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CONTROLLING TRANSVERSE MULTIBUNCH INSTABILITIES IN LINACS OF HIGH ENERGY LINEAR COLLIDERS*

K. A. THOMPSON AND R. D. RUTH

Stanford Linear Accelerator Center

Stanford University, Stanford, California 94309

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ABSTRACT

In this paper, we study multibunch beam breakup, with emphasis on theoretical methods applicable to the design of a linear collider with center-of-mass energy near 1 TeV. One way to significantly improve the luminosity and energy transfer efficiency of such a collider is to accelerate a train of bunches rather than just a single bunch each time the linac accelerating structure is filled with a pulse of RF energy. For the required bunch charges and intensities, the transverse instability due to the wake fields produced in the accelerating structure is very severe unless measures are taken to control it. Therefore, we examine the effects of several methods of reducing this instability: (1) use of damped acceleration cavities, (2) placing the bunches near nodes of the transverse wake fields produced by preceding bunches, (3) introducing a spread (over different cells of the accelerating structure) of the individual mode frequencies in the transverse wake field, and (4) varying the strength of the transverse focusing from bunch to bunch, in such a way as to partially cancel the effects of the wake fields from preceding bunches. We present examples illustrating the effectiveness of these cures, using realistic linear collider design parameters.

1. INTRODUCTION

The design of the next generation of e^+e^- linear colliders, with center-of-mass energy between about 0.5 and 2 TeV, is based upon extensions of conventional RF technology to frequencies above 10 GHz. Even at these frequencies, the power requirements are high, and it is essential to use the available RF energy as efficiently as possible. Furthermore, the luminosity required by high energy physics experiments at these energies is close to 10^{34} cm^{-2} sec^{-1}. For these reasons, it is attractive to accelerate a train of bunches rather than a single bunch on each RF fill, as it is then possible to extract a higher fraction of the available energy and to obtain a luminosity several times higher than in the single bunch case.

However, the passage of intense bunches through a high-frequency accelerating structure leaves behind strong wake fields that influence subsequent bunches in the train. The longitudinal wake fields produce a spread in the energies of the bunches (beam loading), and one must arrange the filling time and bunch spacing to keep the bunch-to-bunch energy variation sufficiently small.[1] The transverse dipole wake fields are responsible for the cumulative beam breakup instability, which is extremely severe in the cases of interest here, unless measures are taken to alleviate it. Each bunch in a closely-spaced train feels the transverse dipole wake produced in the accelerating structure when preceding bunches are slightly off-axis. The spacing between adjacent bunches is only a few RF wavelengths, and the transverse dipole wake in a conventional accelerating structure continues to ring for many multiples of this spacing. Thus, the transverse amplitudes of oscillation of the bunches can grow rapidly as they proceed through many acceleration sections.

Regenerative beam breakup[2,3] is a form of beam breakup that can occur when there is a buildup of deflecting fields in a single acceleration section. This buildup of the fields is due to a "feedback " of energy due to backward-wave components of the fields and/or reflections from the end of the acceleration section. However, the threshold current for regenerative buildup is well above the actual beam current, for the parameters regimes in which we are interested here.

The cumulative beam breakup instability in linacs was first observed in the SLAC linac in 1966,[4] and the first theoretical studies were carried out during the next few years.[3,5,6] A number of subsequent works have treated regimes of the beam breakup differing in various essential respects from that considered here.[7–16] The approach taken by many of these authors, which is most useful for a long beam, is to make a discrete or continuous Laplace transform on the equations of motion (depending on whether bunching of the beam and/or discreteness of the RF cavities are taken into account). The Laplace inversion is then performed analytically using asymptotic methods, to obtain results valid in the steady state limit, in which the bunch number is approaching infinity, or the asymptotic transient limit, in which the blowup has progressed significantly.

Since we are interested in a beam consisting of a relatively short train of bunches, it seems more transparent to remain in the time domain. Furthermore, we wish to know how much transverse blowup of the beam there will be, even when it is only a small factor. We take two different approaches to calculating it. The most general is to numerically integrate a Green function integral representation for the transverse offset of each bunch. The other approach is to derive simple analytic models that illustrate the characteristics of the blowup in some limits of interest and can be compared with the more precise results based on the Green function integrals.

One such limit occurs when the wake is so strongly damped that at each bunch only the wake from the immediately preceding bunch need be taken into account. For this case, a very simple analytic model is derived and is shown to be in good agreement with the more general approach.

In general, the transverse wake consists of a sum of modes of different frequencies. However, the transverse wake in the type of accelerating structure we shall consider tends to be strongly dominated by its fundamental dipole mode. Thus we present a simple analytic model that is valid for bunches placed close to the zero-crossings of such a single-mode wake field.

Several possible methods for alleviating the transverse instability will be studied in this paper:

1. Damping the transverse dipole modes by means of axial slots through the irises of the RF structures, with the slots coupled to radial waveguides.[17]
2. Tuning the frequency of the fundamental transverse dipole mode to place the bunches as near as possible to zero-crossings of the wake fields.
3. Using an RF accelerating structure in which the frequencies of corresponding transverse dipole modes differ from cell to cell, resulting in a reduction of the effective Q of each mode.
4. Using time-varying quadrupoles to introduce a small change in the focusing for different bunches, so that they are not in resonance with each other.

Multibunch beam breakup in very high energy linacs has been previously treated by Yokoya in Ref. 10. He obtains approximate analytic results for the case of a single deflecting mode or several deflecting modes whose frequencies (modulo the bunch frequency) are sufficiently well separated. The bunches are assumed to be not too close to an integer number of half-wavelengths of the (single-mode) wake field frequency. Decker and Wang in Ref. 11 have also studied the cumulative beam-breakup for a single deflecting mode and for two modes of slightly different frequency. Yokoya has studied the use of a spread in the transverse mode frequencies as a cure for the breakup (*i.e.*, the third cure in the above list), but this method by itself is not sufficient to solve the problem in the high-frequency linacs under consideration. We shall emphasize instead the use of damped cavities and placing the bunches near wake field zero-crossings.

The starting conditions for the transverse instability can be initial offsets of one or more bunches, or can arise from misalignments of the RF cavities or focusing elements. In this paper, we shall consider only the breakup due to bunch offsets at the beginning of a linac. Misalignment effects in linacs have been discussed by other authors in Refs. 8 and 10.

Multibunch instabilities are a potential problem in other linear collider subsystems besides the main linacs. In the injector accelerators and preaccelerators, beam breakup is not as severe as in the main linacs, but it is still an issue in their design.[18] Damping rings suitable for a linear collider utilizing multibunching are also being designed.[19,20] The control of coupled-bunch instabilities in such rings is addressed separately from the present work.[21,22]

The organization of the paper is as follows. First, the theory used to calculate the transverse beam blowup is treated, namely the general Green function integrals and the simple models for limiting cases of interest. Next, some possible methods of curing the instability are discussed. Finally, examples for the main linacs of a TeV collider are given, to illustrate the theory and to show the effectiveness of the proposed cures.

2. MULTIPLE BUNCHES WITHOUT ACCELERATION

For reasons that will become clear shortly, it is useful to begin by examining beam breakup without acceleration. We assume an equal charge of N electrons in each bunch and uniform spacing ℓ between adjacent bunches; the bunch spacing ℓ is of course an integer number of RF wavelengths. We use the smooth-focusing approximation $k_n(s) = 1/\beta_n(s)$ for the focusing function of bunch n, where $\beta_n(s)$ is an average beta function which, however, may vary slowly with s.

The bunches are considered to be rigid macroparticles. Single bunch beam blowup is a separate question and can be dealt with using different techniques. It is controlled by opening the irises of the structure, by short bunch lengths, and by using BNS damping[23] to compensate the wake effects. In this paper, we shall only be concerned with the longer range wake fields, which couple each bunch to the bunches which follow it.

Although we are interested here in the dynamics of a train of rigid bunches, it is useful to begin by looking at a treatment of single-bunch beam breakup, in which the structure of the single bunch is modelled as two macroparticles, representing the "head" and "tail" of the bunch. The case of n bunches is a generalization of this simpler problem, when each of the n bunches is regarded as a rigid macroparticle.

The standard treatment of single-bunch beam breakup using two macroparticles[24-26] starts from the equations of motion

$$x_1'' + k_1^2 x_1 = 0 \tag{2.1}$$

$$x_2'' + k_2^2 x_2 = \frac{Ne^2 W_\perp(\ell)}{E} x_1 \quad . \tag{2.2}$$

Here, x_1 and x_2 are the transverse displacements of the two bunches (assumed to be in a single plane), E is the energy of the electrons in the bunches, and primes denote derivatives with respect to longitudinal distance s. The bunch spacing is denoted by ℓ, and $W_\perp(\ell)$ is the transverse dipole wake function at the second bunch due to the first bunch. The wake function $W_\perp$ is a sum of modes of the following form (Ref. 26):

$$W_\perp(z) = \frac{1}{pa^2}\sum_m \frac{2\kappa_m}{K_m}\sin(K_m z)e^{-K_m z/2Q_m} \quad , \tag{2.3}$$

where

$$\begin{aligned} z &= \text{distance behind exciting bunch} \\ K_m = \omega_m/c &= \text{wavenumber of mode } m \\ Q_m &= \text{quality factor of mode } m \\ \kappa_m &= \text{loss factor of mode } m \text{ at the iris} \quad [\text{V/Coul/cell}] \\ p &= \text{cell length} \\ a &= \text{iris radius} \quad . \end{aligned}$$

The units of $W_\perp(z)$, the wake function per unit length, are [V/Coul/m^2]. The wake function $W_\perp(z)$ is multiplied by the charge and transverse displacement of the exciting bunch to get the wake field a distance z behind that bunch. Note that to include the effect of the finite length of a Gaussian bunch, each term in the sum over m in Eq. (2.3) should be multiplied by the factor $e^{-K_m^2\sigma_z^2}$, where σ_z is the rms bunch length. We do not explicitly include these factors, since we assume that the modes included in the long-range wake have wavelengths much longer than the bunch length.

Suppose the focusing is the same for both bunches, that is, $k_1 = k_2 = k$. Then if $x_1(s) = a_1 e^{iks}$ and $x_2(0) = a_1$, the solution $x_2(s)$ for the second bunch satisfies

$$\frac{x_2 - x_1}{a_1} = \frac{Ne^2 W_\perp(\ell)}{2ikE} s e^{iks} \quad . \tag{2.4}$$

Note the linear growth of the envelope of the difference $x_2 - x_1$ with longitudinal distance s.

Now suppose that the focusing of the two bunches is made slightly different, for instance by using time-varying quadrupoles. If $k_1 = k$ and $k_2 = k + \Delta k$, with $\Delta k \ll k$, then

$$\frac{x_2 - x_1}{a_1} \approx \left[1 - \frac{Ne^2 W_\perp(\ell)}{2Ek\Delta k}\right] 2i \sin(\Delta k s/2)\, e^{i(k+(\Delta k/2))s} \quad . \tag{2.5}$$

In this case, the envelope beats with wavelength $4\pi/\Delta k$ instead of growing linearly. If the coefficient in front is made zero by the proper choice of Δk, then there is no growth of the transverse amplitude of the second bunch.

