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Article

Chiral Effects in Hydrodynamics: New Trends

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Abstract: By chiral effects, one understands the manifestations of chiral gauge anomaly and of gravitational chiral anomaly in hydrodynamics. In the last two to three years, our understanding of chiral effects has considerably changed. Here, we present a mini-review of two topics: first, a shift in understanding symmetry, which underlies the chiral magnetic effect and second, the interpretation of the chiral kinematical effect uncovered recently.

Keywords: chiral magnetic effect; chiral vortical effect; kinematical vortical effect; quantum anomalies; relativistic hydrodynamics; gravitational chiral anomaly

1. Introduction

Chiral effects have attracted a lot of attention in the last ten–fifteen years, as seen, e.g., in [1,2]. The beauty of these effects is that they represent a class of quantum loop effects in hydrodynamics which have been traditionally treated as belonging to classical physics (with the exception of the theory of superfluidity which is based on a particular mechanism and this mechanism does not apply to chiral effects).

In particular, the gauge anomaly takes the following form [3]:

$$\partial_\alpha J_5^\alpha = e^2 C_5^{\rm el} \cdot \vec{E} \cdot \vec{B}, \tag{1}$$

where J_α^5 is the axial-vector current, $C_5^{\rm el}$ is a constant which depends on the choice of fundamental constituents, $\vec{E}$ and $\vec{B}$ are electric and magnetic fields, respectively, and e is electromagnetic coupling. By chiral magnetic effect (CME), one understands the electric current flowing in the direction of the magnetic field:

$$\vec{J}^{\rm el} \;=\; e^2 C_5^{\rm el} \cdot \mu_5 \vec{B}, \tag{2}$$

where μ_5 is the axial chemical potential and $C_5^{\rm el}$ is the same constant which enters the definition of the anomaly (1).

The prediction of the CME (2), and subsequently, of a whole class of similar phenomena, stimulated intensive research into effects at the intersection of quantum field theory and hydrodynamics. Much effort has been put into finding experimental manifestations, and collisions of heavy ions (HIC), in which quite extreme conditions can be formed, have become a promising "laboratory". At the moment, however, it is too early to talk about direct observation of the CME in HIC [2,4].

Other examples of systems where conditions are extreme enough for chiral effects to come into play are astrophysical phenomena, such as plasma instability in magnetars [5] and phenomena in the early Universe, where chiral effects can play a role in the formation

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of lepton and baryon asymmetry (see [6–8] and references therein). In particular, the role of the gravitational chiral anomaly in generating the matter–antimatter asymmetry in the early Universe was discussed [9,10]. Note that the physical interpretation of the gravitational anomaly is not as obvious; as for the contribution of gauge fields, however, see [11].

Most of the known chiral effects are associated with either a magnetic field (2) or vorticity, which will be discussed in more detail below. However, in recent years, efforts have been made to also include electric fields and fluid acceleration [12–14]. Of particular interest is the effect of rotation on the QCD phase transition [15–17]. More recently, studies have also appeared to consider the effect of acceleration on the QCD phase transition [18–21].

Another modern development is related to various nonequilibrium effects in electromagnetic and inertial fields [22,23]. In particular, it has been recently shown that dissipation occurs in an accelerated medium [24], which saturates the famous Kovtun–Son–Starinets bound for minimal shear viscosity [25].

From the point of view of phenomenology, the construction of hydrodynamics taking into account spin degrees of freedom [22,26,27] plays a significant role. Another important direction is the study of back reaction phenomena of effects of the type (2), in the context of which the relationship with Beltrami flows and the properties of chaos arise [28].

Let us discuss expression (2) in more detail. As indicated by the e^2 factor, (2) looks to be a Hall-type current. The guess turns true once one takes into account the specific hydrodynamic interaction $\hat{H}_{\text{hydro}}$:

$$\hat{H}_{\text{hydro}} = \hat{H}_{\text{field theory}} - \mu_{\text{el}}\,\hat{Q}_{\text{el}} - \mu_5\,\hat{Q}_5\,, \tag{3}$$

where $\hat{Q}_{\text{el}}$ and $\hat{Q}_5$ are electric and axial charges, and μ_{el}, μ_5 are the corresponding chemical potentials. The use of (3) assumes that the charges are conserved.

