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Q^2 evolution of gluon distribution function inside the hadrons with both shadowing and antishadowing effects

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Abstract. We deliberately attempt to solve the nonlinear Gribov-Levin-Ryskin-Mueller-Qiu,Zhu-Ruan-Shen (GLR-MQ-ZRS) equation in the leading order of QCD to study the Q^2 evolution of gluon distribution function $G(x,Q^2)$, Q^2 is the resolution scale or the four momentum transfered squared and x is the Bjorken variable. Both the shadowing and antishadowing corrections due to gluon recombination processes are incorporated in GLR-MQ-ZRS equation and while solving this equation we have used Regge like behavior in gluon distribution function. We present a phenomenological study on the collective contribution due to both the shadowing and antishadowing effects on the gluon distribution functions at small-x. Our solution of GLR-MQ-ZRS equation is valid in the vicinity of saturation at small-x, where gluon recombination effects cannot be ignored. We have shown comparison of our computed results for different values of strong coupling constant α_s with the PDFs of NNPDF3.0, PDF4LHC and CT14 groups

1. Introduction

Parton Distribution Functions (PDFs) serve as important and basic tools in understanding various standard model processes and in predictions of such processes at the accelerators in LHC, RHIC etc. Consequently, various groups like NNPDF [1], CT [2], MMHT [3], PDF4LHC [4] etc. are sincerely engaged in extracting and global fitting of PDFs. However, at very small momentum fraction x, among the partons, population of gluons become very high. This high population of gluons inside the hadrons lead to many nonlinear effects like saturation [5] and geometrical scaling [6] of the hadronic cross sections. So far, in the description of PDFs, the DGLAP equation [7], which is a linear QCD evolution equation has been extensively used with much phenomenological success. But, the solution of DGLAP equation towards very small-x predicts sharp growth of gluon densities. This would eventually violate (1) the unitarity of physical cross sections [8] and (2) the Froissart bound of the hadronic cross sections at high energies [9]. DGLAP equation has been modified by incorporating correlation among the initial partons and considering various recombination processes by Gribov, Levin, Ryskin (GLR) and later by Mueller, Qiu (MQ) into a new nonlinear equation known as the GLR-MQ equation [10–13]. However, there are certain issues that appear with the GLR-MQ equation: (1) the application of the AGK cutting rule in the GLR-MQ corrections breaks the evolution kernels [14]; (2) the nonlinear term in the GLR-MQ equation violate the momentum conservation [15]; (3) the Double Leading Logarithmic approximation (DLLA) is valid only at small x and the GLR-MQ corrections cannot smoothly connect with the DGLAP equation [16]. These motivations led Zhu and

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his cooperators to re-derive the QCD evolution equation into a new one known as the GLR-MQ-ZRS equation [14–19]. This ZRS version of the GLR-MQ equation consists of an antishadowing term in addition to the shadowing term as in the previous case. The contributions of two-partons-to-two-partons sub processes (i.e., parton recombination) were considered in deriving this equation. In this work, we will perform phenomenological study of this ZRS version of the evolution equation. We will obtain its solution using the Regge like behavior of gluons and then we will perform Q^2 evolution of gluon distribution function. Solutions of these nonlinear equations will play key roles in phenomenological study of saturation phenomena inside the hadrons, which has been already observed in the experiments.

The paper is organized as follows: In section (2), we developed the formalism to obtain Q^2 evolution of gluon distribution function $G(x, Q^2)$ from the GLR-MQ-ZRS equation. In section (3), we discuss our results and then we conclude in section (4).

2. Formalism

In terms of the gluon distribution function $G(x, Q^2)$, the GLR-MQ-ZRS equation at Double Leading Logarithmic Approximation (DLLA) is given by

$$\frac{dG(x,Q^2)}{dt}=\frac{\alpha_s(Q^2)N_c}{\pi}\int_x^1\frac{dx_1}{x_1}G(x_1,Q^2)+\frac{9}{2\pi}\cdot\frac{\alpha_s^2(Q^2)}{R^2Q^2}\cdot\frac{N_c^2}{N_c^2-1}\int_{x/2}^{1/2}\frac{dx_1}{x_1}G^2(x_1,Q^2)$$
$$-\frac{9}{2\pi}\cdot\frac{\alpha_s^2(Q^2)}{R^2Q^2}\cdot\frac{N_c^2}{N_c^2-1}\int_x^{1/2}\frac{dx_1}{x_1}G^2(x_1,Q^2) \qquad (1)$$

