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DERIVATION OF THE GRIBOV-LIPATOV RECIPROCITY RELATION IN A SCALING BOOTSTRAP MODEL*

G. Schierholz†

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

ABSTRACT

The Gribov-Lipatov reciprocity relation $\nu\overline{W}_2(\omega) = -\frac{1}{\omega^3}\,\nu W_2\left(\frac{1}{\omega}\right)$ is shown to be valid in a scaling bootstrap model under certain assumptions.

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† Max Kade Fellow. On leave of absence from II. Institut für Theoretische Physik der Universität, Hamburg, Germany.

Recently, Gribov and Lipatov[1] have calculated the structure functions of deep inelastic electroproduction and annihilation in a pseudoscalar — and vector — gluon model. One of their results is that, in a leading logarithmic approximation, the annihilation and electroproduction structure functions are directly related in their physical regions by the reciprocity relations

$$\overline{W}_1(\omega, q^2) = \frac{1}{\omega} W_1\left(\frac{1}{\omega}, -q^2\right) \tag{1}$$

$$\nu \overline{W}_2(\omega, q^2) = -\frac{1}{\omega^3} \nu W_2\left(\frac{1}{\omega}, -q^2\right). \tag{2}$$

The possibility of such relations is of great interest both experimentally and theoretically as it provides a connection between otherwise distinct physical situations.

Parisi[2] has shown that the reciprocity relations also hold in a $\lambda \phi^4$ theory with negative coupling constant, and Ferrara, Gatto and Parisi[3] state that the reciprocity relations follow directly from group-theoretical considerations employing the conformal group for any renormalizable theory.

Unfortunately, renormalizable theories as γ_5, γ_μ, ϕ^4, etc., violate Bjorken scaling. This raises the question if the reciprocity relations as suggested by renormalizable theories may still be valid in scaling models (such as superrenormalizable theories) which are of particular interest[4]. It is the aim of this note to show that the reciprocity relations can be obtained in a recently proposed bootstrap model[5]. In this model, scaling follows directly from the assumption of (charge) current algebra.

We shall restrict our discussion on the structure function νW_2. For pion targets νW_2 is given by (the generalization to include Regge cuts is straightforward)

$$\nu W_2^{(I)}(\omega) = \tfrac{1}{2}\,\delta^{(I)} \left\{ \sum_{\substack{i \\ \alpha_i^{(I)} > L}} \gamma_i^{(I)}\,\omega^{\alpha_i^{(I)}-1} + \mathrm{Im}\left[\frac{1}{2\pi i} \int_{L-i\infty}^{L+i\infty} \frac{d\ell}{\sin\pi\ell}\, \frac{\omega^{\ell-1}}{D_\ell^{(I)}(0)} - (\omega \leftrightarrow -\omega) \right] \right\} \tag{3}$$

where I labels the isospin in the t-channel ($\delta^{(0)} = 2$, $\delta^{(2)} = -1$) and $D_\ell^{(I)}(0)$ is the denominator function of the (elastic) $\pi\pi$ scattering amplitude at $t = 0$. As is easily seen, νW_2 is analytic except for a cut along the negative real axis (i.e., $-\infty < \omega \leq 0$) so that it can be analytically continued to the physical region of the annihilation channel (i.e., $0 \leq \omega \leq 1$). In the following we shall only report on the case where the annihilation structure function is an analytic continuation in ω of the inelastic scattering function, i.e.

$$\nu \overline{W}_2(\omega) = \nu W_2(\omega) \; . \tag{4}$$

Generally, this need not be true as will be discussed in a subsequent paper. However, if Eq. (4) does not hold, the reciprocity relations (1), (2) are not expected to be valid since they are due to the very same property of the electroproduction structure functions themselves as we shall see. This allows us to separate the discussion of the reciprocity relations from the continuation program.