In the presence of an external gravitational field, the divergence of the axial current J_5^α receives a further contribution [29,30]:

$$\nabla_\alpha J_5^\alpha = C_5^{\text{grav}} R\tilde{R}\,, \tag{4}$$

where ∇_α is the covariant derivative, $R\tilde{R} = (1/2)\epsilon^{\mu\nu\alpha\beta}R^{\gamma\delta}_{\alpha\beta}R_{\gamma\delta\mu\nu}$, $R_{\mu\nu\alpha\beta}$ is the Riemann tensor, and C_5^{grav} is a constant which depends on the spin of the constituent fermions and which is known.

No generalization of Equation (3) to the case of gravitational field is known. Hence, no analogous of (2) is known either. In Section 3, we discuss the recent developments in this direction and consider a novel effect in a vortical and accelerated fluid, suggesting the physical interpretation based on the equivalence principle and the Unruh effect. We will also show that at various stages in the treatment of chiral effects, a symmetry associated with a diffeomorphism arises that unifies the effects in hydrodynamics and gravity. It plays a central role, in particular, in the construction of anomalous hydrodynamics, where, for an ideal fluid, it is necessary to take into account new conserved charges due to which classical analogs of anomalies and anomalous effects appear.

2. From Chiral Symmetry to Diffeomorphism

In this section, we describe briefly the recent progress made in understanding the physics behind the chiral magnetic effect. We discuss the role of the ideal fluid approximation in the conservation of chiral effects, which will lead us to the necessity of introducing additional chemical potentials associated with the corresponding conserved charges. We show that in the framework of such a modified effective field theory for hydrodynamics, anomalies and anomalous effects arise already at the classical level.

Let us first note that there is a well-known analogy between the magnetic field $\vec{B}$ and angular velocity $\vec{\Omega}$. Manifestations of this analogy are known both at the level of general physics, for example, in the analogy between the Coriolis and the Lorentz forces or the existence of the Barnett and Einstein–de Haas effects, as well as in more modern applications, as seen, for example, in [31]. In particular, using this analogy, we can from Equation (2) obtain the so-called chiral vortical effect (CVE):

$$\vec{J}_{5,\mathrm{CVE}} \;=\; \mu_{\mathrm{el}}^2 C_5^{\mathrm{el}} \vec{\Omega}\,. \tag{5}$$

It is worth emphasizing that this axial current exists in absence of $\vec{E}, \vec{B}$. It is not, therefore, affected by the chiral anomaly and is to be conserved. Nevertheless, it is proportional to the constant in front of the gauge anomaly, C_5^{el}. The paradox is strengthened by observing that there is no place for a new Noether current, since there is no free symmetry to associate with such a current. A possible way out is to assume that the conservation of the current (5) is specific for the absence of dissipation, or for an ideal fluid, as seen in [32,33]. Indeed, one can demonstrate that the current (5) is conserved in case of an ideal fluid.

Note that all the expressions that we are obtaining so far are not new. In particular, they are obtained in ref. [34] using a more standard technique but under the same assumption on the absence of dissipation. What we are emphasizing here is that introducing this assumption is a necessary step once we consider loop corrections induced by the effective hydrodynamic interaction (3).

A crucial point is that in case of an ideal fluid, there exists another conserved axial charge (not chiral!); see, e.g., [35]:

$$\mathcal{H}_{\text{fluid helicity}} \;=\; (\text{const}) \int d^3x \; m\vec{v} \cdot (\vec{\nabla} \times \vec{v})\,, \tag{6}$$

or

$$J^{\alpha}_{\text{fluid helicity}} \;=\; (\text{const}) m \epsilon^{\alpha\beta\gamma\delta} u_\beta \partial_\gamma u_\delta\,, \tag{7}$$

where u_α is the fluid 4-velocity. Moreover, the fluid-helicity current (7) is conserved but rather as a consequence of *diffeomorphism* of ideal fluid. Thus, the chiral symmetry, which we started from, is apparently embedded into diffeomorphism and we encounter an enhancement of symmetry. Let us pause here and make a few remarks.

