

Tidal Love Numbers of Fermionic Dark Stars

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Abstract. Dark star is a hypothetical stellar compact object composed of dark particles which clumped together due to self-interaction among dark particles. Tidal Love numbers from binary stars are potential observables for studying the internal structure of the stars. The Tidal Love numbers can be deduced from the gravitational wave of the coalescence or inspiral of binary stars. The ground base interferometer can measure the corresponding gravitational wave. In this work, we study the equation of state, mass-radius relation, electric, magnetic and surficial tidal love numbers of dark stars. We obtain that Tidal Love numbers are not too sensitive to study the role of scalar boson exchange and the impact of the mass of the dark particle in dark matter equation of state.

1. Introduction

It is recently reported that cold collisionless dark matter (CCDM) as the main candidate of dark matter is not too compatible with some astrophysical observations. There are three main problems of CCDM paradigm [1, 2]. The flat density core in dwarf galaxies observations is contradicted with numerical simulation result from CCDM. CCDM also predicts a larger number of satellite galaxies in the Milky Way that have been observed so far. CCDM also predicts massive dwarf galaxies that are too big not to be observed.

Recent discussions have shown that dark matter (DM) with self-interaction can explain not only diverse galactic rotational curves [3] but also can resolve CCDM problems in appropriate range of DM self-interaction strength i.e., in the range of the ratio of total cross section σ to dark particle mass m between [1, 2] $0.1 \text{ cm}^2/\text{g} < \sigma/m < 10 \text{ cm}^2/\text{g}$. The consequence of the existence of DM composed by self-interaction dark particles is the corresponding dark particles may clump together to form a compact object called a dark star. It is also reported that dark stars are not compact enough to mimic black holes [2]. Therefore, dark stars can be distinguishable from black holes when we observe the dark sector of the universe.

The new era of astronomical observations has begun since observations of the binary black hole coalescence GW170814 [4] and the binary neutron star inspiral GW170817 [5] by LIGO and Virgo collaborations. Tidal effect of binary stars is measurable during the early part of evolution before coalescence because the gravitation waveform in this stage is relatively clean [6, 7]. Several studies have shown that the measurement of tidal deformation can be used to extract the information about internal structure of neutron stars [8, 9, 10, 11, 12, 13, 14].

If dark stars exist, the study of the tidal deformation of binary dark stars may shade a light to provide more understanding of the nature of DM. In this paper we study electric tidal love



number k_l and surficial tidal love number h_l for $l = 2, 3, 4$, while for magnetic tidal love number j_2 only for $l = 2$ predicted by fermionic dark stars.

2. Formalism

To describe fermionic DM, we use Lagrangian self-interacting fermionic dark particle where ψ , ϕ , V^μ denotes to dark particle, scalar boson and vector boson fields, respectively and m , m_s , m_v denote their masses with coupling strength g_s for scalar boson and g_v for vector boson [15, 17, 18].

$$\mathcal{L} = \bar{\psi}[\gamma_\mu(i\partial^\mu - g_v V^\mu) - (m - g_s\phi)]\psi + \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2) - \frac{1}{4}V^{\mu\nu}V_{\mu\nu} + \frac{1}{2}m_v^2V^\mu V_\mu \quad (1)$$

where $V^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$. Using standard relativistic mean field (RMF) approximation, we can obtain equation of motion as

$$[(i\gamma_\mu\partial^\mu - m) + g_s\phi_0 + g_v V_0\gamma_0]\psi = 0, \quad (2)$$

$$m_s^2\phi_0 = g_s\langle\bar{\psi}\psi\rangle, \quad (3)$$

$$m_v^2 V_0 = g_v\langle\bar{\psi}\gamma^0\psi\rangle, \quad (4)$$

and for energy-momentum tensor is given by

$$T_{RMF}^{\mu\nu} = \bar{\psi}i\gamma^\mu\partial^\nu\psi - \eta^{\mu\nu}\left[-\frac{1}{2}m_s^2\phi_0^2 + \frac{1}{2}m_v^2V_0^2\right]. \quad (5)$$