As another example, we may consider the case where the two bunches start out with equal but opposite offsets, namely $x_1(0) = -x_2(0) = a_1$. This could also be regarded as a simple model for a bunch that is "crabbed" so that its head and tail start on opposite sides of the linac axis. Then, instead of Eq. (2.4), we obtain

$$\frac{x_2 - x_1}{a_1} = \left[\frac{Ne^2 W_\perp(\ell)}{2ikE} s - 2\right] e^{iks} \quad , \tag{2.6}$$

and, instead of Eq. (2.5),

$$\frac{x_2 - x_1}{a_1} \approx -\left[2\cos(\Delta k s/2) + \frac{Ne^2 W_\perp(\ell)}{2Ek\Delta k} 2i \sin(\Delta k s/2)\right] e^{i(k+(\Delta k/2))s} \quad . \tag{2.7}$$

In this approach, acceleration has not been taken explicitly into account; the energy E of the bunches has been assumed constant. However, we shall see that for a particular choice of focusing function, we can directly use the results obtained from the equations of motion without acceleration if the variables are interpreted appropriately. Thus, we proceed to the case of n bunches without acceleration. The equation of motion for the transverse displacement of bunch n ($n > 1$) ignoring acceleration is

$$x_n'' + k_n^2 x_n = f_n(s) \quad , \tag{2.8}$$

where the driving term is

$$f_n(s) = \frac{Ne^2}{E} \sum_{j=1}^{n-1} W_\perp((n-j)\ell) x_j(s) \quad . \tag{2.9}$$

We look for a solution of the form $x_n(s) = a_n(s) e^{ik_n s}$, which leads to

$$a_n'' + 2ik_n a_n' = f_n(s) e^{-ik_n s} \quad . \tag{2.10}$$

Assuming the variation of a_n with s is sufficiently slow, we may neglect the a_n'' term. Solving for a_n then yields

$$a_n(s) = a_n(0) + \frac{1}{2ik_n} \int_0^s f_n(s') e^{-ik_n s'} ds' \quad , \tag{2.11}$$

so that the solution for x_n is given by

$$x_n(s) = \left[x_n(0) + \frac{Ne^2}{2iEk_n} \int_0^s e^{-ik_n s'} \sum_{j=1}^{n-1} W_\perp((n-j)\ell) x_j(s') ds' \right] e^{ik_n s} \quad . \tag{2.12}$$

3. MULTIPLE BUNCHES WITH ADIABATIC ACCELERATION

Taking acceleration into account, the equation of motion for x_n is

$$\gamma(s)x_n'' + \gamma'(s)x_n' + \gamma(s)k_n^2(s)x_n = F_n(s) \quad , \tag{3.1}$$

where we define

$$F_n(s) \equiv \frac{Ne^2}{mc^2}\sum_{j=1}^{n-1} W_\perp((n-j)\ell)x_j(s) \quad . \tag{3.2}$$

Here m is the rest mass of the electron, and γ is the usual Lorentz factor E/mc^2. The acceleration is assumed to be linear: $\gamma = \gamma_0 + Gs$, with G a constant. We assume the smooth focusing functions $k_n(s)$ vary as the inverse power p of the energy:

$$k(s) = \left(\frac{\gamma_0}{\gamma(s)}\right)^p k_0 \quad . \tag{3.3}$$

Then, the WKB solutions of the homogeneous equation are

$$\begin{aligned} x_n^\pm(s) &= x_n^\pm(0)\left[\frac{\gamma_0 k_n(0)}{\gamma(s)k_n(s)}\right]^{1/2} \exp\left[\pm i\int_0^s k_n(s')ds'\right] \\ &= x_n^\pm(0)\left[\frac{\gamma_0}{\gamma(s)}\right]^{\frac{1-p}{2}} \exp\left[\pm i\int_0^s k_n(s')ds'\right] \quad . \end{aligned} \tag{3.4}$$

Note the presence of the "adiabatic damping factor" $(\gamma_0/\gamma)^{\frac{1-p}{2}}$, due to acceleration. The WKB approximation assumes that the fractional energy change in a betatron wavelength is small, that is, $(\gamma')^2 \ll \gamma^2 k^2$. This is well satisfied for the cases of interest to us.

Now look for a solution to the inhomogeneous equation (3.1) of the form

$$x(s) = u_+(s)x_+(s) + u_-(s)x_-(s) \tag{3.5}$$

(suppressing subscript n for the moment). Without loss of generality we may assume that

$$u'_+x_+ + u'_-x_- = 0 \quad . \tag{3.6}$$

Substituting into the inhomogeneous equation, we obtain

$$\gamma(u'_+x'_+ + u'_-x'_-) = F(s) \quad . \tag{3.7}$$

Thus, we have two simultaneous equations [(3.6) and (3.7)] for u'_+ and u'_-, which we may solve and integrate to obtain

$$u_\pm(s) = u_\pm(0) + \int_0^s \frac{Fx_\mp}{\gamma(x_-x'_+ - x_+x'_-)} ds' \quad . \tag{3.8}$$

It is easy to show that the denominator $\gamma(x_-x'_+ - x_+x'_-) = 2i$. Thus the general solution to the inhomogeneous equation for bunch n is

$$x_n(s) = a_n^+x_n^+(s) + a_n^-x_n^-(s) + \int_0^s G_n(s,s')F_n(s')ds' \quad , \tag{3.9}$$

where a_n^+ and a_n^- are arbitrary constants. The Green function is given by

$$G_n(s,s') = [\gamma(s)\gamma(s')k_n(s)k_n(s')]^{-1/2} \sin\psi_n(s,s') \quad , \tag{3.10}$$

where

$$\psi_n(s,s') \equiv \int_{s'}^s k_n(s'')ds'' \tag{3.11}$$

is the phase advance for bunch n. Let us take the "positive phase" WKB solution as the motion for the first bunch,

$$x_1(s) \;=\; x_1(0)\left(\frac{\gamma_0}{\gamma(s)}\right)^{\frac{1-p}{2}} \exp\left[\psi_1(s,0)\right] \quad , \tag{3.12}$$

and assume $a_n^- = 0$ for all $n > 1$. Then upon substituting the explicit expressions (3.2) and (3.10) for the driving term $F_n(s)$ and the Green function $G_n(s,s')$ into (3.9), we obtain

$$\begin{aligned} x_n(s) \;=\; & x_n(0)\left(\frac{\gamma_0}{\gamma(s)}\right)^{\frac{1-p}{2}} \exp\left[i\psi_n(s,0)\right] \;+ \\ & \frac{Ne^2}{\gamma_0 mc^2 k_n(0)}\left(\frac{\gamma_0}{\gamma(s)}\right)^{\frac{1-p}{2}} \int\limits_0^s \left(\frac{\gamma_0}{\gamma(s')}\right)^{\frac{1-p}{2}} \sin\psi_n(s,s') \\ & \times \sum_{j=1}^{n-1} W_\perp((n-j)\ell)x_j(s')ds' \quad . \end{aligned} \tag{3.13}$$

It is useful to write this in a slightly different form for comparison with later results and for convenience of numerical integration. Upon writing $\sin\psi_n(s,s')$ in terms of exponentials, and dropping a rapidly oscillating term, the solution for the offset of bunch n becomes

$$\begin{aligned} x_n(s) \;=\; & \left[x_n(0) + \frac{Ne^2}{2i\gamma_0 mc^2 k_n(0)} \int\limits_0^s \left(\frac{\gamma_0}{\gamma(s')}\right)^{\frac{1-p}{2}} \exp[-i\psi_n(s',0)]\right. \\ & \left. \times \sum_{j=1}^{n-1} W_\perp((n-j)\ell)x_j(s')ds'\right]\left(\frac{\gamma_0}{\gamma(s)}\right)^{\frac{1-p}{2}} \exp[i\psi_n(s,0)] \quad . \end{aligned} \tag{3.14}$$

3.1. Multiple bunches in the "effective length" representation

Let us assume that the focusing function varies as

$$k(s) = \left(\frac{\gamma_0}{\gamma(s)}\right)^{1/2} k_0 \quad . \tag{3.15}$$

This is a physically reasonable scaling of the focusing function for the following reasons. We would like to focus strongly at the beginning of the linac to control wake field effects. The quadrupole "lens strength" q (*i.e.*, inverse focal length) in, for example, the x plane, scales as

$$q \propto L_{quad} \frac{dB_y}{dx} \frac{1}{\gamma} \quad , \tag{3.16}$$

where L_{quad} is the length of the quadrupole. Since there are practical limits to the magnetic field gradient achievable in quadrupoles, it is most efficient to keep this gradient (here, dB_y/dx) constant near its maximum value. We also assume a FODO lattice (each cell of which consists of a focusing quadrupole, drift space, defocusing quadrupole, and another drift space) with cell length L_{cell} allowed to vary along the linac. The phase advance μ per cell is

$$\sin\frac{\mu}{2} = \frac{qL_{cell}}{4} \quad , \tag{3.17}$$

and the average of the minimum and maximum of the β function in a cell is

$$\bar{\beta} = \frac{L_{cell}}{\sin\mu} \quad . \tag{3.18}$$

Let us also assume that both the phase advance per cell and the "filling fraction" (*i.e.*, the length occupied by quadrupoles divided by the total length of the linac) are kept approximately constant as we go down the linac. Then both L_{quad} and L_{cell} must scale as $\gamma^{1/2}$. In this case, $\bar{\beta} \propto \gamma^{1/2}$, and the corresponding smooth focusing approximation is just Eq. (3.15).

For this choice of focusing function, the result (2.12), derived without acceleration, gives the same result as (3.14) except for adiabatic damping factors, provided we interpret the variables appropriately. In particular, E and the k_n in (2.12) are taken to be the energy and the focusing functions at the beginning of the linac, and s is taken to be not the true distance along the accelerator, but rather an "effective distance." The effective distance is just $\psi(s,0)/k(0)$, where $\psi(s,0)$ is the phase advance in the actual distance from 0 to s along the linac and $k(0)$ is the focusing function at the beginning of the linac, that is,

$$\begin{aligned} s_{eff} = \frac{\psi(s,0)}{k(0)} &= \frac{1}{k(0)} \int_0^s k(s')ds' \\ &= \frac{2\gamma_0^{1/2}}{G}\left[\gamma^{1/2} - \gamma_0^{1/2}\right] \quad . \end{aligned} \tag{3.19}$$

Note that if $\gamma(L) \gg \gamma_0$ at $s = L$, the end of the linac, the effective length of the linac is approximately

$$L_{eff} = 2\left[\frac{\gamma_0}{\gamma(L)}\right]^{1/2} L \quad . \tag{3.20}$$

3.2. Integration of equations of motion

A computer program (LINACBBU) was written to numerically integrate the equations of motion in the effective length approximation (2.12), or for more general focusing, Eq. (3.14). The wake field at the needed bunch spacings is computed from an appropriate input set of transverse dipole modes. The focusing function may be the same for all bunches or may be varied, for instance linearly or sinusoidally, from bunch to bunch.

A number of examples using this program will be given in later sections. In all cases of interest here, the assumption of adiabatic acceleration is an excellent approximation.