First of all, the conserved current (7) is not proportional to the electromagnetic coupling and represents a classical current while the commonly discussed CME current (2) is a quantum, or loop effect. Thus, in the case of an ideal fluid, it seems logical to introduce a chemical potential $\mathcal{M}_5$ conjugated with the fluid-helicity charge $\mathcal{H}_{\text{fluid helicity}}$, as seen in Equation (6)

$$\delta \hat{H}_{\text{hydro}} \;=\; -\,\mathcal{M}_5 \rho_A, \tag{8}$$

where ρ_A is the density of the axial charge (6). Moreover, the introduction of $\mathcal{M}_5$ can serve as a starting point of an alternative phenomenology specific for an ideal fluid (for recent presentation, see, in particular, [36–38]).

Moreover, assuming that the fluid is charged, one can generate from the fluid helicity (6) another charge which is conserved with account of the minimal electromagnetic interaction. To this end, one replaces in Equation (6) the momentum $m\vec{v}$ by the canonical momentum $\vec{\pi}$:

$$m\vec{v} \;\rightarrow\; \vec{\pi} \;=\; m\vec{v} \,+\, q\vec{A}\,, \tag{9}$$

where $\vec{A}$ is the vector-potential of the electromagnetic field and q is the electric charge associated with the fluid of mass m. The actual normalization of q is rather complicated and we follow original papers [36,37], which put $q = 1$. Through substitution (9), we obtain the following:

$$\mathcal{H}_{\text{fluid helicity}} \;=\; (const) \int d^3x\, \vec{\pi} \cdot (\vec{\nabla} \times \vec{\pi})\,. \tag{10}$$

This quantity, total helicity, is conserved and gauge-invariant, as it should be. However, its density is not gauge-invariant and cannot characterize the fluid locally. By careful analysis and central extensions of algebras [36–38], this inconsistency is traded for local, gauge-invariant anomalies, which are amusing *non-relativistic* analogs to the standard chiral anomalies:

$$\partial_\alpha J_5^\alpha \;\sim\; \vec{E} \cdot \vec{B} \quad (\text{as “usual”})\,, \tag{11}$$

$$\partial_\alpha J_{\text{el}}^\alpha \;\sim\; \vec{E}_5 \cdot \vec{B} \equiv\; \vec{\nabla}\mathcal{M}_5 \cdot \vec{B}\,, \tag{12}$$

where electric fields, the standard one, $\vec{E}$, and axial electric field, $\vec{E}_5$, are defined in terms of gradients of the corresponding chemical potentials, $\vec{E} \equiv \vec{\nabla}\mu$, $\vec{E}_5 \equiv \vec{\nabla}\mathcal{M}_5$. The electric fields, $\vec{E}$ ($\vec{E}_5$), exert force on a charged (axially charged) fluid.

Note that according to (12), there is an anomaly in the electromagnetic current. This might seem to represent a fundamental challenge to the theory, but actually, it does not. The point is that electric fields induce accelerated motions and the anomalies (11) in the inertial frames can be canceled by going into the accelerated frame. There is a similar effect in case of gravitational interaction, as demonstrated explicitly in ref. [39].

Here, we summarize our findings in the current section:

- The newly discovered anomalies (11) and (12) are not related to any ultraviolet divergences. Since we started with a non-relativistic current (7), the anomalies (11) and (12) are non-relativistic in nature as well.
- There is a striking similarity between the ideal fluid anomalies (11) and (12) and the standard gauge anomaly (1), associated with the short-distance physics.
- The two sets of the anomalies and hydrodynamic currents are not identical. The non-relativistic anomalies match the commonly known UV anomalies only up to a factor. The symmetries behind are also different.

The overall conclusion to this section is the conjecture that chiral fluid of massless quarks at short distances might look like an ideal non-relativistic fluid at large distances. The reason for the conjecture is that there is matching of the corresponding anomalies (variation of the ’t Hooft consistency condition [40]).

3. Gravitational Anomaly and Hydrodynamics

In this section, we discuss a novel anomalous transport effect—the kinematical vortical effect. It is directly related to the gravitational chiral anomaly, but exists in the absence of real gravitational fields and curvature. We discuss the role of the equivalence principle and the Unruh effect in interpreting this effect.