From the later equation, we can further obtain energy density as

$$\epsilon = \langle T^{00} \rangle = \langle \bar{\psi}i\gamma^0\partial^0\psi \rangle + \frac{1}{2}m_s^2\phi_0^2 - \frac{1}{2}m_v^2V_0^2, \quad (6)$$

and pressure as

$$P = \frac{1}{3}\langle T^{ii} \rangle = \frac{1}{3}\langle \bar{\psi}i\gamma^i\partial^i\psi \rangle - \frac{1}{2}m_s^2\phi_0^2 + \frac{1}{2}m_v^2V_0^2. \quad (7)$$

The corresponding expectation values can be also easily calculated and the results are [15, 18]

$$\langle \bar{\psi}i\gamma^0\partial^0\psi \rangle = g_v V_0 \rho + \frac{\gamma}{(2\pi)^3} \int_0^{k_F} d\mathbf{k} \sqrt{\mathbf{k}^2 + m^{*2}}, \quad (8)$$

$$\langle \bar{\psi}i\gamma^i\partial^i\psi \rangle = \frac{\gamma}{(2\pi)^3} \int_0^{k_F} d\mathbf{k} \frac{\mathbf{k}^2}{\sqrt{\mathbf{k}^2 + m^{*2}}}, \quad (9)$$

$$\langle \bar{\psi}\psi \rangle = \frac{\gamma}{2\pi^2} \int_0^{k_F} dk \frac{k^2 m^*}{\sqrt{k^2 + m^{*2}}}, \quad (10)$$

and

$$\begin{aligned} \langle \bar{\psi}\gamma^0\psi \rangle &= \langle \psi^\dagger\psi \rangle = \sum_s \frac{1}{(2\pi)^3} \int_0^{k_F} d^3k \\ &= \frac{\gamma}{6\pi^2} k_F^3 \equiv \rho, \end{aligned} \quad (11)$$

where $m^* = m - g_s\phi$. Therefore, the boson fields can be expressed as

$$\phi_0 = \frac{g_s}{m_s^2} \langle \bar{\psi}\psi \rangle = \frac{g_s}{m_s^2} \frac{\gamma}{2\pi^2} \int_0^{k_F} dk \frac{k^2 m^*}{\sqrt{k^2 + m^{*2}}}, \quad (12)$$

$$V_0 = \frac{g_v}{m_v^2} \langle \psi^\dagger\psi \rangle = \frac{g_v}{m_v^2} \rho, \quad (13)$$

Table 1. Parameters used in this work to generate the DM EOS. $m_v = m_s \equiv 10$ MeV

EOS	m	α_v	α_s
I	1 GeV	3.0×10^{-3}	2.0×10^{-3}
II		0.7×10^{-3}	0.3×10^{-3}
III	2 GeV	3.0×10^{-3}	2.0×10^{-3}
IV		0.7×10^{-3}	0.3×10^{-3}

$$m^* = m - \frac{g_s^2}{m_s^2} \int_0^{k_F} dk \frac{k^2 m^*}{\sqrt{k^2 + m^{*2}}}, \quad (14)$$

$$\epsilon = \frac{1}{2} m_s^2 \phi_0^2 + \frac{1}{2} m_v^2 V_0^2 + \frac{\gamma}{2\pi^2} \int_0^{k_F} dk k^2 \sqrt{k^2 + m^{*2}}, \quad (15)$$

$$P = -\frac{1}{2} m_s^2 \phi_0^2 + \frac{1}{2} m_v^2 V_0^2 + \frac{1}{3} \left(\frac{\gamma}{2\pi^2} \int_0^{k_F} dk \frac{k^4}{\sqrt{k^2 + m^{*2}}} \right). \quad (16)$$

