



利用格点量子色动力学研究手征平滑过渡温度和手征相变温度

李胜泰 丁亨通

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LI Shengtai, DING Hengtong

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Chiral Crossover and Chiral Phase Transition Temperatures from Lattice QCD

LI Shengtai¹, DING Hengtong^{2,†}

(1. Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China;

2. Key Laboratory of Quark & Lepton Physics (MOE) and Institute of Particle Physics,
Central China Normal University, Wuhan 430079, China)

Abstract: We review our recent studies on chiral crossover and chiral phase transition temperatures in this special issue. We will firstly present a lattice QCD based determination of the chiral crossover transition temperature at zero and nonzero baryon chemical potential μ_B which is $T_{pc} = (156.5 \pm 1.5)$ MeV. At nonzero temperature the curvatures of the chiral crossover transition line are $\kappa_2^B = 0.012(4)$ and $\kappa_4^B = 0.000(4)$ for the 2nd and 4th order of μ_B/T . We will then present a first determination of chiral phase transition temperature in QCD with two degenerate, massless quarks and a physical strange quark. After thermodynamic, continuum and chiral extrapolations we find the chiral phase transition temperature $T_c^0 = 132^{+3}_{-6}$ MeV.

Key words: chiral crossover temperature; chiral phase transition temperature; critical end point; lattice QCD

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1 Introduction

One of ultimate goals of lattice QCD calculations is to map out the QCD phase diagram^[1]. The current understanding of the QCD phase structure at zero baryon chemical potential is sketched in the so-called Columbia plot (see Fig. 1). The horizontal axis of Fig. 1 is the mass of degenerate up and down quarks, while the vertical axis is the mass of the strange quark. The physical point ($m_{u,d}^{\text{phy}}, m_s^{\text{phy}}$) corresponding to a physical pion mass of 140 MeV has been confirmed to possess a crossover type transition but not a true phase transition^[2]. As the strange quark mass keeps its physical quark mass value m_s^{phy} and the light quark masses move towards zero, the chiral phase transition is supposed to become a true phase transition^[3]. Based on recent studies, the chiral phase transition in the chiral limit of light quarks in $N_f=2+1$ QCD is expected to be a second order phase transition belonging to an O(4) universality class^[4-5]. Since the chiral phase transition temperature T_c^0 is a fundamental quantity of QCD and QCD-inspired model calculations predict

that T_c^0 is about 25 MeV smaller than the pseudo-critical temperature T_{pc} at the physical point^[6-7], the determination of T_c^0 is thus important. The chiral phase transition temperature is also relevant to the search for the elusive QCD critical end point (CEP). As seen from Fig. 2, the 2nd order O(4) phase transition in the chiral limit of light quarks will terminate at a tri-critical point ($\mu_B^{\text{tri}}, T_c^{\text{tri}}$) at a sufficiently large baryon chemical potential. It has been found that in the chiral limit the curvature of the transition line is negative up to the 2nd order of μ_B ^[8-9], which suggests $T_c^0 > T_c^{\text{tri}}$. While on the other hand, the crossover transition line at the physical quark mass will end at a critical end point which possesses a 2nd order phase transition belonging to a Z(2) universality class. This critical end point is what people are looking for and is connected to the tri-critical point via a transition line belonging to the Z(2) universality class. The tri-critical temperature T_c^{tri} is expected larger than the critical end point temperature T_c^{CEP} from model studies, *i.e.* $T_c^{\text{tri}} - T_c^{\text{CEP}}(m_q) \propto m_q^{2/5}$ ^[10-12]. So the chiral phase transition T_c^0 can be regarded as an upper bound of

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Biography: LI Shengtai(1990-), male, Dandong, Liaoning, currently working as a postdoc in the Institute of Modern Physics, CAS on lattice QCD at nonzero temperature and density; E-mail: stli@impcas.ac.cn

† Corresponding author: DING Hengtong, E-mail: hengtong.ding@mail.ccnu.edu.cn.

the critical temperature T_c^{CEP} at the CEP. The determination of T_c^0 is thus helpful to constrain the location of the CEP.