Let us start this section with a brief reminder of some of the basics of General Relativity as field theory in flat space. Interaction is built on deviations $h_{\mu\nu}$ of the metric tensor $g_{\mu\nu}$ from its Minkowskian values $\eta_{\mu\nu}$:

$$g_{\mu\nu} \;\approx\; \eta_{\mu\nu} \;+\; h_{\mu\nu}\,. \tag{13}$$

Namely,

$$L_{\text{int}} \;=\; -\,(1/2) \int d^4x\, h_{\mu\nu} T^{\mu\nu}\,, \tag{14}$$

where $T^{\mu\nu}$ is the energy–momentum tensor of matter. As a consequence of the energy–momentum conservation, the interaction Lagrangian (14) is invariant under gauge transformation (analog of $\delta A_\mu = \partial_\mu \Lambda$):

$$\delta h_{\mu\nu} \;=\; \partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu \,, \tag{15}$$

where ϵ_μ are arbitrary (smooth) functions. A gauge-invariant field, analog of $\vec{E}, \vec{B}$, is Riemann tensor $R_{\alpha\beta\gamma\delta}$, which involves $(\partial_\alpha \partial_\beta h_{\mu\nu})$ and $(\partial_\alpha h_{\mu\nu})^2$. Gravitational anomaly is built on the curvature $R_{\alpha\beta\gamma\delta}$, as seen in Equation (4).

As will be shown below, there is an effect associated with gravity, and more specifically, with a gravitational anomaly, even in the limit when there is no real gravitational field with finite curvature. This brings us to a new question, not much explored so far: "In absence of sizable curvature, what is left of the anomaly?". Without being rigorous, let us try to give a preliminary answer: what is left is the equivalence principle. It states that the manifestations of a gravitational field $h_{\mu\nu}$ and of going into a non-inertial frame are identical to each other. It would seem that there are no invariant effects connected with the equivalence principle, considering that it clearly contains non-invariance, with respect to the choice of the reference system ("gauge non-invariance", as stated above)—Christoffel symbols can be turned to zero locally, as is known. However, this does not apply to covariant derivatives, which evidently have the necessary "gauge invariance".

Two basic non-inertial frames, considered by Einstein [41], are accelerated and rotated frames (corresponding $h_{\mu\nu}$ is easy to find out in textbooks). As is noted by Einstein, the effect of rotation (changing the ratio of radius and circumference) can be derived knowing only Lorentz transformations, while analysis of motion in accelerated frame led to a new principle, the equivalence principle. Moreover, the inclusion of quantum effects allows to further appreciate the role of acceleration.

Namely, according to Unruh [42], an observer moving with acceleration a with respect to the Minkowskian vacuum sees thermal distribution of particles with temperature as follows:

$$T_{\text{Unruh}} \;=\; \frac{\hbar a}{2\pi c k_B} \,, \tag{16}$$

where $\hbar$ is the Planck constant, c is the speed of light, and k_B is the Boltzmann constant. On the other hand, an observer at rest sees no particles. For many, it might sound disappointing that the Unruh effect is kind of observer-dependent. In fact, it is dynamical although frame-dependent. Indeed, by virtue of the equivalence principle, accelerated frame is equivalent to a vacuum placed in a (strong) gravitational field resulting in the same acceleration a. Naturally, such a field produces particles and is obviously dynamical. Acceleration, like electric field, produces particles from the vacuum, while rotation, like magnetic field, modifies vacuum currents.

In the case of the gauge theory considered in the preceding section, a special role is played by the chiral–vortical current (5), which is proportional to the constant in front of the gauge anomaly but exists in the absence of electromagnetic fields. One can expect that a similar phenomenon exists also in case of external gravitational field, so that even in the absence of the field, there exists a current determined entirely in terms of kinematical quantities, acceleration and rotation.