where N_C is the number of color charges and R is the correlative radius among interacting gluons. The right hand side (RHS) of equation (1) consists of the linear DGLAP evolution in the first term, the positive antishadowing correction in the second term and the negative shadowing correction in the third term respectively. The size of both the shadowing and antishadowing terms depends on the size of the correlative radius R. If the gluons are populated over the hadronic size then $R \approx 5\ \text{GeV}^{-1}$ and if the gluons are concentrated at hotspots then $R \approx 2\ \text{GeV}^{-1}$.

The small-x behavior of structure functions is so far well explained in terms of Regge-like behavior with much phenomenological success [20–26]. At high energy, for fixed Q^2, the total cross section rises with the increasing total CM energy squared s^2, where $s^2 = Q^2\ (1/x - 1)$ [27]. This high energy behavior of the total hadronic cross sections is reflected from the small-x behavior of the structure functions. In the theoretical description of this behavior, Regge pole exchange picture [28] appears to be quite appropriate. The leading order calculation in ln(1/x) with fixed α_s for moderate Q^2 reveals a steep power-law behavior in the x dependence of the parton densities $xg(x,Q^2)\sim x^{-\lambda_G}$, where $\lambda_G = (4N_c \ln 2)\alpha_s/\pi$ is the Regge intercept [29].

We, therefore, use this model of Regge pole exchange picture and the behavior of the gluon distribution function can be written as follows:

$$G(x,Q^2)=\chi(Q^2)x^{-\lambda_G}=\chi(Q^2)x^{(4N_c\ln2/\pi)\alpha_s} \qquad (2)$$

It is useful to define a variable $t = \ln(Q^2/\Lambda^2)$, Λ being the cutoff parameter. In terms of this variable t, equation (1) then becomes

$$\frac{dG(x,t)}{dt}=\frac{dG(x,t)}{dt}\Big|_{DGLAP}+\frac{9}{2\pi}\cdot\frac{\alpha_s^2(t)}{R^2\Lambda^2e^t}\cdot\frac{N_c^2}{N_c^2-1}\int_{x/2}^{1/2}\frac{dx_1}{x_1}G^2(x_1,t)$$
$$-\frac{9}{2\pi}\cdot\frac{\alpha_s^2(t)}{R^2\Lambda^2e^t}\cdot\frac{N_c^2}{N_c^2-1}\int_x^{1/2}\frac{dx_1}{x_1}G^2(x_1,t) \qquad (3)$$

In this work, we are only concerned with very small-x region, where the gluons are overpopulated and dominates over quarks inside the hadron. Therefore, we can effectively ignore the gluon contribution coming from the quarks. We will only keep the gluon-gluon (P_{gg}) AP kernels [30] in the evolution equation. The DGLAP equation at the leading order in QCD is then given by

$$\begin{aligned}\frac{dG(x,t)}{dt} = \frac{3\alpha_s(t)}{\pi}\Bigg[&\left(\frac{11}{12}-\frac{N_f}{18}+\ln(1-x)\right)G(x,t)\Bigg]\\ &+\int_x^1 dx_1\cdot\left(\left(\frac{x_1 G(x/x_1,t)-G(x,t)}{1-x_1}\right)\right)\\ \frac{+3\alpha_s(t)}{\pi}&\int_x^1 dx_1\left(x_1(1-x_1)+\frac{1-x_1}{x_1}\right)G(x/x_1,t),\end{aligned} \tag{4}$$

where $\alpha_s^{LO}/4\pi = 1/\beta_0 t$, $\beta_0 = 11-(2/3)N_f$. The number of flavors N_f is taken to be 4 in our calculations.

