学 位 論 文

Supersymmetry breaking scale and Gauge coupling unification

（超対称性の破れのスケールとゲージ結合定数の統一）

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Abstract

We study consequences of the threshold effects of supersymmetric (SUSY) and superheavy (GUT) particles to the gauge coupling unification condition in two specific supersymmetric SU(5) models, the minimal model and the missing doublet model with natural doublet-triplet splitting. We focus our main attention on the correction to the estimation of the SUSY breaking scale by the unification condition. We present a consistent treatment of the SU(2)×U(1) breaking mass terms in the SUSY particle threshold effects, as well as that of the top quark threshold effect, which have been ignored in previous works. The GUT threshold effects are constrained by the proton decay experiments and by some theoretical consistency conditions, but they are significant and strongly model dependent. For example, under a certain assumption for the SUSY particle masses, the minimal model favors a large (>1TeV) SUSY breaking scale or high $\alpha_s(m_Z)(> 0.12)$, whereas the missing doublet model allows a low (<1TeV) SUSY breaking scale for $0.11 < \alpha_s(m_Z) < 0.13$. The consequences of these two models in the proton decay experiments are also briefly discussed.

1 Introduction

Recent experiments give precise values of the three gauge couplings in the standard model [1]. These values are consistent [2] with the prediction of the supersymmetric (SUSY) SU(5) grand unification model [3]. It has hence become an important task to study the unification of gauge couplings quantitatively in specific SUSY SU(5) models.

Along these studies, there has been an interesting observation that if more precise value of the strong coupling α_s [4] is obtained, we can estimate the supersymmetry breaking scale m_{SUSY} from the unification condition of gauge couplings. Even though our ignorance on the heavy particle mass spectrum of the GUT scale prevents us from obtaining such information in general [5], the authors of ref.[6], for example, estimated that m_{SUSY} should exceed 10 TeV under a certain constraint on the GUT threshold effects in the minimal SUSY SU(5) model.

In this paper, we study the threshold corrections to the gauge coupling unification condition by SUSY particles and those by the particles with masses of the unification scale, and their effects on the estimation of m_{SUSY}. We show that both threshold effects are very important. We present a consistent treatment of the SU(2)×U(1) symmetry breaking effects in the masses and couplings of the SUSY particles, which have been ignored in previous studies [2,7]. We also study the GUT model dependences in two specific SUSY SU(5) models, the minimal model and the missing doublet model, improving our previous analysis [7]. We find that the experimental limits on proton decays and the theoretical consistency of the GUT model itself give significant constraints on the GUT threshold effects. Because of these constraints on the GUT threshold effects, we can obtain [7,8] nontrivial constraints on m_{SUSY} and $\alpha_s(m_Z)$ from the unification condition without postulating a particular mass spectrum of the GUT scale particles as was done in ref.[7]. We observe that the resulting predictions of the SUSY SU(5) unification condition are very different and distinct in the two GUT models.

The paper is organised as follows. In section 2, we review SUSY SU(5) GUT predictions in the next-to-leading order and define the framework for our study. In section 3, we present the SUSY threshold corrections with a consistent treatment of the SU(2)×U(1) breaking terms in the SUSY sector, as well as a consistent treatment of the top quark threshold effects. In section 4, we review the estimation of the SUSY breaking scale from the gauge coupling unification condition without GUT threshold corrections. In sections 5 and 6, we study GUT threshold corrections and their low energy consequences in two SUSY SU(5) models, the minimal model and the missing doublet model, respectively. We find that the phenomenological consequences of the GUT unification condition are very different in these two models: under a certain assumption for the SUSY particle masses, the minimal model favors large m_{SUSY} or higher $\alpha_s(m_Z)$ while missing doublet model allows small m_{SUSY} for any $\alpha_s(m_Z)$ in the experimentally allowed range. Implications on the proton decay experiments of the two SUSY SU(5) models are also discussed briefly. Section 7 gives our conclusions.

2 SUSY SU(5) GUT predictions in the next-to-leading order

In this section, we show the unification condition of the low-energy gauge coupling constants in SUSY SU(5) models in the next-to-leading order and its implications on the GUT predictions, after preparing the framework for our study. We also review the constraints on the GUT scale parameters from proton decay experiments and from the theoretical consistency of the models.

We first review basic properties of the minimal supersymmetric standard model (MSSM) and the supersymmetric SU(5) model. The minimal SUSY standard model [9] is a simple supersymmetric extension of the minimal standard model (SM)

with two higgs doublets. This model contains the following chiral supermultiplets,

$$(q_L, \tilde{q}_L)_i, \quad (u_R, \tilde{u}_R)_i, \quad (d_R, \tilde{d}_R)_i,$$
$$(l_L, \tilde{l}_L)_i, \quad (e_R, \tilde{e}_R)_i, \quad (i = 1 - 3)$$
$$(\tilde{h}_1, h_1), \quad (\tilde{h}_2, h_2),$$

and the gauge supermultiplets,

$$(g, \tilde{g}), \quad (W, \tilde{W}) \quad (B, \tilde{B}).$$

All the particles with tilder are the superpartners of the particles in the standard model. All of them have masses in the order of the SUSY breaking scale.

The higgs scalars particles in the minimal SUSY standard model need special care. There are the following five mass eigenstates of the physical higgs scalars in this model:

$$(h^0, H^0, P^0, H^{\pm}). \tag{2.1}$$

In the case where m_{P^0} is much greater than m_Z, the fields h^0 and $(H^0, P^0, H^{\pm})$ physically split. In this limit, h^0 is almost the usual higgs scalar in the minimal standard model, which is responsible to the breaking of SU(2)×U(1) gauge symmetry. On the other hand, the remaining four scalars $(H^0, P^0, H^{\pm})$ become very heavy and almost independent of SU(2)×U(1) breaking physics. Then these fields form a complete SU(2)×U(1) doublet H whose masses are nearly degenerate. We call these four scalar fields as "extra higgs scalar doublet" in this paper. In the minimal supergravity model, m_{P^0} increases with the SUSY breaking scale in order to realize the radiative breaking of SU(2)×U(1) gauge symmetry. Therefore we treat these four scalar higgses as members of the SUSY particles in this paper. In the case where the SUSY breaking scale is low, the state is more complicated. Fortunately, we show in section 3 that in more consistent treatment, only the charged scalar $H^{\pm}$ is relevant to the SUSY threshold corrections.

The SUSY SU(5) model is a grand unification of the minimal SUSY standard model. This model contains at least the following chiral superfields: $(\bar{f}_i(\bar{5}), F_i(10))(i = 1-3)$, quarks and leptons; Σ, the higgs field which breaks SU(5) gauge symmetry to SU(3)×SU(2)×U(1) and $(\phi, \bar{\phi})$, the higgs fields which break SU(2)×U(1) gauge symmetry to $U(1)_{EM}$. This model also contains a vector supermultiplet which consists of the standard model gauge fields and superheavy supermultiplet X. We use m_X, the mass of X, as a GUT unification scale. For the SUSY SU(5) models treated in this paper, the low-energy effective theory is the minimal SUSY standard model.

We second fix our framework to study the unification condition of the low energy gauge coupling constants in the SUSY SU(5) models. In principle, we can derive all physical consequences directly from the SUSY GUT. But in many low energy physics such as gauge interactions of massless particles, these predictions suffer from large logarithmic factors $\ln(m_{GUT}/m_Z)$ in $\overline{\text{MS}}$ renormalization scheme since heavy particles in loop graphs do not decouple in this scheme. To solve this problem, the effective gauge theory approach [10,11] is usually used.

The effective gauge theory approach is summarized as follows. If we want to calculate low energy physics in a theory with heavy particles, we first obtain the "effective gauge theory" by integrating out all heavy particles in the initial "full" theory, then calculate physical quantities in the effective theory. The matching condition between both theories are given by the relation between coupling constants. As for low energy gauge interactions, the large logarithmic factors which appear in calculations in the full theory are absorbed into gauge couplings of the effective theory if $\overline{\text{MS}}$ renormalization scale is taken to be sufficiently low.

We use the following procedure to study the unification condition of the low energy gauge couplings in the SUSY SU(5) model.

(1) start from $\alpha_5(m_X)$ in the SUSY SU(5) model,

(2) derive $\alpha_i(m_X)_{\text{MSSM}}$ by integrating out all superheavy particles to obtain the minimal SUSY standard model,

(3) derive $\alpha_i(m_Z)_{\text{MSSM}}$ from the renormalization group equations in the minimal SUSY standard model, and

(4) derive $\alpha_i(m_Z)_{\text{SM}}$ by integrating out all SUSY particles to obtain the minimal standard model.

As a result, the unification condition of $\alpha_i(m_Z)_{\text{SM}}$ takes the following form:

$$\frac{1}{\alpha_i(m_Z)_{\text{SM}}} = \frac{1}{\alpha_5(m_X)} + (\ln\frac{m_X}{m_Z}, \alpha_j \ln\frac{m_X}{m_Z}\ldots) + (1, \alpha_j, \ldots). \tag{2.2}$$

The second term on the right hand side of (2.2) represents a contribution from the running of $\alpha_i(\mu)_{\text{MSSM}}$ between m_X and m_Z while the third term represents that from the matching conditions of gauge couplings. Since $\ln(m_X/m_Z)$ is of the order of α_j^{-1} in GUT, we conclude that in order to study SUSY GUT predictions in the next-to-leading order for given values of $\alpha_i(m_Z)_{\overline{\text{MS}}}$ as defined in the minimal standard model, we need the 2-loop renormalization group equations for the minimal SUSY standard model couplings, and the 1-loop matching condition between the full SUSY GUT model couplings and the minimal SUSY standard model couplings, and that between the minimal SUSY standard model and the minimal standard model.

The 2-loop renormalization group equations [12] for the gauge coupling constants $g_i(\mu)_{\overline{\text{MS}}} = (4\pi\alpha_i(\mu)_{\overline{\text{MS}}})^{1/2}$ are

$$\frac{d}{d\ln\mu} g_i(\mu) = \frac{b_i}{8\pi^2} g_i^3 + \sum_{j=1}^{3} \frac{b_{ij}}{(8\pi^2)^2} g_i^3 g_j^2, \qquad i = 1,2,3, \tag{2.3}$$

where the coefficients b_i and b_{ij} in the minimal SUSY standard model are

$$b_1 = 33/10, \quad b_2 = 1/2, \quad b_3 = -3/2, \tag{2.4}$$

and

$$b_{ij} = \begin{pmatrix} 199/100 & 27/20 & 22/5 \\ 9/20 & 25/4 & 6 \\ 11/20 & 9/4 & 7/2 \end{pmatrix}, \tag{2.5}$$

respectively. The solution of (2.3) is expressed as

$$\frac{\pi}{\alpha_i(m_Z)_{\overline{\text{MS}}}} = \frac{\pi}{\alpha_i(m_X)_{\overline{\text{MS}}}} + b_i \ln\frac{m_X}{m_Z} + \delta_i(2). \qquad i = 1,2,3. \tag{2.6}$$

Here $\delta_i(2)$ represents the correction from the 2-loop contribution in the running of the gauge coupling constants between the scales m_X and m_Z. Their explicit forms are shown in the last of this section.

The 1-loop matching condition between the couplings of the full SUSY GUT and those of the minimal SUSY standard model and that between MSSM and the minimal standard model are essentially obtained by using the effective gauge theory approach. In order to show their explicit forms, we first review the matching condition between the gauge coupling in the effective theory and that in the full theory in general case. At 1-loop order, the matching condition between these gauge couplings is expressed as follows [10,11,13]:

$$\begin{aligned} \frac{\pi}{\alpha(\mu)_{eff}} &= \frac{\pi}{\alpha(\mu)_{full}} - \sum_j^{S,F} b(j) \ln \frac{m_j}{\mu} \\ &- \sum_j^{V} b(j) \left(\ln \frac{m_j}{\mu} - c \right). \end{aligned} \tag{2.7}$$

Here the sum is taken for all the particles (scalars(S), fermions(F) and vectors(V)) in the full theory which decouple in the effective theory. The coefficient $b(j)$ is the contribution of the particle j to the 1-loop renormalization group equation of $\alpha(\mu)_{eff}$, which is determined by the gauge representation of j in the effective theory. Note that eq.(2.7) is an improved relation by 1-loop renormalization group equation. Eq.(2.7) agrees with the result of the step approximation in the 1-loop renormalization group equation at μ =(particle mass) apart from the constant c.

Special remark is needed for the threshold correction from the heavy vector particles. In (2.7), there is an additional constant factor c which depends on the renormalization scheme. For example, $c = 1/21$ holds [10,11] in the $\overline{\mathrm{MS}}$ scheme. It is not appropriate in studies of the SUSY GUTs where the supersymmetry is almost exact at the unification scale, since the threshold effect of the heavy vector boson and that of its superpartners (fermion and scalar) takes different forms in this case. We can avoid this problem by using $\overline{\mathrm{DR}}$ (modified dimensional reduction) scheme [13] in the SUSY GUTs, which preserves the supersymmetry in reguralization. In this scheme, $c = 0$ hold in the 1-loop level, therefore we obtain the manifestly supersymmetric form for the threshold effects.