After solving those integrals, Eqs. (12-16) can be written in more compact forms. The corresponding final expressions for effective mass, energy density and pressure are

$$m^* = m - \frac{g_s^2}{m_s^2} \frac{m^{*3} x^3}{2\pi^2} \chi(x), \quad (17)$$

$$\epsilon = m^{*4} \xi(x) + \left[\frac{2}{9\pi^3} m^{*6} x^6 \right] \left[\frac{\alpha_v}{m_v^2} + \frac{9}{4} \frac{\alpha_s}{m_s^2} (\chi(x))^2 \right], \quad (18)$$

$$P = m^{*4} \Psi(x) + \left[\frac{2}{9\pi^3} m^{*6} x^6 \right] \left[\frac{\alpha_v}{m_v^2} - \frac{9}{4} \frac{\alpha_s}{m_s^2} (\chi(x))^2 \right]. \quad (19)$$

where $\alpha_i \equiv g_i^2/4\pi$, and exact form of those integrals are

$$\begin{aligned} \chi(x) &\equiv \frac{1}{x^3} \left[x\sqrt{x^2+1} - \ln(x + \sqrt{x^2+1}) \right], \\ \xi(x) &\equiv \frac{1}{8\pi^2} \left[x\sqrt{x^2+1} (2x^2+1) - \ln(x + \sqrt{x^2+1}) \right], \\ \Psi(x) &\equiv \frac{1}{8\pi^2} \left[x\sqrt{x^2+1} \left(\frac{2}{3}x^2 - 1 \right) + \ln(x + \sqrt{x^2+1}) \right], \end{aligned} \quad (20)$$

Note that $\gamma = 2$ for spin degeneracy factor and $x = k_F/m^*$. Eqs. (17-20) are used to generate equation of state (EOS) as parametric function of x . In calculation, we use parameters shown in table 1. Those parameters yield ratios of cross section to dark particle mass which are still inside the $0.1 \text{ cm}^2/\text{g} < \sigma/m < 10 \text{ cm}^2/\text{g}$ constraint. The EOS results are shown in figure 1.

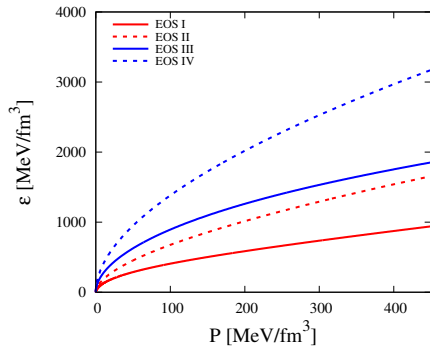


Figure 1. EOSs for each parameter set used in this work.

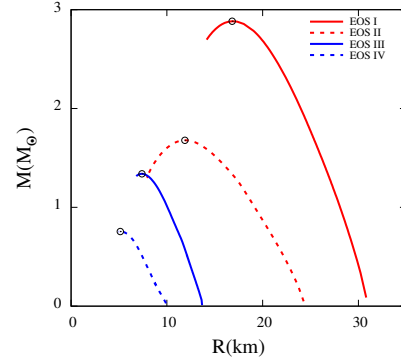


Figure 2. Mass-Radius relation for each EOS used in this work. M_{\odot} is solar mass.

3. Mass and Radius

Mass and radius are macroscopic observable that characterize astrophysical compact object. We can calculate them by solving Tolman-Oppenheimer-Volkoff(TOV) equations bellow

$$\frac{dP(r)}{dr} = -\frac{GM(r)\epsilon(r)}{r^2} \left(1 + \frac{P(r)}{\epsilon(r)}\right) \left(1 + \frac{4\pi r^3 P(r)}{M(r)}\right) \left(1 - \frac{2GM(r)}{r}\right)^{-1}, \quad (21)$$