4. VERY STRONGLY DAMPED WAKE

In cases where the wake field is strongly damped, a bunch will only see a significant wake from the immediately preceding bunch, and we can use a simple "daisy chain" model to estimate the transverse blowup of each bunch in the train. Let us assume that the focusing function is the same for all bunches. Then, the equations of motion in the effective length approximation are

$$\begin{aligned} x_1'' + k^2 x_1 &= 0 \\ x_n'' + k^2 x_n &= \frac{Ne^2 W_\perp(l)}{E} x_{n-1} \qquad (n > 1) \quad . \end{aligned} \tag{4.1}$$

We assume $x_1(s) = a_1 e^{iks}$ where a_1 is a constant, and look for solutions $x_n(s) = a_n(s)e^{iks}$. Neglecting the a_n'' terms, we obtain

$$a_n' = -i\sigma a_{n-1} \quad , \tag{4.2}$$

where

$$\sigma \equiv \frac{Ne^2 W_\perp(l)}{2kE} \quad . \tag{4.3}$$

It is straightforward to show that the solution is

$$a_n(s) = \sum_{j=0}^{n-1} \frac{(-i\sigma s)^j}{j!} a_{n-j}(0) \quad , \tag{4.4}$$

and if we take as initial condition $a_n(0) = 1$ for all n, this simplifies to

$$a_n(s) = \sum_{j=0}^{n-1} \frac{(-i\sigma s)^j}{j!} \quad . \tag{4.5}$$

We see that $(a_2 - a_1)$ grows linearly with effective distance s, as we noted earlier, and in general, for sufficiently large σs, the amplitude of oscillation of bunch n would grow approximately as s^{n-1}. However, for the strongly damped wakes we are considering in this model, σs does not necessarily become large in the distances s of interest. It is apparent that the criterion for little or no blowup in the linac is $|\sigma L_{eff}| < 1$, that is,

$$\frac{Ne^2 |W_\perp(l)| L_{eff}}{2kE} < 1 \quad , \tag{4.6}$$

where L_{eff} is the effective length of the linac. Recall that k and E are the values of focusing function and energy at the beginning of the linac.

The results of the daisy chain model, where applicable, will be compared with the integration of the equations of motion in the section on numerical results. We will see that for the main linacs of a TeV collider, with highly damped acceleration cavities, we in fact have $|\sigma L_{eff}| \sim 1$. Thus for n greater than a few, $a_n(s)$ is approximately $e^{-i\sigma s}$, and there is almost no blowup for bunches beyond the first few in the train.

5. LINEARIZED WAKE-ZERO-CROSSING MODEL

Let us now consider the case where the bunches are placed near the zero-crossings of a single mode wake function, given by

$$W_{\perp}(z) \;=\; W_1 \sin(K_1 z) e^{-K_1 z/2Q} \quad . \tag{5.1}$$

For simplicity, we use the effective length formalism, and we assume a focusing function k that is the same for all bunches. The bunch spacing ℓ is assumed to be close to an integer number q of half-wavelengths of the wake function. We define the quantity Δ by

$$\Delta \;\equiv\; \pi q - K_1 \ell \quad . \tag{5.2}$$

Then, provided that $(n-1)|\Delta| \ll 1$, we can approximate the driving term for bunch n, given by Eq. (2.9), by expanding the wake function to first order about the zero-crossings:

$$f_n \;=\; \frac{Ne^2}{E} \sum_{j=1}^{n-1} \left.\frac{dW_{\perp}}{dz}\right|_{z_{n-j}} \delta z_{n-j}\, x_j \quad . \tag{5.3}$$

Here

$$z_{n-j} \;\equiv\; \frac{q(n-j)\pi}{K_1} \tag{5.4}$$

is the position of the bunch-j wake zero-crossing that is nearest to bunch n, and

$$\delta z_{n-j} \;=\; (n-j)\frac{\Delta}{K_1} \tag{5.5}$$

is the distance of bunch n from this nearby zero-crossing. It is convenient to express Δ in terms of the fractional deviation of the wavenumber K_1 from the nearby wavenumber K_1^0 for which the bunches would be exactly at the zero-crossings. So, suppose $K_1 = K_1^0 + \delta K_1$. Using the fact that $K_1^0 \ell = q\pi$, we can write

$$\Delta = -\frac{\delta K_1}{K_1^0} \cdot q\pi \quad . \tag{5.6}$$

Thus, the condition for the validity of this linearized wake-zero-crossing model is

$$q\pi(n-1)\left|\frac{\delta K_1}{K_1^0}\right| \ll 1 \quad . \tag{5.7}$$

The derivatives of the wake function at the zero-crossings of interest are just

$$\left.\frac{dW}{dz}\right|_{K_1 z = q(n-j)\pi} = (-1)^{q(n-j)} W_1 K_1 e^{-q(n-j)\pi/2Q} \quad , \tag{5.8}$$

and so the driving term is

$$f_n = \frac{Ne^2 W_1 \Delta}{E} \sum_{j=1}^{n-1} (n-j)[-e^{-\pi/2Q}]^{q(n-j)} x_j \quad . \tag{5.9}$$

Substituting into Eq. (2.11), we obtain the bunch amplitude function

$$a_n(s) = a_n(0) - iA(\Delta) \sum_{j=1}^{n-1} (n-j) B^{n-j} \int_0^s a_j(s') ds' \quad , \tag{5.10}$$

where

$$A(\Delta) \equiv \frac{Ne^2 W_1}{2kE} \Delta \tag{5.11}$$

and

$$B \equiv [-e^{-\pi/2Q}]^q \quad . \tag{5.12}$$

5.1. Initial offset of first bunch only

Suppose the first bunch starts with offset $a_1 = \hat{x}$ and all the other bunches start with offset $a_n(0) = 0$. It is easy to see that in this case the solution for a_n will be of the form

$$a_n(s) \;=\; \hat{x}B^{n-1}\sum_{j=0}^{n-1} C_j^n \frac{(-iAs)^j}{j!} \quad , \tag{5.13}$$

where the C_j^n are constant coefficients defined for $j = 0$ to $n-1$. Since $a_1 = \hat{x}$, we have $C_0^1 = 1$. Upon substituting a_j of the form given in Eq. (5.13) into Eq. (5.10) and simplifying, we obtain for $n > 1$

$$a_n(s) \;=\; \hat{x}B^{n-1}\sum_{j=1}^{n-1}\sum_{k=j}^{n-1}(n-k)C_{j-1}^k \frac{(-iAs)^j}{j!} \quad . \tag{5.14}$$

Comparing Eqs. (5.13) and (5.14), we obtain

$$C_0^n \;=\; 0 \quad , \qquad (n \;>\; 1) \quad , \tag{5.15}$$

and, for $j > 0$, the recursion relation

$$C_j^n \;=\; \sum_{k=j}^{n-1}(n-k)C_{j-1}^k \quad . \tag{5.16}$$

5.2. Equal initial offset of all bunches

It is only somewhat more complicated to treat the case where all bunches start out with the same offset $a_n(0) = \hat{x}$. Then one can see that the solution will be of the form

$$a_n(s) = \hat{x}\left[1 + \sum_{j=1}^{n-1}\sum_{\ell=j}^{n-1} C_{j\ell}^n B^\ell \frac{(-iAs)^j}{j!}\right] \quad , \tag{5.17}$$

where the $C_{j\ell}^n$ are another set of constant coeffients, defined for $j = 1$ to $n-1$, and for $\ell = j$ to $n-1$. When we substitute a_j of this form into Eq. (5.10), we obtain after some manipulation

$$a_n(s) = \hat{x}\left[1 + \sum_{j=1}^{n-1}\sum_{\ell=j}^{n-1}\sum_{k'=j-1}^{\ell-1} (n-\ell)B^{n-\ell+k'} C_{j-1,k'}^\ell B^\ell \frac{(-iAs)^j}{j!}\right] \quad , \tag{5.18}$$

where we have defined the "initial" coefficients

$$\begin{aligned} C_{00}^n &= 1 \\ C_{0\ell}^n &= 0 \quad , \qquad (\ell > 0) \quad . \end{aligned} \tag{5.19}$$

Comparing Eqs. (5.17) and (5.18), we find that

$$C_{j\ell}^n = \sum_{k'=j}^{\ell} (n-\ell)B^{n-2\ell+k'-1} C_{j-1,k'-1}^\ell \quad . \tag{5.20}$$

In order to have a relation among the coefficients that is independent of B, the exponent of B must be zero. This yields the recursion relation

$$C_{j\ell}^n = \sum_{k=j}^{\ell} (\ell-k+1) C_{j-1,k-1}^\ell \quad , \qquad (j \geq 1) \quad , \tag{5.21}$$

which together with Eq. (5.19) determines the desired coefficients $C_{j\ell}^n$.

Low-Q limit. Actually, for a damped wake the requirement $(n-1)\Delta \ll 1$ may be more stringent than necessary. We really only need $(n_d - 1)\Delta \ll 1$, where n_d is the number of bunch spacings at which the wake field has damped to a negligible value.

Obviously, we recover the daisy chain result when Q is low enough. In the case where the wake is negligible beyond one bunch spacing and each bunch is close to a zero-crossing of the wake from the preceding bunch, we can write the criterion (4.6) for little blowup as

$$\frac{Ne^2|W_1\Delta|e^{-q\pi/2Q}L_{eff}}{2kE} < 1 \quad , \qquad (\Delta \ll 1) \quad . \tag{5.22}$$

<u>High-Q limit.</u> For a single-mode wake field with little damping ($Q \to \infty$), we have $B \approx (-1)^q$. Thus, for the case $a_1 = \hat{x}$, and $a_n(0) = 0$ for $n > 1$, the amplitude function of bunch n is

$$a_n(s) \;=\; \hat{x}(-1)^{q(n-1)} \sum_{j=1}^{n-1} C_j^n \frac{(-iAs)^j}{j!} \quad . \tag{5.23}$$

For the case $a_n(0) = \hat{x}$ for all n, we have

$$a_n(s) \;=\; \hat{x}\left[1 + \sum_{j=1}^{n-1} \frac{(-iAs)^j}{j!} \sum_{\ell=j}^{n-1} (-1)^{q\ell} C_{j\ell}^n\right] \quad . \tag{5.24}$$

Note that for a given j, the terms in the sum over ℓ are positive for q even and alternating in sign for q odd. Thus for a given value of A, we expect the amplitudes to be larger when the bunch-to-bunch spacing ℓ is an even number of half-wavelengths of the wake function than when it is an odd number of half-wavelengths. This just reflects the fact that the contributions to the driving force on bunch n due to the $n-1$ preceding bunches are the same sign (alternate in sign) for q even (odd), because the corresponding wake function values are the same sign (alternate signs). Remember that we assumed the bunches start out with the same offset, and since the focusing function was assumed the same for all the bunches, they will tend to stay in phase.

A similar effect (odd q versus even q) also occurs in Eq. (5.18). Although it is most pronounced for longer range wakes, the effect can be seen in our examples even when the wake is significant for a distance of only a few bunch spacings. Illustrations will be given in later sections, where we will also compare this model with results from the LINACBBU program.

6. CURES FOR THE TRANSVERSE INSTABILITY

We now turn to the study of ways to prevent multi-bunch beam breakup in linacs. The four cures that we shall study are damping the transverse wake, minimizing the wake effects by placing the bunches close to nodes of the wake field, introducing a spread in the frequency of corresponding transverse dipole modes, and varying the focusing to partly cancel the wake force at the bunches (BNS damping). As noted earlier, a cell-to-cell frequency spread in the transverse modes is present in the existing SLAC linac and has also been examined by Yokoya in the context of a next-generation collider (Ref. 10). We also note that BNS damping is used to control single bunch emittance growth, in essentially all extant designs for a next-generation collider.[27] We shall emphasize the usefulness of the first two cures, in a very high energy collider utilizing multibunching.

6.1. DAMPED CAVITIES

Theoretical and experimental studies show that it is possible to construct damped acceleration cavities that significantly reduce the $Q's$ of the transverse dipole wake modes (Ref. 17). One way to construct such cavities is to cut axial slots through the irises of the structure and couple these slots to radial waveguides. Transverse mode Q's as low as 10 can be obtained in this way. Measurements have shown that there is no significant adverse effect of such slots on the accelerating mode. Another type of damped cavity has side-coupled slots that go into the cavity without cutting the irises. These slots perturb the accelerating mode to some extent, but do not transmit it. The Q's of the transverse modes can be as low as about 40 in this case.