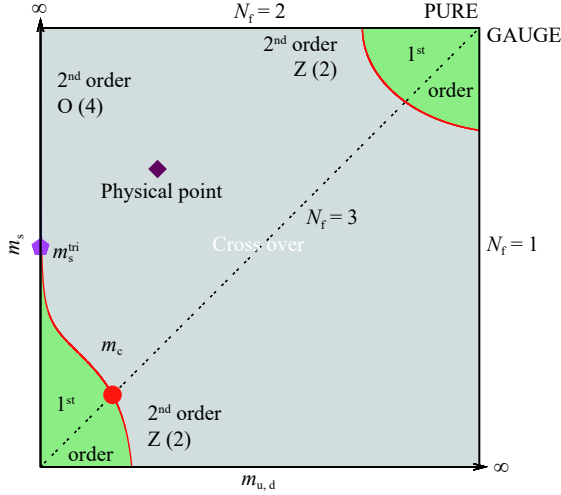


Fig. 1 (color online) QCD phase structure in the mass quark plane^[1].

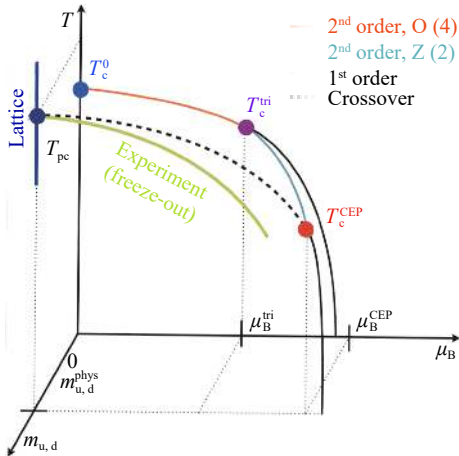


Fig. 2 (color online) QCD phase structure in the 3-D plane of temperature (T), quark mass ($m_{u,d}$) and baryon chemical potential (μ_B)^[13].

In the next two sections we will summarize our recent studies on the determination of chiral crossover transition temperature at small baryon chemical potentials, and the chiral phase transition temperature.

2 QCD transition at the physical quark mass and small baryon chemical potentials

Since the transition at the physical point is not a true phase transition there is no unique "critical" temperature whose definition can be arbitrary. While the chiral phase transition region of 3-flavor QCD in the lower-left corner of the Columbia plot (see Fig. 1) is small and far away from the physical point^[14–17], the 2nd order O(4) transition line in the chiral limit of

light quarks should have more influence to the thermodynamics at the physical point. Based on the remnant scaling behavior at the physical point we define the chiral crossover temperature through the peak/inflection points of many chiral observables such that all of them converge to a single value in the chiral limit, *i.e.* the chiral phase transition temperature. The continuum extrapolation of the chiral crossover transition temperature is shown in Fig. 3. Note that the results do not reply on critical exponents of any universality class and the determination of chiral crossover temperature is done, *e.g.* by extracting the peak location of light quark susceptibilities and inflection point of light quark chiral condensates etc at each lattice cutoff. It is not expected that the transition temperature extracted via these various chiral observables will converge to a rather precise point or value of $156.5 \pm 1.5 \text{ MeV}$ even at the physical values of light quarks. In Fig. 4 we show the dependence of chiral crossover transition temperature on the baryon chemical potential up to the order of $(\mu_B/T)^4$. The transition line at nonzero μ_B is parameterized as

$$T_{pc}(\mu_B) = T_{pc}(\mu_B = 0) \left[1 - \kappa_2^B \left(\frac{\mu_B}{T_{pc}(0)} \right)^2 - \kappa_4^B \left(\frac{\mu_B}{T_{pc}(0)} \right)^4 \right], \quad (1)$$

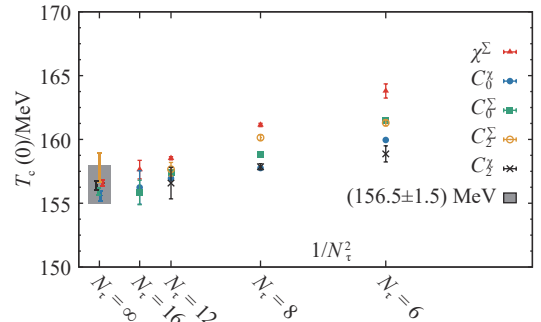


Fig. 3 (color online) The continuum extrapolation of chiral crossover transition temperature at the physical pion mass^[18].