$$\frac{dM(r)}{dr} = 4\pi\epsilon(r)r^2, \quad (22)$$

where $M(r)$ denotes the profile of a stellar mass, r denotes radius, G is gravitational constant. These equations need EOS as an input. In this work, we plot $\epsilon(r)$ and $P(r)$ from Eqs. (18) and (19) as parametric data so we get the plot line in figure 1. To make our numerical problem easier, we can approximate exact function of $\epsilon(P)$ with some polinomial using fitting method. This method is more efficient rather than call $\epsilon(x)$ and $P(x)$ for each iteration which is contains m^* as a self consistent function that need a lot of process to get the result. So TOV equations become

$$\frac{dP(r)}{dr} = -\frac{GM(r)\epsilon(P(r))}{r^2} \left(1 + \frac{P(r)}{\epsilon(P(r))}\right) \left(1 + \frac{4\pi r^3 P(r)}{M(r)}\right) \left(1 - \frac{2GM(r)}{r}\right)^{-1}, \quad (23)$$

$$\frac{dM(r)}{dr} = 4\pi\epsilon(P(r))r^2, \quad (24)$$

For solving TOV equations numerically, Runge-Kutta method is used which is require initial values in the center of the stars. We use $P(0) = P_c$ and $M(0) \approx 0$ as pressure and mass at the center of the star. We don't know the exact value of P_c so we variate that. We determine the star radius R when $P(R) \approx 0$ (Pressure in the surface of the stars approach zero), and in this point we obtain the star total mass $M = M(R)$ and radius R of the stars. As we mention earlier, we variate P_c until maximum mass is reached. After reaching maximum mass, larger P_c value tends to lower the star masses. In this region, the star is unstable. The mass-radius results are shown in figure 2.

4. Tidal Love Numbers

There is a possibility to extract information of the internal structure of astronomical compact stars via gravitational wave through tidal deformation properties of the stars in inspiral binary

stars during coalescence. Tidal deformability is a response parameter of mass quadrupole moment of a star Q_{ij} which is affected by external gravitational field from another star (tidal field) \mathcal{E}_{ij} of binary stars.

$$Q_{ij} = -\lambda \mathcal{E}_{ij}, \quad (25)$$

where parameter λ depends on the EOS via a relation

$$\lambda = \frac{2}{3} k_2 R^5 \quad (26)$$

where k_2 are dimensionless quantity called love number for $l = 2$. This effect becomes important over time because the orbit radius of binary stars decrease gradually due to energy loss due to gravitational waves. When each star becomes closer to each other, gravitational attraction becomes stronger and stars will be deformed. So quantity k_2 so on for higher order k_3 and k_4 provide important information which is obtained from gravitational wave [16].

To calculate love numbers k_l for ($l = 2, 3, 4$), we solve differential equation below [16]

$$r \frac{dy_l(r)}{dr} + y_l(r)^2 + y_l(r)F(r) + r^2 Q_l(r) = 0, \quad (27)$$

here $l = 2, 3, 4, \dots$. Note that the explicit form of $F(r)$ and $Q_l(r)$ is

$$F(r) = 1 - 4\pi r^2 G(\epsilon(r) - P(r)) \left(1 - 2 \frac{GM(r)}{r}\right)^{-1}, \quad (28)$$

$$\begin{aligned} Q_l(r) = & \left[4\pi r^2 G \left(5\epsilon(r) + 9P(r) + \frac{\epsilon(r) + P(r)}{(\partial P / \partial \epsilon)(r)} \right) - l(l+1) \right] \left(1 - 2 \frac{GM(r)}{r}\right)^{-1} \\ & - 4 \left(\frac{GM(r)}{r} \right)^2 \left(1 + \frac{4\pi r^3 P}{M(r)} \right)^2 \left(1 - 2 \frac{GM(r)}{r}\right)^{-2}. \end{aligned} \quad (29)$$

Because those equation need variable with dependence of r , we should solve equation (27) and TOV equation simultaneously with Runge Kutta method. Initial value $y_l(0) = l$ where $l = 2, 3, 4$ is used.