The $Q's$ obtained for the higher-order modes should be at least as low as the Q of the fundamental. For simplicity in the numerical computations, we will generally take the $Q's$ to be the same for all modes. Another option is to assume that there is "equal damping" of all modes. That is, given a value of Q for the fundamental mode, assume that all modes damp as $e^{-\alpha z}$, where α is the damping rate of the fundamental mode:

$$\alpha = \frac{K_1}{2Q_1} \quad . \tag{6.1}$$

Obviously, if the fundamental dipole mode is sufficiently dominant, it does not make much difference whether we assume equal $Q's$, equal damping of the higher-order modes, or individual Q's for each mode.

6.2. Tuning the Frequency of the Fundamental Transverse Mode

The transverse dipole wake for the accelerating structure considered here is indeed strongly dominated by its fundamental mode and has zero-crossings that are approximately equally spaced. Figure 1 shows the dipole wake computed using the program TRANSVRS[28] for a disk-loaded structure designed to operate at 11.4 GHz. The structure has a cell length of 8.75 mm, internal cell radius of 11.2 mm, and a relatively large iris radius of 5.2 mm. This structure has no slots to damp the transverse modes. However, assuming that such slots damp higher-order transverse modes at least as much as they damp the fundamental transverse mode, and that this fundamental mode dominates the others, the slotted structures will have a damped wake with nearly periodic zero-crossings throughout the extent of the wake.

Therefore, it is possible to place all the bunches in a train near zero-crossings of the wake field, if the ratio of the frequency of the fundamental dipole mode to the frequency of the accelerating RF is appropriately tuned. The condition that this be so is just

$$\frac{1}{2} q \lambda_{W_\perp} = m \lambda_{rf} = \ell \quad , \tag{6.2}$$

where ℓ is the bunch spacing, m and q are integers, and λ_{rf} and $\lambda_{W_\perp}$ are the wavelengths of the RF and the fundamental dipole wake mode.

6.3. Spread in frequency of each transverse dipole mode

One might also consider an RF structure in which the frequencies of corresponding transverse dipole modes differ from cell to cell. This is the case, for example, in the existing SLAC linac, where the mode frequency spread is a few percent.[29] The frequency spread results in a reduction of the effective Q of each mode.

In the main linacs of a linear collider, this method is a partial cure at best. Note that the design and construction of an accelerating structure incorporating both damping slots coupled to radial waveguides and a cell-to-cell mode frequency spread would probably be rather complicated.

6.4. Bunch-to-bunch variation of transverse focusing

By the use of a system of time-varying quadrupoles in addition to the main system of quadrupoles, we could introduce a small spread in the focusing functions k_n of the bunches. This is essentially the BNS damping mechanism (Ref. 23) applied to multiple bunches. If the focusing increment at a given bunch is chosen appropriately, one can at least partially cancel the wake force due to the preceding bunches [*cf.* Eq. (2.5)]. It is not practical to use this method by itself to control the wake field effects of multiple bunches because, for the parameter regimes we will be considering, the required spread in the values of the k_n would be large. The resulting chromatic phase advance differences create complications with orbit correction, as will be discussed later in the examples. Note also that a given bunch feels the wakes from all the preceding bunches, which means that the choice of an optimum bunch-to-bunch focusing spread is not as simple as in the two-bunch case.

7. CHOICE OF MAIN LINAC PARAMETERS

7.1. Linac RF frequency

In the choice of RF frequency, there is a tradeoff between less power consumption at higher frequencies and lower transverse wake fields at lower frequencies. We shall consider some examples of main linacs with accelerating frequency of 11.4 and 17.1 GHz. The optimization depends on many factors other than just the need to control beam breakup.[30]

7.2. SCALING OF FOCUSING FUNCTION

As discussed earlier, a focusing function scaling as the inverse of the square root of the energy is a reasonable choice physically, and is also particularly convenient to analyze. We shall assume this scaling in all our examples of TeV collider main linacs.

7.3. RELATION BETWEEN BUNCH CHARGE AND BUNCH SPACING

Keeping the bunch-to-bunch energy variation as small as possible imposes a relation between the number of particles per bunch, N, and the bunch spacing ℓ (Ref. 1):

$$\ell \approx cT_f \frac{\eta_0}{2} e^{\tau} \quad , \tag{7.1}$$

where T_f is the filling time and τ is the ratio of the filling time to the attenuation time of the RF structure. The single-bunch loading is

$$\eta_0 \equiv \frac{4Ne\kappa_0}{\mathcal{E}_z} \quad , \tag{7.2}$$

where κ_0 is the loss parameter of the accelerating mode and $\mathcal{E}_z$ is the acceleration gradient. Thus, for given parameters of the accelerating structure, we have a relation between the bunch spacing ℓ and the number N of particles per bunch.

8. EXAMPLES OF MAIN LINACS AT 17.1 GHZ

For illustration, let us first consider a main linac accelerating frequency of 17.1 GHz.

Table 1. Parameters for main linacs at 17.1 GHz.

Number of bunches	10
Number of particles per bunch	1.67×10^{10}
Bunch spacing ℓ	$24\lambda_{rf} \approx 42.0$ cm
Initial energy of linac	18 GeV
Final energy of linac	500 GeV
Linac length	3000 m
Initial beta function	3.2 m ($k_0 = 0.3125\ \mathrm{m}^{-1}$)

The parameter set used is shown in Table 1. Each linac accelerates 10 bunches per RF fill, to an energy of 0.5 TeV. The bunch spacing and bunch charge should be chosen to make the energy of each bunch as nearly the same as possible, in accordance with Eq. (7.1). Taking $T_f = 60$ nsec, $\tau = 0.6$, $\kappa_0 = 430$ V/pC/m, and $\mathcal{E}_z = 186$ MeV/m in this equation gives

$$\ell \approx (0.25\ \mathrm{m})\frac{N}{10^{10}} \quad . \tag{8.1}$$

The single bunch loading, from Eq. (7.2), is

$$\eta_0 \approx (1.5 \times 10^{-2})\frac{N}{10^{10}} \quad . \tag{8.2}$$

In the present examples, we shall take ℓ to be 24 RF cycles (about 42 cm) and $N = 1.67 \times 10^{10}$; this gives $\eta_0 = 2.5\%$.

We shall examine all four of the cures for beam breakup discussed earlier. In all our examples we assume that the first cure, damped acceleration cavities, is used to reduce the Q's of the transverse modes to values well below 100. Except for very low Q's, it is still necessary to also apply at least one of the other cures. We shall try to give a representative selection of examples, with realistic parameters for main linacs at 17.1 GHz in this section and at 11.4 GHz in the next section.

In considering the first two cures, we have a two-dimensional parameter space to explore, namely:

1. The Q value of the modes of the transverse dipole wake (taken to be the same for all the modes).
2. The frequency of the fundamental transverse dipole mode (in our computations, the frequencies of the other modes will be assumed unchanged).

The RF wavelength at 17.1 GHz is 1.75 cm, and the wavelength of the fundamental mode of the unmodified transverse dipole wake (Figure 1) is 1.36 cm. If the frequency of the fundamental mode is shifted slightly, so that its wavelength is 1.31 cm, then Eq. (6.2) is satisfied with $q = 64$, and we have

$$\frac{\lambda_{rf}}{\lambda_{W_\perp}} = \frac{4}{3} \quad . \tag{8.3}$$

When this relation is satisfied, the frequency of the fundamental transverse mode is 477.85 m^{-1}, which we shall denote by K_1^0. In Figure 2, we show "tuning curves" of the maximum transverse amplitude x_{max} in the bunch train as a function of the frequency of the fundamental transverse dipole mode, for values of $Q = 20$ to 50. The value of x_{max} is the maximum of the amplitudes reached by all bunches as they travel down the linac, normalized by dividing out the adiabatic damping factor

$(\gamma_0/\gamma)^{1/4}$. The central frequency K_1^0, at which $\lambda_{rf}/\lambda_{W_\perp} = 4/3$, is 477.85 m^{-1}. The range about the central frequency shown in the figure is $\pm 1\%$. If we take $x_{max} \leq 2$ as a figure of merit, then for $Q = 50$ we would have to tune to within about 0.26% of the central frequency K_1^0.

Of course, the lower the Q, the less sharply defined is the frequency of the mode; the full width at half-maximum of the resonance around the central frequency K_1^0 is $\Gamma \equiv K_1^0/Q$ (and the central frequency is shifted slightly from that of the undamped mode). Therefore, it is also of interest to compare the ratio R of the tuning tolerance for a given Q to the full width Γ of the resonance at that Q:

$$R \equiv \frac{\Delta K_1^0}{\Gamma} \quad . \tag{8.4}$$

Table 2. Tuning parameters for the fundamental transverse dipole mode for 17.1 GHz main linacs.

Q	ΔK_1^0 (m^{-1})	$\Gamma = K_1^0/Q$ (m^{-1})	$\Delta K_1^0/K_1^0$	$R \equiv \Delta K_1^0/\Gamma$
20	—	23.9	—	—
30	—	15.9	—	—
35	3.26	13.7	0.68%	24%
40	2.10	11.9	0.44%	18%
45	1.53	10.6	0.32%	14%
50	1.26	9.56	0.26%	13%

In Table 2, we show the full-width tuning tolerance ΔK_1^0 for the criterion $x_{max} \leq 2$, the full-width Γ of the resonance peak, the tuning tolerance expressed as a percentage of the undamped central frequency, and the ratio R. The parameters used and the values of Q tabulated are those used in Figure 2. For all these values of Q, the tolerance on tuning is at least 10% of the bandwidth of the resonance; this should be straightforward to do. For sufficiently strong damping (Q = 20 and 30), the blowup is less than a factor of 2, even without tuning the value of the fundamental transverse mode to put the bunches near wake zero-crossings. However, for the larger Q's, some tuning of the frequency of this mode would be necessary to keep the blowup small.

In Figure 3, we show the effects of introducing the third cure. In this example, there is a total spread of 2% in the frequency of each transverse mode, distributed uniformly over 200 values. Except for this frequency spread, the parameters used are the same as in Figure 2. For Q's of 40 or less, no tuning of the fundamental transverse mode frequency is required to keep the blowup less than a factor of two. For the higher values of Q shown, some tuning would be required. Recall that Q's of 40 or so are obtainable without slotting the irises. Thus, for these parameters, an acceptable solution is possible without slotting the irises, provided that we either tune the fundamental transverse mode frequency or introduce at least a 2% spread in the transverse mode frequencies.

Finally, in Figure 4, we illustrate the effect of the fourth cure, namely, a variation in the strength of the focusing function at each bunch to partially cancel the wake effects. From Eq. (2.5), we see that there will be exact cancellation of the part of the wake that is due to the immediately preceding bunch when

$$\Delta k_{adj} = \frac{Ne^2 W_\perp(\ell)}{2Ek_0} \quad . \tag{8.5}$$

Here, Δk_{adj} is the difference between the focusing functions of adjacent bunches. Multiplying this by 9 gives the total spread Δk_{tot} over all 10 bunches. For $Q = 40$

and for a frequency of the fundamental transverse mode 0.6% above K_1^0, we obtain $\Delta k_{tot} = 10.8\%$. We show the results with this value in Figure 4(b). However, the phase advance difference due to a focusing spread can introduce complications. In our example, the total phase advance in the main linacs is about 90π. Thus, for a focusing spread of 1%, the spread in phase advance is already significant compared to 2π. In such a case, the amplitude of betatron oscillations must be smaller than the transverse bunch dimensions or there must be position control of individual bunches at the end of the linac, to keep the bunches from missing each other at the interaction point. Figure 4(a) shows the case of a total spread of 1% linearly distributed over the bunches. We see that for the smaller value of Δk_{tot} shown in Figure 4(a), there is no appreciable increase in the tuning tolerance compared to Figure 2. Thus, in order to obtain a significant effect on the tuning tolerance, it would be necessary to introduce a spread so large that there would be significant "chromatic" phase advance differences among the bunches.

