nu a} - \eta^\mu\bar{\xi}^{\nu a}) \\ &- h^{\mu a}(\partial_\mu e^a - \eta_\mu e^a) + \lambda^a(\partial_\mu\bar{\xi}^{\mu a} - \eta_\mu\bar{\xi}^{\mu a}) - w^a\lambda^a\Big]. \end{aligned} \quad (6.9)$$

Therefore, under the FFBRST transformation, given in **Chapter two**, the generating functional changes as

$$\begin{aligned} Z^L &= \int \mathcal{D}\phi' \; e^{iS_{ext}[\phi']} = \int J(\phi)\mathcal{D}\phi \; e^{iS_{ext}[\phi]} \\ &= \int \mathcal{D}\phi \exp\Big(i\int d^4x \left[\mathcal{L}_{ext} + (-D_\mu C^a)(\partial^\mu\bar{C}^a - \eta^\mu\bar{C}^a) - B^a(\partial^\mu\bar{C}^a - \eta^\mu\bar{c}^a)\right. \\ &- \omega^a(\partial^\mu\xi^a_\mu - \eta^\mu\xi^a_\mu) - [(D_\mu\phi)^a + f^{abc}C^b\xi^c_\mu][\partial^\mu\bar{\phi}^a + \eta^\mu\bar{\phi}^a] - h^{\mu a}[(\partial^\nu B^a_{\mu\nu}) \\ &- (\eta^\nu B^a_{\mu\nu})] + [-(D_\mu\xi_\nu - D_\nu\xi_\mu)^a - f^{abc}B^b_{\mu\nu}C^c \\ &+ f^{abc}\epsilon_{\mu\nu\rho\sigma}\partial^\rho\cdot\bar{\xi}^{\sigma b}\phi^c](\partial^\mu\bar{\xi}^{\nu a} - \eta^\mu\bar{\xi}^{\nu a}) - h^{\mu a}(\partial_\mu e^a - \eta_\mu e^a) \\ &+ \left.\lambda^a(\partial_\mu\bar{\xi}^{\mu a} - \eta_\mu\bar{\xi}^{\mu a}) - \omega^a\lambda^a\Big)\right] \\ &= \int \mathcal{D}\phi \; e^{iS'_{ext}[\phi]} = Z^A, \end{aligned} \quad (6.10)$$

where Z^A is the generating functional for the axial-gauge. Here, in the intermediate steps, we have utilized the relation which we have used in **Chapter two**. This is nothing but the generating functional of the BF model in the axial-gauge condition. This shows that the field-dependent BRST transformation changes generating functional from *one* gauge to *another*.

6.4 An application of FFBRST transformation to generate gaugeon modes in the BF model

In this section, we study the Yokoyama gaugeon formalism to analyse the quantum gauge freedom for the BF model. To analyse the gaugeon formalism for the BF model, let us start with the effective Lagrangian density for (3+1) dimensional theory in the Landau-gauge

$$\begin{aligned}
\mathcal{L}_Y &= -\frac{1}{4}\int d^4x[\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}B^a_{\rho\sigma} - (B^a\partial^\mu A^a_\mu + \bar{C}^a\partial^\mu(D_\mu C)^a + h^{a\nu}(\partial^\mu B^a_{\mu\nu}) \\
&+ w^a\partial^\mu\xi^a_\mu + h^a_\mu(\partial^\mu e^a) + w^a\lambda^a + (\partial^\mu\bar{\xi}^a_\mu)\lambda^a - (\partial^\mu\bar{\phi}^a)[(D_\mu\phi)^a + f^{abc}C^b\xi^c_\mu] \\
&- (\partial^\mu\bar{\xi}^{a\nu})[(D_\mu\xi_\nu)^a - (D_\nu\xi_\mu)^a + f^{abc}B^b_{\mu\nu}C^c] \\
&+ \frac{1}{2}f^{abc}\epsilon^{\mu\nu\rho\sigma}(\partial_\mu\bar{\xi}^a_\nu)(\partial_\rho\bar{\xi}^b_\sigma)\phi^c)\partial^\nu\rho^{\mu a}) + \epsilon(Y^{\star a}_\nu + \alpha^a h^a_\nu)^2 \\
&- (\partial_\mu Y^{\star a}_\nu - \partial_\nu Y^{\star a}_\mu)\partial^\mu Y^{a\nu} - \partial_\mu Y^{\star a}\partial^\mu Y^a + \frac{\epsilon}{2}(Y^{\star a} + \alpha^a B^a)^2], \qquad (6.11)
\end{aligned}$$

where Y^a_ν and $Y^{\star a}_\nu$ are the gaugeon fields, respectively. For introducing gaugeon fields in the BF model, we choose Θ' as

$$\begin{aligned}
\Theta' &= \int d^4x[-\bar{\xi}^a_\mu\frac{h^a_\mu}{(h^b_\sigma)^2}(Y^*_\nu)^2 - \epsilon(\alpha^a)^2\bar{\xi}^a_\mu\frac{h^a_\mu}{(h^b_\sigma)^2}(h^a_\nu)^2 - \epsilon\bar{\xi}^a_\mu\frac{h^a_\mu}{(h^b_\sigma)^2}(Y^*_\nu h^\nu) \\
&- \bar{\xi}^a_\mu\frac{h^a_\mu}{(h^b_\sigma)^2}(\partial_\alpha Y^{\star a}_\nu - \partial_\nu Y^{\star a}_\alpha)\partial^\alpha Y^{\nu a} + \bar{\xi}^a_\mu\frac{h^a_\mu}{(h^b_\sigma)^2}(\partial_\nu Y^{\star a}\partial^\nu Y^a) \\
&+ \frac{\epsilon}{2}(Y^{\star a} + \alpha^a B^a)^2]. \qquad (6.12)
\end{aligned}$$

Using this Θ', the expression of the Jacobian J, is given as

$$\begin{aligned}
J &= \exp i\int d^4x[(\partial_\mu Y^{\star a}_\nu - \partial_\nu Y^{\star a}_\mu)\partial^\mu Y^{\nu a} + \frac{\epsilon}{2}(Y^{\star a}_\nu + \alpha^a h^a_\nu)^2 \\
&- \partial_\mu Y^{\star a}\partial^\mu Y^a + \frac{\epsilon}{2}(Y^{\star a} + \alpha^a B^a)^2]. \qquad (6.13)
\end{aligned}$$

After using the condition (2.6), (2.7) and (2.8), given in **Chapter two**, we get

$$\int \mathcal{D}\phi\ e^{iS_{eff}+S_1[\phi(x,1),1]} = \int \mathcal{D}\phi\ e^{S^Y_{eff}}, \qquad (6.14)$$

where S_{eff}^{Y} is the effective action of the gaugeon formalism for the BF model.

6.5 Conclusions

In this **Chapter**, we have discussed the BRST quantization of BF model in the Landau-gauge. Further, to analyse the quantum gauge freedom for the BF model, we have introduced the gaugeon modes into the Lagrangian density. Remarkably, the resulting Lagrangian density admits the quantum gauge transformation under which a natural shift occurs for gauge parameter. We have removed these unphysical modes by introducing two subsidiary conditions. Furthermore, we have investigated the generalized BRST transformation by making the transformation parameter finite and field dependent. Such generalized transformations are not symmetry of the functional measure. We have calculated the explicit form of the Jacobian of functional measure under the generalized BRST transformation. For an specific choice of finite field-dependent parameter, it connects the Landau-gauge to the axial-gauge. Further, we have also shown that, for an appropriate choice of the infinitesimal field-dependent parameter, gaugeon modes appear from the Jacobian under the generalized BRST transformation.

Chapter 7

A superspace description of Chern-Simons theory in BV formulation

In this **Chapter**, we study the superspace formulation of Chern-Simons theory in field/anti-field formulation.

7.1 Chern-Simons effective theory

We start this section by considering the Lagrangian density of the CS theory in (2 + 1) flat space-time dimensions, given as

$$\mathcal{L}_{CS} = -\mathrm{Tr}\left[\frac{k}{4\pi}\epsilon^{\mu\nu\rho}\left(A_\mu\partial_\nu A_\rho - \frac{2i}{3}A_\mu A_\nu A_\rho\right)\right], \tag{7.1}$$

where k is the inverse of the coupling constant and A_μ is a gauge field in the Lie-algebraic space. The Lagrangian density (7.1) is gauge invariant under the following gauge transformation

$$\delta A_\mu = D_\mu\theta = \partial_\mu\lambda + i[\lambda, A_\mu]. \tag{7.2}$$

Here λ is a Lie algebra-valued local parameter. It is well known that there are gauge redundancy due to gauge symmetry. In order to fix the redundancy of gauge freedom in the CS theory, we adopt here the axial-gauge.

$$\eta^\mu A_\mu = 0, \tag{7.3}$$

where n^μ is an arbitrary constant vector. To implement this gauge condition (7.3) into the CS theory, we add the following expressions for gauge-fixing and corresponding ghost terms to the classical Lagrangian density:

$$\begin{aligned} \mathcal{L}_{gf} &= \mathrm{Tr}\left(B\ \eta^\mu A_\mu\right), \\ \mathcal{L}_{gh} &= -\mathrm{Tr}\left(\bar{C}\ \eta^\mu D_\mu C\right), \end{aligned} \tag{7.4}$$

where C and $\bar{C}$ are the FP ghost and anti-ghost fields respectively. Now, the effective Lagrangian density for CS theory, in axial gauge, is given by

$$\mathcal{L} = \mathcal{L}_{CS} + \mathcal{L}_{gf} + \mathcal{L}_{gh}, \tag{7.5}$$

which remains invariant under the following nilpotent BRST transformations:

$$\begin{aligned} \delta_b A_\mu &= D_\mu C = \partial_\mu C + i[C, A_\mu], \\ \delta_b C &= iC^2, \\ \delta_b \bar{C} &= B, \\ \delta_b B &= 0. \end{aligned} \tag{7.6}$$

Here we have followed the adjoint representation where the notation C^2 is defined as

$$C^2 \equiv i f^{abc} C^b C^c.$$

The combination of the gauge-fixing and ghost terms are BRST exact and hence, can be written in terms of the BRST variation of gauge-fixing fermion, $\Psi = \left(\eta^{\mu\nu}\bar{C} n_\mu A_\nu\right)$, as follows

$$\mathcal{L}_{gf} + \mathcal{L}_{gh} = \mathrm{Tr}[s\left(\eta^{\mu\nu}\bar{C} n_\mu A_\nu\right)], \tag{7.7}$$

where $\eta^{\mu\nu}$ is the Minkowski metric of the 3-dimensional flat space-time. For the Landau (covariant) gauge choice, the expression of gauge-fixing and ghost terms respectively are:

$$
\begin{aligned}
\mathcal{L}'_{gf} &= \mathrm{Tr}\left(B\partial^\mu A_\mu\right), \\
\mathcal{L}'_{gh} &= -\mathrm{Tr}\left(\bar{C}\partial^\mu D_\mu C\right).
\end{aligned} \tag{7.8}
$$

Now, adding the above gauge-fixing and ghost terms in the Lagrangian density of the kinetic part of CS theory, the total Lagrangian density in the Landau gauge becomes

$$
\mathcal{L}' = \mathcal{L}_{CS} + \mathcal{L}'_{gf} + \mathcal{L}'_{gh} = \mathcal{L}_{CS} + \mathrm{Tr}[s\left(\eta^{\mu\nu}\bar{C}\partial_\mu A_\nu\right)], \tag{7.9}
$$

which also remains invariant under the same set of BRST transformation (7.6).