Since the low energy physics is usually described in the $\overline{\mathrm{MS}}$ scheme, we should convert the gauge couplings $\alpha_i(\mu)_{\overline{\mathrm{MS}}}$ to $\alpha_i(\mu)_{\overline{\mathrm{DR}}}$ at some scale below the unification scale. Their matching condition is [14]

$$\frac{\pi}{\alpha_i(\mu)_{\overline{\mathrm{DR}}}} = \frac{\pi}{\alpha_i(\mu)_{\overline{\mathrm{MS}}}} - c_i, \tag{2.8}$$

where the constant c_i is determined by the gauge group for α_i. For example, $c_i = N/12$ holds for SU(N) ($N \geq 2$) gauge coupling whereas $c_i = 0$ for U(1) gauge coupling.

Using these results, we present general forms of the 1-loop matching condition between the SUSY SU(5) model and the minimal SUSY standard model, and that between the minimal SUSY standard model and the minimal standard model.

The SUSY threshold corrections to the gauge coupling constants are obtained by evaluating eq.(2.7) at $\mu = m_Z$ in the case where we can ignore all SU(2)×U(1) breaking terms in the SUSY sector. They are then expressed as

$$\begin{aligned} \frac{\pi}{\alpha_i(m_Z)_{\mathrm{SM}}} &= \frac{\pi}{\alpha_i(m_Z)_{\mathrm{MSSM}}} + \delta_i(light), \\ \delta_i(light) &= -\sum_j b_i(j) \ln \frac{m_j}{m_z}, \qquad i = 1,2,3. \end{aligned} \tag{2.9}$$

$\delta_i(light)$ represents the threshold correction from all superpartners and the extra higgs scalar doublet H =(H^0, P^0, $H^{\pm}$), which we call as the SUSY sector particles in this paper. The sum is taken for all SU(3)×SU(2)×U(1) gauge multiplets of the SUSY sector particles. The values of $b_i(j)$ are determined by the SU(3)×SU(2)×U(1) gauge group representations of the particles and they are listed in Table 1. This correction depends on the mass spectra of the SUSY sector particles. The forms of the SUSY threshold corrections including the SU(2)×U(1) breaking terms are presented in section 3.

The GUT threshold corrections are obtained by evaluating eq.(2.7) at $\mu = m_X$ in the $\overline{\mathrm{DR}}$ scheme. For convenience, we regard the conversion factors from the $\overline{\rm DR}$ to the $\overline{\rm MS}$ scheme as parts of the GUT threshold effects. The total GUT threshold corrections are then expressed as

$$\frac{\pi}{\alpha_i(m_X)_{\overline{\rm MS}}} = \frac{\pi}{\alpha_5(m_X)_{\overline{\rm DR}}} + \delta_i(GUT),$$
$$\delta_i(GUT) = c_i - \sum_j b_i(j) \ln \frac{m_j}{m_X}, \qquad (2.10)$$

$\delta_i(GUT)$ represents the threshold correction by the particles with masses of the unification scale or, more precisely, all the particles in the full SUSY GUT which decouples in the minimal SUSY standard model, which we call as the GUT sector particles in this paper, including the conversion factor c_i from the $\overline{\rm DR}$ to the $\overline{\rm MS}$ scheme given in eq.(2.8). The sum is taken for all GUT sector particles. The sum is taken for all SU(3)×SU(2)×U(1) gauge multiplets of the GUT sector particles. The coefficients $b_i(j)$ are determined in the same manner as in the SUSY threshold effects. This correction depends on the details of the GUT model, especially on the masses of all the GUT sector particles with a nontrivial SU(3)×SU(2)×U(1) quantum number. Note that the three gauge coupling constants $\alpha_i(\mu)$ in the minimal SUSY standard model are not unified at one scale unless all GUT sector particles masses are degenerate.

By combining the 2-loop effect (2.6) in the running of the couplings and the 1-loop threshold effects (2.9) and (2.10), the final form of the unification condition of the $\overline{\rm MS}$ gauge coupling constants is as follows:

$$\frac{\pi}{\alpha_i(m_Z)_{\overline{\rm MS}}} = \frac{\pi}{\alpha_5(m_X)_{\overline{\rm DR}}} + b_i \ln \frac{m_X}{m_Z} + \Delta_i, \qquad i = 1,2,3. \qquad (2.11)$$
$$\Delta_i = \delta_i(2) + \delta_i(light) + \delta_i(GUT), \qquad i = 1,2,3. \qquad (2.12)$$

Here the gauge couplings on the left hand side of eq.(2.11) are those of the minimal standard model and the coupling on the right hand side is that of the full SUSY SU(5) model. The GUT coupling α_5 is renormalized in the $\overline{\rm DR}$ scheme. The terms Δ_i represent the total next-to-leading corrections. The definition of each term in (2.12) is shown in (2.6), (2.9) and (2.10), respectively.

Starting from eq.(2.11), the following SUSY GUT predictions for $s^2(m_Z)_{\overline{\rm MS}} \equiv (\sin^2\theta_W)(m_Z)_{\overline{\rm MS}}$ (the weak mixing angle), m_X and $\alpha_5(m_X)_{\overline{\rm DR}}$ are obtained.

$$s^2(m_Z)_{\overline{\rm MS}} = s^{2(0)}(m_Z) + \frac{\alpha(m_Z)}{\pi}\delta_s, \qquad (2.13)$$
$$\ln m_X = \ln m_X^{(0)} + \delta_X, \qquad (2.14)$$
$$\frac{\pi}{\alpha_5(m_X)_{\overline{\rm DR}}} = \frac{\pi}{\alpha_5(m_X)^{(0)}} + \delta_5, \qquad (2.15)$$

where

$$\delta_s = \frac{5}{3d}\left[(b_2-b_1)\Delta_3 + (b_1-b_3)\Delta_2 + (b_3-b_2)\Delta_1\right], \qquad (2.16)$$
$$\delta_X = \frac{1}{d}\left(\frac{8}{3}\Delta_3 - \Delta_2 - \frac{5}{3}\Delta_1\right), \qquad (2.17)$$
$$\delta_5 = \frac{1}{d}\left[b_3(\Delta_2 + \frac{5}{3}\Delta_1) - (b_2 + \frac{5}{3}b_1)\Delta_3\right], \qquad (2.18)$$

with

$$d \equiv b_2 + \tfrac{5}{3}b_1 - \tfrac{8}{3}b_3 = 10, \qquad (2.19)$$
$$\alpha^{-1} = \alpha_2^{-1} + \tfrac{5}{3}\alpha_1^{-1}. \qquad (2.20)$$

In the above formulae,

$$s^{2(0)}(m_Z) = \frac{1}{5} + \frac{7\alpha(m_Z)}{15\alpha_3(m_Z)}, \qquad (2.21)$$
$$\ln \frac{m_X^{(0)}}{m_Z} = \frac{\pi}{10\alpha(m_Z)} - \frac{4\pi}{15\alpha_3(m_Z)}, \qquad (2.22)$$
$$\frac{1}{\alpha_5(m_X)^{(0)}} = \frac{3}{20}\left(\frac{1}{\alpha(m_Z)} + \frac{4}{\alpha_3(m_Z)}\right), \qquad (2.23)$$

are the SUSY SU(5) model predictions in the leading order. Obviously, each next-to-leading order correction term δ is decomposed into 2-loop, SUSY sector and GUT sector correction factors via eq.(2.12), for example, as

$$\delta_s = \delta_s(2) + \delta_s(light) + \delta_s(GUT). \qquad (2.24)$$

In studying the consequences of these relations, we can make use of the following three types of informations. First, we have three measured values of the standard model coupling constants $\alpha_i(m_Z)_{\overline{\rm MS}}$ extracted from recent precision experiments. Second, we have the lower mass limits of X, the superheavy gauge boson of the SU(5) model that mediates proton decays via dimension-6 operators ($p \to e^+ + \pi^0$ etc.), and also that of $\tilde{D}$, the color triplet higgsino that mediates proton decays via dimension-5 operators ($p \to \bar{\nu} + K^+$ etc.). These bounds on the GUT sector particle masses are obtained as consequences of the non-observation of corresponding proton decays. Third, we should impose a certain theoretical consistency conditions on the SUSY GUT model. One of them is the "Planck mass limit", the condition that all masses of the GUT model particles should not exceed the Planck scale, $m_P \simeq 10^{19}$GeV. This is needed in order that the SUSY GUT model without gravity makes sense. In addition, we impose the finiteness of the unified gauge coupling $\alpha_5(m_j)$ on all the superheavy particle mass shells. Finally, in later sections we briefly study the consequences of the "weak higgs coupling condition" that dimensionless couplings in the superpotential should not diverge until the Planck scale. The last two conditions are needed in order for our perturbative estimation to be valid quantitatively.

In this paper, we use the following values of the gauge coupling constants that are obtained by recent experiments [1,15].

$$1/\alpha(m_Z)_{\overline{\rm MS}} = 127.9 \pm 0.12 + \frac{8}{9\pi}\ln\frac{m_t}{m_Z}, \tag{2.25}$$

$$\sin^2\theta_W(m_Z)_{\overline{\rm MS}} = \left[1 + \frac{8\alpha_{5q}(m_Z)}{9\pi}\ln\frac{m_t}{m_Z}\right]^{-1} \times \left(0.2325 \pm 0.0007 + \frac{\alpha_{5q}(m_Z)}{3\pi}\ln\frac{m_t}{m_Z}\right), \tag{2.26}$$

$$1/\alpha_s(m_Z)_{\overline{\rm MS}} = (0.12 \pm 0.01)^{-1} + \frac{1}{3\pi}\ln\frac{m_t}{m_Z}, \tag{2.27}$$

where we have explicitly shown the logarithmic dependences of the standard model $\overline{\rm MS}$ couplings on the top quark mass m_t. These logarithmic m_t dependences are obtained by starting from the experimentally given value of the effecting mixing angle $\bar{s}^2$ (see section 3 for details) with the assumption that there is no order m_Z^2/m_t^2

contribution in the observables at the scale m_Z. We have explicitly checked that the order m_Z^2/m_t^2 effects on these couplings are negligibly small at the 1-loop level for $m_t > 100$GeV. In eq.(2.26), $\alpha_{5q}(m_Z)$ stands for $\alpha(m_Z)_{\overline{\rm MS}}$ in eq.(2.25) at $m_t = m_Z$, which is the $\overline{\rm MS}$ coupling in the absence of the top quark. It is worth noting that these logarithm m_t dependences exactly cancel in the SU(5) prediction of $s^2(m_Z)$ and m_X [16,7]. This can easily be shown by substituting eqs.(2.25–2.27) directly into eq.(2.21,2.22). We can hence safely use the values at $m_t = m_Z$ in these predictions. Even for $\alpha_5(m_X)$, the m_t dependence is not significant for our study. In some existing studies [6], this cancellation of the logarithmic m_t dependence in the SU(5) GUT predictions has not been taken account properly.

In the following, we use very rough lower limits of m_X and m_D from the upper limit of proton decay rates, $\tau(p \to e^+\pi^0) > 5.5 \times 10^{32}$years and $\tau(p \to \bar{\nu}K^+) > 1.0 \times 10^{32}$years [17],

$$m_X > 10^{15}\text{GeV}[18], \qquad m_D > 10^{16}\text{GeV}[19,20]. \tag{2.28}$$

These conservative limits are sufficient to obtain all the results of our analysis.

We comment on the validity of the lower mass limits (2.28). Exactly, the lower limits of the proton partial life times $\tau(p \to e^+\pi^0)$ and $\tau(p \to \bar{\nu}K^+)$ are not directly related to the lower mass limits of the superheavy particles X and D. In fact, they give constraints on the combinations $m_X^4/\alpha_5(m_X)^2$ and $m_D^2 m_{SUSY}^2$, respectively. If these additional factors are largely modified by the threshold effects, the mass limits (2.28) can become looser. In this paper, however, we can safely ignore these effects. The reason for each case is as follows. For proton decays mediated by X, the threshold correction of α_5 is of the $O(\alpha_5)$ whereas that of m_X is of the $O(\alpha_5^0)$, so we can use the limit (2.28) in the next-to-leading order calculations. For proton decays mediated by $\tilde{D}$, the argument is more involved. The breakdown of the limit (2.28) may occur when m_{SUSY} becomes very large. But we will show in sections 5 and 6 that the lower limit of m_D increases with m_{SUSY} by the gauge coupling unification condition. So the limit (2.28) is proved to be valid *a posteriori*.

Finally, we show the explicit form of the 2-loop correction factor $\delta_i(2)$ as used in our analysis. We use the one-iteration approximation,

$$\delta_i(2) = \sum_{j=1}^{3} \frac{b_{ij}}{2b_j} \ln \left[1 + \frac{b_j}{\pi} \alpha_5(m_X)^{(0)} \ln \frac{m_X^{(0)}}{m_Z} \right], \tag{2.29}$$

in the following numerical analyses. We ignore any threshold effects of the GUT, SUSY and top quark sector in eq.(2.29) since they form a part of the next-to-next-to-leading order corrections. Then all $\delta_i(2)$ are simple constants. Here we give the numerical values of $\delta(2)$'s for the case $\alpha(m_Z) = 1/127.9$ and $\alpha_3(m_Z) = 0.12$,

$$\frac{\alpha}{\pi}\delta_s(2) \simeq 0.0030, \tag{2.30}$$

$$\delta_X(2) \simeq -0.22, \tag{2.31}$$

$$\delta_5(2) \simeq -0.67. \tag{2.32}$$

These corrections are not negligible in the estimation of the SUSY breaking scale, as will be seen in later sections.