8.1. Comparison with daisy chain model

In the 17.1 GHz examples just given, the number of e-foldings of the wake between bunches is about

$$\frac{K_1^0 \ell}{2Q} \approx \frac{100}{Q} \quad , \tag{8.6}$$

which gives about three e-foldings, for $Q = 35$. This is a case in which it would be reasonable to apply the daisy chain model, which only takes account of the wake between adjacent bunches. In Figure 5, the results of the daisy chain model are compared with the results of the program LINACBBU. We have included twenty bunches in the train to illustrate the fact that there is no blowup for bunches sufficiently far back in the train. The wavenumber of the first wake mode is taken to be 475 m^{-1}, $Q = 35$, and other parameters are as given in Table 1. For bunch 2, the agreement is of course exact. For bunches 3 through 9, there is some observable discrepancy, due to the effects of wakes at more than one bunch spacing. However, the overall agreement is very good. Note that in this example $|\sigma L_{eff}| \approx 2.5$, where σ is defined in Eq. (4.3) and L_{eff} is the effective length.

9. EXAMPLES OF MAIN LINACS AT 11.4 GHZ

Taking $T_f = 80$ nsec, $\tau = 0.4$, $\kappa_0 = 190$ V/pC/m, and $\mathcal{E}_z = 186$ MeV/m in Eq. (7.2) gives the single-bunch loading

$$\eta_0 \approx (6.7 \times 10^{-3}) \frac{N}{10^{10}} \quad , \tag{9.1}$$

and, from (7.1), the relation between bunch charge and bunch spacing

$$\ell \approx (0.12 \text{ m}) \frac{N}{10^{10}} \quad . \tag{9.2}$$

For a given charge Ne per bunch, the single-bunch loading and bunch spacing are roughly half of what they were for the 17.1 GHz example. Since the bunch spacing is closer (and also the wake frequencies are lower), the wake extends over more bunches than in the 17.1 GHz case, for a given value of Q. Thus, the daisy chain model would not be applicable except for extremely low Q's. However, the effect of odd versus even q, where q is the number of half-wavelengths of the dominant wake mode between bunches, is more significant and can be used to advantage.

It is also of interest to examine the trade-off between the number of bunches and the charge per bunch. Suppose we fix the total charge nNe accelerated per RF pulse, *i.e.,* the total charge in the bunch train. For larger N and smaller n, we would obtain more luminosity per pulse. However, it may be necessary to go to smaller N and larger n, to help alleviate the problem of pair production at the interaction point.[31]

Table 3. Parameters for main linacs at 11.4 GHz.

Number of bunches	20
Number of particles per bunch	0.88×10^{10}
Bunch spacing ℓ	$4\lambda_{rf} \approx 10.5$ cm
Initial energy of linac	18 GeV
Final energy of linac	500 GeV
Linac length	3000 m
Initial beta function	3.2 m ($k_0 = 0.3125$ m^{-1})

We first consider an example with parameters as shown in Table 3. The linac length, energy, and focusing function are the same as for our 17.1 GHz examples. The bunch spacing is chosen to be four RF wavelengths, and the corresponding bunch charge from Eq. (9.2) is 0.88×10^{10}. The single-bunch loading from Eq. (9.1) is about 0.6%. With the lower charge per bunch and weaker wake fields, it is possible to have more bunches; for this example, we have chosen 20 bunches. In order to take advantage of odd q, we have tuned the frequency of the fundamental transverse mode so that there are 5.5 wavelengths of this mode between bunches (*i.e.*, $q = 11$). The resulting tuning curves, for $Q = 20$ to 60 are depicted in Figure 6. As in the 17.1 GHz examples, a spread of $\pm 1\%$ about the central frequency is shown. We see that the tuning tolerances are comparable to, although somewhat less tight than, those in the 17.1 GHz example of Figure 2.

9.1. Comparison of even versus odd q

For comparison with Figure 6, we show in Figure 7 the tuning curves for exactly the same parameters, except that we have tuned the fundamental transverse mode frequency to make q even. In particular, $q = 10$ in Figure 7(a), and $q = 12$ in Figure 7(b). As expected, the tuning tolerances for both these cases are tighter than in the $q = 11$ case shown in Figure 6.

9.2. Examples with same total charge but different spacing

Next we examine two cases that are identical to that in Figure 6 except that the same total charge is distributed differently among bunches, while still satisfying Eq. (9.2). In the first case [Figure 8(a)], we choose $\ell = 8\lambda_{rf} \approx 0.21$ m. The charge per bunch, in accord with (9.2), is 1.75×10^{10}. To keep the total charge in the train the same, we choose the number of bunches $n = 10$. We tune the frequency of the fundamental transverse dipole mode so that we have 10.5 wavelengths of this mode per bunch spacing. In the second case [Figure 8(b)], we choose $\ell = 3\lambda_{rf} \approx 0.0789$ m, resulting in $N = 0.65 \times 10^{10}$ and $n = 27$. The frequency of the fundamental transverse mode is tuned to have 3.5 wavelengths per bunch spacing. Note that in both of these cases, we have kept q odd, as it is in the example of Figure 6. We see that for the given values of Q, the tuning tolerances for the three cases shown in Figures 6, 8(a), and 8(b) are not drastically different, although those in Figure 8(a) are somewhat tighter than for the other two. Figure 8(a) has the smaller number of bunches, with more charge per bunch and larger bunch spacing. In all three cases, there is less than one e-folding of the wake between bunches, and we are in a regime where the effect of greater charge per bunch dominates the exponential decrease of the wake, to produce somewhat larger blowup in Figure 8(a).

9.3. Comparison with wake-zero-crossings model

The quantitative agreement between the wake-zero-crossings model and the LINACBBU program is good, provided the bunches are near enough to the zero-crossings. Consider the case $N = 1.75 \times 10^{10}$, $n = 10$, $\ell = 8\lambda_{rf} \approx 0.21$ m, with a single-mode wake damped to $Q = 60$ and with frequency tuned to 299.1059 m^{-1}, which is 0.15% above the point where $\lambda_{rf}/\lambda_{W_\perp} = 10/8$. From Eq. (5.7), we expect the wake-zero-crossing model to give reasonably good agreement for

$$\left|\frac{\delta K_1}{K_1^0}\right| < 0.18\% \quad . \tag{9.3}$$

This is borne out by Figure 9, which compares the results from the program LINACBBU with the prediction of the wake-zero-crossing model. The solid curve shows the transverse oscillation of the bunch as a function of effective length, according to LINACBBU. The dotted curve shows the envelope of bunch oscillation, according to the wake-zero-crossing model. The motion of the second, sixth, and tenth bunch in the train are shown. The simple linearized model overestimates the wake slightly, leading to the small observable discrepancy with the LINACBBU result.

10. CONCLUSIONS AND ACKNOWLEDGMENTS

We have demonstrated that it is possible to control the potentially severe multibunch beam breakup in the main linacs of a TeV linear collider. The solution that seems most generally applicable for the regime of interest is a combination of two cures: (1) using damped acceleration cavities that reduce the Q's of transverse dipole modes, and (2) tuning the frequency of the fundamental transverse dipole mode so that bunches may be placed near wake zero-crossings. Simple analytic models for the limit of a very strongly damped wake and for the limit of bunches in the linear region about the zero-crossings were presented and were shown to agree well with the results obtained from the Green function integrals, in their respective regimes of validity. The other two cures examined, namely (3) a cell-to-cell variation of transverse mode frequencies, and (4) a bunch-to-bunch spread in in the focusing function, could also be useful in combination with one or both of the first two cures.

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FIGURE CAPTIONS

1) The dipole wake for a disk-loaded structure designed to operate at 11.4 GHz. Ninety modes have been included. The wavenumber of the fundamental mode is about 308 m^{-1} and the zero-crossings are nearly equally spaced at half the corresponding wavelength.

2) Maximum transverse amplitude x_{max} of all bunches, normalized by dividing out the adiabatic damping factor $(\gamma_0/\gamma)^{1/4}$, as a function of the frequency of the fundamental transverse dipole mode, for values of $Q = 20$ to 50, at 17.1 GHz accelerating frequency. The central frequency K_1^0, where $\lambda_{rf}/\lambda_{W_\perp} = 4/3$, is 477.85 m^{-1}. The spread shown about K_1^0 is $\pm$ 1.0%.

3) Maximum transverse amplitude x_{max} (normalized) of all bunches as a function of the frequency of the fundamental transverse dipole mode, for values of $Q = 40$ to 70, at 17.1 GHz accelerating frequency, with a spread in each transverse mode frequency of 2%. The central frequency K_1^0, where $\lambda_{rf}/\lambda_{W_\perp} = 4/3$, is 477.85 m^{-1}. The spread shown about K_1^0 is $\pm$ 1.0%.

4) Maximum transverse amplitude x_{max} (normalized) of all bunches as a function of the frequency of the fundamental transverse dipole mode, for values of $Q = 20$ to 50, with nonzero, linearly distributed spread in the focusing functions over the bunches. In (a), $\Delta k/k_0 = 1\%$, and in (b), $\Delta k/k_0 = 10.8\%$.

5) Comparison of the results of the daisy chain model (plotted as □'s) with the results of the program LINACBBU (plotted as X's). In each case, the value of the envelope function $|a_n(s)|$ at the end of the linac, for each bunch number n, is plotted. The transverse offset $x_n(s) = a_n(s)e^{iks}$.

6) Maximum transverse amplitude x_{max} (normalized) of all bunches as a function of the frequency of the fundamental transverse dipole mode, for values of $Q = 20$ to 60, at 11.4 GHz accelerating frequency. The central frequency, where $\lambda_{rf}/\lambda_{W_\perp} = 5.5/4$, is 328.52 m^{-1}. The spread shown about K_1^0 is $\pm$ 1.0%.

7) Maximum transverse amplitude x_{max} (normalized) of all bunches as a function of the frequency of the fundamental transverse dipole mode, for values of $Q = 20$ to 60, at 11.4 GHz accelerating frequency. In (a), the central frequency, where $\lambda_{rf}/\lambda_{W_\perp} = 5/4$, is 298.66 m^{-1}. In (b), the central frequency, where $\lambda_{rf}/\lambda_{W_\perp} = 6/4$, is 358.39 m^{-1}. The spread about the central frequency is $\pm$ 1.0% in both cases.

8) Maximum transverse amplitude x_{max} (normalized) of all bunches as a function of the frequency of the fundamental transverse dipole mode, for values of $Q = 20$ to 60, at 11.4 GHz accelerating frequency. In (a), $\ell = 0.21$ m, $N = 1.75 \times 10^{10}$, $n = 10$, and the central frequency, where $\lambda_{rf}/\lambda_{W_\perp} = 10.5/8$, is 313.59 m^{-1}. In (b), $\ell = 0.0789$ m, $N = 0.65 \times 10^{10}$, $n = 27$, and the central frequency K_1^0, where $\lambda_{rf}/\lambda_{W_\perp} = 3.5/3$, is 278.75 m^{-1}. The spread shown about K_1^0 is $\pm$ 1.0% in both cases.

9) Comparison between wake-zero-crossing model and program LINACBBU, for an example at 11.4 GHz accelerating frequency, with $\ell = 0.21$ m, $N = 1.75 \times 10^{10}$, $n = 10$. The wake includes a single mode with $Q = 60$ and frequency 299.1059 m^{-1}, which is 0.15% above the point where $\lambda_{rf}/\lambda_{W_\perp} = 10/8$. Results are given for the second, sixth, and tenth bunch in the train. The solid line shows the bunch offset obtained from LINACBBU (normalized by dividing out the adiabatic damping factor) as a function of effective distance

along the linac. The dotted line shows the envelope of the bunch offset, from the wake-zero-crossing model.