In applications to a liquid, notably quark–gluon plasma, acceleration and rotation 4-vectors are given by the following:

$$a_\mu \;=\; u^\alpha \nabla_\alpha u_\mu, \quad \omega_\mu \;=\; \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} u^\nu \nabla^\rho u^\sigma \quad (u^\nu \text{ is } 4-\text{velocity})\,. \tag{17}$$

In the limit of flat space, the covariant derivative becomes the ordinary one $\nabla \to \partial$. Moreover, for massless constituents and on dimensional grounds, the "kinematical" axial current is determined in terms of two constants, $\lambda_{1,2}$:

$$J_5^{\alpha,\text{kinematic}} = -\frac{1}{24\pi^2}\Big(\lambda_1\omega^\mu\omega_\mu + \lambda_2 a_\mu a^\mu\Big)\omega^\alpha \tag{18}$$

It is proven in ref. [43] that a difference between the coefficients λ_1 and λ_2 is related to the coefficient C_5^{grav} in front of the gravitational anomaly (4):

$$\frac{\lambda_2-\lambda_1}{768\pi^2} = C_5^{\text{grav}}. \tag{19}$$

This relation is called kinematical vortical effect (KVE) [43]. An attempt to generalize (19) to the case of presence of cosmological constant is made in [44].

By a direct calculation, the relation (19) has been verified both in case of spin-1/2 [43] and spin-3/2 [45] constituents, seen as follows:

$$\lambda_1 = 1\,,\ \ \lambda_2 = 3\,,\ \ C_5^{\text{grav}} = 1/(384\pi^2) \qquad (\text{spin } 1/2)\,, \tag{20}$$

$$\lambda_1 = 53\,,\ \ \lambda_2 = 15\,,\ \ C_5^{\text{grav}} = -19(384\pi^2) \qquad (\text{spin } 3/2)\,. \tag{21}$$

We have not explained yet the physical idea behind the derivation of (19). In fact, it is related to the notion of duality between statistical and gravitational approaches, which is introduced in the next section.

Here, we add an interpretation of the kinematical vortical effect (19). The key element is the observation that the current (19) refers to a non-inertial frame, accelerated and rotated. Thus, we start with Minkowski space and no current at all. Let us now move on to the accelerated frame of reference, for which the Unruh effect is observed. Due to the Unruh effect, we find there a sample of particles with temperature (16). We calculate then the axial current on that sample of particles. Elements of this picture can readily be traced in the actual calculation. The factorization of the two steps (going to the non-inertial frame and calculation of the current) is, however, an oversimplification. There is also the subtle point discussed above that rotation does not produce work. For this reason, the "Unruh sample of particles" is determined by acceleration alone, not by rotation.

4. Duality of Statistical and Gravitational Approaches

This section is devoted to a discussion of a certain new duality between the "statistical" and "gravitational" approaches. The same quantities can be calculated either within the framework of quantum field theory in curved space or in ordinary space, but taking into account the modification of statistical distributions by new terms.

Properties of fluids in equilibrium are evaluated statistically in terms of density operator, or effective interaction:

$$\hat{H}_{\text{eff}} = -\vec{\Omega}\cdot\hat{\vec{M}} - \vec{a}\cdot\hat{\vec{K}}\,, \tag{22}$$

where $\hat{\vec{M}}$ is the operator of angular momentum and $\hat{\vec{K}}$ is the operator of boost (for details, see, e.g., [13]).

In field theory, gravitational interaction is described by fundamental interaction Lagrangian (14):

$$\delta\mathcal{L}_{\text{fund}} = -\frac{1}{2}T^{\alpha\beta}h_{\alpha\beta}\,, \tag{23}$$

where $T^{\alpha\beta}$ is the energy–momentum tensor of matter, $h_{\alpha\beta}$ is the gravitational potential accommodating the same $\vec{\Omega}$, $\vec{a}$ as (22). Furthermore, one evaluates "external probes", $<\hat{T}^{\alpha\beta}>, <\hat{J}_5^\alpha>$ within both approaches, statistical and gravitational. For the same values of a_μ, ω_μ, the expectation is that results turn out to be the same. The expectations are confirmed on many examples [14].