3 SUSY and top threshold corrections

In this section, we study the threshold corrections to the gauge coupling unification from the SUSY sector particles or, in other words, from all superpartners and the extra higgs scalar doublet in the minimal standard SUSY model. We present the correct treatment of the SU(2)×U(1) breaking terms in this sector, which has not been shown clearly in the previous studies. The correct treatment of the top quark threshold corrections is also presented.

First, in the case where all SU(2)×U(1) breaking terms can be ignored in the SUSY sector, the form of the SUSY threshold corrections are expressed as eq.(2.9). Therefore we can safely use eq.(2.9) if the SUSY breaking scale is sufficiently large.

In the case where SUSY breaking scale is relatively low, O(100GeV), we cannot ignore SU(2)×U(1) breaking masses and mixings in the SUSY sector. We should properly incorporate this effect. The difficulty arises in this case since the

low energy effective theory where SUSY sector particles are integrated out is no more SU(2)×U(1) invariant and we cannot justify the usual effective gauge theory approach for the matching condition of s^2, α_2 and α_1.

In order to treat this SU(2)×U(1) breaking threshold corrections properly, we return to the derivation of the $\overline{\rm MS}$ couplings $\alpha_2(m_Z)$ and $\alpha(m_Z)$ from experimental data. At present, most accurate measurements of the weak mixing angle are performed at LEP from the leptonic and b-quark forward-backward asymmetries and from the τ lepton polarization. These experiments determine the effective mixing angle $\bar{s}^2(m_Z^2)$, which represents the effective coupling of fermions to on-shell Z, including process-independent contributions from loop graphs. This effective coupling is expressed in terms of the $\overline{\rm MS}$ couplings in the minimal SUSY standard model as follows [21,22]:

$$\begin{aligned} \frac{\pi}{\bar{\alpha}(q^2)} &= \frac{\pi}{\alpha(\mu)_{\rm MSSM}} + \left[\frac{1}{4}{\rm Re}\Pi^{QQ}_{T,\gamma}(q^2) + \frac{\Pi^{3Q}_T(0)}{2m_W^2}\right]_{SM} + \frac{1}{4}{\rm Re}\Pi^{QQ}_{T,\gamma}(q^2)_{SUSY} \\ &= \frac{\pi}{\alpha(\mu)_{\rm SM}} + \left[\frac{1}{4}{\rm Re}\Pi^{QQ}_{T,\gamma}(q^2) + \frac{\Pi^{3Q}_T(0)}{2m_W^2}\right]_{SM}, \end{aligned} \tag{3.1}$$

$$\begin{aligned} \frac{\pi\bar{s}^2(q^2)}{\bar{\alpha}(q^2)} &= \frac{\pi}{\alpha_2(\mu)_{\rm MSSM}} + \left[\frac{1}{4}{\rm Re}\Pi^{3Q}_{T,\gamma}(q^2) + \frac{\Pi^{3Q}_T(0)}{2m_W^2}\right]_{SM} + \frac{1}{4}{\rm Re}\Pi^{3Q}_{T,\gamma}(q^2)_{SUSY} \\ &= \frac{\pi}{\alpha_2(\mu)_{\rm SM}} + \left[\frac{1}{4}{\rm Re}\Pi^{3Q}_{T,\gamma}(q^2) + \frac{\Pi^{3Q}_T(0)}{2m_W^2}\right]_{SM}. \end{aligned} \tag{3.2}$$

Here Π^{QQ}_T and Π^{3Q}_T are 1-loop self energy corrections in the $\overline{\rm MS}$ scheme with which the transverse parts of the γ and $\gamma - Z$ mixing propagators are expressed as

$$\Pi^{\gamma\gamma}_T(q^2) = \frac{\alpha}{4\pi}\Pi^{QQ}_T(q^2), \tag{3.3}$$

$$\Pi^{\gamma Z}_T(q^2) = \frac{\alpha}{4\pi sc}[\Pi^{3Q}_T(q^2) - s^2\Pi^{QQ}_T(q^2)], (c^2 \equiv 1 - s^2) \tag{3.4}$$

the subscript ",γ" means removal of the residue at $q^2 = 0$,

$$\Pi_{T,\gamma}(q^2) = \frac{\Pi_T(q^2) - \Pi_T(0)}{q^2}, \tag{3.5}$$

and $\bar{\alpha}(q^2)$ is the effective electromagnetic coupling [21] which agrees with the fine structure constant at $q^2 = 0$. Note that in(3.1) and (3.2), only a vector particle W

contribute to $\Pi_T^{3Q}(0)$. From these equations we can obtain the relation between the $\overline{\rm MS}$ gauge couplings in the minimal standard model and that in the minimal SUSY standard model:

$$\delta_\alpha(\mu) \equiv \frac{\pi}{\alpha(\mu)_{\rm SM}} - \frac{\pi}{\alpha(\mu)_{\rm MSSM}} = \frac{1}{4}{\rm Re}\Pi_{T,\gamma}^{QQ}(q^2)_{\rm SUSY}, \tag{3.6}$$

$$\delta_{\alpha W}(\mu) \equiv \frac{\pi}{\alpha_2(\mu)_{\rm SM}} - \frac{\pi}{\alpha_2(\mu)_{\rm MSSM}} = \frac{1}{4}{\rm Re}\Pi_{T,\gamma}^{3Q}(q^2)_{\rm SUSY}, \tag{3.7}$$

provided that the effective couplings $\bar{\alpha}(q^2)$ and $\bar{s}^2(q^2)$ are independent of the SUSY sector physics. These differences represent all contributions from the SUSY sector particles. Note that only the charged particles enter in δ_α and $\delta_{\alpha W}$. This property simplifies our discussion since the most complicated parts in the minimal SUSY standard model, neutralinos and neutral higgs scalars, do not enter.

As for $\alpha(\mu)_{\overline{\rm MS}}$, we may obtain the same result as in the effective $U(1)_{EM}$ gauge theory approach by evaluating eq.(3.6) at $q^2 = 0$. On the other hand, the result (3.7) for $\alpha_2(\mu)_{\overline{\rm MS}}$ is not familiar. $\Pi_{T,\gamma}^{3Q}(q^2)_{SUSY}$ is determined by masses, charges and effective diagonal couplings to W^3-vector boson, of the SUSY sector particles and is unambiguous even in the presence of SU(2)×U(1) breaking. We can safely interpret $\delta_{\alpha W}$ as the SUSY threshold correction of $\alpha_2(\mu)_{\overline{\rm MS}}$. We note in passing that at $\mu = m_Z$, $\alpha_2(m_Z)_{SM}$ is $\alpha(m_Z)/s^2(m_Z)_{\overline{\rm MS}}$ in eqs.(2.25, 2.26).

The explicit form of δ_α and $\delta_{\alpha w}$ is as follows:

$$\begin{aligned}\delta_\alpha(m_Z) &= -\sum_j b_Q(j)\ln\frac{m_j}{m_Z} \\ &= -\frac{2}{9}\ln\frac{m_{\tilde{t}_1}m_{\tilde{t}_2}}{m_Z^2} - \frac{2}{9}\ln\frac{m_{\tilde{c}_L}m_{\tilde{c}_R}}{m_Z^2} - \frac{2}{9}\ln\frac{m_{\tilde{u}_L}m_{\tilde{u}_R}}{m_Z^2} \\ &\quad -\frac{1}{18}\ln\frac{m_{\tilde{b}_L}m_{\tilde{b}_R}}{m_Z^2} - \frac{1}{18}\ln\frac{m_{\tilde{s}_L}m_{\tilde{s}_R}}{m_Z^2} - \frac{1}{18}\ln\frac{m_{\tilde{d}_L}m_{\tilde{d}_R}}{m_Z^2} \\ &\quad -\frac{2}{3}\ln\frac{m_{\tilde{W}_1}m_{\tilde{W}_2}}{m_Z^2} - \frac{N_g}{6}\ln\frac{m_{\tilde{l}_L^-}m_{\tilde{l}_R^-}}{m_Z^2} - \frac{1}{6}\ln\frac{m_{H^\pm}}{m_Z},\end{aligned} \tag{3.8}$$

and

$$\begin{aligned}\delta_{\alpha w}(m_Z) &= -\sum_j b_{3Q}(j)\ln\frac{m_j}{m_Z}\end{aligned}$$

$$\begin{aligned}&= -\frac{1}{6}\ln\frac{m_{\tilde{t}_1}}{m_Z} - \frac{\sin^2\phi_{\tilde{t}}}{6}\ln\frac{m_{\tilde{t}_1}}{m_{\tilde{t}_2}} - \frac{1}{6}\ln\frac{m_{\tilde{c}_L}}{m_Z} - \frac{1}{6}\ln\frac{m_{\tilde{u}_L}}{m_Z} \\ &\quad -\frac{1}{12}\ln\frac{m_{\tilde{b}_L}}{m_Z} - \frac{1}{12}\ln\frac{m_{\tilde{s}_L}}{m_Z} - \frac{1}{12}\ln\frac{m_{\tilde{d}_L}}{m_Z} \\ &\quad -\frac{2}{3}\ln\frac{m_{\tilde{W}_1}}{m_Z} - \frac{1}{3}\ln\frac{m_{\tilde{W}_2}}{m_Z} - \frac{1}{6}(\sin^2\phi_L + \sin^2\phi_R)\ln\frac{m_{\tilde{W}_2}}{m_{\tilde{W}_1}} \\ &\quad -\frac{N_g}{12}\ln\frac{m_{\tilde{l}_L^-}}{m_Z} - \frac{1}{12}\ln\frac{m_{H^\pm}}{m_Z}.\end{aligned} \tag{3.9}$$

In eq.(3.9), $\phi_{\tilde{t}}$, $\phi_{L,R}$ are the mixing angles between mass and gauge eigenstates defined as

$$\begin{pmatrix}\tilde{t}_1 \\ \tilde{t}_2\end{pmatrix} = \begin{pmatrix}\cos\phi_{\tilde{t}} & \sin\phi_{\tilde{t}} \\ -\sin\phi_{\tilde{t}} & \cos\phi_{\tilde{t}}\end{pmatrix}\begin{pmatrix}\tilde{t}_L \\ \tilde{t}_R\end{pmatrix} \tag{3.10}$$

$$\begin{pmatrix}\tilde{W}_{1L}^- \\ \tilde{W}_{2L}^-\end{pmatrix} = \begin{pmatrix}\cos\phi_L & \sin\phi_L \\ -\sin\phi_L & \cos\phi_L\end{pmatrix}\begin{pmatrix}\tilde{W}_L^- \\ \tilde{H}_{1L}^-\end{pmatrix} \tag{3.11}$$

$$\begin{pmatrix}\tilde{W}_{1R}^- \\ \tilde{W}_{2R}^-\end{pmatrix} = \begin{pmatrix}\cos\phi_R & \sin\phi_R \\ -\epsilon\sin\phi_R & \epsilon\cos\phi_R\end{pmatrix}\begin{pmatrix}\tilde{W}_R^- \\ \tilde{H}_{2R}^-\end{pmatrix}, \tag{3.12}$$

where ϵ is a sign factor which is irrelevant to our discussion. The form of δ_α agrees with the matching condition of the effective QED coupling constant.

In the derivation of eq.(3.9) from eq.(3.7), we have made an approximation in ignoring the deviation of $\bar{\alpha}(m_Z^2)$ by the SUSY sector particles and in replacing $\Pi_{T,\gamma}^{3Q}(m_Z^2)$ by $\Pi_{T,\gamma}^{3Q}(0)$. The difference made by these approximation is obtained by regarding the quantities $\bar{\alpha}(0)$ and $\bar{s}^2(m_Z^2)$ as independent of the SUSY sector physics. In this case, $\delta_{\alpha W}(m_Z)$ is explicitly expressed as

$$\begin{aligned}\delta_{\alpha W}(m_Z) &= \frac{1}{4}{\rm Re}\Pi_{T,\gamma}^{3Q}(0)_{\rm SUSY} + \frac{1}{4}{\rm Re}\left[\Pi_{T,\gamma}^{3Q}(m_Z^2) - \Pi_{T,\gamma}^{3Q}(0)\right]_{\rm SUSY} \\ &\quad -\frac{\bar{s}^2(m_Z^2)}{4}{\rm Re}\left[\Pi_{T,\gamma}^{QQ}(m_Z^2) - \Pi_{T,\gamma}^{QQ}(0)\right]_{\rm SUSY},\end{aligned} \tag{3.13}$$

with $\mu = m_Z$. The first term on the right hand side of (3.13) corresponds to eq.(3.9) while the last two terms represent the difference made by these approximation. They are finite and of the order of $(m_Z/m_{SUSY})^2$. In fact, deviations of this order also appear in the derivation of $\bar{s}^2(m_Z)$ from the experimental data. These effects are expected to be small for $m_{SUSY} \gg m_Z$. In this paper, we have ignored these additional effects. We will show the complete analysis of the SUSY threshold correction including these effects elsewhere.