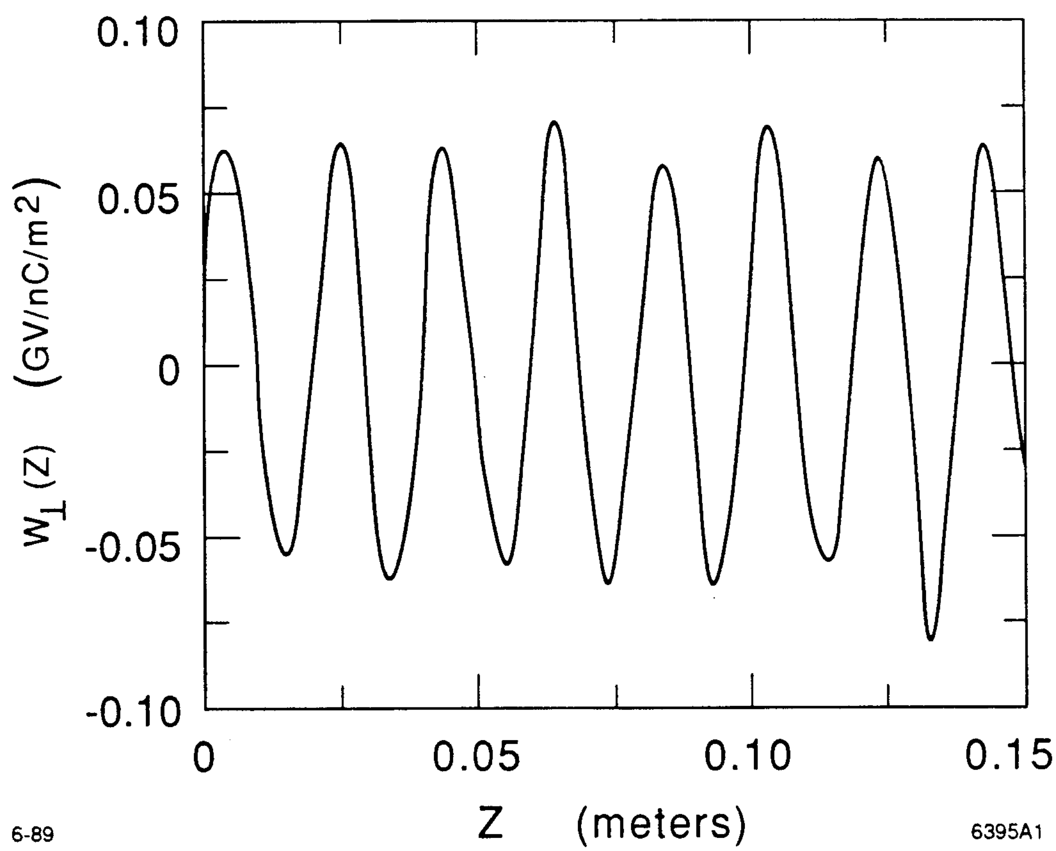
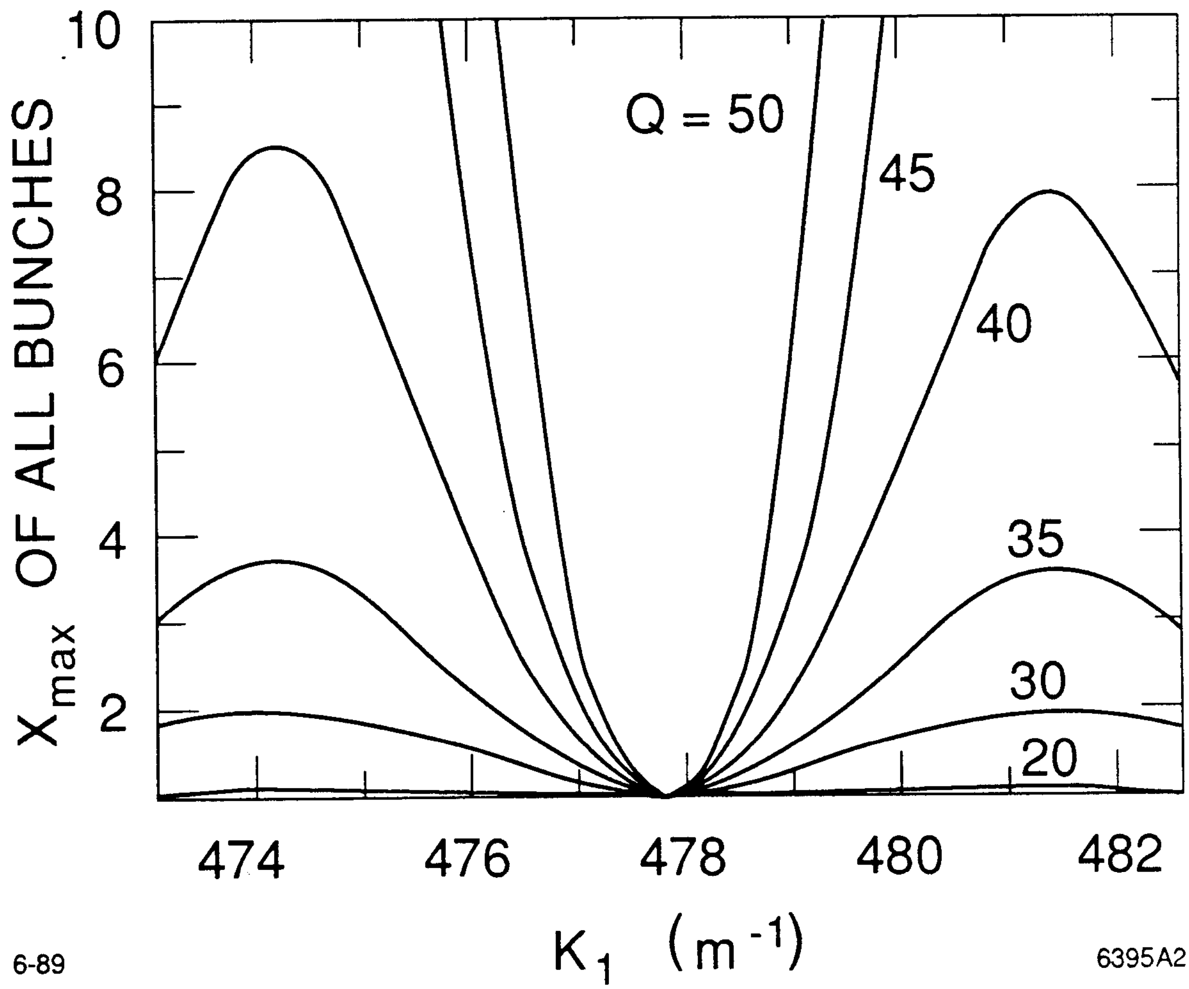
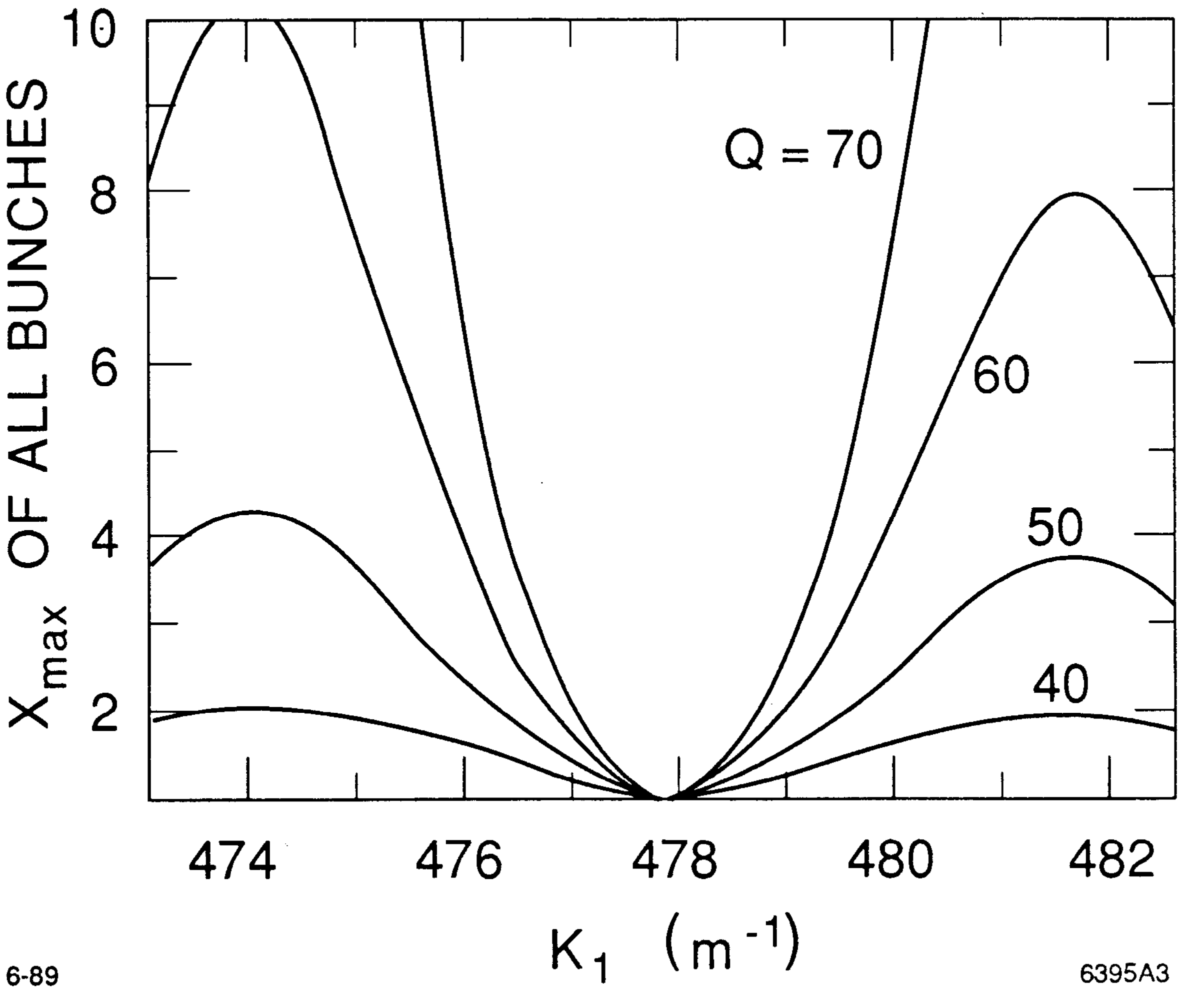

0.10
0.05
0
-0.05
-0.10
$W_\perp(Z)$ (GV/nC/m^2)
0
0.05
0.10
0.15
Z (meters)
6-89
6395A1

Fig. 1



10
8
6
4
2
X_{max} OF ALL BUNCHES
Q = 50
45
40
35
30
20
474
476
478
480
482
K_1 (m^{-1})
6-89
6395A2

Fig. 2



10
8
6
4
2
X_{max} OF ALL BUNCHES
Q = 70
60
50
40
474
476
478
480
482
K_1 (m^{-1})
6-89
6395A3