7.2 Extended BRST invariant Lagrangian density

In this section, we analyse the extended BRST transformations for the CS theory in BV formulation by extending all the fields from its original value by tilde fields. The advantage of doing so is that the anti-fields get identifications naturally. We start the analysis by shifting all the fields from their original values as

$$
A_\mu \longrightarrow A_\mu - \tilde{A}_\mu \quad C \longrightarrow C - \tilde{C} \quad \bar{C} \longrightarrow \bar{C} - \tilde{\bar{C}} \quad B \longrightarrow B - \tilde{B}. \tag{7.10}
$$

Under such shifting of fields, the Lagrangian density (7.5) and (7.9), respectively, also get shifted as follows:

$$
\tilde{\mathcal{L}} = \mathcal{L}(A_\mu - \tilde{A}_\mu, C - \tilde{C}, \bar{C} - \bar{C}, B - \tilde{B}), \tag{7.11}
$$

$$
\tilde{\mathcal{L}}' = \mathcal{L}'(A_\mu - \tilde{A}_\mu, C - \tilde{C}, \bar{C} - \bar{C}, B - \tilde{B}). \tag{7.12}
$$

This shifted Lagrangian density remains invariant under the BRST transformation in tandem with shift symmetry transformation, commonly known as extended BRST transformation. The extended BRST symmetry transformations of Lagrangian density (7.2) admit the following extended BRST symmetry transformations

$$\begin{aligned} s_b A_\mu &= \psi_\mu, \\ s_b \tilde{A}_\mu &= \psi_\mu - D_\mu(C - \tilde{C}), \\ s_b C &= \epsilon, \\ s_b \tilde{C} &= \epsilon - i(C - \tilde{C})^2, \\ s_b \bar{C} &= \bar{\epsilon}, \\ s_b \tilde{\bar{C}} &= \bar{\epsilon} - (B - \tilde{B}), \\ s_b B &= \rho, \\ s_b \tilde{B} &= \rho, \end{aligned} \tag{7.13}$$

where $\psi_\mu, \epsilon, \bar{\epsilon}$ and ρ are the ghost fields associated with the shift symmetry for $A_\mu, C, \tilde{C}$ and B, respectively. To preserve the nilpotency of extended BRST symmetry (7.13), the ghost fields are required to have the following BRST transformation:

$$\begin{aligned} s_b \psi &= 0, \\ s_b \epsilon &= 0, \\ s_b \bar{\epsilon} &= 0, \\ s_b \rho &= 0. \end{aligned} \tag{7.14}$$

To make our theory ghost-free, we are required to introduce the anti-fields corresponding to each fields $A^\star_\mu, C^\star, \bar{C}^\star$ and $B^\star$ of the theory. The BRST symmetric transformations of the anti-fields are given by

$$
\begin{aligned}
s_b A^\star_\mu &= -\zeta_\mu, \\
s_b C^\star &= -\sigma, \\
s_b \bar{C}^\star &= -\bar{\sigma}, \\
s_b B^\star &= -\bar{\upsilon},
\end{aligned} \tag{7.15}
$$

where $\zeta_\mu, \sigma, \bar{\sigma}$ and $\bar{\upsilon}$ are the Nakanishi-Lautrup type auxiliary fields corresponding to the shifted fields $\tilde{A}_\mu, \tilde{C}, \tilde{\bar{C}}$ and $\tilde{B}$, which have the following BRST symmetry transformation:

$$
\begin{aligned}
s_b \zeta_\mu &= 0, \\
s_b \sigma &= 0, \\
s_b \bar{\sigma} &= 0, \\
s_b \bar{\upsilon} &= 0.
\end{aligned} \tag{7.16}
$$

Now, our original theory can be recovered by adding a gauge-fixing term corresponding to the shift symmetry in such a way that all the tilde fields vanish. In order to do so, we add the following gauge-fixed Lagrangian density in the shifted Lagrangian density (7.2):

$$
\begin{aligned}
\tilde{\mathcal{L}}_{gf+gh} &= \mathrm{Tr}\Big[-\zeta^\mu \tilde{A}_\mu - A^\star_\mu[\psi^\mu - D^\mu(C - \tilde{C})] - \sigma\tilde{\bar{C}} + C^\star[\bar{\epsilon} - (B - \tilde{B})] \\
&- \bar{\sigma}\tilde{C} + \bar{C}^\star[\epsilon - i(C - \tilde{C})^2] - \bar{\upsilon}\tilde{B} - B^\star\rho\Big].
\end{aligned} \tag{7.17}
$$

The gauge-fixed Lagrangian density $\tilde{\mathcal{L}}_{gf+gh}$, given in (7.17), is also invariant under the extended BRST symmetry transformation (7.13). By performing equations of motion of auxiliary fields in the above expression, we obtain

$$
\tilde{\mathcal{L}}_{gf+gh} = \mathrm{Tr}\left[-A^\star_\mu(\psi^\mu - D^\mu C) + C^\star(\bar{\epsilon} - B) + \bar{C}^\star(\epsilon - iC^2) - B^\star\rho\right]. \tag{7.18}
$$

Since the gauge-fixing and ghost terms of the Lagrangian density (7.18) are BRST exact, these can be expressed in terms of a general gauge-fixing fermion Ψ as

$$\begin{aligned}\mathrm{Tr}(s_b\Psi) &= \mathrm{Tr}\left[s_bA_\mu\frac{\delta\Psi}{\delta A_\mu}+s_bC\frac{\delta\Psi}{\delta C}+s_b\bar{C}\frac{\delta\Psi}{\delta\bar{C}}+s_bB\frac{\delta\Psi}{\delta B}\right],\\ &= \mathrm{Tr}\left[-\frac{\delta\Psi}{\delta A_\mu}\psi_\mu+\frac{\delta\Psi}{\delta C}\epsilon+\frac{\delta\Psi}{\delta\bar{C}}\bar{\epsilon}-\frac{\delta\Psi}{\delta B}\rho\right].\end{aligned} \tag{7.19}$$

After adding equations (**??**), (7.18) and (7.19) together, we get the complete effective Lagrangian density for the CS theory in the axial-gauge which respects the extended BRST symmetry. This complete Lagrangian density is

$$\begin{aligned}\mathcal{L}_{eff} &= \tilde{\mathcal{L}}+\mathcal{L}_{gf}+\mathcal{L}_{gh}+\tilde{\mathcal{L}}_{gf+gh},\\ &= \tilde{\mathcal{L}}+\mathrm{Tr}\left[\left(-A^\star_\mu-\frac{\delta\Psi}{\delta A^\mu}\right)\psi^\mu+\left(\bar{C}^\star+\frac{\delta\Psi}{\delta C}\right)\epsilon+\left(C^\star+\frac{\delta\Psi}{\delta\bar{C}}\right)\bar{\epsilon}\right.\\ &+ \left.\left(-B^\star-\frac{\delta\Psi}{\delta B}\right)\rho+A^\star_\mu D^\mu C-C^\star B-i\bar{C}^\star C^2\right].\end{aligned} \tag{7.20}$$

We can obtain the anti-fields by using equation of motion of the ghost fields corresponding to the shift symmetry as follows:

$$\begin{aligned}A^\star_\mu &= \frac{\delta\Psi}{\delta A^\mu},\\ \bar{C}^\star &= -\frac{\delta\Psi}{\delta C},\\ C^\star &= -\frac{\delta\Psi}{\delta\bar{C}},\\ B^\star &= \frac{\delta\Psi}{\delta B}.\end{aligned} \tag{7.21}$$

The anti-fields, which we have obtained from the gauge-fixing fermions Ψ (corresponding to axial-gauge (given in 7.7)), have the following explicit expressions:

$$\begin{aligned} A^\star_\mu &= \eta_{\mu\nu}\bar{C}n^\nu, \\ \bar{C}^\star &= 0, \\ C^\star &= \eta^{\mu\nu}n_\mu A_\nu, \\ B^\star &= 0. \end{aligned} \tag{7.22}$$

Plugging these values of the anti-ghost fields into (7.20), we recover the Lagrangian density of our original CS theory in axial-gauge. The expressions of the anti-fields for the gauge-fixing fermion corresponding to the Landau gauge (given in 7.9), are given by

$$\begin{aligned} A^\star_\mu &= -\eta_{\mu\nu}\partial^\nu\bar{C}, \\ \bar{C}^\star &= 0, \\ C^\star &= \eta^{\mu\nu}\partial_\mu A_\nu, \\ B^\star &= 0. \end{aligned} \tag{7.23}$$

From these values of the anti-fields (7.23), we can recover the Lagrangian density of the CS theory in the Landau gauge.

7.3 Extended BRST invariant superspace description

In this section, we study the extended BRST invariant CS theory in a superspace labelled by the coordinates (x, θ) where θ is Grassmannian in nature and x_μ is usual space-time in (2+1) dimensions. A superspace description for the extended BRST invariant theory is obtained by defining the superfields of the form:

$$
\begin{aligned}
A_\mu(x,\theta) &= A_\mu + \theta\psi_\mu, \\
\tilde{A}_\mu(x,\theta) &= \tilde{A}_\mu + \theta[\psi_\mu - D_\mu(C-\tilde{C})], \\
\chi(x,\theta) &= C + \theta\epsilon, \\
\tilde{\chi}(x,\theta) &= \tilde{C} + \theta[\epsilon - i(C-\tilde{C})^2], \\
\bar{\chi}(x,\theta) &= \bar{C} + \theta\bar{\epsilon}, \\
\tilde{\bar{\chi}}(x,\theta) &= \tilde{\bar{C}} + \theta[\bar{\epsilon} - (B-\tilde{B}), \\
B(x,\theta) &= B + \theta\rho, \\
\tilde{B}(x,\theta) &= \tilde{B} + \theta\rho.
\end{aligned} \tag{7.24}
$$

The expressions of the super-anti-fields, using the extended BRST transformation, corresponding to the anti-fields are defined by

$$
\begin{aligned}
\tilde{A}^\star_\mu(x,\theta) &= A^\star_\mu - \theta\zeta_\mu, \\
\tilde{\chi}^\star(x,\theta) &= C^\star - \theta\sigma, \\
\tilde{\bar{\chi}}^\star(x,\theta) &= \bar{C}^\star - \theta\bar{\sigma}, \\
\tilde{B}^\star(x,\theta) &= B^\star - \theta\bar{\upsilon}.
\end{aligned} \tag{7.25}
$$

We calculate the following expressions by utilizing the above superfields and super-anti-fields:

$$
\begin{aligned}
\frac{\delta(\tilde{A}^\star_\mu \tilde{A}^\mu)}{\delta\theta} &= -A^\star_\mu[\psi^\mu - D^\mu(C-\tilde{C})] - \zeta_\mu\tilde{A}^\mu, \\
\frac{\delta(\tilde{\bar{\chi}}^\star \tilde{\chi})}{\delta\theta} &= \bar{C}^\star[\epsilon - i(C-\tilde{C})^2] - \bar{\sigma}\tilde{C}, \\
\frac{\delta(\tilde{\bar{\chi}}\tilde{\chi}^\star)}{\delta\theta} &= -\sigma\tilde{\bar{C}} + C^\star[\bar{\epsilon} - (B-\tilde{B})], \\
\frac{\delta(\tilde{B}^\star \tilde{B})}{\delta\theta} &= -B^\star\rho - \bar{\upsilon}\tilde{B}.
\end{aligned} \tag{7.26}
$$