When we know the value of $y_l = y_l(R)$, we can calculate k_l from following equations

$$\begin{aligned} k_2 = \frac{8}{5} (1 - 2C)^2 C^5 [2C(y_2 - 1) - y_2 + 2] & \left\{ 2C(4(y_2 + 1)C^4 + (6y_2 - 4)C^3 + (26 - 22y_2)C^2 \right. \\ & \left. + 3(5y_2 - 8)C - 3y_2 + 6) - 3(1 - 2C)^2 (2C(y_2 - 1) - y_2 + 2) \ln \left(\frac{1}{1 - 2C} \right) \right\}^{-1} \end{aligned} \quad (30)$$

$$\begin{aligned} k_3 = \frac{8}{7} (1 - 2C)^2 C^7 [2(y_3 - 1)C^2 - 3(y_3 - 2)C + y_3 - 3] & \left\{ 2C[4(y_3 + 1)C^5 + 2(9y_3 - 2)C^4 \right. \\ & - 20(7y_3 - 9)C^3 + 5(37y_3 - 72)C^2 - 45(2y_3 - 5)C + 15(y_3 - 3)] - 15(1 - 2C)^2 \\ & \left. \times (2(y_3 - 1)C^2 - 3(y_3 - 2)C + y_3 - 3) \ln \left(\frac{1}{1 - 2C} \right) \right\}^{-1} \end{aligned} \quad (31)$$

$$\begin{aligned} k_4 = \frac{32}{147} (1 - 2C)^2 C^9 [12(y_4 - 1)C^3 - 34(y_4 - 2)C^2 + 28(y_4 - 3)C - 7(y_4 - 4)] & \\ \times \left\{ 2C[8(y_4 + 1)C^6 + (68y_4 - 8)C^5 + (1284 - 996y_4)C^4 + 40(55y_4 - 116)C^3 \right. & \\ + (5360 - 1910y_4)C^2 + 105(7y_4 - 24)C - 105(y_4 - 4)] - 15(1 - 2C)^2 & \\ \times [12(y_4 - 1)C^3 - 34(y_4 - 2)C^2 + 28(y_4 - 3)C - 7(y_4 - 4)] \ln \left(\frac{1}{1 - 2C} \right) & \left. \right\}^{-1}, \end{aligned} \quad (32)$$