To discuss the effect of these corrections to the GUT predictions, we identify these formula as

$$\delta_2(light) = \delta_{\alpha w}(m_Z), \tag{3.14}$$

$$\begin{aligned} \delta_1(light) &= \frac{3}{5}(\delta_\alpha(m_Z) - \delta_{\alpha w}(m_Z)) \\ &\equiv -\sum_j b_1(j) \ln \frac{m_j}{m_Z} \end{aligned} \tag{3.15}$$

and substitute into the formulas in section 2. Note that $\delta_X(light)$ and $\delta_5(light)$ are expressed only in terms of $\delta_3(light)$ and δ_α, as can be easily checked. We can easily check that these results reduce to those without SU(2)×U(1) breaking if all SUSY particle masses are much larger than m_Z.

The top quark threshold effects which are presented in (2.25–2.27) are also obtained in the same manner as the SUSY threshold effect. To show this, we rewrite eq.(3.2) as

$$\frac{\pi \bar{s}^2(q^2)}{\bar{\alpha}(q^2)} = \frac{\pi}{\alpha_2(\mu)_{\rm SM}} + \left[\frac{1}{4}{\rm Re}\Pi^{3Q}_{T,\gamma}(q^2) + \frac{\Pi^{3Q}_T(0)}{2m_W^2}\right]_{5q} + \frac{1}{4}{\rm Re}\Pi^{3Q}_{T,\gamma}(q^2)_{top}, \tag{3.16}$$

where the subscript $5q$ means the contribution of the minimal standard model except for the top quark (the standard higgs scalar does not contribute at 1-loop level since it is neutral). Therefore all the dependences of $\alpha_2(\mu)_{\rm SM}$ on m_t are contained in the last term of eq.(3.16). The logarithmic dependence of the numerator of (2.26) on m_t is then obtained from (3.16) with the approximation to replace $\Pi^{3Q}_{T,\gamma}(m_Z^2)$ by $\Pi^{3Q}_{T,\gamma}(0)$. The other logarithmic dependences in (2.25–2.27) are easily obtained by usual effective gauge theory approach.

The typical SUSY particle mass spectrum expected in the minimal supergravity model is parametrized by 5 parameters ($m_{1/2}$, m_0, $\tan\beta$, A_t, m_μ) as follows [23,24,25].

$$m_{\tilde{g}} \simeq 2.7 m_{1/2} \tag{3.17}$$

$$m^2_{\tilde{u}_L} \simeq m_0^2 + 6m_{1/2}^2 + (\frac{1}{2} - \frac{2}{3}s^2) m_Z^2 \cos 2\beta \tag{3.18}$$

$$m^2_{\tilde{d}_L} \simeq m_0^2 + 6m_{1/2}^2 + (-\frac{1}{2} + \frac{1}{3}s^2) m_Z^2 \cos 2\beta \tag{3.19}$$

$$m^2_{\tilde{u}_R} \simeq m_0^2 + 6m_{1/2}^2 + \frac{2}{3}s^2 m_Z^2 \cos 2\beta \tag{3.20}$$

$$m^2_{\tilde{d}_R} \simeq m_0^2 + 6m_{1/2}^2 - \frac{1}{3}s^2 m_Z^2 \cos 2\beta \tag{3.21}$$

$$m^2_{\tilde{e}_L} \simeq m_0^2 + 0.5m_{1/2}^2 + (-\frac{1}{2} + s^2) m_Z^2 \cos 2\beta \tag{3.22}$$

$$m^2_{\tilde{e}_R} \simeq m_0^2 + 0.15m_{1/2}^2 - s^2 m_Z^2 \cos 2\beta \tag{3.23}$$

$$m_{\tilde{W}_{1,2}} = \begin{pmatrix} M_2 \simeq 0.79 m_{1/2} & \sqrt{2} m_W \cos\beta \\ \sqrt{2} m_W \sin\beta & \mu \end{pmatrix} \tag{3.24}$$

$$m^2_{\tilde{t}_L - \tilde{t}_R} = -m_t(A_t + \mu \cot\beta) \tag{3.25}$$

$$m_{\tilde{h}} = |m_\mu|. \tag{3.26}$$

The meaning of the parameters are as follows. $m_{1/2}$, m_0 are the soft SUSY breaking terms at the unification scale. $\tan\beta$ is a ratio of the vacuum expectation values of two higgs doublets in the minimal SUSY standard model. m_μ is the supersymmetric higgsino mass. A_t is the soft SUSY breaking correction to the coupling of $\tilde{t}_L$-$\tilde{t}_R$-(higgs scalar). M_2 is the SU(2) gauge symmetric mass of the wino. The mass of the charged higgs scalar, $m_{H^\pm}$, is a complicated function of other parameters and we have treated it as an independent parameter in this paper. We have not shown the masses of sneutrino and neutralinos in above list since they do not appear in eqs.(3.8,3.9) and hence are irrelevant for our discussion on the SUSY threshold corrections with SU(2)×U(1) breaking. In the numerical analysis in this paper, we always use the assumption $m_{1/2} = m_0$ and $M_2 = m_\mu = m_{H^\pm} \equiv m_{SUSY}$.

In fig. 1, we show the numerical effect of the inclusion of SU(2)×U(1) breaking in SUSY sector for $\delta_s(\tilde{g}, \tilde{W}_i)$, assuming $m_\mu = \pm M_2$. We can clearly see that the SU(2)×U(1) breaking effect is significant in the low M_2 region and it decreases as M_2 grows, as naively expected.

4 Estimation of the SUSY breaking scale without GUT threshold effects

In this section, we review the estimation of the SUSY breaking scale from the unification condition of gauge couplings in the case where the GUT threshold effects is not included [2].

First, for most of the later numerical analyses, we present simpler form of $\delta(light)$'s expressed with one parameter m_{SUSY}, which is usually defined as one of the masses of the SUSY sector particles. Then the SUSY threshold corrections take the following simple forms:

$$\delta_s(light) = -\frac{19}{60}\ln\frac{m_{SUSY}}{m_Z} + C_{sl} \tag{4.1}$$

$$\delta_X(light) = -\frac{7}{60}\ln\frac{m_{SUSY}}{m_Z} + C_{Xl} \tag{4.2}$$

$$\delta_5(light) = \frac{73}{40}\ln\frac{m_{SUSY}}{m_Z} + C_{5l}. \tag{4.3}$$

The parameters C_l's represent the effect of the mass splitting among SUSY sector particles and depend on the model of the mass spectrum in the SUSY sector. The explicit forms of C_l's are expressed in terms of the masses of the SUSY sector particles as follows:

$$\begin{aligned} C_{sl} &= \frac{7}{15}\ln\frac{m_{\tilde{g}}}{m_{SUSY}} - \frac{1}{30}(16 - 5(\sin^2\phi_L + \sin^2\phi_R))\ln\frac{m_{\tilde{W}_1}}{m_{SUSY}} \\ &\quad -\frac{1}{30}(6 + 5(\sin^2\phi_L + \sin^2\phi_R))\ln\frac{m_{\tilde{W}_2}}{m_{SUSY}} - \frac{1}{20}\ln\frac{m_{H^\pm}}{m_{SUSY}} \\ &\quad +\frac{N_g}{60}\ln\frac{m_{\tilde{e}_R}^2 m_{\tilde{d}_R}^3}{m_{\tilde{e}_L}^3 m_{\tilde{d}_L}^2} - \frac{1}{12}\ln\frac{m_{\tilde{u}_L} m_{\tilde{c}_L}}{m_{\tilde{u}_R} m_{\tilde{c}_R}} - \frac{1}{12}(1 - 2\sin^2\phi_{\tilde{t}})\ln\frac{m_{\tilde{t}_1}}{m_{\tilde{t}_2}}, \end{aligned} \tag{4.4}$$

$$\begin{aligned} C_{Xl} &= -\frac{4}{15}\ln\frac{m_{\tilde{g}}}{m_{SUSY}} + \frac{1}{15}\ln\frac{m_{\tilde{W}_1} m_{\tilde{W}_2}}{m_{SUSY}^2} + \frac{1}{60}\ln\frac{m_{H^\pm}}{m_{SUSY}} \\ &\quad +\frac{N_g}{60}\ln\frac{m_{\tilde{e}_L} m_{\tilde{e}_R}}{m_{\tilde{d}_L} m_{\tilde{d}_R}}, \end{aligned} \tag{4.5}$$

$$\begin{aligned} C_{5l} &= \frac{3}{5}\ln\frac{m_{\tilde{g}}}{m_{SUSY}} + \frac{1}{10}\ln\frac{m_{\tilde{W}_1} m_{\tilde{W}_2}}{m_{SUSY}^2} + \frac{1}{40}\ln\frac{m_{H^\pm}}{m_{SUSY}} \\ &\quad +N_g\left(\frac{1}{40}\ln\frac{m_{\tilde{e}_L} m_{\tilde{e}_R}}{m_{SUSY}^2} + \frac{7}{120}\ln\frac{m_{\tilde{d}_L} m_{\tilde{d}_R}}{m_{SUSY}^2}\right) \\ &\quad +\frac{5}{60}\left(\ln\frac{m_{\tilde{u}_L} m_{\tilde{u}_R}}{m_{SUSY}^2} + \ln\frac{m_{\tilde{c}_L} m_{\tilde{c}_R}}{m_{SUSY}^2} + \ln\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_{SUSY}^2}\right). \end{aligned} \tag{4.6}$$

In above results, we have included the effects of the SU(2)×U(1) breaking terms in the SUSY sector, following eq.(3.9). C_l's are constants in the case where we can make an assumption that SU(2)×U(1) breaking is negligible and that all masses of superparticles are proportional to one parameter m_{SUSY}. This assumption is valid as long as m_{SUSY} is sufficiently larger than m_Z.

In realistic models, where colored SUSY sector particles tend to be heavier than non-colored particles (see (3.17–3.26)), C_{sl} is positive while C_{Xl} is negative. In section 5, we will see that both constants increase the estimation of m_{SUSY} [26] in this case. In the terms C_{sl} and C_{Xl}, the contribution of squarks and sleptons cancel each other to leave only the constant factor reflecting their mass ratios and $O(m_Z/m_{SUSY})^2$ terms, as seen in eq.(4.4) and (4.5). The reason is that the squarks and sleptons form complete SU(5) multiplets.

If we take the SUSY mass spectrum as expected from the minimal supergravity model, as given in (3.17–3.26), we can make estimates for these constant factors. For $m_{1/2} = m_0$ and $m_{H^\pm} = m_\mu = M_2 \equiv m_{SUSY}$, we obtain the following numerical values

$$C_{sl} = 0.60,\ C_{Xl} = -0.41,\ C_{5l} = 1.82, \tag{4.7}$$

ignoring SU(2)×U(1) breaking terms. In the above assumption for the SUSY masses, the main contribution to eq.(4.7) comes from the mass splitting between gluino and wino. The constants of eq.(4.7) can be as large as or even larger than the logarithmic terms in eqs.(4.1) to (4.3). Therefore the neglect of these constant terms from the SUSY mass splittings does not lead to realistic estimates in this class of models [2,26].

We use the values (4.3) of the constant factors C_l's in the analysis in section 5 in the studies of the minimal SUSY SU(5) model, where the preferred value of m_{SUSY} is large. In section 6 in the studies of the missing doublet model, where we find that small value of m_{SUSY} is allowed, we include SU(2)×U(1) breaking effects in the constants C_l's via eqs.(4.4)–(4.6).

Here we show the estimation of m_{SUSY} from low energy experimental data in the case where GUT threshold corrections are unknown. From eqs.(2.13,4.1), we obtain

$$\ln \frac{m_{SUSY}}{m_Z} = \frac{60}{19}\left[-\frac{\pi}{\alpha(m_Z)}[s^2(m_Z) - s^{(0)2}(m_Z)] + C_{sl} + \delta_s(2) + \delta_s(GUT)\right]. \quad (4.8)$$

We can see that in (4.8), the dependence of m_{SUSY} on α_s, C_{sl}, $\delta_s(2)$ and $\delta_s(GUT)$ is very significant. The constant $C_{sl} = 0.60$ given in (4.7) multiplies the estimation of m_{SUSY} by about 7. The 2-loop correction $\delta_s(2)$ given in eq.(2.30) multiplies m_{SUSY} by about 50. These corrections are very large [2,26]. The resulting estimation of m_{SUSY} without GUT threshold effects ($\delta_s(GUT) = 0$) is shown in fig.2 with two dashed lines. Their typical values for several values of $\alpha_s(m_Z)$ are as follows:

$$\left\{\begin{array}{lll} 23\text{TeV} < m_{SUSY} < 130\text{TeV} & \text{for} & \alpha_s(m_Z) = 0.11, \\ 820\text{GeV} < m_{SUSY} < 4.9\text{TeV} & \text{for} & \alpha_s(m_Z) = 0.12, \\ 50\text{GeV} < m_{SUSY} < 300\text{GeV} & \text{for} & \alpha_s(m_Z) = 0.13. \end{array}\right. \quad (4.9)$$

These results essentially agree with those in [2]. The uncertainty of m_{SUSY} for fixed α_s, about a factor 6, comes from the experimental error of s^2 in (2.26).

Since the GUT threshold correction $\delta_s(GUT)$ is a part of the next-to-leading corrections, its effect to the estimation of m_{SUSY} may be as significant as the 2-loop and SUSY threshold corrections. In later sections, we will explicitly show that this is indeed the case.