By combining all the equations of above expressions (7.26), we get

$$\begin{aligned}\mathrm{Tr}\left[\frac{\delta}{\delta\theta}(\tilde{A}^{\star}_{\mu}\tilde{A}^{\mu}+\tilde{\bar{\chi}}^{\star}\tilde{\chi}+\tilde{\bar{\chi}}\tilde{\chi}^{\star}+\tilde{B}^{\star}\tilde{B})\right] &= \mathrm{Tr}\Big[-A^{\star}_{\mu}[\psi^{\mu}-D^{\mu}(C-\tilde{C})]-\zeta_{\mu}\tilde{A}^{\mu}\\ &+ \bar{C}^{\star}[\epsilon-i(C-\tilde{C})^{2}]-\bar{\sigma}\tilde{C}-\sigma\tilde{\bar{C}}\\ &+ C^{\star}[\bar{\epsilon}-(B-\tilde{B})]-B^{\star}\rho-\bar{v}\tilde{B}\Big], \qquad (7.27)\end{aligned}$$

which is nothing but the shifted gauge-fixed Lagrangian density $\tilde{\mathcal{L}}_{gf+gh}$ (7.17). Now, we can define the general super-gauge-fixing fermion in superspace formulation as follows,

$$\Phi(x,\theta)=\Psi(x)+\theta(s\Psi). \qquad (7.28)$$

Further, the above equation can be expressed in the following way:

$$\Phi(x,\theta)=\Psi(x)+\theta\left(-\frac{\delta\Psi}{\delta A_{\mu}}\Psi_{\mu}+\frac{\delta\Psi}{\delta C}\epsilon+\frac{\delta\Psi}{\delta\bar{C}}\bar{\epsilon}-\frac{\delta\Psi}{\delta B}\rho\right). \qquad (7.29)$$

Hence, the original gauge-fixed Lagrangian density in the superspace can be defined as the left derivative of the super-gauge-fixing fermion with respect to θ, i.e. $\mathrm{Tr}\left[\frac{\delta\Phi(x,\theta)}{\delta\theta}\right]$. We write the effective Lagrangian density for the CS theory in the superspace as follows

$$\mathcal{L}_{gen} = \tilde{\mathcal{L}}+\mathrm{Tr}\left[\frac{\delta}{\delta\theta}(\tilde{A}^{\star}_{\mu}\tilde{A}^{\mu}+\tilde{\bar{\chi}}^{\star}\tilde{\chi}+\tilde{\bar{\chi}}\tilde{\chi}^{\star}+\tilde{B}^{\star}\tilde{B}+\Phi)\right]. \qquad (7.30)$$

This compact expression indicates the BV Lagrangian density of the extended CS theory in superspace which is invariant under the extended BRST transformations.

7.4 Extended anti-BRST Lagrangian density

In this section, we construct the expression of the extended anti-BRST transformation under which the extended Lagrangian density remains invariant. This is given by

$$\begin{aligned}
\bar{s}_b A_\mu &= A^\star_\mu + D_\mu(\bar{C} - \tilde{\bar{C}}), \\
\bar{s}_b \tilde{A}_\mu &= A^\star_\mu, \\
\bar{s}_b C &= C^\star + (B - \tilde{B}), \\
\bar{s}_b \tilde{C} &= C^\star, \\
\bar{s}_b \bar{C} &= (\bar{C}^\star - \tilde{\bar{C}}^\star) + i(\bar{C} - \tilde{\bar{C}})^2, \\
\bar{s}_b \tilde{B} &= B^\star, \\
\bar{s} B &= B^\star.
\end{aligned} \tag{7.31}$$

Under the extended anti-BRST symmetry transformations, the expression of ghost fields associated with the shift symmetry transform as

$$\begin{aligned}
s_{ab}\psi_\mu &= \zeta_\mu, \\
s_{ab}\epsilon &= \sigma, \\
s_{ab}\bar{\epsilon} &= \bar{\sigma}, \\
s_{ab}\rho &= \bar{\upsilon}.
\end{aligned} \tag{7.32}$$

The BRST transformations of the anti-fields of the auxiliary fields associated with the shift symmetry, are given by

$$
\begin{aligned}
s_{ab}\zeta_\mu &= 0,\\
s_{ab}A^\star_\mu &= 0,\\
s_{ab}\sigma &= 0,\\
s_{ab}C^\star &= 0,\\
s_{ab}\bar{\sigma} &= 0,\\
s_{ab}\bar{C}^\star &= 0,\\
s_{ab}\bar{\upsilon} &= 0,\\
s_{ab}B^\star &= 0.
\end{aligned}
\tag{7.33}
$$

In axial-gauge as well as Landau-gauge of the CS theory, the anti-gauge-fixing fermions are defined respectively as

$$
\begin{aligned}
\bar{\Psi} &= \eta^{\mu\nu} C n_\mu A_\nu,\\
\bar{\Psi}' &= \eta^{\mu\nu} C \partial_\mu A_\nu.
\end{aligned}
\tag{7.34}
$$

The anti-BRST variations of these gauge-fixing fermions give the corresponding gauge-fixing and ghost parts of the complete Lagrangian density.

7.5 Extended BRST and anti-BRST invariant superspace

In this section, we write the extended BRST and anti-BRST invariant Lagrangian density in the superspace with the help of two Grassmannian coordinates θ and $\bar{\theta}$. Requiring the field strength to vanish along the unphysical directions θ and $\bar{\theta}$, we determine the superfields in the following forms:

$$
\begin{aligned}
A_\mu(x,\theta,\bar\theta) &= A_\mu(x) + \theta\psi_\mu + \bar\theta[A^\star_\mu + D_\mu\bar C] + \theta\bar\theta[\zeta_\mu + \partial_\mu\bar\epsilon],\\
\tilde A_\mu(x,\theta,\bar\theta) &= \tilde A_\mu(x) + \theta[\psi_\mu - D_\mu(C-\tilde C)] + \bar\theta A^\star_\mu + \theta\bar\theta\zeta_\mu,\\
\chi(x,\theta,\bar\theta) &= C(x) + \theta\epsilon + \bar\theta[C^\star + (B-\tilde B)] + \theta\bar\theta\sigma,\\
\tilde\chi(x,\theta,\bar\theta) &= \tilde C(x) + \theta[\epsilon - iCC] + \bar\theta C^\star + \theta\bar\theta\sigma,\\
\bar\chi(x,\theta,\bar\theta) &= \bar C(x) + \theta\bar\epsilon + \bar\theta[\bar C^\star + i\bar C\bar C] + \theta\bar\theta\bar\sigma,\\
\tilde{\bar\chi}(x,\theta,\bar\theta) &= \tilde{\bar C}(x) + \theta[\bar\epsilon - B] + \bar\theta\bar C^\star + \theta\bar\theta\bar\sigma,\\
B(x,\theta,\bar\theta) &= B(x) + \theta\rho + \bar\theta B^\star + \theta\bar\theta\bar\upsilon,\\
\tilde B(x,\theta,\bar\theta) &= \tilde B(x) + \theta\rho + \bar\theta B^\star + \theta\bar\theta\bar\upsilon.
\end{aligned}
\tag{7.35}
$$

Using above expressions of superfields, we calculate the following relation:

$$
\begin{aligned}
-\frac{1}{2}\mathrm{Tr}\left[\frac{\partial}{\partial\bar\theta}\frac{\partial}{\partial\theta}(\tilde A_\mu\tilde A^\mu + \tilde\chi\tilde{\bar\chi} + \tilde B\tilde B)\right] &= \mathrm{Tr}\Big[-\zeta^\mu\tilde A_\mu - A^\star_\mu[\psi^\mu - D^\mu(C-\tilde C)] - \sigma\tilde{\bar C}\\
&+ C^\star[\bar\epsilon - (B-\tilde B)] - \bar\sigma\tilde C + \bar C^\star[\epsilon - i(C-\tilde C)^2]\\
&- \bar\upsilon\tilde B - B^\star\rho\Big],\\
&= \tilde{\mathcal{L}}_{gf+gh},
\end{aligned}
\tag{7.36}
$$

which is nothing but the shifted gauge-fixed Lagrangian density. It is obvious that the above gauge-fixed Lagrangian density is invariant under both the extended BRST as well as the anti-BRST transformations. Now, we define the general super-gauge-fixing fermion for the extended BRST and the anti-BRST invariant theory as follows

$$
\Phi(x,\theta,\bar\theta) = \Psi(x) + \theta(s_b\Psi) + \bar\theta(s_{ab}\Psi) + \theta\bar\theta(s_b s_{ab}\Psi). \tag{7.37}
$$

This yields the original gauge-fixing and ghost part of the effective Lagrangian density as $\mathrm{Tr}\left[\frac{\partial}{\partial\theta}\left[s_b(\bar\theta)\Phi(x,\theta,\bar\theta)\right]\right]$. Hence, in the general gauge, the complete Lagrangian density for the extended BRST and anti-BRST invariant CS theory, can now be given by

$$\mathcal{L}_{gen} = \; = \; \tilde{\mathcal{L}} - \frac{1}{2}\mathrm{Tr}\left[\frac{\partial}{\partial\bar{\theta}}\frac{\partial}{\partial\theta}(\tilde{A}_\mu\tilde{A}^\mu + \tilde{\chi}\tilde{\bar{\chi}} + \tilde{B}\tilde{B})\right] + \mathrm{Tr}\left[\frac{\partial}{\partial\theta}\left[s_b(\bar{\theta})\Phi(x,\theta,\bar{\theta})\right].\right] \tag{7.38}$$

We can remove all the shifted fields from the above expression by using equations of motion for the auxiliary fields and also we can obtain the exact expressions of the anti-fields by integrating out the ghost fields for the shift symmetry.

7.6 Conclusions

The (2+1) dimensional CS theory is subject of interest because of its some intriguing as well as interacting properties. For example, the Green function for the model in axial-gauge shows the unique and exact solution of the Ward identities without reference to any action principle. In this **Chapter**, we have considered the $(2+1)$ dimensional CS theory, in both the axial-gague and the Landau-gauge and have attempted to describe the extended BRST and anti-BRST invariant (including shift symmetry) CS theory in the BV formulation. We have shown that the anti-fields arise naturally in such formulation. We have further provided the superspace and superfield description of this CS theory. We have shown that the BV Lagrangian density for such CS theory can be written in a manifestly extended BRST invariant manner in a superspace with one fermionic coordinate (θ). However, a superspace with two Grassmannian coordinates (θ) and ($\bar{\theta}$) are required for a manifestly covariant formulation of the extended BRST and extended anti-BRST invariant BV Lagrangian density for the CS theory in any arbitrary gauge.

Chapter 8

The quantum description of BF model in superspace

In this **Chapter**, we attempt to provide a superspace version of the BF model in the BV formulation. For this purpose, we first consider the BRST invariant BF model in the Landau gauge. Further, we extend the BRST symmetry of the theory by including the shift symmetry. The advantage of making such analysis is that the anti-fields get their own identifications naturally. Further, we describe the extended BRST invariant BF model in the superspace using only *one* Grassmannian coordinate together with $(3 + 1)$ space-time coordinates. However, for both the extended BRST and extended anti-BRST invariant BF model, we require two Grassmannian coordinates.