For example, one can evaluate the energy density $\epsilon_{s=1/2}$ of massless fermions with spin 1/2 as a function of independent temperature and acceleration:

$$\epsilon_{s=1/2} = \Big(\frac{7\pi^2 T^4}{60} + \frac{|a|^2 T^2}{24} - \frac{17|a|^4}{960\pi^2}\Big). \tag{24}$$

Note that the energy density $\epsilon_{s=1/2}$ is vanishing at the Unruh temperature, $T = T_U$, and is positive at $T > T_U$. Moreover, Equation (24) has been derived both within gravitational [46] and thermodynamic approaches [47,48]. However, while within the statistical approach, the integral over energies of states converges at $E \sim T$. On the other hand, in the gravitational case, to derive (24), one needs to introduce a UV cut off.

Moreover, the two theories that we are discussing, theory of equilibrium and of gravitational interaction, satisfy a standard set of conditions to be satisfied for two theories to be dual:

- Two theories are to have the same symmetry pattern, and indeed, both ideal fluid and gravity are diffeomorphic-invariant.
- Infrared vs. ultraviolet sensitivity. Statistics is not valid at short distances, while field theory does need UV regularization. Thus, the two approaches are complementary to each other.
- Both theories allow to evaluate a common set of observables.

As a result, evaluation of the kinematical effect can be rewritten as regularization, via acceleration, of gravitational chiral anomaly. Such a regularization is infrared-sensitive, unlike the standard derivation of the anomalous divergence of the current, which is UV-sensitive. To elaborate how general this observation is and whether it could be developed into a novel type of regularization is a task for the future.

In the most recent times, development has been focused on the role of the vacuum effects, as seen, e.g., [49,50] and references therein. The point is that consideration of acceleration and vorticity assumes the use of non-inertial frames, or of General Relativity. Moreover, on the quantum level, the vacuum state in non-inertial frames is not the same as in inertial frames. For example, the last term in the r.h.s. of Equation (24), proportional to $|a|^4$, survives in the limit of vanishing temperature T and represents a vacuum, or Casimir effect.

To reiterate, we started with evaluating the quantum correction to the energy–momentum tensor associated with thermal gas of massless fermions but ended up with fixing a pure vacuum contribution, independent of the temperature. Moreover, the total energy, or the sum of the thermal and vacuum terms, vanishes at the Unruh temperature, $T = T_U$. This cancellation of the total energy is straightforward to appreciate in this particular case. Indeed, let us start from the Minkowskian vacuum and normalize the energy density to zero; next, go to an accelerated frame. We know that in the accelerated frame, there is gas at the Unruh temperature, but with the change of coordinates, the energy–momentum tensor transforms according to the standard rules. In particular, if all components of a tensor vanish in one reference frame, then the tensor vanishes in another frame as well and we come to a sum rule:

$$\epsilon_{Unruh} + \epsilon_{Casimir} = 0 \ , \tag{25}$$

which agrees with (24). Note that the region $T < T_U$, when the negative vacuum contribution begins to dominate, is of interest; in particular, there are indications of the formation of corresponding states in the heavy ion collisions [18].

In the general case, however, separation of vacuum terms might turn out to be a complicated problem.

5. Conclusions

It might be fair to say that originally (see, e.g., [1,2]), the literature on chiral effects in gauge theories was dominated by discussion of the prospects of discovering novel types of phenomenology. Possible manifestations of gravitational anomaly received much less attention because of apparent lack of applications.

The development which we concentrated on here tends more to the theoretical side. In the case of gauge anomalies, we emphasized that there exist two types of anomalies and chiral currents, depending on whether one starts from short or long distances. It might be worth mentioning that in both cases, the anomaly is not considered as a mechanism of production of particles, in contradistinction to popular models of chiral effects, as seen, e.g., in [1,2]. In the case of gravitational anomaly, we considered it mostly within the context of the bulk/boundary duality which is inherent to theories with diffeomorphism, as seen, e.g., in [51]. The duality between equilibrium theory and external-field gravity introduced in [47] is probably the simplest example of this more general phenomenon.

Despite our emphasis on more theoretical issues, these results have phenomenological implications. In particular, we argued that consistency of radiative corrections to chiral currents requires absence of dissipation. Indeed, the quark–gluon plasma is both chiral and close to ideal fluid. Moreover, evaluations of currents in an accelerated frame might be relevant in the future for experiments with the quark–gluon plasma, since the plasma is produced in an accelerated state.

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