5 GUT threshold effects in the minimal SUSY SU(5) model

In order to discuss GUT threshold corrections, we should fix the SUSY GUT model and derive the masses and the SU(3)×SU(2)×U(1) gauge representations of

all the GUT sector particles. In this and the next sections, we study the GUT threshold effect and its consequences in two specific SUSY SU(5) models, the minimal model and the missing doublet model, respectively. The consequences to the estimation of the SUSY breaking scale are mainly discussed. The results of this section essentially agrees with those of refs.[8,6].

We start by studying the minimal SUSY SU(5) model [3]. First, we obtain masses and gauge representations of the GUT sector particles. The superpotential of the GUT sector particles is

$$\mathcal{W}_{\rm MM} = M_{24}{\rm Tr}(\Sigma^2) + \lambda_1{\rm Tr}(\Sigma^3) + \lambda_2\overline{\phi}\Sigma\phi + M_5\overline{\phi}\phi, \quad (5.1)$$

with three chiral supermultiplets: $\Sigma(24)$, $\phi(5)$ and $\overline{\phi}(\overline{5})$. By choosing the SU(3)× SU(2)×U(1) symmetric vacuum, $\langle\Sigma\rangle = V_{24}$ diag (-2, -2, -2, 3, 3)$/2\sqrt{15}$ with $V_{24} = -4\sqrt{15}M_{24}/(3\lambda_1)$ and $\langle\phi\rangle = \langle\overline{\phi}\rangle = 0$, we find the mass spectra of Tables 2 and 3, after making the fine tuning

$$2(\lambda_2/\lambda_1)M_{24} - M_5 = 0 \quad (5.2)$$

which is necessary for keeping the Higgs doublets in ϕ and $\overline{\phi}$ massless. The (3, 2, ±5/6) components of Σ combine with the corresponding (X, Y) components of the gauge multiplet to make the super gauge-Higgs multiplet given in Table 2 with the common mass $m_X^2 = (5/6)g_5^2V_{24}^2$. The rest of Σ has either the mass $5M_{24}$ ($\equiv m_\Sigma$) or M_{24}, as listed in Table 3. The triplet components of ϕ and $\overline{\phi}$, shown as D in Table 3, have a common mass $m_D = (5/3)M_5$.

The GUT threshold corrections in minimal SUSY SU(5) model are then found as follows:

$$\delta_s(GUT) = \frac{1}{60} - \frac{1}{10}\ln\frac{m_\Sigma}{m_X} + \frac{3}{10}\ln\frac{m_D}{m_X}, \quad (5.3)$$

$$\delta_X(GUT) = \frac{1}{20} - \frac{3}{10}\ln\frac{m_\Sigma}{m_X} - \frac{1}{10}\ln\frac{m_D}{m_X}, \quad (5.4)$$

$$\delta_5(GUT) = -\frac{7}{40} + \frac{21}{20}\ln\frac{m_\Sigma}{m_X} + \frac{7}{20}\ln\frac{m_D}{m_X}. \quad (5.5)$$

At first sight, one might argue that we can learn nothing about these corrections since they contain many unknown mass parameters [5]. But we already have lower limits of m_D and m_X by proton decay experiments. Together with the upper limits from the theoretical consistencies, as discussed in section 2, we can make use of these limits to obtain nontrivial constraints on the GUT sector mass spectrum.

To see the GUT threshold correction to the low energy physics, we derive two useful relations from eqs.(2.13–2.22, 4.1–4.2, 5.3–5.4).

$$\ln\frac{m_D}{m_X^{(0)}} = \frac{5}{6}\ln\frac{m_{SUSY}}{m_Z} + \frac{3\pi}{\alpha(m_Z)}[s^2(m_Z) - s^{2(0)}(m_Z)] - 3(C_{sl}+C_{sh}) + (C_{Xl}+C_{Xh}), \tag{5.6}$$

and

$$\ln\frac{m_\Sigma m_X^2}{m_X^{(0)3}} = -\frac{2}{3}\ln\frac{m_{SUSY}}{m_Z} - \frac{\pi}{\alpha(m_Z)}[s^2(m_Z) - s^{2(0)}(m_Z)] + (C_{sl}+C_{sh}) + 3(C_{Xl}+C_{Xh}), \tag{5.7}$$

where

$$C_{sh} = \frac{1}{60} + \delta_s(2), \tag{5.8}$$

$$C_{Xh} = \frac{1}{20} + \delta_X(2), \tag{5.9}$$

are sum of the constant factors from the GUT threshold corrections (5.3-5.5) (the $\overline{\rm DR}$ to $\overline{\rm MS}$ conversion factor only in the minimal model) and the 2-loop running effects. The identities (5.6) and (5.7) have first been noted clearly in ref.[8]. Eq.(5.6) shows that for fixed $\alpha_i(m_Z)_{\overline{\rm MS}}$ values, the SUSY breaking scale m_{SUSY} is determined only by the value of m_D. We show in fig. 2 the relation of m_{SUSY} and $\alpha_s(m_Z)$ for various upper limit values of m_D. The upper limit value of m_D in fig.2 is achieved in the case where s^2 takes its maximal value allowed in (2.26). The uncertainty of m_{SUSY} from the experimental error $\Delta(s^2) = 0.0014$ for fixed values of m_D and $\alpha_s(m_Z)$ is about a factor of 7.6. On the other hand, if we multiply m_D by 10, m_{SUSY} is multiplied by about 16. This means that even if we have exact values of the low energy gauge couplings and a definite mass spectrum in the SUSY sector,

we cannot determine the SUSY breaking scale by the gauge coupling unification condition without informations of the GUT sector.

We first note that in the minimal SUSY SU(5) model, we can obtain the lower limit of m_{SUSY},

$$\ln\frac{m_{SUSY}}{m_Z} \geq \frac{6}{5}\ln\frac{(m_D)_{\rm min}}{m_X^{(0)}} - \frac{18\pi}{5\alpha(m_Z)}[s^2(m_Z) - s^{2(0)}(m_Z)] + \frac{18}{5}(C_{sl}+C_{sh}) - \frac{6}{5}(C_{Xl}+C_{Xh}), \tag{5.10}$$

by imposing the constraint $m_D > (m_D)_{\rm min}$ from proton decay experiments. Using the conservative limit $(m_D)_{\rm min} = 10^{16}$GeV, we find that the GUT threshold effects increase the estimation of m_{SUSY} in most cases, as seen in fig.2. We find that under the assumption (4.7) for the SUSY sector masses, the minimal SUSY SU(5) model favors high $\alpha_s(m_Z)$(>0.12) if m_{SUSY} <1TeV is satisfied. This result is severe for the naturalness condition, which is typically expressed as m_{SUSY} <1TeV. To be precise, there is also the upper limit of m_{SUSY} from the Planck mass limit of m_D, $m_D < m_P$. But this upper limit is beyond the frame of fig.2.

In addition, we comment on the relation between m_X and m_{SUSY}. The relation between the combination $m_\Sigma m_X^2$ and m_{SUSY} is given in eq.(5.7). Unfortunately, m_Σ has no constraint except for the Planck limit, $m_\Sigma < m_P$. This constraint gives no useful information on the relation between m_{SUSY} and m_X beyond the present experimental limits. Therefore any values of m_X in the range from 10^{15}GeV to m_P are theoretically allowed.

The authors of ref.[8,20] have shown that if one further imposes the "weak higgs coupling condition" that the higgs couplings λ_1 and λ_2 in (5.1) should not diverge until the Planck scale, the mass ratios m_D/m_X and m_Σ/m_X should not exceed a value about 3, therefore strong constraint on m_X for given value of m_{SUSY} is obtained. This study is beyond the scope of this paper. We only quote their result that the proton decays mediated by X cannot be comparable to or dominant over the proton decays mediated by $\tilde{D}$ in the minimal model if m_{SUSY} is smaller than 1TeV.

6 GUT threshold effects in the missing doublet SUSY SU(5) model

As stated in the last section, the minimal SUSY SU(5) model has a problem of fine tuning, eq.(5.2), to arrange for the huge gauge hierarchy. One way to avoid the fine tuning problem is the missing partner mechanism. In this section, we study a SUSY SU(5) model which realize this mechanism, the missing doublet model [27].

As in the last section, we first obtain masses and the SU(3)×SU(2)×U(1) gauge representations of all the GUT sector particles. The superpotential of the missing doublet model for the GUT sector is

$$\mathcal{W}_{\rm MDM} = M_{75}{\rm Tr}(\Sigma^2) + \lambda_1 {\rm Tr}(\Sigma^3) + \lambda_2 \Phi\Sigma\phi + \lambda_3 \bar{\Phi}\Sigma\bar{\phi} + M_{50}\bar{\Phi}\Phi, \tag{6.1}$$

with chiral supermultiplets $\Sigma(75)$, $\phi(5)$, $\bar{\phi}(\bar{5})$, $\Phi(50)$ and $\bar{\Phi}(\overline{50})$. Σ is now a 75 chiral supermultiplet which acquires a vacuum expectation value

$$\begin{aligned}
\langle\Sigma_{ij}^{kl}\rangle &= -\epsilon_{ijm}\epsilon^{klm}V_{75}, \\
\langle\Sigma_{i\alpha}^{j\beta}\rangle &= \delta_i^j\delta_\alpha^\beta V_{75}, \\
\langle\Sigma_{\alpha\beta}^{\gamma\delta}\rangle &= -3\epsilon_{\alpha\beta}\epsilon^{\gamma\delta}V_{75}, \\
V_{75} &= M_{75}/(4\lambda_1),
\end{aligned} \tag{6.2}$$

that uniquely breaks gauge symmetry to SU(3)×SU(2)×U(1) (see the Appendix for notation and detail).

The (3, 2, ±5/6) components of Σ combine with the corresponding (X, Y) components of the gauge multiplet to make the super gauge-Higgs multiplet of Table 2, just as in the minimal model, with the common mass $m_X^2 = 24g_5^2V_{75}^2$. All the remaining components of Σ acquire masses proportional to M_{75}, as listed in Table 4. The (8, 3, 0) component has the largest mass $20M_{75}$, which we denote as m_Σ. All the supermultiplets of $\Phi(50)$ and $\overline{\Phi}(\overline{50})$ obtain the common mass $m_\Phi = M_{50}$, except

for their (3, 1, ±1/3) components ($\bar{D}_\Phi$, $D_{\bar{\Phi}}$) that couple with the triplet components (D_ϕ, $\bar{D}_{\bar{\phi}}$) of ϕ and $\overline{\phi}$ to form two massive states D_1 and D_2, as is explained below. The doublet components of ϕ and $\bar{\phi}$ remain massless since $\Phi(50)$ and $\bar{\Phi}(\overline{50})$ have no (1, 2, ±1/2) components, as is clearly seen in the decomposition of 50 into SU(3)×SU(2)×U(1) representations.

$$\begin{aligned}
50 &= (6,3,+1/3) + (8,2,-1/2) + (3,2,+7/6) + (\bar{6},1,-4/3) \\
&\quad +(\bar{3},1,+1/3)(\equiv \bar{D}) + (1,1,+2).
\end{aligned} \tag{6.3}$$

The triplet-doublet splitting is then naturally achieved in this model.

The quadratic part of the superpotential for the triplet components of ϕ, $\overline{\phi}$, Φ and $\overline{\Phi}$ is as follows from (6.1):

$$\begin{aligned}
\mathcal{W}(D,\bar{D}) &= (\bar{D}_{\bar{\phi}},\ \bar{D}_\Phi)\mathcal{M}\begin{pmatrix} D_\phi \\ D_{\bar{\Phi}} \end{pmatrix} \\
&= (\bar{D}_1,\ \bar{D}_2)\mathcal{M}_d\begin{pmatrix} D_1 \\ D_2 \end{pmatrix},
\end{aligned} \tag{6.4}$$

where $\mathcal{M}$ and $\mathcal{M}_d$ are the mass matrices in the gauge and mass eigenstates basis, respectively,

$$\mathcal{M} = \begin{pmatrix} 0 & \lambda_3 V_{75} \\ \lambda_2 V_{75} & M_{50} \end{pmatrix}, \tag{6.5}$$

$$\mathcal{M}_d = V\mathcal{M}U^\dagger = {\rm diag}(m_{D_1}, m_{D_2}), \tag{6.6}$$

$$m_{D_1} < m_{D_2}.$$

The two unitary matrices U and V are needed to obtain the two Dirac supermultiplets D_1 and D_2 as the mass eigenstates. The two mass eigenvalues

$$m_{D_{1,2}} = |(M_{50}^2 + (\lambda_2+\lambda_3)^2V_{75}^2)^{1/2} \mp (M_{50}^2 + (\lambda_2-\lambda_3)^2V_{75}^2)^{1/2}|/2, \tag{6.7}$$

satisfy the following useful identity

$$m_{D_1}m_{D_2} = |\lambda_2\lambda_3|V_{75}^2. \tag{6.8}$$

All the five parameters of the superpotential (6.1) are independent, and we can take the five physical masses m_X, m_Σ, m_{D_1}, m_{D_2} and m_Φ as the parameters of the model.