8.1 BRST invariant BF model

In this section, we discuss the preliminaries of the BF model with its BRST invariance. In this view, the BF model in $(3+1)$ flat space-time dimensions is given by the following gauge invariant Lagrangian density

$$\mathcal{L}_0 = -\frac{1}{4}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}B^a_{\rho\sigma} \tag{8.1}$$

where $B^a_{\rho\sigma}$ and $F^a_{\mu\nu}$ are the two-form field and field strength tensor for the vector field, respectively. In order to remove the discrepancy due to gauge symmetry, the gauge-fixing and ghost terms are given by

$$
\begin{aligned}
\mathcal{L}_{gf+gh} &= +(B^a\partial^\mu A^a_\mu + \bar{C}^a\partial^\mu(D_\mu C)^a + h^{a\nu}(\partial^\mu B^a_{\mu\nu}) + \omega^a\partial^\mu\xi^a_\mu + h^a_\mu(\partial^\mu e^a) \\
&+ \omega^a\lambda^a + (\partial^\mu\bar{\xi}^a_\mu)\lambda^a - (\partial^\mu\bar{\phi}^a)[(D_\mu\phi)^a + f^{abc}C^b\xi^c_\mu] \\
&- (\partial^\mu\bar{\xi}^{a\nu})[(D_\mu\xi_\nu)^a - (D_\nu\xi_\mu)^a + f^{abc}B^b_{\mu\nu}C^c] \\
&+ \frac{1}{2}f^{abc}\epsilon^{\mu\nu\rho\sigma}(\partial_\mu\bar{\xi}^a_\nu)(\partial_\rho\bar{\xi}^b_\sigma)\phi^c),
\end{aligned} \tag{8.2}
$$

where fields $(C^a, \xi^a_\mu), (\bar{C}^b\bar{\xi}^c_\mu)$ and (B^a, h^a_μ) are the ghosts, antighost and the multipliers fields, respectively, while the fields $\phi^a, \bar{\phi}^a$ and ω^a are taken into account to remove further degeneracy due to the existence of zero modes in the transformations. The effective Lagrangian density $(\mathcal{L}_0 + \mathcal{L}_{gf+gh})$ for the BF model is given by the following expression

$$
\begin{aligned}
\mathcal{L}_{eff} &= -\frac{1}{4}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}B^a_{\rho\sigma} + (B^a\partial^\mu A^a_\mu + \bar{C}^a\partial^\mu(D_\mu c)^a + h^{a\nu}(\partial^\mu B^a_{\mu\nu}) + \omega^a\partial^\mu\xi^a_\mu + h^a_\mu(\partial^\mu e^a) \\
&+ \omega^a\lambda^a + (\partial^\mu\bar{\xi}^a_\mu)\lambda^a - (\partial^\mu\bar{\phi}^a)[(D_\mu\phi)^a + f^{abc}C^b\xi^c_\mu] \\
&- (\partial^\mu\bar{\xi}^{a\nu})[(D_\mu\xi_\nu)^a - (D_\nu\xi_\mu)^a + f^{abc}B^b_{\mu\nu}C^c] \\
&+ \frac{1}{2}f^{abc}\epsilon^{\mu\nu\rho\sigma}(\partial_\mu\bar{\xi}^a_\nu)(\partial_\rho\bar{\xi}^b_\sigma)\phi^c)
\end{aligned} \tag{8.3}
$$

The Lagrangian density (with gauge-fixing and ghost term) of the BF model possesses following BRST symmetry

$$
\begin{aligned}
&s_b A^a_\mu = -(D_\mu C)^a \\
&s_b C^a = \frac{1}{2}f^{abc}C^bC^c \\
&s_b \xi^a_\mu = (D_\mu\phi)^a + f^{abc}C^b\xi^c_\mu \\
&s_b \phi^a = f^{abc}C^b\phi^c \\
&s_b B^a_{\mu\nu} = -(D_\mu\xi_\nu - D_\nu\xi_\mu)^a - f^{abc}B^b_{\mu\nu}C^c + f^{abc}\epsilon_{\mu\nu\rho\sigma}(\partial^\rho\bar{\xi}^{b\sigma})\phi^c \\
&s_b \bar{\xi}^a_\mu = h^a_\mu \\
&s_b \bar{C}^a = B^a \\
&s_b \bar{\phi}^a = \omega^a \\
&s_b e^a = \lambda^a \\
&s_b(h^a_\mu, B^a, \omega^a, \lambda^a) = 0.
\end{aligned} \tag{8.4}
$$

The Lagrangian density of the gauge-fixing and ghost terms of this model is BRST exact and, hence, can be written in terms of the BRST variation of the generic gauge-fixing fermionic field

$$\Psi = \left(\bar{C}^a \partial^\mu A^a_\mu + \bar{\xi}^{a\mu} \partial^\nu B^a_{\mu\nu} + \bar{\phi}^a \partial^\mu \xi^a_\mu + e^a \omega^a + e^a \partial^\mu \bar{\xi}^a_\mu\right), \tag{8.5}$$

as follows

$$\mathcal{L}_{gf+gh} = s_b \left(\bar{C}^a \partial^\mu A^a_\mu + \bar{\xi}^{a\nu} \partial^\mu B^a_{\mu\nu} + \bar{\phi}^a \partial^\mu \xi^a_\mu + e^a \omega^a + e^a \partial^\mu \bar{\xi}^a_\mu\right), \tag{8.6}$$

where s_b is the BRST symmetry transformations (8.4).

8.2 Extended BRST invariant Lagrangian density

In this section, we analyze the extended BRST transformations for the BF model in BV formulation. The advantage of doing so is that the anti-fields get identification naturally. We start the analysis by shifting all the fields from their original value as follows

$$\begin{aligned}
&B^a_{\mu\nu} \longrightarrow B^a_{\mu\nu} - \tilde{B}^a_{\mu\nu}, \quad A^a_\mu \longrightarrow A^a_\mu - \tilde{A}^a_\mu, \quad C^a \longrightarrow C^a - \tilde{C}^a, \\
&\bar{C}^a \longrightarrow \bar{C}^a - \tilde{\bar{C}}^a, \quad B^a \longrightarrow B^a - \tilde{B}^a, \quad \xi^a_\mu \longrightarrow \xi^a_\mu - \tilde{\xi}^a_\mu, \\
&\bar{\xi}^a_\mu \longrightarrow \bar{\xi}^a_\mu - \tilde{\bar{\xi}}^a_\mu, \quad \phi^a \longrightarrow \phi^a - \tilde{\phi}^a, \quad \bar{\phi}^a \longrightarrow \bar{\phi}^a - \tilde{\bar{\phi}}^a, \\
&h^a_\mu \longrightarrow h^a_\mu - \tilde{h}^a_\mu, \quad e^a \longrightarrow e^a - \tilde{e}^a, \quad \omega^a \longrightarrow \omega^a - \tilde{\omega}^a, \\
&\lambda^a \longrightarrow \lambda^a - \tilde{\lambda}^a.
\end{aligned} \tag{8.7}$$

The Lagrangian density of the BF model also gets shifted under such shifting of the fields, respectively, and is given by:

$$\begin{aligned}\tilde{\mathcal{L}} &= \mathcal{L}(A_\mu^a - \tilde{A}_\mu^a, \quad C^a - \tilde{C}^a, \quad \bar{C}^a - \tilde{\bar{C}}^a, \quad B^a - \tilde{B}^a, \\ &\quad \xi_\mu^a - \tilde{\xi}_\mu^a, \quad \bar{\xi}_\mu^a - \tilde{\bar{\xi}}_\mu^a, \quad \phi^a - \tilde{\phi}^a, \quad \bar{\phi}^a - \tilde{\bar{\phi}}^a, \\ &\quad h_\mu^a - \tilde{h}_\mu^a, \quad e^a - \tilde{e}^a, \quad \omega^a - \tilde{\omega}^a, \quad \lambda^a - \tilde{\lambda}^a). \end{aligned} \tag{8.8}$$

The shifted Lagrangian density (8.8) remains invariant under the following extended BRST symmetry transformations

$$\begin{aligned} sA_\mu^a &= \psi_\mu^a, \\ s\tilde{A}_\mu^a &= \psi_\mu^a - D_\mu(C - \tilde{C})^a, \\ sC^a &= \epsilon^a, \\ s\tilde{C}^a &= \epsilon^a - \frac{1}{2} f^{abc} C^b \xi_\mu^a, \\ s\bar{C}^a &= \bar{\epsilon}^a, \\ s\tilde{\bar{C}}^a &= \bar{\epsilon}^a - (B - \tilde{B})^a, \\ sB^a &= \chi^a, \\ s\tilde{B}^a &= \chi^a, \\ s\phi^a &= M^a, \\ s\tilde{\phi}^a &= M^a - f^{abc} C^b \phi^c, \\ s\bar{\phi}^a &= \bar{M}^a, \\ s\tilde{\bar{\phi}}^a &= M^a - \omega^a, \\ se^a &= N^a, \\ s\tilde{e}^a &= N^a - \lambda^a, \\ s\xi_\mu^a &= L_\mu^a, \\ s\tilde{\xi}_\mu^a &= L_\mu^a - [(D_\mu \phi)^a + f^{abc} C^b \xi_\mu^c], \\ s\bar{\xi}_\mu^a &= \bar{L}_\mu^a, \\ s\tilde{\bar{\xi}}_\mu^a &= L_\mu^a - h_\mu^a, \end{aligned} \tag{8.9}$$

where $\psi_\mu^a, \epsilon^a, \bar{\epsilon}^a, \chi^a, M^a, \bar{M}^a, N^a, L_\mu^a$, and $\bar{L}_\mu^a$ are the ghost fields corresponding to the shift symmetry for $A_\mu^a, C^a, \bar{C}^a, B^a, \phi^a, \bar{\phi}^a, e^a, \xi_\mu^a$ and $\bar{\xi}_\mu^a$, respectively. The nilpotency of the extended BRST symmetry (8.9), leads to the BRST transformation for the ghost field as:

$$
\begin{aligned}
s\psi_\mu^a &= 0, \\
s\epsilon^a &= 0, \\
s\bar{\epsilon}^a &= 0, \\
s\chi^a &= 0, \\
sM^a &= 0, \\
s\bar{M}^a &= 0, \\
sN^a &= 0, \\
sL_\mu^a &= 0, \\
s\bar{L}_\mu^a &= 0.
\end{aligned} \tag{8.10}
$$