Fig. 3

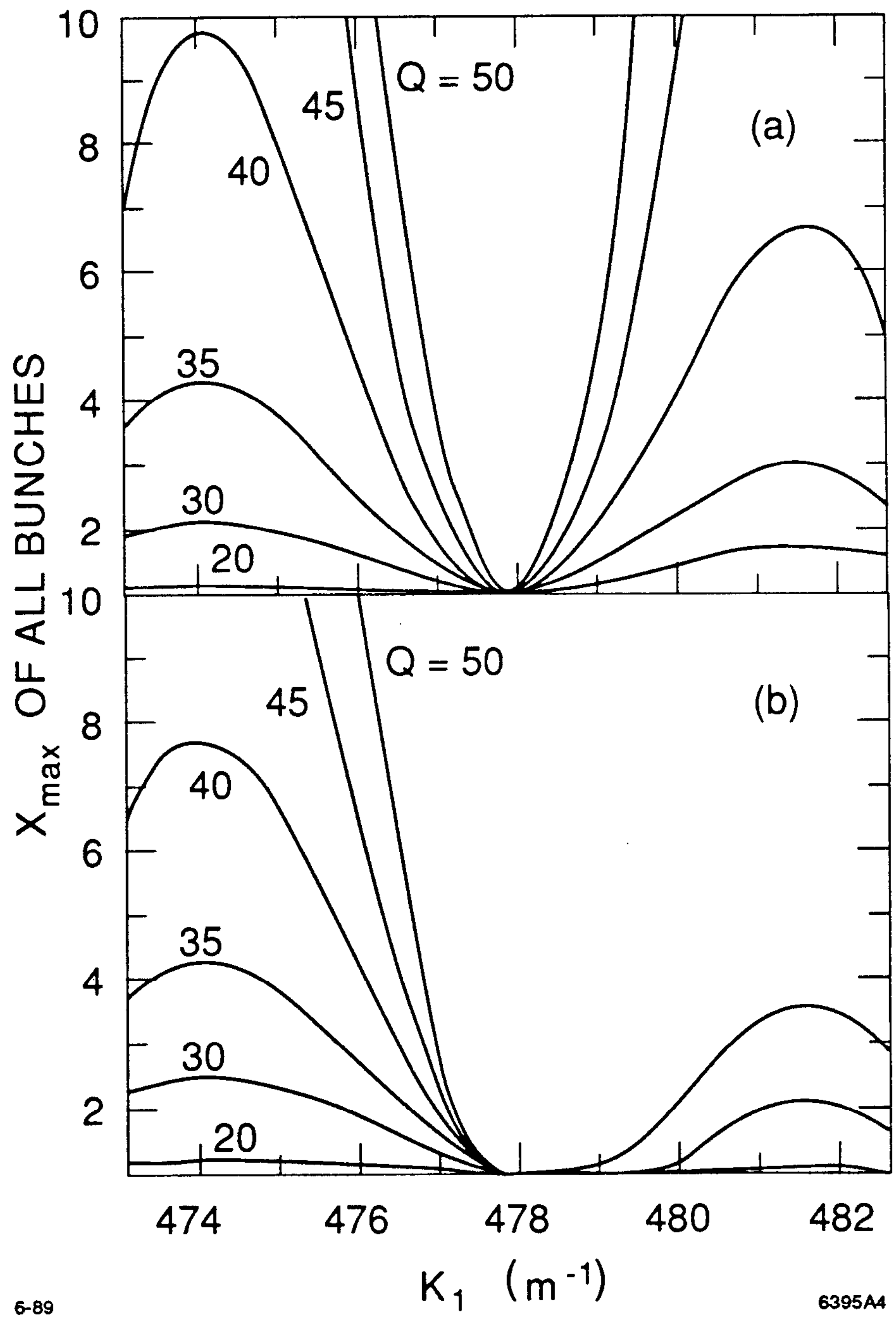

10
8
6
4
2
10
8
6
4
2
474
476
478
480
482
X_{max} OF ALL BUNCHES
K_1 (m^{-1})
Q = 50
45
40
35
30
20
(a)
Q = 50
45
40
35
30
20
(b)
6-89
6395A4

Fig. 4

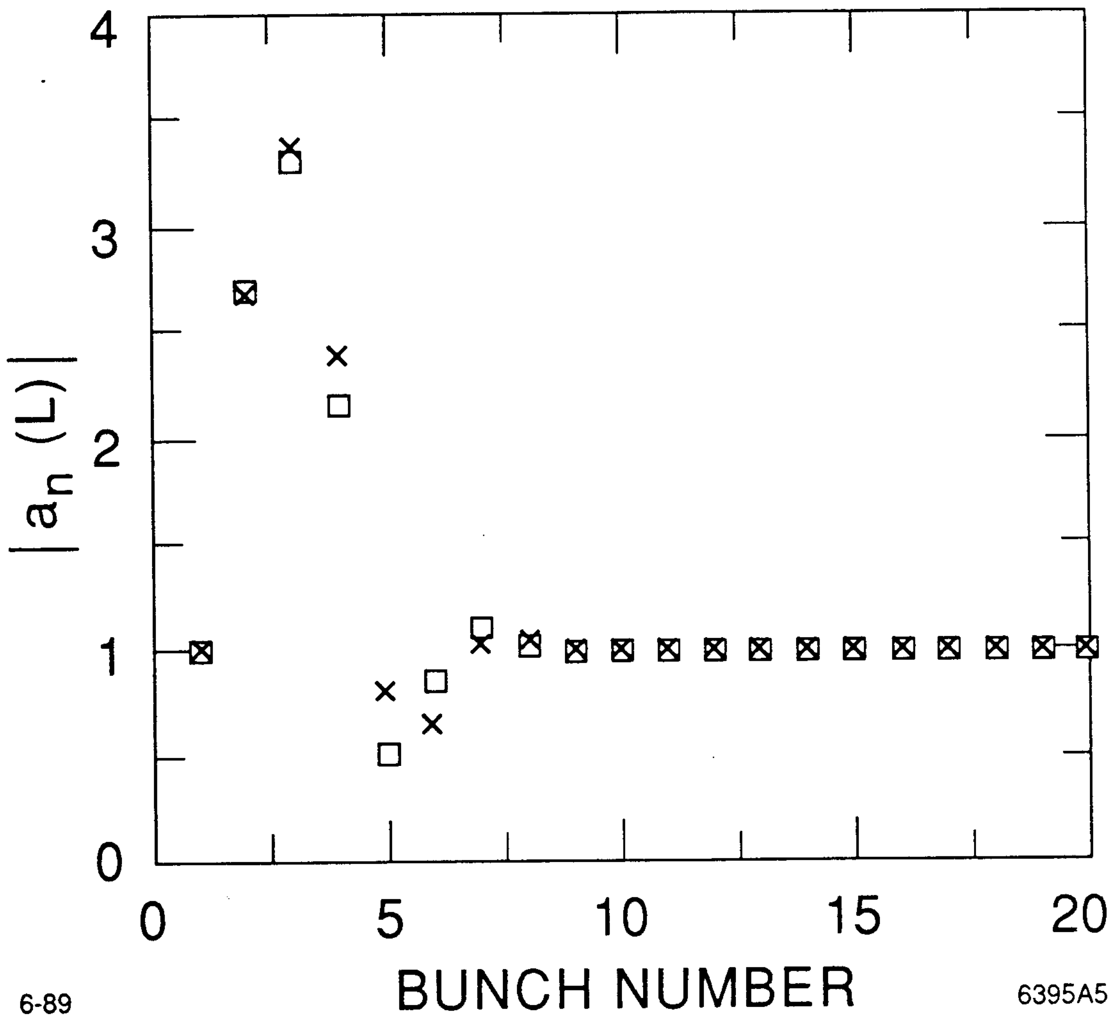
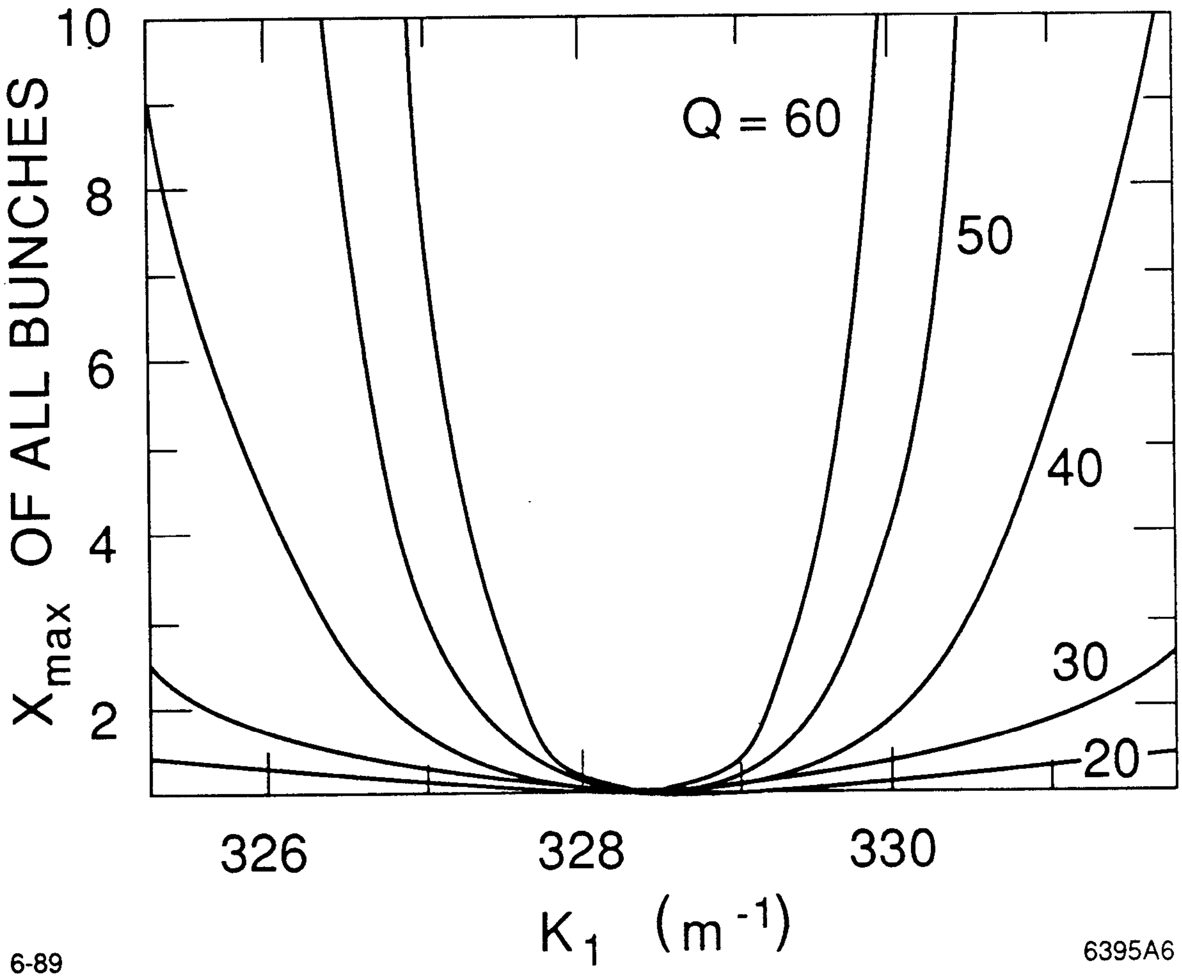
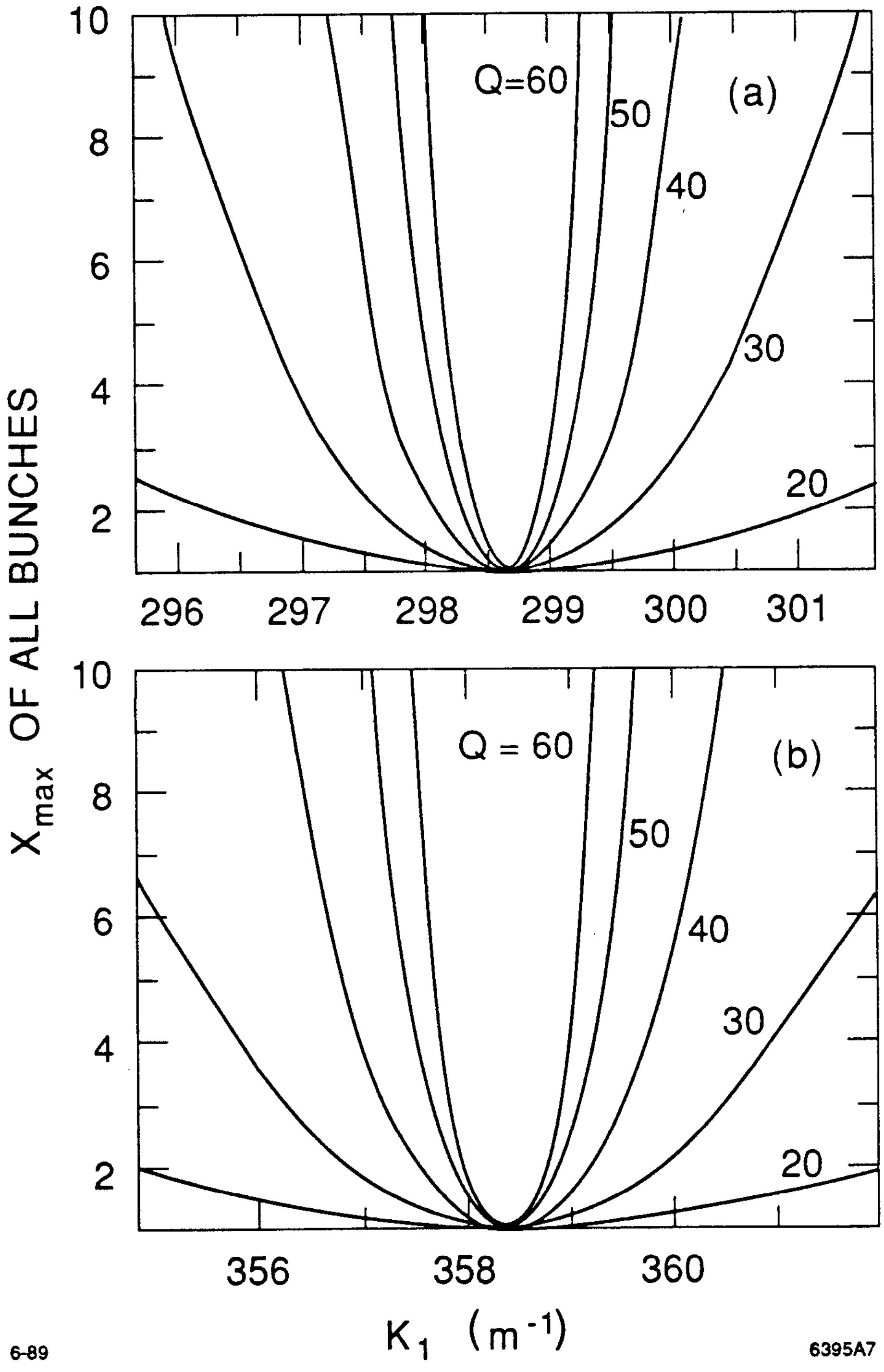