Using the Regge ansatz given in equation (2) and the DGLAP evolution given in equation (4), and on integrating explicitly, the equation (3) can be written in a simplified form as

$$\frac{dG(x,t)}{dt} = P(x,\alpha_s)G(x,t)+\frac{\kappa}{(8N_c\ln2/\pi)\alpha_s}\left[2^{(8N_c\ln2/\pi)\alpha_s}-1\right]\frac{G^2(x,t)}{t^2e^t}, \tag{5}$$

where the functions involved are

$$\begin{aligned}P(x,\alpha_s)=\frac{12}{\beta_0}\Bigg[&\frac{11}{12}-\frac{N_f}{18}+\ln(1-x)+\int_x^1 dx_1\frac{x_1^{(4N_c\ln2/\pi)\alpha_s+1}-1}{1-x_1}\Bigg]\\ &\left(+\int_x^1\left(x_1(1-x_1)+\frac{1-x_1}{x_1}\right)x_1^{(4N_c\ln2/\pi)\alpha_s}\,dx_1\right)\end{aligned} \tag{6}$$

and

$$\kappa = \frac{144\pi N_c^2}{N_c^2-1}\cdot\frac{1}{\beta_0^2R^2\Lambda^2} \tag{7}$$

Equation (5) can be solved appropriately for which the solution has the following form

$$G(x,t)=\frac{e^{P(x,\alpha_s)\ln t}}{C+\frac{\kappa}{(8N_c\ln2/\pi)\alpha_s}\left(2^{(8N_c\ln2/\pi)\alpha_s}-1\right)\Gamma[P(x,\alpha_s)-1,t]}, \tag{8}$$

where C is the constant of integration, it is to be R obtained using proper initial conditions. The incomplete gamma function is given by $\Gamma[a,x]=\int_x^\infty z^{a-1}e^{-z}dz.$

To obtain Q^2 evolution of gluon distribution function, we will consider a suitable input at a lower value $t = t_0$ (or $Q^2 = Q_0^2$). We will take the input G(x, t_0) from the PDFs of NNPDF3.0 group [1]. Thus, the t or Q^2 evolution of gluon distribution function for fixed values of x becomes

$$G(x,t)=\frac{e^{P(x,\alpha_s)\ln t}G(x,t_0)}{e^{P(x,\alpha_s)\ln t_0}+\kappa G(x,t_0)\frac{\left(2^{(8N_c\ln 2/\pi)\alpha_s}-1\right)}{(8N_c\ln 2/\pi)\alpha_s}\left(\Gamma[P(x,\alpha_s)-1,t]-\Gamma[P(x,\alpha_s)-1,t_0]\right)}, \quad (9)$$