In order to make the present BRST-invariant theory ghost-free, we introduce the anti-fields $A_\mu^{\star a}, C^{\star a}, \bar{C}^{\star a}, B^{\star a}, \xi_\mu^{\star a}, \bar{\xi}_\mu^{\star a}, \phi^{\star a}, \bar{\phi}^{\star a}$, and $e^{\star a}$ corresponding to the ghost fields $\psi_\mu^a, \epsilon^a, \bar{\epsilon}^a, \chi^a, M^a, \bar{M}^a, N^a, L_\mu^a$ and $\bar{L}_\mu^a$, respectively. The BRST transformations of these anti-fields are

$$
\begin{aligned}
sA_\mu^{\star a} &= -\zeta_\mu^a, \\
sC^{\star a} &= -\sigma^a, \\
s\bar{C}^{\star a} &= -\bar{\sigma}^a, \\
sB^{\star a} &= -\varpi^a, \\
s\phi^{\star a} &= -\upsilon^a, \\
s\bar{\phi}^{\star a} &= -\bar{\upsilon}^a, \\
se^{\star a} &= -\tau^a, \\
s\xi_\mu^{\star a} &= -\kappa_\mu^a, \\
s\bar{\xi}_\mu^{\star a} &= -\bar{\kappa}_\mu^a,
\end{aligned} \tag{8.11}
$$

where $\zeta^a_\mu, \sigma^a, \bar{\sigma}^a$ and $\bar{\upsilon}^a$ are the Nakanishi-Lautrup type auxiliary fields corresponding to the shifted fields $A^{\star a}_\mu, C^{\star a}, \bar{C}^{\star a}, B^{\star a}, \phi^{\star a}, \bar{\phi}^{\star a}, e^{\star a}, \xi^{\star a}_\mu$, and $\bar{\xi}^{\star a}_\mu$, respectively. These auxiliary fields possesses the following BRST transformations

$$\begin{aligned} s\zeta^a_\mu &= 0, \\ s\sigma^a &= 0, \\ s\bar{\sigma}^a &= 0, \\ s\varpi^a &= 0, \\ s\upsilon^a &= 0, \\ s\bar{\upsilon}^a &= 0, \\ s\tau^a &= 0, \\ s\kappa^a_\mu &= 0, \\ s\bar{\kappa}^a_\mu &= 0. \end{aligned} \tag{8.12}$$

We can recover our original Lagrangian density for the BF model by adding the gauge-fixing Lagrangian density corresponding to the shift symmetry in such a way such that all the tilde fields vanish. We achieve this by adding the following gauge-fixed term in the shifted Lagrangian density (8.8):

$$\begin{aligned} \tilde{\mathcal{L}}_{gf+gh} &= \Big[-\zeta^{\mu a}\tilde{A}^a_\mu - A^{\star a}_\mu[\psi^{\mu a} - D^\mu(C-\tilde{C})^a] - \sigma^a\tilde{C}^a + C^{\star a}[\epsilon^a - \frac{1}{2}f^{abc}C^b\xi^c_\mu] \\ &- \bar{\sigma}^a\tilde{\bar{C}}^a + \bar{C}^{\star a}[\bar{\epsilon}^a - (B^a - \tilde{B}^a)] - \upsilon^a\tilde{\phi}^a - \phi^{\star a}(M^a - f^{abc}C^b\phi^c_\mu) - \bar{\upsilon}^a\tilde{\bar{\phi}}^a \\ &- \bar{\phi}^{\star a}(M^a - \omega^a) - \tau^a\tilde{e}^a + e^{\star a}(N^a - \lambda^a) - \varpi^a\tilde{B}^a + B^{\star a}\xi^a - \kappa^{a\mu}\tilde{\xi}^a_\mu \\ &+ \xi^{\star a\mu}(L^a_\mu - [D_\mu\phi + f^{abc}C^b\xi^c_\mu]) - \bar{\kappa}^{a\mu}\tilde{\bar{\xi}}^a_\mu + \bar{\xi}^{\star a\mu}(\bar{L}^a_\mu - h^a_\mu)\Big]. \end{aligned} \tag{8.13}$$

The Lagrangian density $\tilde{\mathcal{L}}_{gf+gh}$ given, in the above equation (8.13), is also invariant under the extended BRST symmetry transformations. If we perform equations of motion of auxiliary fields in the above expression, we obtain

$$
\begin{aligned}
\tilde{\mathcal{L}}_{gf+gh} &= \Big[-A^{\star a}_{\mu}[\psi^{\mu a} - D^{\mu}(C-\tilde{C})^{a}] - +C^{\star a}[\epsilon^{a} - \frac{1}{2}f^{abc}C^{b}\xi^{c}_{\mu}] \\
&- \bar{C}^{\star a}[\bar{\epsilon}^{a} - (B^{a} - \tilde{B}^{a})] - \phi^{\star a}(M^{a} - f^{abc}C^{b}\phi^{c}_{\mu}) \\
&- \bar{\phi}^{\star a}(M^{a} - \omega^{a}) + e^{\star a}(N^{a} - \lambda^{a}) + B^{\star a}\xi^{a} \\
&+ \xi^{\star a\mu}(L^{a}_{\mu} - [D_{\mu}\phi^{a} + f^{abc}C^{b}\xi^{c}_{\mu}]) + \bar{\xi}^{\star a\mu}(\bar{L}^{a}_{\mu} - h^{a}_{\mu})\Big]\,. \qquad (8.14)
\end{aligned}
$$

The gauge-fixing and FP-ghost terms of the Lagrangian density are BRST exact and it can be expressed in terms of a general gauge-fixing fermion Ψ as

$$
\begin{aligned}
(s_b\Psi) &= \Big[s_bA^{a}_{\mu}\frac{\delta\Psi}{\delta A^{a}_{\mu}} + s_bC^{a}\frac{\delta\Psi}{\delta C^{a}} + s_b\bar{C}^{a}\frac{\delta\Psi}{\delta\bar{C}^{a}} + s_bB^{a}\frac{\delta\Psi}{\delta B^{a}} + s_b\xi^{a}_{\mu}\frac{\delta\Psi}{\delta\xi^{a}_{\mu}} + s_b\bar{\xi}^{a}_{\mu}\frac{\delta\Psi}{\delta\bar{\xi}^{a}_{\mu}} \\
&+ s_b\phi^{a}\frac{\delta\Psi}{\delta\phi^{a}} + s_b\bar{\phi}^{a}\frac{\delta\Psi}{\delta\bar{\phi}^{a}} + s_be^{a}\frac{\delta\Psi}{\delta e^{a}}\Big], \\
&= \Big[-\frac{\delta\Psi}{\delta A^{a}_{\mu}}\psi^{a}_{\mu} + \frac{\delta\Psi}{\delta C^{a}}\epsilon^{a} + \frac{\delta\Psi}{\delta\bar{C}^{a}}\bar{\epsilon}^{a} - \frac{\delta\Psi}{\delta B^{a}}\chi^{a} - \frac{\delta\Psi}{\delta\xi^{a}_{\mu}}L^{a}_{\mu} - \frac{\delta\Psi}{\delta\bar{\xi}^{a}_{\mu}}\bar{L}^{a}_{\mu} \\
&- \frac{\delta\Psi}{\delta\phi^{a}}M^{a} - \frac{\delta\Psi}{\delta\bar{\phi}^{a}}\bar{M}^{a} - \frac{\delta\Psi}{\delta e^{a}}N^{a}\Big], \qquad (8.15)
\end{aligned}
$$

After integrating out the auxiliary fields which set the tilde fields to zero, we have the following effective Lagrangian density for the BF model in the Landau-gauge

$$
\begin{aligned}
\mathcal{L}_{eff} &= \tilde{\mathcal{L}} + \mathcal{L}_{gf+gh} + \tilde{\mathcal{L}}_{gf+gh}, \\
&= \tilde{\mathcal{L}} + \Big[\Big(-A^{\star a}_{\mu} - \frac{\delta\Psi}{\delta A^{\mu a}}\Big)\psi^{\mu a} + \Big(\bar{C}^{\star a} + \frac{\delta\Psi}{\delta C^{a}}\Big)\epsilon^{a} + \Big(C^{\star a} + \frac{\delta\Psi}{\delta\bar{C}^{a}}\Big)\bar{\epsilon}^{a} \\
&+ \Big(B^{\star a} + \frac{\delta\Psi}{\delta B^{a}}\Big)\bar{\chi}^{a} + \Big(\xi^{\star a}_{\mu} + \frac{\delta\Psi}{\delta\xi^{a}_{\mu}}\Big)L^{a}_{\mu} + \Big(\bar{\xi}^{\star a}_{\mu} + \frac{\delta\Psi}{\delta\bar{\xi}^{a}_{\mu}}\Big)\bar{L}^{a}_{\mu} \\
&+ \Big(\phi^{\star a} + \frac{\delta\Psi}{\delta\phi^{a}}\Big)M^{a} + \Big(\bar{\phi}^{\star a} + \frac{\delta\Psi}{\delta\bar{\phi}^{a}}\Big)\bar{M}^{a} + \Big(-e^{\star a} - \frac{\delta\Psi}{\delta e^{a}}\Big)N^{a} \\
&+ A^{\star a}_{\mu}D^{\mu}C^{a} - \frac{C^{\star a}}{2}f^{abc}C^{b}\xi^{c}_{\mu} + C^{\star a}B^{a} + \xi^{\star a}_{\mu}[(D_{\mu}\phi)^{a} + f^{abc}C^{b}\xi^{c}_{\mu}] \\
&+ \phi^{\star a}f^{abc}C^{b}\xi^{c}_{\mu}\Big], \qquad (8.16)
\end{aligned}
$$

where Ψ refers the gauge-fixing fermion corresponding to the Landau-gauge. Integrating out the ghost fields associated with shift symmetry, we obtain

$$\begin{aligned}
A_\mu^{a\star} &= \frac{\delta\Psi}{\delta A^{\mu a}}, \\
\bar{C}^{\star a} &= -\frac{\delta\Psi}{\delta C^a}, \\
C^{\star a} &= -\frac{\delta\Psi}{\delta \bar{C}^a}, \\
B^{\star a} &= \frac{\delta\Psi}{\delta B^a}, \\
\xi_\mu^{\star a} &= \frac{\delta\Psi}{\delta \xi_\mu^{\star a}}, \\
\bar{\xi}_\mu^{\star a} &= \frac{\delta\Psi}{\delta \bar{\xi}_\mu^{\star a}}, \\
\phi^{\star a} &= \frac{\delta\Psi}{\delta \phi^a}, \\
\bar{\phi}^{\star a} &= \frac{\delta\Psi}{\delta \bar{\phi}^a}, \\
e^{\star a} &= \frac{\delta\Psi}{\delta e^a}. \qquad (8.17)
\end{aligned}$$

For a particular choice of the gauge-fixing fermion Ψ, corresponding to the Landau-gauge given in (8.5), the expression of anti-ghost fields yield

$$\begin{aligned}
A_\mu^{\star a} &= \partial_\mu \bar{C}^a, \\
\bar{C}^{\star a} &= \partial_\mu A^{\mu a}, \\
C^{\star a} &= 0 \\
B^{\star a} &= 0, \\
\xi_\mu^{\star a} &= \partial_\mu \bar{\phi}^a, \\
\bar{\xi}_\mu^{\star a} &= -\partial^\nu B_{\mu\nu}^a - \partial_m u e^a, \\
\phi^{\star a} &= 0, \\
\bar{\phi}^{\star a} &= -\partial^\mu \xi_\mu^a, \\
e^{\star a} &= \omega^a + \partial^\mu \xi_\mu^a. \qquad (8.18)
\end{aligned}$$

Plugging these expressions of the anti-ghost fields in (8.16), we recover the Lagrangian density of our original BF model in the Landau-gauge.