Fig. 5



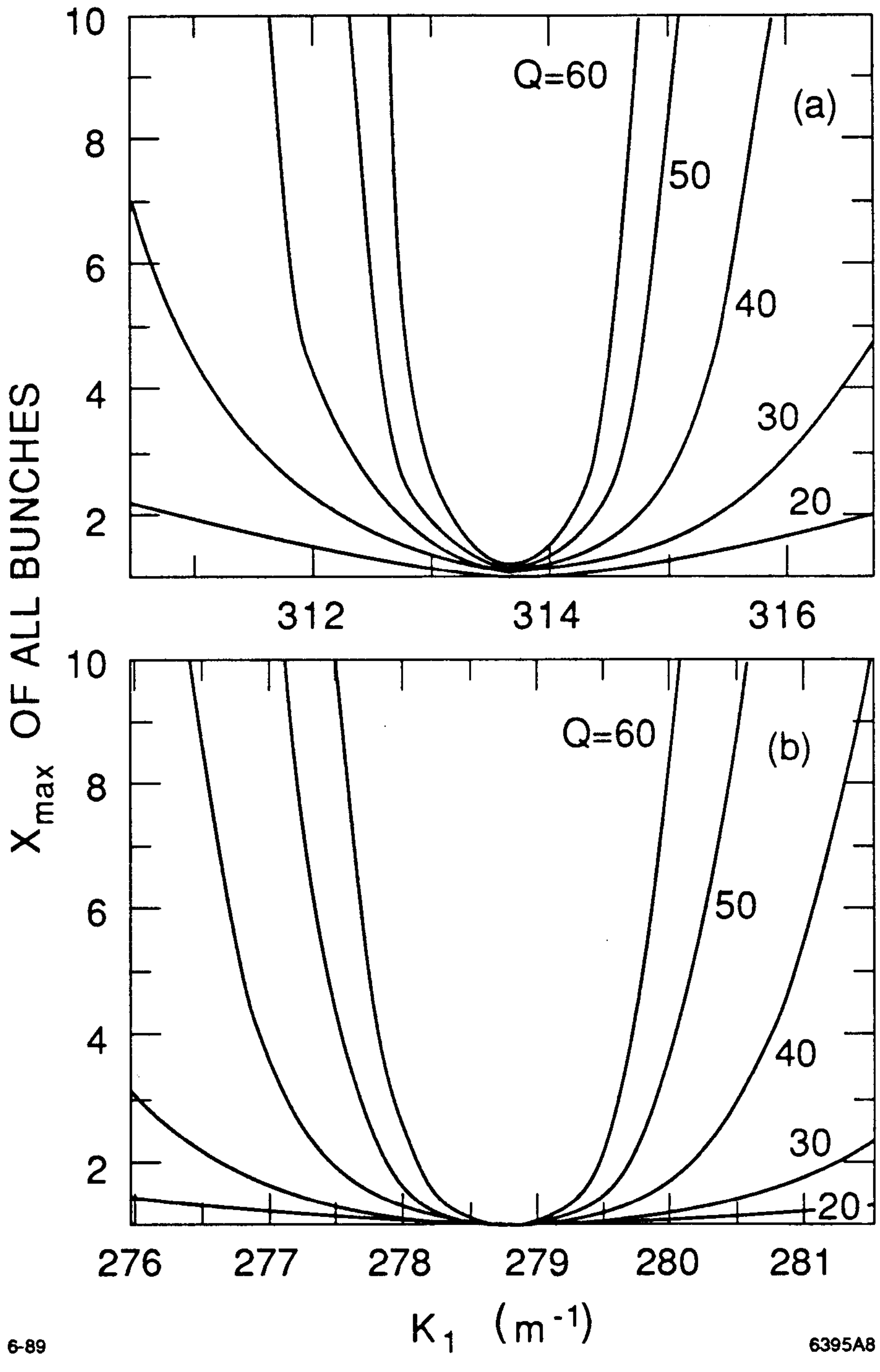
10
8
6
4
2
X_{max} OF ALL BUNCHES
Q = 60
50
40
30
20
326
328
330
K_1 (m^{-1})
6-89
6395A6

Fig. 6



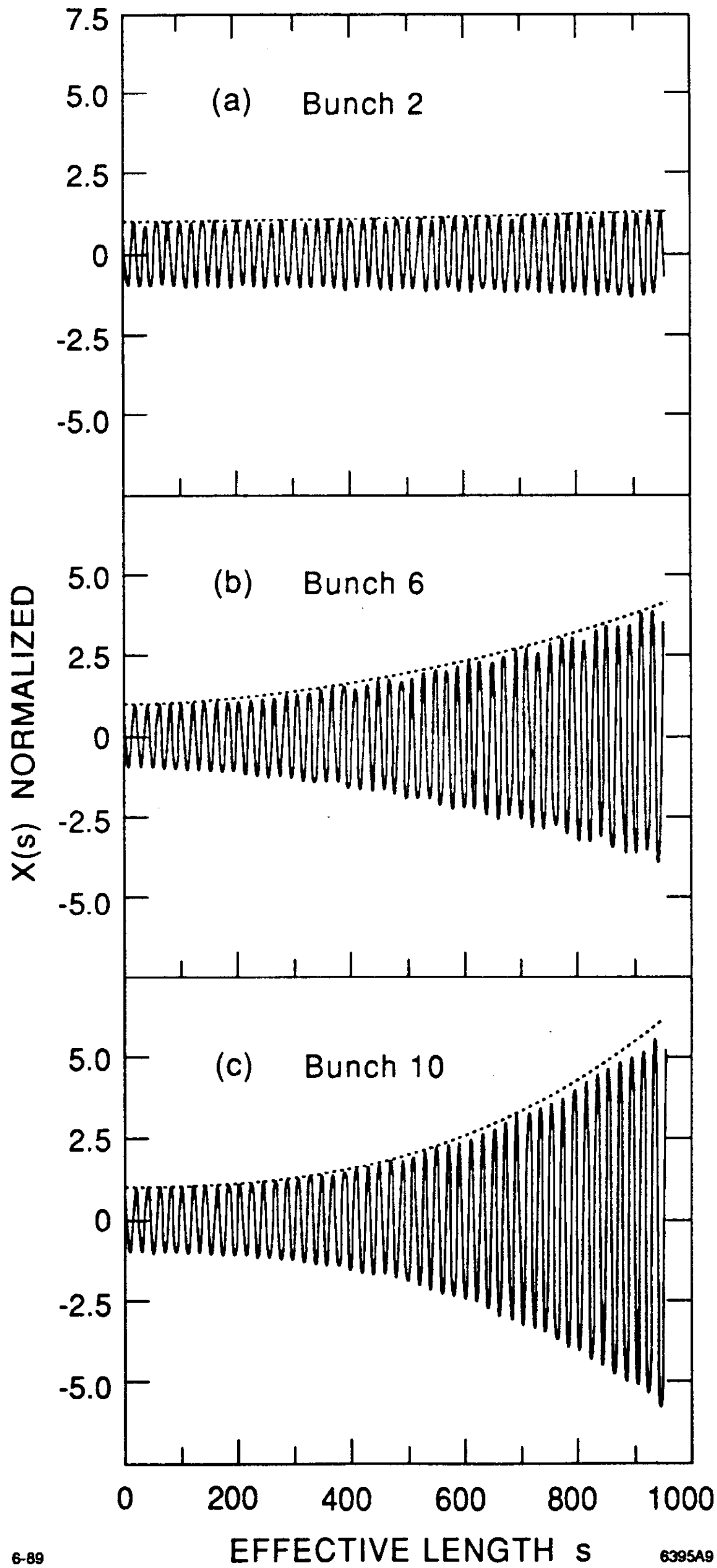
Q=60
50
40
30
20
(a)
296
297
298
299
300
301
10
8
6
4
2
Q = 60
50
40
30
20
(b)
356
358
360
X_{max} OF ALL BUNCHES
K_1 (m^{-1})
6-89
6395A7

Fig. 7



10
8
6
4
2
Q=60
50
40
30
20
(a)
312
314
316
X_{max} OF ALL BUNCHES
(b)
276
277
278
279
280
281
K_1 (m^{-1})
6-89
6395A8

Fig. 8



(a) Bunch 2
(b) Bunch 6
(c) Bunch 10
X(s) NORMALIZED
7.5
5.0
2.5
0
-2.5
-5.0
0
200
400
600
800
1000
EFFECTIVE LENGTH s
6-89
6395A9

Fig. 9