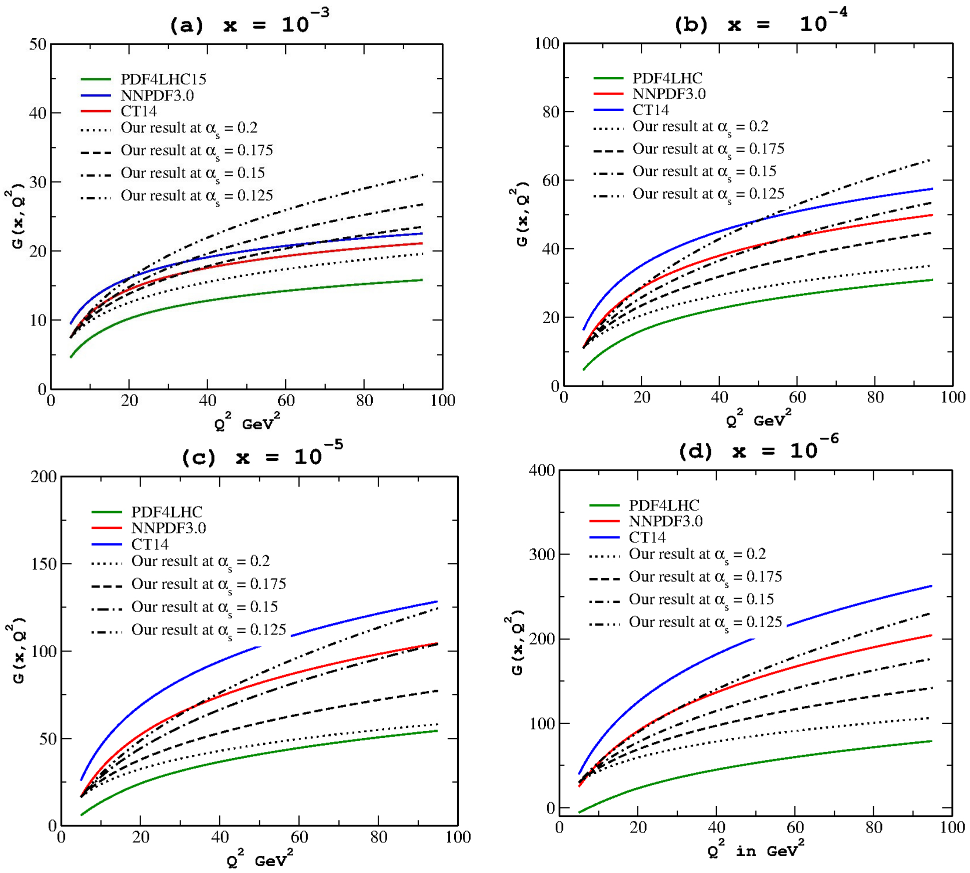

Figure 1. Q^2 -evolution of $G(x, Q^2)$ for different values of α_s viz., $\alpha_s = 0.125, 0.150, 0.175$ and 0.2 at four fixed values of small-x, viz., $x = 10^{-3}$, 10^{-4}, 10^{-5} and 10^{-6} respectively. Figure shows comparison of our results with that of the global fits by various groups. The correlative radius R is taken to be 5 GeV^{-1} and the cutoff parameter $\Lambda = 300$ *MeV*.

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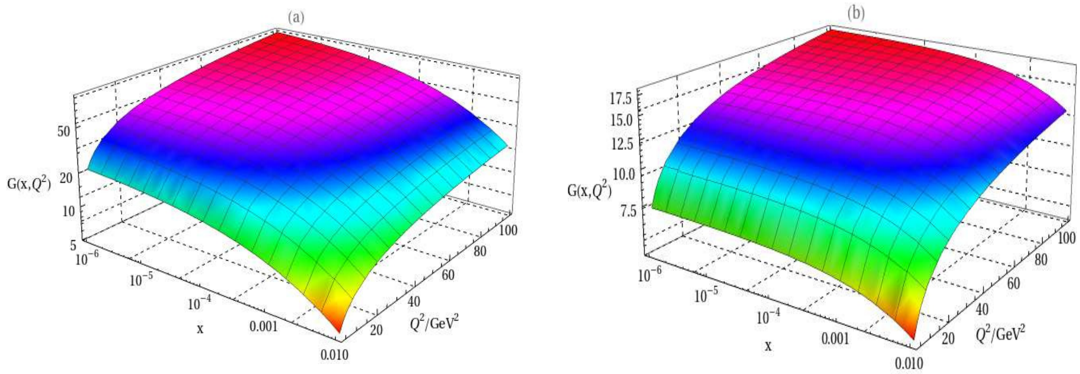

Figure 2. Evolution of $G(x,Q^2)$ in x and Q^2 space at (a) $\alpha_s = 0.15$ and (b) $\alpha_s = 0{:}2$.

3. Result and Discussion

In this paper, we have obtained the Q^2 evolution of gluon distribution function $G(x, Q^2)$ by solving the GLR-MQ-ZRS equation in leading order of QCD, using Regge like behavior of gluons. The kinematic range of our phenomenological study is $10^{-3} \geq x \geq 10^{-6}$ and $5 \leq Q^2 \leq 100$ GeV^2 respectively. Our solution is valid in the vicinity of saturation scale where the gluon density is very high and the nonlinear effects such as gluon recombination cannot be ignored. Fig. 1(a-d) represents the Q^2 evolution of $G(x, Q^2)$ from the solution in Eq. (9) at four small values of x, 10^{-3} , 10^{-4} , 10^{-5} and 10^{-6} respectively. The input gluon distribution is taken at an input value of Q_0^2= 5 GeV^2 from the PDFs of the NNPDF3.0 global PDF group. Our predicted evolution of $G(x, Q^2)$ is compared with the results of PDF4LHC [4] , NNPDF3.0 [1] and CT14 [2] global groups. We have used APFEL [31, 32] tool with the LHAPDF6 [33] library of evolution in order to extract the data of PDFs from these global groups. From the figure, we can observe that on increasing the value of α_s, the gluons evolve slowly with Q^2 . $G(x, Q^2)$ at $\alpha_s = 0.125$ rises steeper as compared to $G(x, Q^2)$ at $\alpha_s = 0.2$. On the other hand, we also notice that for smaller values of x, the evolution from our solutions are tamed down due to the nonlinear corrections. At very small value of $x = 10^{-6}$, the evolution from our solution at $\alpha_s = 0.125$ agrees well with the result of NNPDF3.0 in a quite broad range of Q^2 . Thus, it can be inferred that for smaller values of x, α_s has to be kept small.