Now, let us assume that

$$D_\ell^{(I)}(0) = D_{-\ell-1}^{(I)}(0) \tag{5}$$

in the right half-plane $\mathrm{Re}\,\ell \geq -2$. Then νW_2 can be rewritten in the form[6]

$$\nu W_2^{(I)}(\omega) = \tfrac{1}{4}\,\delta^{(I)} \left\{ \sum_{\substack{i \\ -\frac{1}{2} \leq \alpha_i^{(I)} \leq 1}} \gamma_i^{(I)} \left(\omega^{\alpha_i^{(I)} - 1} - \omega^{-\alpha_i^{(I)} - 2} \right) + \mathrm{Im} \left[\frac{1}{2\pi i} \int_{-2-i\infty}^{-2+i\infty} \frac{d\ell}{\sin \pi \ell} \, \frac{\omega^{\ell-1} + \omega^{-\ell-2}}{D_\ell^{(I)}(0)} - (\omega \longleftrightarrow -\omega) \right] \right\} \tag{6}$$

which explicitly satisfies

$$\nu W_2(\omega) = -\frac{1}{\omega^3}\,\nu W_2\!\left(\frac{1}{\omega}\right) . \tag{7}$$

This is still valid if $D_\ell^{(I)}(0)$ involves branch points. In that case terms like

$$\frac{1}{2\pi i} \int_{\alpha_C^{(I)}}^{-\alpha_C^{(I)}-1} \frac{d\ell}{\sin \pi \ell}\, \omega^{\ell-1}\, \mathrm{Disc}\left[\frac{1}{D_\ell^{(I)}(0)} \right] - (\omega \longleftrightarrow -\omega) \tag{8}$$

have to be included in the square brackets of Eq. (6) corresponding to various Regge cuts.

It is evident that $D_\ell^{(I)}(0)$ will be symmetric under $\ell \to -\ell - 1$ if this is the case for the pion partial-wave amplitude[7] $f_\ell^{(I)}(t)$. For relativistic Coulomb scattering (the only case which has been solved in a closed form with this respect) it has been shown that $f_\ell^{(I)}(t)$ obeys this symmetry.[8] In the general case we write the pion scattering amplitude in the form (omitting isospin and signature indices)

$$A(s,t=0) = -\frac{1}{2i}\int_{1-i\infty}^{1+i\infty}\frac{d\lambda}{\sin\pi\lambda}(2\lambda+1)f_\lambda(0)P_\lambda(-z_{t=0}) \ . \tag{9}$$

By employing the Froissart-Gribov definition of the partial-wave amplitude

$$f_\ell(t) = \frac{1}{2}\int_{-1}^{+1} dz\, P_\ell(z)\,A(s,t) - \sin\pi\ell\,\frac{1}{\pi}\int_{-\infty}^{-1} dz\, Q_\ell(-z)\,A(s,t) \ , \tag{10}$$

Eq. (9) leads to the following expression for $f_\ell(0)$:

$$f_\ell(0) = \frac{1}{2\pi i}\int_{1-i\infty}^{1+i\infty} d\lambda\,(2\lambda+1)\,\frac{f_\lambda(0)}{(\ell-\lambda)(\ell+\lambda+1)} \ , \quad \mathrm{Re}\,\ell > 1 \ . \tag{11}$$

It should be remarked that the Froissart-Gribov projection (10) is only justified upon the assumption of a Mandelstam representation for $A(s,t)$ which we shall postulate here. Now assume that $f_\lambda(0)$ is analytic in the right half-plane $\mathrm{Re}\,\lambda \geq -2$ except for isolated poles. Eq. (11) can then be written

$$f_\ell(0) = \sum_{\substack{i \\ \alpha_i > -2}} \frac{(2\alpha_i+1)\beta_i}{(\ell-\alpha_i)(\ell+\alpha_i+1)} + \frac{1}{2\pi i}\int_{-2-i\infty}^{-2+i\infty} d\lambda\,(2\lambda+1)\,\frac{f_\lambda(0)}{(\ell-\lambda)(\ell+\lambda+1)} \tag{12}$$

so that $f_\ell(0) = f_{-\ell-1}(0)$ is explicitly fulfilled for $\mathrm{Re}\,\ell \geq -2$. This is even true in the more general situation where f_λ has branch points. In that case the background integral in Eq. (12) will enclose a number of branch cuts but allows the same conclusion.

We conclude that the reciprocity relation (7) holds if the pion scattering amplitude can be described in terms of Regge poles and branch cuts down to Re $\ell \geq -2$ which seems not to be an unreasonable assumption.

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