8.3 Extended BRST invariant superspace description

In this section, the Lagrangian density of the BF model, which is invariant under the extended BRST transformation, is described in a superspace by including one extra Grassmannian coordinates θ in the (3+1) space-time dimensions. In order to give superspace description for the extended BRST invariant theory, we fist define the superfields of the form:

$$
\begin{aligned}
A^a_\mu(x,\theta) &= A^a_\mu + \theta\psi^a_\mu,\\
\tilde{A}^a_\mu(x,\theta) &= \tilde{A}^a_\mu + \theta[\psi_\mu - D_\mu(C-\tilde{C})]^a,\\
\chi^a(x,\theta) &= C^a + \theta\epsilon^a,\\
\tilde{\chi}^a(x,\theta) &= \tilde{C}^a + \theta[\epsilon^a - \frac{1}{2}f^{abc}C^b\xi^c_\mu],\\
\bar{\chi}^a(x,\theta) &= \bar{C}^a + \theta\bar{\epsilon}^a,\\
\tilde{\bar{\chi}}^a(x,\theta) &= \tilde{\bar{C}}^a + \theta[\bar{\epsilon}^a - (B-\tilde{B})^a],\\
B^a(x,\theta) &= B^a + \theta\chi^a,\\
\tilde{B}^a(x,\theta) &= \tilde{B}^a + \theta\chi^a,\\
\xi^a_\mu(x,\theta) &= \xi^a_\mu + \theta L^a_\mu,\\
\tilde{\xi}^a_\mu(x,\theta) &= \tilde{\xi}^a_\mu + \theta[L^a_\mu - (D_\mu\phi^a + f^{abc}C^b\xi^c_\mu)],\\
\bar{\xi}^a_\mu(x,\theta) &= \bar{\xi}^a_\mu + \theta\bar{L}^a_\mu,\\
\tilde{\bar{\xi}}^a_\mu(x,\theta) &= \tilde{\bar{\xi}}^a_\mu + \theta(\bar{L}^a_\mu - h^a_\mu),\\
\phi^a(x,\theta) &= \phi^a + \theta M^a,\\
\tilde{\phi}^a(x,\theta) &= \tilde{\phi}^a + \theta(M^a - f^{abc}C^b\phi^c),\\
\bar{\phi}^a(x,\theta) &= \bar{\phi}^a + \theta\bar{M}^a,\\
\tilde{\bar{\phi}}^a(x,\theta) &= \tilde{\bar{\phi}}^a + \theta(\bar{M}^a - \omega^a),\\
e^a(x,\theta) &= e^a + \theta N^a,\\
\tilde{e}^a(x,\theta) &= \tilde{e}^a + \theta(N^a - \lambda^a).
\end{aligned}
\tag{8.19}
$$

The super anti-fields in superspace are given as follows

$$\begin{aligned}
\tilde{A}^{\star a}_{\mu}(x,\theta) &= \tilde{A}^{\star a}_{\mu} - \theta\zeta^{a}_{\mu}, \\
\tilde{\chi}^{\star a}(x,\theta) &= \tilde{C^{\star a}} - \theta\sigma^{a}, \\
\tilde{\bar{\chi}}^{\star a}(x,\theta) &= \tilde{\bar{C}^{\star a}} - \theta\bar{\sigma}^{a}, \\
\tilde{B}^{\star a}(x,\theta) &= \tilde{B}^{\star a} - \theta\varpi^{a}, \\
\tilde{\xi}^{\star a}_{\mu}(x,\theta) &= \tilde{\xi}^{\star a}_{\mu} - \theta\kappa^{a}_{\mu}, \\
\tilde{\bar{\xi}}^{\star a}_{\mu}(x,\theta) &= \tilde{\bar{\xi}}^{\star a}_{\mu} - \bar{\kappa}^{a}_{\mu}, \\
\tilde{\phi}^{\star a}(x,\theta) &= \tilde{\phi}^{\star a} - \theta\upsilon^{a}, \\
\tilde{\bar{\phi}}^{\star a}(x,\theta) &= \tilde{\bar{\phi}}^{\star a} - \theta\bar{\upsilon}^{a}, \\
\tilde{e}^{\star a}(x,\theta) &= \tilde{e}^{\star a} - \theta\tau^{a}. \qquad (8.20)
\end{aligned}$$

We obtain the following expressions with the help of these superfields and super-anti-fields

$$\begin{aligned}
\frac{\delta(\tilde{A}^{\star a}_{\mu}\tilde{A}^{\mu a})}{\delta\theta} &= -A^{\star a}_{\mu}[\psi^{\mu a} - D^{\mu}(C-\tilde{C})^{a}] - \zeta^{a}_{\mu}\tilde{A}^{\mu a}, \\
\frac{\delta(\tilde{\bar{\chi}}^{\star a}\tilde{\chi}^{a})}{\delta\theta} &= C^{\star a}[\epsilon^{a} - \frac{1}{2}f^{abc}C^{b}\xi^{c}_{\mu}] - \sigma^{a}\tilde{C}^{a}, \\
\frac{\delta(\tilde{\bar{\chi}}^{a}\tilde{\chi}^{\star a})}{\delta\theta} &= -\bar{\sigma}^{a}\tilde{\bar{C}}^{a} + \bar{C}^{\star a}[\bar{\epsilon}^{a} - (B-\tilde{B})^{a}], \\
\frac{\delta(\tilde{B}^{\star a}\tilde{B}^{a})}{\delta\theta} &= -B^{\star a}\chi^{a} - \varpi^{a}\tilde{B}^{a}, \\
\frac{\delta(\tilde{\xi^{a\star}_{\mu}}\tilde{\chi}^{\mu a})}{\delta\theta} &= +\xi^{\star a}_{\mu}[L^{a}_{\mu} - (D^{\mu}\phi^{a} + f^{abc}C^{b}\xi^{c}_{\mu})] - \kappa^{\mu a}\tilde{\xi}^{a}_{\mu}, \\
\frac{\delta(\tilde{\bar{\xi}}^{\star a}_{\mu}\tilde{\bar{\xi}}^{\mu a})}{\delta\theta} &= -\bar{\xi}^{a\star}_{\mu}[L^{\mu a} - L^{\mu a}] - \bar{\kappa}^{a}_{\mu}\tilde{\bar{\xi}}^{\mu a}, \\
\frac{\delta(\tilde{\phi}^{\star a}\tilde{\phi}^{a})}{\delta\theta} &= -\tilde{\phi}^{a}\upsilon^{a} + \phi^{\star a}(M^{a} - f^{abc}C^{b}\phi^{c}), \\
\frac{\delta(\tilde{\bar{\phi}}^{\star a}\tilde{\bar{\phi}}^{a}}{\delta\theta} &= -\tilde{\bar{\phi}}^{a}\bar{\upsilon}^{a} + \bar{\phi}^{\star a}(M^{a} - \omega^{a}), \\
\frac{\delta(\tilde{e}^{\star a}\tilde{e}^{a})}{\delta\theta} &= e^{\star a}[N^{a} - \lambda^{a}] - \tilde{e}^{a}\tau^{a}. \qquad (8.21)
\end{aligned}$$

Combining all the equation given in (8.21), we find that

$$\begin{aligned}
& \frac{\delta}{\delta\theta}(\tilde{A}^{a\star}_{\mu}\tilde{A}^{a\mu} + \tilde{\bar{\chi}}^{a\star}\tilde{\chi}^{a} + \tilde{\bar{\chi}}^{a}\tilde{\chi}^{a\star} + \tilde{B}^{a\star}\tilde{B}^{a} + \tilde{\xi}^{a\star}_{\mu}\tilde{\xi}^{a\mu} + \tilde{\bar{\xi}}^{a\star}_{\mu}\tilde{\bar{\xi}}^{a\mu} + \tilde{\phi}^{a\star}\tilde{\phi}^{a} + \tilde{\bar{\phi}}^{a\star}\tilde{\bar{\phi}}^{a} + \tilde{e}^{a\star}\tilde{e}^{a}) \\
& = -\zeta^{a\mu}\tilde{A}^{a}_{\mu} - A^{a\star}_{\mu}[\psi^{a\mu} - (D^{\mu} - \tilde{D}^{\mu})(C - \tilde{C})^{a}] - \bar{\sigma}^{a}\tilde{C}^{a} - \sigma^{a}\tilde{\bar{C}}^{a} - \upsilon^{a}\tilde{\phi}^{a} \\
+ \;& \bar{C}^{a\star}[\epsilon^{a} - \frac{1}{2}f^{abc}(C^{b} - \tilde{C}^{b})(\xi^{a}_{\mu} - \tilde{\xi}^{a}_{\mu})] + C^{a\star}[\bar{\epsilon}^{a} - (B^{a} - \tilde{B}^{a})] - \tau^{a}\tilde{e}^{a} \\
- \;& \phi^{\star a}\left[M^{a} - f^{abc}(C^{b} - \tilde{C}^{b})(\phi^{c} - \tilde{\phi}^{c})\right] - \bar{\upsilon}^{a}\tilde{\bar{\phi}}^{a} - \bar{\phi}^{\star a}(\bar{M}^{a} - \omega^{a} + \tilde{\omega}^{a}) \\
- \;& e^{\star a}[N^{a} - \lambda^{a} + \tilde{\lambda}^{a}] - \varpi^{a}\tilde{B}^{a} - B^{\star a}\chi^{a} - \kappa^{a\mu}\tilde{\xi}^{a}_{\mu} - \bar{\kappa}^{a\mu}\tilde{\bar{\xi}}^{a}_{\mu} + \bar{\xi}^{\star a\mu}\left[\bar{L}^{a}_{\mu} - (h^{a}_{\mu} - \tilde{h}^{a}_{\mu})\right] \\
+ \;& \xi^{\star a\mu}\left(L^{a}_{\mu} - \left[(D_{\mu} - \tilde{D}_{\mu})(\phi - \tilde{\phi})^{a} + f^{abc}(C^{b} - \tilde{C}^{b})(\xi^{c}_{\mu} - \tilde{\xi}^{c}_{\mu})\right]\right), \qquad (8.22)
\end{aligned}$$

which is nothing but the shifted gauge-fixed Lagrangian density $\tilde{\mathcal{L}}_{gf+gh}$ given in (8.13). Now, we define the general super-gauge-fixing fermion written in superspace formulation as follows

$$\Phi(x,\theta) = \Psi(x) + \theta(s\Psi), \qquad (8.23)$$

which can further be expressed as:

$$\begin{aligned}
\Phi(x,\theta) \;=\; & \Psi(x) + \theta\left[-\frac{\delta\Psi}{\delta A^{a}_{\mu}}\psi^{a}_{\mu} + \frac{\delta\Psi}{\delta C^{a}}\epsilon^{a} + \frac{\delta\Psi}{\delta\bar{C}^{a}}\bar{\epsilon}^{a} - \frac{\delta\Psi}{\delta B^{a}}\chi^{a} - \frac{\delta\Psi}{\delta\xi^{a}_{\mu}}L^{a}_{\mu}\right. \\
- \;& \left.\frac{\delta\Psi}{\delta\bar{\xi}^{a}_{\mu}}\bar{L}^{a}_{\mu} - \frac{\delta\Psi}{\delta\phi^{a}}M^{a} - \frac{\delta\Psi}{\delta\bar{\phi}^{a}}\bar{M}^{a} - \frac{\delta\Psi}{\delta e^{a}}N^{a}\right]. \qquad (8.24)
\end{aligned}$$

So, the original gauge-fixed Lagrangian density in the superspace can be defined as the left derivative of the super-gauge-fixing fermion with respect to θ as $\left[\frac{\delta\Phi(x,\theta)}{\delta\theta}\right]$. Hence, the effective Lagrangian density for the BF model, in general gauge in the superspace, is now given by

$$\begin{aligned}
\mathcal{L}_{gen} \;=\; & \tilde{\mathcal{L}} + \left[\frac{\delta}{\delta\theta}(\tilde{A}^{\star a}_{\mu}\tilde{A}^{\mu a} + \tilde{\bar{\chi}}^{\star a}\tilde{\chi}^{a} + \tilde{\bar{\chi}}^{a}\tilde{\chi}^{\star a} + \tilde{B}^{\star a}\tilde{B}^{a} + \tilde{\chi}^{\star a}_{\mu}\tilde{\chi}^{\mu a} + \tilde{\bar{\chi}}^{\star a}_{\mu}\tilde{\bar{\chi}}^{\mu a}\right. \\
+ \;& \left.\tilde{\phi}^{\star a}\tilde{\phi}^{a} + \tilde{\bar{\phi}}^{\star a}\tilde{\bar{\phi}}^{a} + \tilde{e}^{\star a}\tilde{e}^{a}) + \Phi)\right]. \qquad (8.25)
\end{aligned}$$

This compact expression indicates that the BV action of the extended BF model, in superspace, is invariant under the extended BRST transformation.