In figure 2(a-b), a three dimensional representation of the evolution of gluon distribution function $G(x, Q^2)$ is shown with respect to x and Q^2 at (a) $\alpha_s = 0.15$ and (b) $\alpha_s = 0.2$. It can be clearly seen, $G(x, Q^2)$ rises higher at $\alpha_s = 0.15$ as compared to $G(x, Q^2)$ at $\alpha_s = 0.2$.

In figure 3(a-b), the sensitivity of the correlation radius R is checked on our results. We plot the Q^2 evolution of $G(x, Q^2)$ for two different values of R viz., 2 GeV^{-1} and 5 GeV^{-1} at (a) $\alpha_s = 0.125$ and (b) $\alpha_s = 0.2$ respectively. From the two figures 3(a) and 3(b), it can be observed that R is less sensitive to our results at $\alpha_s = 0.2$; but, a significant deviation of the results can be observed at $\alpha_s = 0.125$ as R is changed from 2 to 5 GeV^{-1}. We can also see that the sensitivity of R becomes more towards the smaller values of x; for instance in figure 3(a), at $x = 10^{-3}$ the deviation of the evolution is very insignificant whereas at $x = 10^{-6}$, the results deviate significantly from each other as R is changed from 2 to 5 GeV^{-1} .

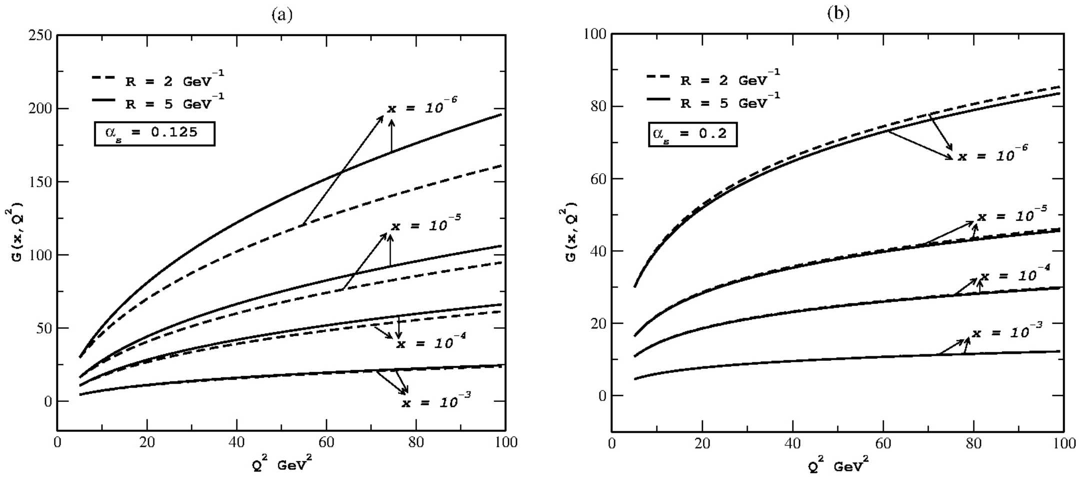

Figure 3. Q^2 -evolution of $G(x, Q^2)$ for different values of correlative radius R viz., 2 and 5 GeV^{-1} respectively. The evolution of $G(x, Q^2)$ is shown for four fixed values of x viz., 10^{-3}, 10^{-4}, 10^{-5} and 10^{-6} respectively at (a) $\alpha_s = 0.125$ and (b) $\alpha_s = 0.2$.

4. Conclusion

In conclusion, we have performed phenomenological analysis on the Q^2 evolution of gluon distribution function $G(x, Q^2)$ from the GLR-MQ-ZRS equation at leading order in QCD. Our results seem to be compatible with the PDFs of various global groups such as NNPDF3.0, CT14 and PDF4LHC. Therefore, we conclude that our solutions are valid in the kinematic range of $10^{-3} \geq x \geq 10^{-6}$ and $5 \leq Q^2 \leq 100$ GeV^2 respectively. We also conclude that, towards smaller values of x, α_s must be kept small and R becomes more sensitive.

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