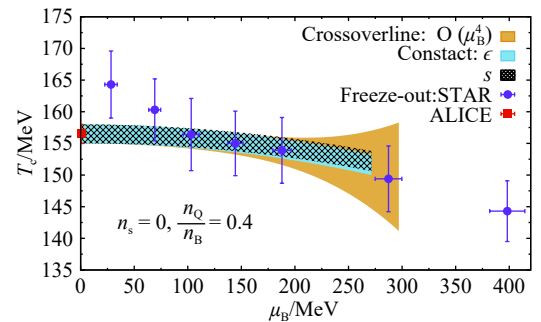


Fig. 4 (color online) Chiral crossover transition line in (2+1)-flavor QCD^[18].

we found $\kappa_2^B=0.012(4)$ and $\kappa_4^B=0.000(4)$. These results are consistent with other studies on the lattice^[19–22]. Also shown in Fig. 4 are the freeze-out temperatures T_f obtained at RHIC and LHC energies. The freeze-out temperature at the LHC energy obtained from particle yields is in very good agreement with chiral crossover temperature at vanishing baryon chemical potential obtained from lattice QCD computations, while T_f at RHIC energies are also in good agreement to QCD transition line at μ_B larger than about 80 MeV and the 200 GeV data point is about 1.5 sigma away from the transition line.

3 QCD transition towards the chiral limit

To determine the chiral phase transition line we start by introducing the subtracted chiral condensates and their susceptibilities which are free from UV divergences,

$$M = \frac{m_s \left(\langle \bar{\psi}\psi \rangle_1 - \frac{2m_l}{m_s} \langle \bar{\psi}\psi \rangle_s \right)}{f_K^4},$$

$$\chi_M \equiv \frac{\partial M}{\partial H} \equiv \frac{m_s^2 \chi_{1,\text{subtot}}}{f_K^4}, \quad (2)$$

where H represents the breaking field of chiral symmetry and is defined as the ratio of the light quark mass to the strange quark mass, *i.e.* $H = m_l/m_s$. We have normalized the chiral observables by multiplying proper powers of the kaon decay constant $f_K = 156.1/\sqrt{2}$ MeV.

As shown in Fig. 5, the pseudo-critical temperature $T_{pc}(H)$ (peak location) decreases with decreasing pion mass, and the peak height at each pion mass $\chi_M(T_{pc}, H)$ increases with decreasing pion mass which is consistent with the O(4) scaling relation.

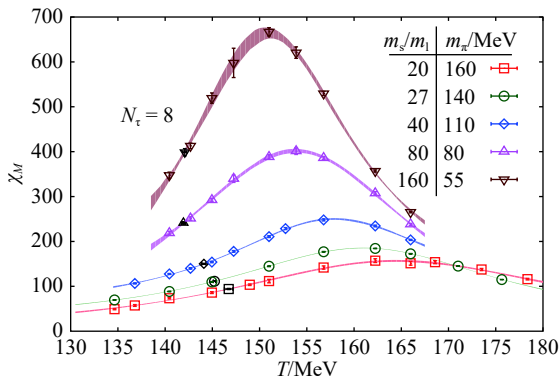


Fig. 5 (color online) Chiral susceptibilities obtained from (2+1)-flavor QCD with various quark masses as a function of temperature on $N_\tau = 8$ lattices^[23].

To determine the chiral phase transition temperature T_c^0 , we employ a novel estimators $T_{60}(V, H)$ which was introduced in Refs. [4, 5, 23] and denotes the temperature satisfying $\chi_M(T_{60}, H, V) = 0.6 \times \chi_M(T_{pc}, H, V)$ with $T_{60} < T_{pc}$, and $T_{60}(V, H)$ will converge to T_c^0 in the chiral limit and thermodynamic limit according to the following O(4) scaling relation,

$$T_X(H, L) = T_c^0 \left(1 + \left(\frac{z_X(z_L)}{z_0} \right) H^{1/\beta\delta} \right) + c_X H^{1-1/\delta+1/\beta\delta}, \quad X = 60, \delta. \quad (3)$$