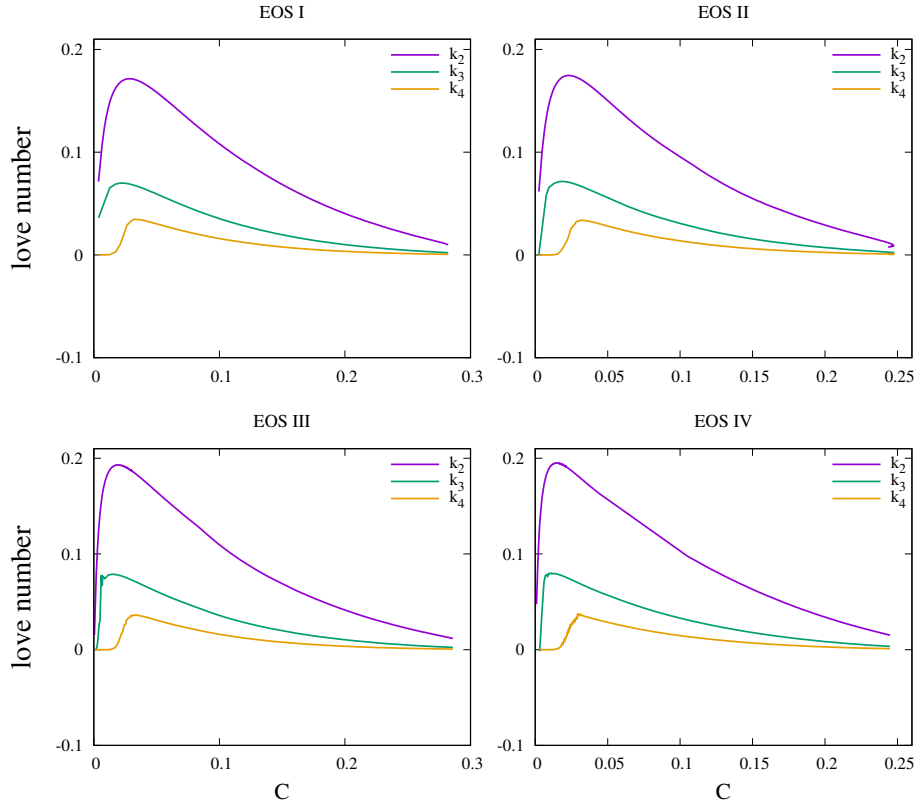


Figure 3. Electric Love numbers for each EOS used.

where $C = GM/R$ is a compactness. We calculate the surfical love number h_l which describe deformation of the body's surface in a multipole expansion. For a perfect fluid, the relation between surfical love number h_l and tidal love number k_l is given by [16]

$$h_l = \Gamma_1 + 2\Gamma_2 k_l, \quad (33)$$

where

$$\Gamma_1 = \frac{l+1}{l-1}(1-C)F(-l, -l, -2l; 2C) - \frac{2}{l-1}F(-l, -l-1, -2l; 2C) \quad (34)$$

$$\Gamma_2 = \frac{l}{l+2}(1-C)F(l+1, l+1, 2l+2; 2C) + \frac{2}{l+2}F(l+1, l, 2l+2; 2C). \quad (35)$$

Here $F(a, b, c; z)$ is the hyper-geometric function. We also calculate the magnetic tidal love number j_l , but only for $l = 2$ case with the final expression is given as [16]

$$j_2 = \{96C^5(2C-1)(y_2-3)\} \left\{ 5(2C(12(y_2+1)C^4 + 2(y_2-3)C^3 + 2(y_2-3)C^2 + 3(y_2-3)C - 3y_2 + 9) + 3(2C-1)(y_2-3)\ln(1-2C)) \right\}^{-1}. \quad (36)$$

5. Result and Discussion

Solving Eqs.(17-20) by using the parameter sets shown in table 1, we can obtain the EOSs which are shown in figure 1. It can be observed that EOS IV and EOS II are stiffer than those of EOS