We should be careful when imposing the limit from experiments of the proton decays mediated by dimension-5 operators (2.28) in the missing doublet model since both D_1 and D_2 mediate this decay. Fortunately, as we will show below, this limit is represented by just one combination of the mass parameters, which we denote by $m_D(eff)$. To prove this, let us evaluate the magnitude of the dimension-5 operator which mediates proton decays by making use of the property that only ϕ and $\bar{\phi}$ couples to quarks and leptons.* Then the operator is proportional to

$$\sum_{i=1}^{2} \frac{(U^\dagger)_{1i}(V)_{i1}}{m_{D_i}} = (\mathcal{M}^{-1})_{11} = \frac{m_\Phi}{m_{D_1} m_{D_2}} \equiv \frac{1}{m_D(eff)} \tag{6.9}$$

if the momentum square of the propagator of D_i is negligible as compared to m_{D_i}'s. Hence eq.(2.28), the lower limit of m_D in the minimal model is also the lower limit of $m_D(eff)$ in the missing doublet model. Note that the Planck scale upper limit is not applicable for $m_D(eff)$ since this quantity is not a physical mass of a particle.

We find that the use of $m_D(eff)$ instead of m_{D_i}'s is very useful for our discussion. The reason is that with $m_D(eff)$, the GUT threshold correction in the missing doublet model is expressed in the form very similar to that in the minimal model.

The next-to-leading order correction terms, $\delta(light)$ and $\delta(2)$ are common to the minimal model since these are determined by the low energy effective theory, the minimal SUSY standard model. The GUT threshold corrections of the missing doublet model are as follows:

$$\delta_s(GUT) = -3.00 - \frac{1}{10}\ln\frac{m_\Sigma}{m_X} + \frac{3}{10}\ln\frac{m_D(eff)}{m_X} \tag{6.10}$$

$$\delta_X(GUT) = 0.72 - \frac{3}{10}\ln\frac{m_\Sigma}{m_X} - \frac{1}{10}\ln\frac{m_D(eff)}{m_X} \tag{6.11}$$

*Here we have ignored possible direct coupling of $\bar{\Phi}$ to quarks and leptons [27].

$$\delta_5(GUT) = -6.28 + \frac{221}{20}\ln\frac{m_\Sigma}{m_X} + \frac{7}{20}\ln\frac{m_D(eff)}{m_X} + \frac{35}{2}\ln\frac{m_\Phi}{m_X}. \tag{6.12}$$

We first comment on the fact that the coefficients of the logarithms in eqs.(6.10,6.11) are the same as those of eqs.(5.3,5.4) in the minimal model, despite the significant difference between these models. This property has already been pointed out in ref.[11], in the context of non-SUSY GUT models. The reason is that in both models, the $(3, 2, \pm 5/6)$ components of Σ, which are absorbed into the heavy gauge supermultiplets (X, Y), and two $(1, 2, \pm 1/2)$ components of ϕ, $\bar{\phi}$ (and Φ, $\bar{\Phi}$ in the missing doublet model) are splitted from complete SU(5) multiplets in the GUT sector. Therefore, the remaining logarithmic contributions to δ_s and δ_X should be the same in both models. Second, we note that the large negative constant factors in eqs.(6.10) and (6.12) come from the mass splitting inside $\Sigma(75)$, see Table 4. For example, the contribution of $\Sigma(75)$ to δ_s is as follows:

$$\begin{aligned}\delta_s(\Sigma) &= -\frac{43}{10}\ln\frac{m_\Sigma}{m_X} + \frac{19}{10}\ln\frac{0.8m_\Sigma}{m_X} + \frac{16}{10}\ln\frac{0.4m_\Sigma}{m_X} + \frac{7}{10}\ln\frac{0.2m_\Sigma}{m_X} \\ &= -\frac{1}{10}\ln\frac{m_\Sigma}{m_X} - 3.02.\end{aligned} \tag{6.13}$$

We can clearly see that the coefficients of the logarithms cancel whereas the constant terms add up. The corresponding constant terms are absent in the minimal model, see eqs.(5.3–5.5), since all the mass eigenstates of $\Sigma(24)$ with non-trivial SU(3)×SU(2)×U(1) quantum numbers have the common mass, $5M_{24}$, there (see Table 3). Due to these two facts, we can perform the discussion of GUT threshold corrections which is parallel to but distinct from that in the minimal model.

The relations analogous to eqs.(5.6,5.7) in the minimal model hold also in the missing doublet model with the replacement

$$m_D \to m_D(eff), \tag{6.14}$$

with

$$C_{sh} = -3.00 + \delta_s(2), \tag{6.15}$$

$$C_{Xh} = 0.72 + \delta_X(2). \tag{6.16}$$

Because of the large constant terms in eqs.(6.15, 6.16), we find that the missing doublet model favors smaller m_{SUSY} and lower $\alpha_s(m_Z)$ than those in the minimal model, for the same m_D or $m_D(eff)$ value. Explicitly, the relation

$$m_D(eff)_{\rm MDM} = 1.7 \times 10^4 (m_D)_{\rm MM} \tag{6.17}$$

holds for the common m_{SUSY} and $\alpha_s(m_Z)$. In order to satisfy the non-observation of the SUSY sector particles at colliders, which is expressed typically as $m_{SUSY} > m_Z/2$, we need very high values of $m_D(eff)$, as seen in fig.3. Consequently, in this model we cannot expect to observe proton decays mediated by the dimension-5 operators in the near future. In other words, observation of the proton decay expected from the dimensional 5 operators in the near future is almost sufficient to rule out the missing doublet model. As for the SUSY breaking scale, lower m_{SUSY} is allowed for $\alpha_s(m_Z) > 0.11$ after imposing $m_D(eff) > 10^{16}$GeV, in contrast to the result in the minimal model.

In the missing doublet model, we should impose a further condition that the gauge coupling constant $\alpha_5(\mu)_{\overline{\rm DR}}$ should be finite at all the GUT particle masses in the theory. This condition is non-trivial in the missing doublet model or, in general, models which contain large number of higgs supermultiplets in the GUT sector, since these models are asymptotically non-free, in contrast to the minimal model. In the missing doublet model,

$$\frac{\pi}{\alpha_5(m_j)_{\overline{\rm DR}}} = \frac{\pi}{\alpha_5(m_X)_{\overline{\rm DR}}} - 26 \ln \frac{m_j}{m_X} \tag{6.18}$$

holds.

In fig.4, we show the allowed region on the parameter space $(m_\Phi,\, m_X)$ for several values of m_{SUSY} and $\alpha_s(m_Z)$ together with the corresponding values of $m_D(eff)$, after imposing the following conditions: measured values of the standard model gauge coupling $\alpha_i(m_Z)$, lower limits of m_X and $m_D(eff)$ from the proton decay experiments (lower frame of fig.4), Planck mass limits (upper frame and

vertical line) and the finiteness of $\alpha_5(\mu)$ at all the GUT particle masses (skew and curved lines). Among the remaining two parameters in the missing doublet model, m_Σ is dependent on m_X via the analogous relation of eq.(5.7), while m_{D_2} is set to its lowest value for given (m_X, m_Φ) to obtain the most general conditions. The lowest value is obtained by choosing $\lambda_2 = \lambda_3$ in eq.(6.7). Explicitly,

$$\min(m_{D_2}) = \frac{m_\Phi}{2}\left((1 + 4m_D(eff)/m_\Phi)^{1/2} + 1\right) \tag{6.19}$$

holds. We have imposed the finiteness of $\alpha_5(m_j)$ on $m_j = m_X$, m_Σ and m_{D_2}. The finiteness of $\alpha_5(m_\Phi)$ is not needed since $m_{D_2} > m_\Phi$ always holds from eq.(6.19).

The allowed region of $(m_\Phi,\, m_X)$ is rather narrow. We can see that in fig.4, the allowed parameter region gets narrower as m_{SUSY} and $\alpha_s(m_Z)$ increase. Indeed, from the finiteness of $\alpha_5(m_j)$, we can obtain the upper limit of m_{SUSY} since for too large m_{SUSY} and, consequently, for large $m_D(eff)$ there remains no allowed parameter region. We can explicitly prove this by showing that the following three constraints $m_{D_1}m_{D_2} < m_P^2$, $m_X > (m_X)_{\rm min} = 10^{15}$GeV and $\alpha_5(\sqrt{m_{D_1}m_{D_2}})^{-1} > 0$ can be simultaneously satisfied only in the case where

$$\begin{aligned}\ln\frac{m_{SUSY}}{m_Z} \;<\; & \frac{1}{119}\Bigg[6\Big(\frac{\pi}{\alpha_5(m_X)^{(0)}} + C_{5l} + C_{5h}\Big) + 54\ln\frac{m_P}{m_X^{(0)}} - 150\ln\frac{(m_X)_{\rm min}}{m_X^{(0)}} \\ & +96(C_{Xl} + C_{Xh}) - \frac{375\pi}{\alpha(m_Z)}(s^2(m_Z) - s^{2(0)}(m_Z)) + 375(C_{sl} + C_{sh})\Bigg].\end{aligned} \tag{6.20}$$

Here C_{5h} is defined as

$$C_{5h} = -6.28 + \delta_5(2). \tag{6.21}$$

The upper border of the allowed range of $(m_{SUSY},\, \alpha_s(m_Z))$ from this constraint is shown in fig.3 with a dashed line. The resulting upper limit of m_{SUSY} is below 1TeV for $\alpha_s(m_Z) > 0.13$, making a sharp contrast to the result in the minimal model.

It is interesting to study consequences of the finiteness of $\alpha_5(\mu)$ at the Planck scale m_P, although this condition is not necessary for theoretical consistency of the model. However, we find that by imposing this condition, the upper limit of m_{SUSY} (6.20) is not modified although the allowed region of (m_Φ, m_X) is largely restricted. The reason is that in the case where m_{SUSY} is near its maximal limit, m_{D_2} is near the Planck scale. In other words, the finiteness condition of $\alpha_5(m_P)$ has been implicitly included in eq.(6.20).

Another important consequence is that the allowed value of m_X is much lower than that of $m_D(eff)$, as seen in fig.4. This means that in the missing doublet model, proton decays are expected to occur dominantly by the dimension-6 operators. This also makes a strong contrast to the prediction of the minimal model.

Here we comment on the relation between m_{SUSY} and m_X. In contrast to the case in the minimal model, the upper limit of m_{SUSY} and m_X is closely related via eq.(6.20), without further constraints such as the weak higgs coupling condition. For example, if we can improve the lower limit of m_X by a factor 10, the upper limit of m_{SUSY} is reduced by a factor about 18.

Finally, we comment on the consequence of the weak higgs coupling condition in this model. As can be seen, for the triplet higgses, only the combination $m_{D_1}m_{D_2} = m_D(eff)m_\Phi$, not $m_D(eff)$ and m_{D_i} themselves, is subject to this conditions:

$$\frac{m_{D_1}m_{D_2}}{m_X^2} = \frac{|\lambda_2\lambda_3|}{24g_5^2}. \tag{6.22}$$

If we impose the weak higgs coupling condition to λ_2 and λ_3, the above mass ratio should not be much larger than $O(1)$. We can obtain the lowest allowed value of this mass ratio for given m_{SUSY} from the finiteness of $\alpha_5(m_j)$. Explicitly, from the finiteness of $\alpha_5(\sqrt{m_{D_1}m_{D_2}})$ and $\alpha_5(m_\Sigma)$,

$$\begin{aligned}\ln\frac{m_{D_1}m_{D_2}}{m_X^2} &> \frac{1}{129}\left[-18\left(\frac{\pi}{\alpha_5(m_X)^{(0)}}+C_{5l}+C_{5h}\right)+293\ln\frac{m_{SUSY}}{m_Z}\right.\\ &\left.+\frac{1029\pi}{\alpha(m_Z)}(s^2(m_Z)-s^{2(0)}(m_Z))-1029(C_{sl}+C_{sh})\right]\end{aligned} \tag{6.23}$$

follows. We find that this mass ratio should exceed 10^2 in wide range of $(\alpha_s(m_Z), m_{SUSY})$, whose upper border is shown in fig.3 with a dotted line. So if $\alpha_s(m_Z)$ is higher than 0.12, the weak higgs coupling condition is likely to be violated in the missing doublet model. But since light SUSY breaking scale is concerned in this case, a definite conclusion may require a more complete analysis including the non-logarithmic SUSY mass corrections to the present experimental observables.

7 Conclusion

In this paper, we have studied SUSY and GUT threshold effects on the predictions of the two supersymmetric SUSY SU(5) models, the minimal model and the missing doublet model. The effect of these corrections on the estimation of the SUSY breaking scale m_{SUSY} from low energy experimental data and the unification condition is discussed in detail.

For the SUSY threshold corrections, we have reviewed the effects of the mass splitting within SUSY sector particles. We have also presented a correct treatment of the effect of SU(2)×U(1) breaking in the SUSY particle mass spectrum, which has been ignored in previous studies. We have shown that this effect is significant for the low m_{SUSY} case ($m_{SUSY} \sim m_Z$) and vanishes as increasing m_{SUSY} ($m_{SUSY} \gg m_Z$), as naively expected.