8.4 Extended anti-BRST Lagrangian density

In this section, we construct the extended anti-BRST transformation under which the extended Lagrangian density remains invariant as follows

$$
\begin{aligned}
s_{ab}A_\mu^a &= A_\mu^{\star a} + D_\mu(\bar{C} - \tilde{\bar{C}})^a, \\
s_{ab}\tilde{A}_\mu^a &= A_\mu^{\star a}, \\
s_{ab}C^a &= C^{\star a} - \frac{1}{2} f^{abc} C^b \xi_\mu^c, \\
s_{ab}\tilde{C}^a &= C^{\star a}, \\
s_{ab}\bar{C}^a &= \bar{C}^{\star a} - (B - \tilde{B})^a, \\
s_{ab}\tilde{\bar{C}}^a &= \bar{C}^{\star a}, \\
s_{ab}\tilde{B}^a &= B^{\star a}, \\
s_{ab}B^a &= B^{\star a} + \chi^a \\
s_{ab}\xi_\mu^a &= \chi_\mu^{\star a} - [D_\mu\phi^a + f^{abc} C^b \xi_\mu^c], \\
s_{ab}\tilde{\xi}_\mu^a &= \xi_\mu^{\star a}, \\
s_{ab}\bar{\xi}_\mu^a &= \bar{\xi}_\mu^{\star a} - h_\mu^a, \\
s_{ab}\tilde{\bar{\xi}}_\mu^a &= \bar{\xi}_\mu^{\star a}, \\
s_{ab}\phi^a &= \phi^{\star a} - f^{abc} C^b \phi^c, \\
s_{ab}\tilde{\phi}^a &= \phi^{\star a}, \\
s_{ab}\bar{\phi}^a &= \bar{\phi}^{\star a} - \omega^a, \\
s_{ab}\tilde{\bar{\phi}}^a &= \bar{\phi}^{\star a}, \\
s_{ab}e^a &= e^{\star a} - \lambda^a, \\
s_{ab}\tilde{e}^a &= e^{\star a}.
\end{aligned} \tag{8.26}
$$

The ghost fields associated with the shift symmetry, under the extended anti-BRST symmetry, transforms as

$$
\begin{aligned}
\bar{s}\psi_\mu^a &= \zeta_\mu^a, \\
\bar{s}\epsilon^a &= \sigma^a, \\
\bar{s}\bar{\epsilon}^a &= \bar{\sigma}^a, \\
\bar{s}\chi^a &= \varpi^a, \\
\bar{s}L_\mu^a &= \kappa_\mu^a, \\
\bar{s}\bar{L}_\mu^a &= \bar{\kappa}_\mu^a, \\
\bar{s}M^a &= \upsilon^a, \\
\bar{s}\bar{M}^a &= \bar{\upsilon}^a, \\
\bar{s}N^a &= \tau^a.
\end{aligned}
\tag{8.27}
$$

The nilpotency of above transformations demands the auxiliary and anti-ghost fields associated with the shift symmetry transform as

$$
\begin{aligned}
\bar{s}\zeta_\mu^a = 0, &\quad \bar{s}A_\mu^{\star a} = 0, \\
\bar{s}\sigma^a = 0, &\quad \bar{s}C^{\star a} = 0, \\
\bar{s}\bar{\sigma}^a = 0, &\quad \bar{s}\bar{C}^{\star a} = 0, \\
\bar{s}\varpi^a = 0, &\quad \bar{s}B^{\star a} = 0, \\
\bar{s}\kappa_\mu^a = 0, &\quad \bar{s}\xi_\mu^{\star a} = 0, \\
\bar{s}\bar{\kappa}_\mu^a = 0, &\quad \bar{s}\bar{\xi}_\mu^{\star a} = 0, \\
\bar{s}\upsilon^a = 0, &\quad \bar{s}\phi^{\star a} = 0, \\
\bar{s}\bar{\upsilon}^a = 0, &\quad \bar{s}\bar{\phi}^{\star a} = 0, \\
\bar{s}\tau^a = 0, &\quad \bar{s}e^{\star a} = 0.
\end{aligned}
\tag{8.28}
$$

The anti-gauge-fixing fermions for the BF model in the Landau-gauge ($\bar{\Psi}$) is defined by

$$\bar{\Psi} = \eta^{\mu\nu} C n_\mu A_\nu. \tag{8.29}$$

The anti-BRST variation of the gauge-fixing fermion gives the corresponding gauge-fixing and ghost parts of the effective Lagrangian density.

8.5 Extended BRST and anti-BRST invariant superspace

The extended BRST and anti-BRST invariant Lagrangian density can be written in superspace with the help of two Grassmannian coordinates θ and $\bar{\theta}$. Requiring the field strength to vanish along the unphysical direction θ and $\bar{\theta}$. We have the super expansions along $(\theta, \bar{\theta})$ directions of the following superfields

$$\begin{aligned}
A^a_\mu(x,\theta,\bar{\theta}) &= A^a_\mu(x) + \theta\psi^a_\mu + \bar{\theta}[A^{\star a}_\mu + D_\mu \bar{C}]^a + \theta\bar{\theta}[\zeta^a_\mu + \partial_\mu \epsilon^a], \\
\tilde{A}^a_\mu(x,\theta,\bar{\theta}) &= \tilde{A}^a_\mu(x) + \theta[\psi^a_\mu - D_\mu(C - \tilde{C})^a] + \bar{\theta} A^{a\star}_\mu + \theta\bar{\theta}\zeta^a_\mu, \\
C^a(x,\theta,\bar{\theta}) &= C^a(x) + \theta\epsilon^a + \bar{\theta}[C^{\star a} - \frac{1}{2} f^{abc} C^b \xi^c_\mu] + \theta\bar{\theta}\sigma^a, \\
\tilde{C}^a(x,\theta,\bar{\theta}) &= \tilde{C}^a(x) + \theta[\epsilon^a - \frac{1}{2} f^{abc} C^b \xi^c_\mu] + \bar{\theta} C^{\star a} + \theta\bar{\theta}\sigma^a, \\
\bar{\chi}^a(x,\theta,\bar{\theta}) &= \bar{C}^a(x) + \theta\bar{\epsilon}^a + \bar{\theta}[\bar{C}^{\star a} - (B - \tilde{B})^a] + \theta\bar{\theta}\bar{\sigma}^a, \\
\tilde{\bar{\chi}}^a(x,\theta,\bar{\theta}) &= \tilde{\bar{C}}^a(x) + \theta[\bar{\epsilon}^a - (B - \tilde{B})^a] + \bar{\theta}\bar{C}^{\star a} + \theta\bar{\theta}\bar{\sigma}^a, \\
B^a(x,\theta,\bar{\theta}) &= B^a(x) + \theta\chi^a + \bar{\theta}(B^{\star a} + \chi^a) + \theta\bar{\theta}\varpi^a, \\
\tilde{B}^a(x,\theta,\bar{\theta}) &= \tilde{B}^a(x) + \theta\chi^a + \bar{\theta} B^{\star a} + \theta\bar{\theta}\varpi^a, \\
\xi^a_\mu(x,\theta,\bar{\theta}) &= \xi^a_\mu(x) + \theta L^a_\mu + \bar{\theta}[\xi^{\star a}_\mu - (D_\mu\phi + f^{abc} C^b \xi^c_\mu)] + \theta\bar{\theta}\xi^{\star a}_\mu, \\
\tilde{\xi}^a_\mu(x,\theta,\bar{\theta}) &= \tilde{\xi}^a_\mu(x) + \theta[\xi^{\star a}_\mu - (D_\mu\phi + f^{abc} C^b \xi^c_\mu)] + \bar{\theta}\xi^\star_\mu + \theta\bar{\theta}\kappa^a_\mu, \\
\bar{\xi}^a_\mu(x,\theta,\bar{\theta}) &= \bar{\xi}^a_\mu(x) + \theta\bar{L}^a_\mu + \bar{\theta}(\bar{\xi}^{\star a}_\mu - h^a_\mu) + \theta\bar{\theta}\bar{\kappa}^a_\mu, \\
\tilde{\bar{\xi}}^a_\mu(x,\theta,\bar{\theta}) &= \tilde{\bar{\xi}}^a_\mu(x) + \theta(\bar{L}^a_\mu - h^a_\mu) + \bar{\theta}\bar{\xi}^{\star a}_\mu + \theta\bar{\theta}\bar{\kappa}^a_\mu,
\end{aligned}$$

$$\begin{aligned}
\phi^a(x,\theta,\bar\theta) &= \phi^a(x) + \theta M^a + \bar\theta(\phi^{\star a} - f^{abc}C^b\phi^c) + \theta\bar\theta\upsilon^a, \\
\tilde\phi^a(x,\theta,\bar\theta) &= \tilde\phi^a(x) + \theta(M^a - f^{abc}C^b\phi^c) + \bar\theta\phi^{\star a} + \theta\bar\theta\upsilon^a, \\
\bar\phi^a(x,\theta,\bar\theta) &= \bar\phi^a(x) + \theta\bar M^a + \bar\theta(\bar\phi^{\star a} - \omega^a) + \theta\bar\theta\bar\upsilon^a, \\
\tilde{\bar\phi}^a(x,\theta,\bar\theta) &= \tilde{\bar\phi}^a(x) + \theta(\bar M^a - \omega^a) + \bar\theta\bar\phi^{\star a} + \theta\bar\theta\bar\upsilon^a, \\
e^a(x,\theta,\bar\theta) &= e^a(x) + \theta N^a + \bar\theta(e^{\star a} - \lambda^a) + \theta\bar\theta\tau, \\
\tilde e^a(x,\theta,\bar\theta) &= \tilde e^a(x) + \theta(N^a - \lambda^a) + \bar\theta e^{\star a} + \theta\bar\theta\tau^a.
\end{aligned} \tag{8.30}$$