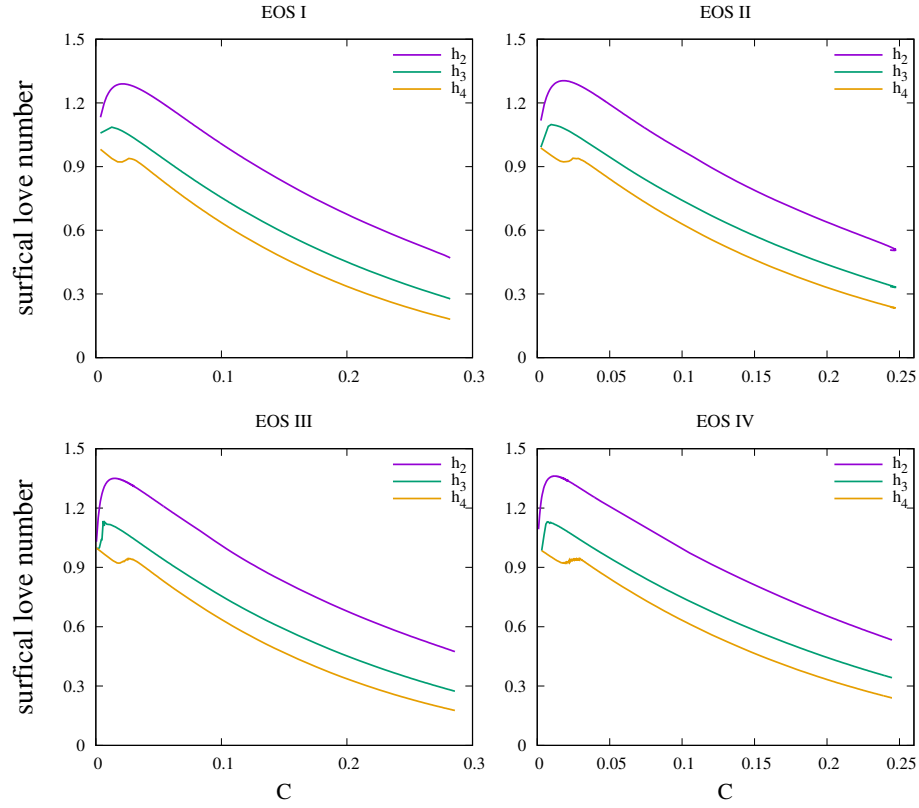


Figure 4. Surficial Love numbers for each EOS used.

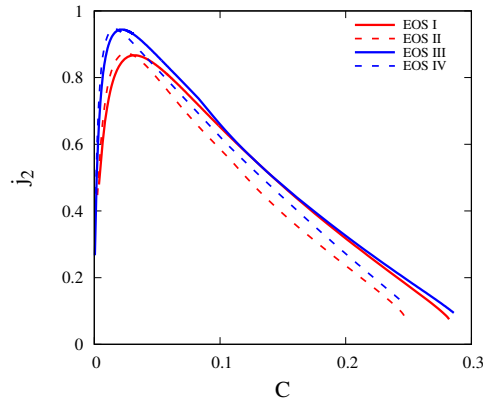


Figure 5. Magnetic tidal Love number for each EOS used.

III and EOS I at high densities but they have the same behavior at low densities. It can be also seen that for heavier DM mass, the EOS tends to be softer than that of small DM mass. The mass-radius relation results of the corresponding EOSs are shown in figure 2. It is obvious that the maximum mass reduces significantly if the DM mass increases and the EOS is softened.

We solve Eqs.(27-29) alongside with TOV equations to obtain the electric love number k_l

for ($l = 2, 3, 4$). $l = 2, 3, 4$ represent quadrupole, octupole and hexadecapole deformations, respectively. In figure 3, we show the electric tidal Love numbers predicted by the EOSs in table 1. For heavier dark particle mass k_2 peak tends to shift slightly to the larger compactness and the k_2 peak magnitude becomes higher. This result can be understood because if the EOS is softer, the star can deform more easily. Note that the stiffness of the EOS can be tuned by increasing or decreasing dark particle mass or by adjusting the boson coupling constants. We also can see that for larger compactness, all k_l tend to go zero as compactness is closer the compactness of black hole $C = 0.5$. We can also see this similar pattern for higher order Love numbers.

Using Eq. (33) we can also calculate the surficial Love numbers where the results are shown in figure 4. It can be observed that similar to the ones of electric tidal Love numbers, the difference in EOS is not also significantly reflected in surficial Love numbers.

It can be seen in figure 5, for magnetic tidal love numbers, the peak shift depends strongly to the dark particle mass. However, at relative high compactness region, the magnetic tidal Love numbers with the same coupling constant strength tend to coincide one another.

6. Conclusion

In this work, we study the effect of dark particle mass and scalar boson coupling exchange to the EOS, mass-radius relation, and tidal Love numbers properties of fermionic dark stars. We calculate and discuss the electric tidal Love numbers, surficial tidal Love numbers, and magnetic tidal Love numbers. We obtain that tidal love numbers are not too sensitive in respect to the differences of the used EOSs. Therefore, it seems that they are not quite sensitive to investigate the dark particle mass and scalar boson coupling of fermionic dark stars.

7. Acknowledgments

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