For the GUT threshold corrections, we have studied the model dependence of these corrections for the two SUSY SU(5) models, the minimal model and the missing doublet model which solves the fine-tuning problem. We have shown that the GUT threshold corrections in the missing doublet model contain large constant terms generated by the mass splitting within a superheavy higgs multiplet $\Sigma(75)$, whereas the coefficients of the logarithmic terms generated by the mass splitting among different SU(5) multiplets are mostly common to those of the minimal model. Due to these large constant terms and the asymptotical non-freedom of the missing doublet model, the low energy consequences of these two SUSY SU(5) models are found to be very different.

We have focused our attention mainly on the estimation of the SUSY breaking scale from the gauge coupling unification condition. We have used the following constraints for the threshold effects: the gauge coupling constants at the scale m_Z, the proton decay experiments, and some theoretical consistency conditions such as the Planck mass limit and the finiteness of the unified gauge coupling constant $\alpha_5(m_j)_{\overline{\rm DR}}$ at all the GUT particle masses m_j. Under these constraints, we have shown that the GUT threshold corrections are significant and strongly model dependent. For example, we have found that under a certain assumption for the SUSY particle masses, the minimal model favors large m_{SUSY}($>$1TeV) or high $\alpha_s(m_Z)$ (> 0.12), whereas the missing doublet model allows low m_{SUSY} ($<$1TeV) for $0.11 < \alpha_s(m_Z) < 0.13$. Moreover, m_{SUSY} should be less than 1TeV in the latter model if $\alpha_s(m_Z)$ is very large (> 0.13).

We have also found that the main proton decay mode is different in both models. In the minimal model, we have checked that m_X has no relation to the values of $(\alpha_s(m_Z),\ m_{SUSY})$ in experimentally allowed regions, but it has already been found [20] that the decays mediated by dimension-5 operators, $p \to \bar{\nu} + K^+$ etc., are expected to dominate if we impose the "weak higgs coupling condition" that the higgs couplings in the superpotential do not diverge until the Planck scale.

On the contrary, in the missing doublet model, we have shown that the decays mediated by dimension-6 operators, $p \to e^+ + \pi^0$ etc., are expected to dominate if we impose the conditions that m_{SUSY} is larger than $m_Z/2$ and that $\alpha_5(\mu)_{\overline{\rm DR}}$ is finite at all the GUT particle masses in the model. In addition, the upper limit of m_{SUSY} is sensitive to the value of m_X in this model. These results have been obtained without imposing the weak higgs coupling condition.

We have also shown that in the missing doublet model, the higgs couplings in the superpotential should be very large for wide range of $(\alpha_s(m_Z),\ m_{SUSY})$. So, the weak higgs coupling condition is likely to be violated in this model if $\alpha_s(m_Z)$ is

higher than 0.12.

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Appendix

In this Appendix, we derive the mass spectrum of the $\Sigma(75)$ supermultiplet in the missing doublet model, which is listed in Table 4.

The multiplet $\Sigma(75)$ is represented by a SU(5) tensor Σ^{cd}_{ab} $(a, b, \cdots = 1-5)$ with the constraints $\Sigma^{cd}_{ab} = -\Sigma^{cd}_{ba} = -\Sigma^{dc}_{ab}$ and $\Sigma^{ad}_{ab} = 0$. The superpotential of $\Sigma(75)$ is contained in eq.(6.1),

$$\mathcal{W}(\Sigma) = M\mathrm{Tr}(\Sigma^2) + \lambda\mathrm{Tr}(\Sigma^3). \tag{A.1}$$

For simplicity, we abbreviate M_{75}, λ_1 in (6.1) as M, λ, respectively. We fix the definition of the $\mathrm{Tr}(\Sigma^2)$ and $\mathrm{Tr}(\Sigma^3)$ in eq.(A.1) as follows:

$$\mathrm{Tr}(\Sigma^2) \;=\; \Sigma^{cd}_{ab}\Sigma^{ab}_{cd}, \tag{A.2}$$

$$\mathrm{Tr}(\Sigma^3) \;=\; \Sigma^{cd}_{ab}\Sigma^{ef}_{cd}\Sigma^{ab}_{ef}, \tag{A.3}$$

These definitions are unique up to total normalization factors.

To obtain the mass spectrum of $\Sigma(75)$, we decompose Σ^{cd}_{ab} into SU(3)$\times$SU(2) $\times$U(1) representations. The results are

$$\begin{aligned}\Sigma(75) \;=\; & (H^{AM}(8,3,0), H_i(3,1,+5/3), H^i(\bar{3},1,-5/3), \\ & H_{(ij)\alpha}(6,2,+5/6), H^{(ij)\alpha}(\bar{6},2,-5/6), H(1,1,0),\end{aligned}$$

$$H^A(8,1,0), H_{i\alpha}(3,2,-5/6), H^{i\alpha}(\bar{3},2,+5/6)), \tag{A.4}$$

$$\begin{aligned}
\Sigma_{ij}^{kl} &= \epsilon_{ijm}\epsilon^{kln}(-\frac{1}{3\sqrt{2}}\delta_n^m H + \frac{1}{\sqrt{3}}(\tau^A)_n^m H^A) && \text{(A.5)}\\
\Sigma_{ij}^{k\alpha} &= \epsilon_{ijl}(H^{(lk)\alpha} - \frac{1}{\sqrt{6}}\epsilon^{lkm}\epsilon^{\alpha\beta}H_{m\beta}) && \text{(A.6)}\\
\Sigma_{ij}^{\alpha\beta} &= \epsilon_{ijk}\epsilon^{\alpha\beta}H^k && \text{(A.7)}\\
\Sigma_{i\alpha}^{j\beta} &= \frac{1}{3\sqrt{2}}\delta_i^j\delta_\alpha^\beta H + \frac{1}{2\sqrt{3}}\delta_\alpha^\beta(\tau^A)_i^j H^A + \frac{1}{2}(\tau^A)_i^j(\sigma^M)_\alpha^\beta H^{AM} && \text{(A.8)}\\
\Sigma_{i\alpha}^{\beta\gamma} &= \sqrt{2/3}\epsilon^{\beta\gamma}H_{i\alpha} && \text{(A.9)}\\
\Sigma_{\alpha\beta}^{\gamma\delta} &= -\frac{1}{\sqrt{2}}\epsilon_{\alpha\beta}\epsilon^{\gamma\delta}H && \text{(A.10)}
\end{aligned}$$

and their hermite conjugates for the gauge representations, not for the chiralities. Here the letters $(i, j\cdots)$, $(\alpha, \beta\cdots)$, A and M represent SU(3) fundamental (1–3), SU(2) fundamental (1,2), SU(3) adjoint (1–8) and SU(2) adjoint (1–3) indices, respectively. The matrices τ^A and σ^M are the generator of SU(3) and SU(2), respectively, with the normalization $\mathrm{Tr}(\tau^A\tau^B) = 2\delta^{AB}$ etc. The normalization factors in eqs.(A.5–A.10) are chosen so that the left hand sides represent the properly normalized fields. Among these fields, the scalar component of H acquires a vacuum expectation value V_{75},

$$\langle H\rangle = 3V/\sqrt{2}, \qquad V = M/(4\lambda), \tag{A.11}$$

where V is normalized to be consistent with (6.2).

Now we are ready to derive mass spectra of $\Sigma(75)$ after SU(5) breaking. First, the quadratic term $\mathrm{Tr}(\Sigma^2)$, which gives SU(5) symmetric masses, is expressed in terms of the component fields H's as follows:

$$\begin{aligned}
\mathrm{Tr}\Sigma^2 &= 4(H^{AM}H^{AM} + H^2 + H^AH^A)\\
&\quad +8(H_iH^i + H_{(ij)\alpha}H^{(ij)\alpha} + H_{i\alpha}H^{i\alpha}). && \text{(A.12)}
\end{aligned}$$

After SU(5) breaking (A.11), also the cubic term $\mathrm{Tr}(\Sigma^3)$ contributes to the mass spectrum. After the replacement $\Sigma \to \Sigma - \langle\Sigma\rangle$, the quadratic superpotential

is as follows:

$$\begin{aligned}
\mathcal{W}^{(2)}(\Sigma) &= M\mathrm{Tr}(\Sigma^2) + 3\lambda\Sigma_{ab}^{cd}\Sigma_{cd}^{ef}\langle\Sigma_{ef}^{ab}\rangle\\
&= (4M + 24\lambda V)(H^{AM})^2 + (8M - 96\lambda V)H_\alpha H^\alpha\\
&\quad +(8M + 0\lambda V)H_{(ij)\alpha}H^{(ij)\alpha} + (4M - 32\lambda V)H^2\\
&\quad +(4M - 8\lambda V)(H^A)^2 + (8M - 32\lambda V)H_{i\alpha}H^{i\alpha}. && \text{(A.13)}
\end{aligned}$$

After substituting (A.11) into (A.13), we obtain the mass spectrum of Σ listed in Table 4. The fields $H_{i\alpha}$ and $H^{i\alpha}$ receive no mass term from the superpotential. These fields are absorbed into the heavy vector supermultiplet (X, Y).

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Tables

Table 1 SU(3)×SU(2)×U(1) quantum numbers, masses and β-function coefficients $b_i(j)$ of the SUSY sector particles j in the minimal standard model. SU(2)×U(1) breaking is not included.

j	R	$b_1(j)$	$b_2(j)$	$b_3(j)$	*comments*
$\tilde{g}$	(8,1,0)	0	0	1	
$\tilde{W}$	(1,3,0)	0	2/3	0	
$\tilde{h}$	$(1,2,\pm1/2)$	1/5	1/3	0	
$\tilde{q}_L$	$(3,2,+1/6)$	1/60	1/4	1/6	
$\tilde{u}_R$	$(3,1,+2/3)$	2/15	0	1/12	
$\tilde{d}_R$	$(3,1,-1/3)$	1/30	0	1/12	for 1 generation
$\tilde{l}_L$	$(1,2,-1/2)$	1/20	1/12	0	
$\tilde{e}_R$	$(1,1,-1)$	1/10	0	0	
H	$(1,2,\pm1/2)$	1/20	1/12	0	extra higgs doublet

Table 2 the heavy multiplets in the supersymmetric SU(5) model gauge sector.

j	R	*mass*	$b_1(j)$	$b_2(j)$	$b_3(j)$
X,Y	$(3,2,\pm5/6)$	m_X	-35/4	-21/4	-7/2
$\tilde{X},\tilde{Y}$	$(3,2,\pm5/6)$	m_X	10/3	2	4/3
H_X,H_Y	$(3,2,\pm5/6)$	m_X	5/12	1/4	1/6
sum		m_X	-5	-3	-2

Table 3 Higgs sector of the minimal model with $\Sigma(24)$, $\phi(5)$ and $\bar{\phi}(\bar{5})$.

j	R	*mass*	$b_1(j)$	$b_2(j)$	$b_3(j)$	*comments*
$H_{(8,1)}$, $\bar{H}_{(8,1)}$	(8,1,0)	m_Σ	0	0	3/2	
$H_{(1,3)}$, $\bar{H}_{(1,3)}$	(1,3,0)	m_Σ	0	1	0	in Σ
$H_{(1,1)}$, $\bar{H}_{(1,1)}$	(1,1,0)	$0.2m_\Sigma$	0	0	0	
D, $\bar{D}$	$(3,1,\pm1/3)$	m_D	1/5	0	1/2	in ϕ and $\bar{\phi}$

Table 4 Higgs sector of the missing doublet model with $\Sigma(75)$, $\phi(5)$, $\bar{\phi}(\bar{5})$, $\Phi(50)$ and $\bar{\Phi}(\overline{50})$.

j	R	*mass*	$b_1(j)$	$b_2(j)$	$b_3(j)$	*comments*
$H_{(8,3)}$, $\bar{H}_{(8,3)}$	(8,3,0)	m_Σ	0	8	9/2	
$H_{(3,1)}$, $\bar{H}_{(3,1)}$	$(3,1,\pm5/3)$	$0.8m_\Sigma$	5	0	1/2	
$H_{(6,2)}$, $\bar{H}_{(6,2)}$	$(6,2,\pm5/6)$	$0.4m_\Sigma$	5	3	5	in Σ
$H_{(1,1)}$, $\bar{H}_{(1,1)}$	(1,1,0)	$0.4m_\Sigma$	0	0	0	
$H_{(8,1)}$, $\bar{H}_{(8,1)}$	(8,1,0)	$0.2m_\Sigma$	0	0	3/2	
D_1, $\bar{D}_1$	$(3,1,\pm1/3)$	m_{D_1}	1/5	0	1/2	in
D_2, $\bar{D}_2$	$(3,1,\pm1/3)$	m_{D_2}	1/5	0	1/2	ϕ, $\bar{\phi}$, Φ, $\bar{\Phi}$
H_{50}, $\bar{H}_{50}$		m_Φ	173/10	35/2	17	$(\Phi,\bar{\Phi})-(\bar{D}_\Phi,D_{\bar{\Phi}})$

Figure Captions

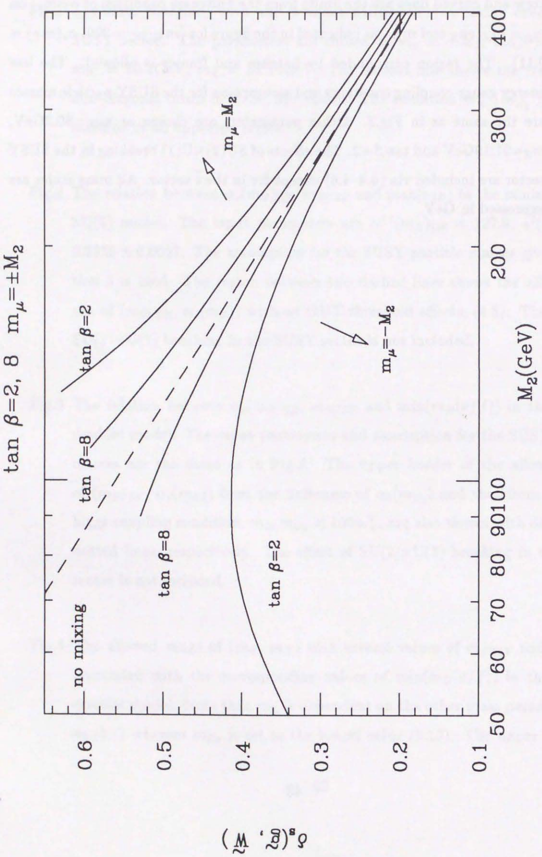
Fig.1 δ_s(gluino, chargino) with and without SU(2)×U(1) breaking effects in the SUSY sector. The parameters are chosen as $m_\mu = \pm M_2$, $\tan\beta$ =2 and 8, m_W = 80.2GeV, m_Z = 91.19GeV. The dashed line shows the result when non-diagonal terms in (3.24) are absent. The condition $m_{\tilde{W}_1}, m_{\tilde{W}_2} > m_Z/2$ is satisfied in all appeared region.