With the help of above expression (8.30) of superfields, we are able to establish the following relation

$$\begin{aligned}
&- \frac{1}{2}\left[\frac{\partial}{\partial\bar\theta}\frac{\partial}{\partial\theta}(\tilde A^a_\mu\tilde A^{\mu a} + \tilde\chi^a\tilde{\bar\chi}^a + \tilde B^a\tilde B^a + \tilde\xi^a_\mu\tilde\xi^{\mu a} + \tilde{\bar\xi}^a_\mu\tilde{\bar\xi}^{\mu a} + \tilde\phi^a\tilde\phi^a + \tilde{\bar\phi}^a\tilde{\bar\phi}^a + \tilde e^a\tilde e^a\right] \\
&= \Big[-\zeta^{\mu a}\tilde A^a_\mu - A^{\star a}_\mu[\psi^{\mu a} - D^\mu(C-\tilde C)^a] - \sigma^a\tilde C^a + C^{\star a}[\epsilon^a - \frac{1}{2}f^{abc}C^b\xi^c_\mu] \\
&- \bar\sigma^a\tilde{\bar C}^a + \bar C^{\star a}[\bar\epsilon - (B^a - \tilde B^a)] - \upsilon^a\tilde\phi^a - \phi^{\star a}(M^a - f^{abc}C^b\phi^c_\mu) - \bar\upsilon^a\tilde{\bar\phi}^a \\
&- \bar\phi^{\star a}(M^a - \omega^a) - \tau^a\tilde e^a + e^{\star a}(N^a - \lambda^a) - \varpi^a\tilde B^a + B^{\star a}\xi^a - \kappa^{a\mu}\tilde\xi^a_\mu \\
&+ \xi^{\star a\mu}(L^a_\mu - [D_\mu\phi^a + f^{abc}C^b\xi^c_\mu]) - \bar\kappa^{a\mu}\tilde{\bar\xi}^a_\mu + \bar\xi^{\star a\mu}(\bar L^a_\mu - h^a_\mu)\Big].
\end{aligned} \tag{8.31}$$

which is nothing but the shifted gauge-fixed Lagrangian density. Being the $\theta\bar\theta$ component of a super field, this gauge-fixed Lagrangian density is manifestly invariant under the extended BRST and anti-BRST transformation. Now, we define the general super-gauge-fixing fermion for the extended BRST (and the anti-BRST invariant) theory as follows

$$\Phi(x,\theta,\bar\theta) = \Psi(x) + \theta(s_b\Psi) + \bar\theta(\bar s_b\Psi) + \theta\bar\theta(s_b\bar s_b\Psi), \tag{8.32}$$

which yields the original gauge-fixing and ghost part of the effective Lagrangian density as $\mathrm{Tr}\left[\frac{\partial}{\partial\theta}\left[s_b(\bar\theta)\Phi(x,\theta,\bar\theta)\right]\right]$. Therefore, the complete Lagrangian density for the extended BRST and anti-BRST invariant BF model in the general gauge can now be given by

$$\begin{aligned}\mathcal{L}_{gen} = &= -\frac{1}{2}\left[\frac{\partial}{\partial\bar{\theta}}\frac{\partial}{\partial\theta}(\tilde{A}^a_\mu\tilde{A}^{\mu a} + \tilde{C}^a\tilde{C}^a + \tilde{\bar{C}}^a\tilde{\bar{C}}^a + \tilde{B}^a\tilde{B}^a + \tilde{\xi}^a_\mu\tilde{\xi}^{\mu a} + \tilde{\bar{\xi}}^a_\mu\tilde{\bar{\xi}}^{\mu a}\right] \\ &+ \left[\tilde{\phi}^a\tilde{\phi}^a + \tilde{\bar{\phi}}^a\tilde{\bar{\phi}}^a + \tilde{e}^a\tilde{e}^a + \frac{\partial}{\partial\theta}\left[s(\bar{\theta})\Phi(x,\theta,\bar{\theta})\right]\right]. \end{aligned} \tag{8.33}$$

Performing equations of motion of auxiliary fields, the shift fields can be removed from the above expression and by integrating out the ghost fields for the shift symmetry, we obtain the exact expressions for the anti-fields.

8.6 Conclusions

The $(3+1)$ dimensional BF model is subject of great interest due to its topological nature and its intriguing properties.

In the present **Chapter**, we have considered $(3+1)$ dimensional BF model in the Landau-gauge and then we have extended the BRST and anti-BRST invariant (including the shift symmetry) BF model in the BV formulation. The anti-fields corresponding to each field naturally arise. Further, we have provided the superfield description of the BF model in the superspace where we show that the BV Lagrangian density for the BF model can be expressed in a manifestly extended BRST invariant manner in a superspace by considering *one* additional Grassmannian (fermionic) coordinate. However, we need two additional Grassmannian coordinate to express *both* the extended BRST and extended anti-BRST invariant BV Lagrangian density of the BF model in superspace.

Chapter 9

Conclusions

In this thesis, we have studied the fundamentals of gauge theories. We have studied the generalization of BRST symmetry by making the transformation parameter finite and field-dependent which is known as the FFBRST transformation. We have discussed the applications of FFBRST transformation in different models. We have emphasized the quantum gauge transformation and the superspace description of gauge theories also. We have started this thesis with the brief introduction about the fundamentals of gauge theories, BRST formulation and its importance in gauge theories and we have build-up the whole thesis (total chapters 9) based on these central theme.

In introduction **Chapter**, we have mentioned various generalizations of the BRST transformation. In these generalizations the FFBRST transformation has been found most important and interesting. The interesting feature of the FFBRST transformation is that this is also the symmetry of the effective action and is nilpotent. However, under this transformation, the path integral measure of the generating functional of a given theory is *not* invariant due to finiteness of the parameter. The Jacobian of the path integral measure is not unity under such transformation and leads to a factor (e^{iS_1}) in the generating functional where S_1 is local functional of the fields. This Jacobian (e^{iS_1}) extends the effective action of the theory. In this way, the FFBRST transformation is extremely useful in connecting various different effective actions in different gauges.

In **Chapter two**, we have provided the basic formulations of the gauge theories which are the building blocks of this thesis. Specifically, we have outlined the methodology of the FFBRST transformation by making the transformation parameter finite and field dependent. Further, we have presented a method to evaluate the Jacobian of functional measure under the FFBRST transformation. We have also discussed the basic idea of field/anti-field (BV) formulation within this **Chapter**.

In **Chapter three**, we have shown that the 3-form Abelian gauge theory, in non-covariant gauge, can be obtained from the *same* theory in covariant gauge with the help of the FFBRST transformation. Usual BRST transformations have been generalized to the case of Abelian 3-form gauge theory in covariant gauge. The generating functional for this theory is shown to be connected to that of the non-covariant-(axial) gauge through FFBRST transformation. We have established this connection by constructing an appropriate FFBRST parameter. However, the various non-covariant gauges like Coulomb-gauge, light-cone-gauge, planer-gauge and temporal-gauge can also be obtained under such formulation. Thus, such investigations enable us to study the 3-form Abelian gauge theory in the non-covariant gauge. We have further demonstrated that these results, within the framework of the BV formulation and established these results at the quantum level, too.

In **Chapter four**, we have shown that one can generate the Lowenstein-Zimmermann mass term for QED_3 using FFBRST transformation. Here, we have studied the BRST symmetry for the ultraviolet finite, superrenormalizable theory of massless QED_3. Further, we have generalized the BRST symmetry of the theory by making the transformation parameter finite and field-dependent. The fascinating feature of FFBRST transformation is that, under the change of variables, it leads to a non-trivial Jacobian for the path integral measure of generating functional. We have computed the Jacobian for the FFBRST transformation with appropriate finite field-dependent parameter. Remarkably, we have found that the Lowenstein Zimmermann mass term, together with the external sources for massless QED_3, emerge naturally within the functional integral through the Jacobian of a single FFBRST transformation. The notable feature of FFBRST symmetry is that, any gauge invariant (BRST exact) quantity, can be generated through the FFBRST symmetry. Although these Lowenstein-Zimmermann terms are mass terms but are gauge invariant, too. Thus, we have seen that the extra physical degrees of freedom emerge due to the non-linear BRST transformation (where the parameter exhibits the extra physical degrees of freedom due to the mass terms). Though, we have illustrated our results for QED_3 theory only, these certainly are not limited to a particular theory. In fact, it is a more general result and can be applied to any gauge theory to obtain gauge invariant mass terms and their dynamics in a useful manner.

On the other hand, quantum gauge symmetry has been discussed in **Chapter five**. Starting from the most general gauge-fixing Lagrangian density (including the gaugeon fields), we have presented a general form of the BRST symmetric gaugeon formalism for the reducible gauge theory. This most general gauge-fixing Lagrangian possesses the quantum gauge symmetry under which the Lagrangian density remains *form* invariant. The theory contains two gauge parameters in which one gets shifted by the quantum gauge transformation. By introducing the FP ghost and ghost for the ghost terms corresponding to the gaugeon fields, we have given a BRST symmetric gaugeon formalism for the Abelian 2-form gauge theory. The BRST symmetry enables us to improve the Yokoyama's subsidiary conditions by replacing these to a *single* Kugo-Ojima type subsidiary condition. The quantum gauge transformation commutes with the BRST transformation. As a result, the BRST charge is invariant and thus the physical subspace is also gauge invariant. We have studied FFBRST formulation in the context of gaugeon, too. Finally, using FFBRST, we have shown that the gaugeon fields can be generated in the reducible gauge theory (i.e. Abelian 2-form gauge theory).

In **Chapter six** we have discussed the BRST quantization of BF model in Landau-gauge. Furthermore, we have investigated the FFBRST transformation by making the transformation parameter finite and field dependent. Such generalized transformations are *not* the symmetry transformation for the functional measure. We have calculated the explicit form of the Jacobian of functional under generalized BRST transformation. For an specific choice of infinitesimal field-dependent parameter, it connects the Landau-gauge to axial-gauge. The connection of gauges are very important for the computational purposes.

In **Chapter seven**, we have constructed the BV Lagrangian density of CS theory in superspace. We have considered (2+1) dimensional CS theory in both the axial-gauge and the Landau-gauge and have attempted to describe the extended BRST and anti-BRST invariant (including shift symmetry) CS theory in the BV formulation. We have shown that the anti-fields arise naturally in such formulation. We have further provided a superspace and the superfield description of CS theory. We have shown that the BV Lagrangian density for this CS theory can be written in a manifestly extended BRST invariant manner in a superspace with one Grassmannian (fermionic) coordinate. However, a superspace with two Grassmannian coordinates are required for a manifestly covariant formulation of the extended BRST and extended anti-BRST invariant BV Lagrangian density for the CS theory in any arbitrary gauge.

The quantum description of BF model in superspace has been discussed in **Chapter eight**, where we have considered (3+1) dimensional BF model in the Landau-gauge and then we have shifted the Lagrangian density by shifting all the fields to obtain the extended BRST and anti-BRST invariant BF model in the BV formulation. The anti-fields corresponding to each field naturally arise. Further, we have provided the superfield description of the BF model in superspace where we have shown that the BV Lagrangian density for the BF model can be written in a manifestly extended BRST invariant manner in a superspace by considering *one* additional Grassmannian coordinate. However, we need *two* Grassmannian coordinates to express both the extended BRST and extended anti-BRST invariant BV Lagrangian density of the BF model in the superspace formulation.

In this thesis, we have made an attempt to extend the FFBRST formulation by incorporating it in different field theoretic models. We believe that this formulation will find many more new applications in future. In particular, it might be helpful in removing the discrepancy of the anomalous dimension calculation for the gauge invariant operators. Exploring this formulation in the context of different field theoretic models (having spontaneous symmetry breaking) will *also* be very exciting.

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