Fig.2 The relation between $\alpha_s(m_Z)_{\overline{\rm MS}}$, m_{SUSY} and $\max(m_D)$ in the minimal SUSY SU(5) model. The input parameters are $\alpha^{-1}(m_Z)_{\overline{\rm MS}}$ = 127.9, $s^2(m_Z)_{\overline{\rm MS}}$ = 0.2325 ± 0.0007. The assumption for the SUSY particle masses given in section 3 is used. The region between two dashed lines shows the allowed values of (m_{SUSY}, $\alpha_s(m_Z)$) without GUT threshold effects, (4.8). The effect of SU(2)×U(1) breaking in the SUSY sector is not included.

Fig.3 The relation between $\alpha_s(m_Z)_{\overline{\rm MS}}$, m_{SUSY} and $\min(m_D(eff))$ in the missing doublet model. The input parameters and assumption for the SUSY particle masses are the same as in Fig.2. The upper border of the allowed range of (m_{SUSY}, $\alpha_s(m_Z)$) from the finiteness of $\alpha_5(m_{D_2})$ and that from the weak higgs coupling condition, $m_{D_1}m_{D_2} < 100m_X^2$, are also shown with dashed and dotted lines, respectively. The effect of SU(2)×U(1) breaking in the SUSY sector is not included.

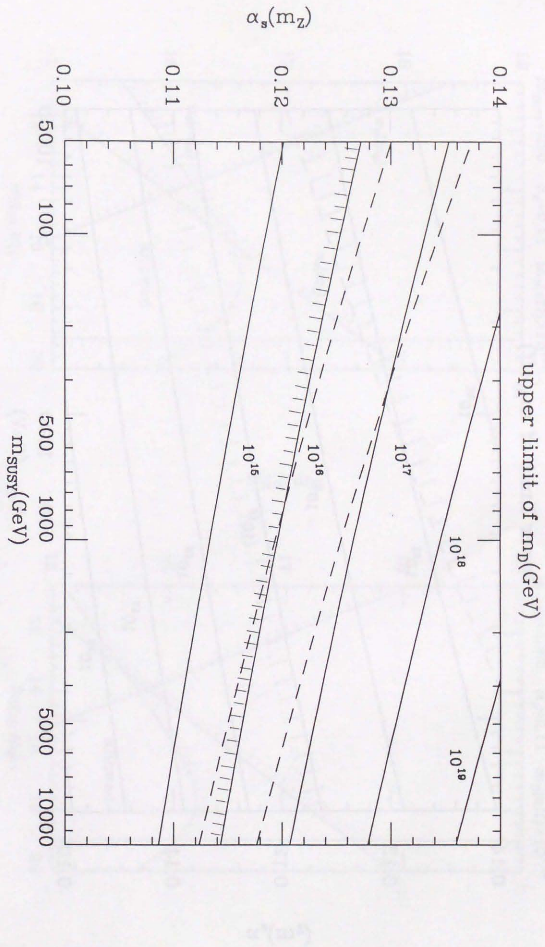
Fig.4 The allowed range of (m_Φ, m_X) with several values of m_{SUSY} and $\alpha_s(m_Z)$, associated with the corresponding values of $\min(m_D(eff))$ in the missing doublet model. Note that m_Σ is dependent on the other mass parameters via eq.(5.7) whereas m_{D_2} is set to the lowest value (6.19). The upper frames of

m_X and the vertical lines represent the Planck mass limits. The lower frame of m_X comes from eq.(2.28), the limit from proton decay experiments. The skew and curved lines are the limits from the finiteness condition of $\alpha_5(m_j)$ on $m_j = m_X$, m_Σ and m_{D_2}, as indicated in the figure for (m_{SUSY} = 200, $\alpha_s(m_Z)$ = 0.11). The region surrounded by hatches and frames is allowed. The low energy gauge coupling constants and assumption for the SUSY particle masses are the same as in Fig.2. Other parameters are chosen as m_W=80.2GeV, m_Z=91.19GeV and $\tan\beta$=2. The effects of SU(2)×U(1) breaking in the SUSY sector are included via (4.4–4.6) except for in the $\tilde{t}$ sector. All mass scales are expressed in GeV.



tan β=2, 8 m_μ=±M_2
no mixing
tan β=2
tan β=8
tan β=8
tan β=2
m_μ=M_2
m_μ=−M_2
0.6
0.5
0.4
0.3
0.2
0.1
50
60
70
80
90100
200
300
400
M_2(GeV)
δ_s(Q, W)

Fig. 1



upper limit of m_D(GeV)
α_s(m_Z)
0.10
0.11
0.12
0.13
0.14
50
100
500
1000
5000
10000
m_SUSY(GeV)
10^15
10^16
10^17
10^18
10^19

Fig. 2

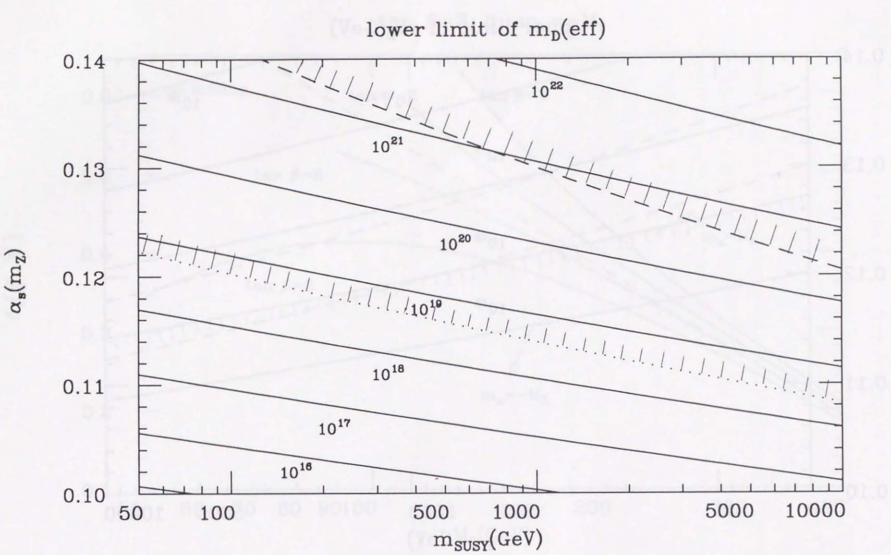

lower limit of m_D(eff)
$\alpha_s(m_Z)$
0.14
0.13
0.12
0.11
0.10
50
100
500
1000
5000
10000
m_{SUSY}(GeV)
10^{22}
10^{21}
10^{20}
10^{19}
10^{18}
10^{17}
10^{16}

Fig. 3

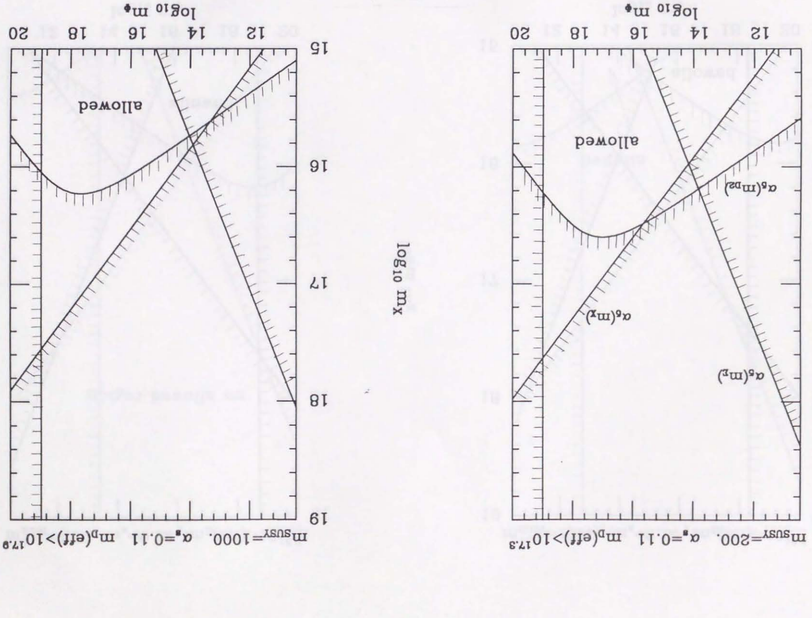
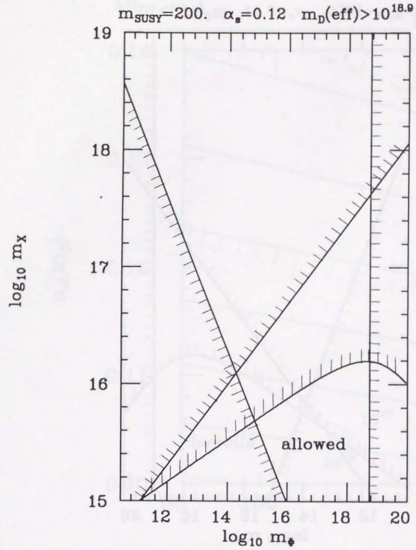

m_{SUSY}=200. α_s=0.11 m_D(eff)>$10^{17.3}$
m_{SUSY}=1000. α_s=0.11 m_D(eff)>$10^{17.9}$
$\log_{10} m_X$
$\log_{10} m_\Phi$
allowed
$\alpha_s(m_{D2})$
$\alpha_s(m_D)$
$\alpha_s(m_X)$

Fig.4



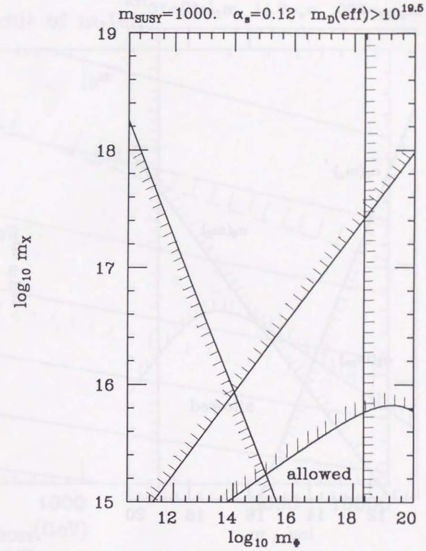
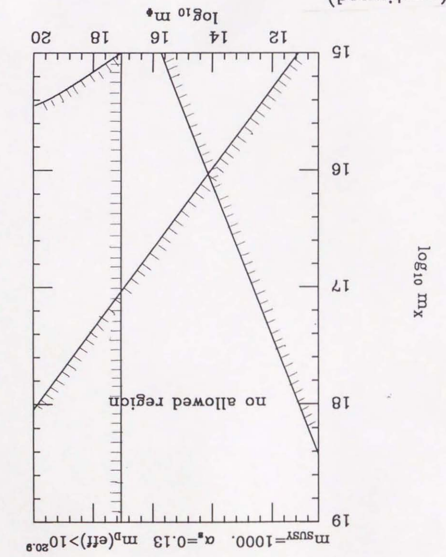


Fig.4 (continued)



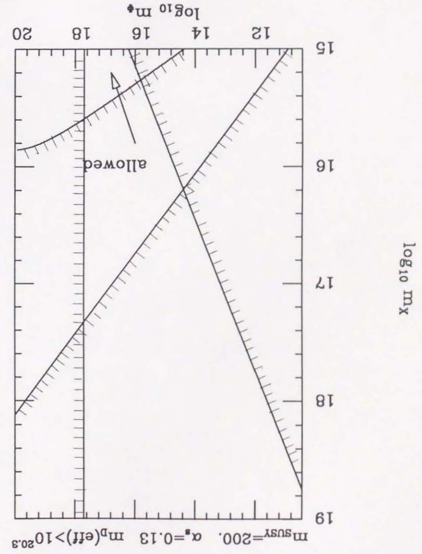


Fig.4 (continued)