The advantage of using T_{60} as well as another quantity T_δ as estimators for the chiral phase transition temperature is that these estimators are insensitive to the universality class. Based on the above O(4) scaling relation, we have performed thermodynamic limit, continuum limit and chiral limit extrapolation of chiral phase transition temperature^[23]. As shown in Fig. 6, we perform the chiral limit using the data points $T_{60}(V \rightarrow \infty, N_\tau \rightarrow \infty)$, $T_{pc}(V \rightarrow \infty, N_\tau \rightarrow \infty)$. We have considered two types of the continuum limit, leaving out or including our coarsest lattice $N_\tau=6$. Here the black square (triangle) points give the thermodynamic extrapolated and continuum extrapolated results of chiral crossover temperature $T_{pc}(H)$ obtained from simulations with lattices with $N_\tau = 6, 8, 12$ ($N_\tau = 8, 12$), where the data points at $m_l = m_s/27$ (the physical point) stands for the transition temperature $T_{pc}(1/27) = 156.5(1.5)$ MeV. The decreasing behavior of the pseudo-critical temperature with decreasing light quark mass obeys the following O(4) scaling relation,

$$T_p(H) = T_c^0 \left(1 + \frac{z_p}{z_0} H^{1/\beta\delta} \right). \quad (4)$$

Based on Eq. (4) we extrapolated the data points to the chiral limit as shown in Fig. 6 in which the color blocks show the critical temperature and its error bar in the chiral limit. The black star (circle) points in Fig. 6 show the results of $T_{60}(H)$ in the thermodynamic limit and continuum limit. Since T_{60} is already close to chiral phase transition temperature T_c^0 , it gives more reliable estimate of T_c^0 comparing that obtained by T_{pc} . We extrapolated the T_{60} and another estimate T_δ to the chiral limit based on the O(4) scaling relation as shown in Eq. 3. This gives the value of the chiral phase transition temperature $T_c^0 = 132_{-6}^{+3}$, where the asymmetric error bar stands for the uncertainties from continuum extrapolations of two estimators of the chiral phase transition temperature of by either including or discarding results obtained on the

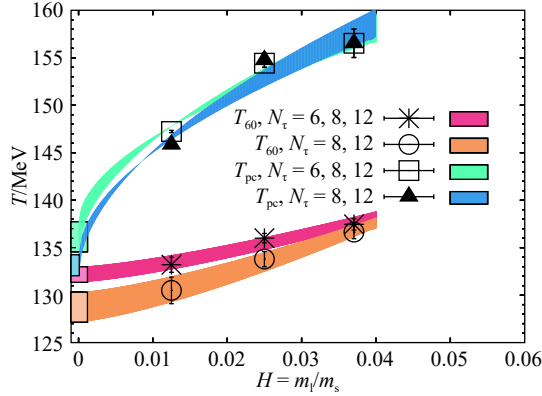


Fig. 6 (color online) Extrapolation of the transition temperature to the chiral limit using T_{pc} and T_{60} .

coarsest ($N_\tau=6$) lattices MeV^[23].

4 Summary & outlook

In this proceedings we reported our recent studies on the chiral crossover transition temperature and chiral phase transition temperature. We determined the chiral crossover transition temperature at zero baryon chemical potential to be 156.5 ± 1.5 MeV in the continuum limit, and the curvatures for the chiral crossover transition line at nonzero values of μ_B to be $\kappa_2^B = 0.012(4)$ and $\kappa_4^B = 0.000(4)$. We also presented a first lattice QCD determination of chiral phase transition temperature in the thermodynamic, continuum and chiral limit, $T_c^0 = 132_{-6}^{+3}$ MeV.

While consistent results on the chiral crossover transition temperature at small baryon density has been obtained from other Lattice QCD groups^[19, 21], the determination of chiral phase transition temperature at small and nonzero baryon density in the continuum limit is absent although the curvature of chiral phase transition line at small baryon density has been studied using the p4fat3 action^[8] and recently the HISQ action^[9] at finite lattice cutoffs. It would be interesting to determine the chiral phase transition temperature also at nonzero baryon density based on the studies reported in Ref. [23] to narrow down the critical temperature at the critical end point in the $T-\mu_B$ plane of the QCD phase diagram.

Towards the chiral limit there still remain quite a lot unknowns in understanding the Columbia plot (cf. Fig. 1), *e.g.* order of the transition and the universality class in the chiral limit of light quarks in $N_f=3$ and $N_f=2+1$ QCD^[17]. It has been found that in $N_f=3$ QCD the critical quark mass where the crossover transition ends at a second order line belonging to the $Z(2)$ universality class reduces with improved discretization schemes and finer lattice spacing^[14–16, 24–25], and further studies towards the continuum limit

would be needed to map out the region of 1st order chiral phase transition which could shed light on the possible origin of the critical end point in the $T-\mu_B$ phase diagram^[26–28]. Another important and unresolved aspect of QCD at nonzero temperature is the fate of axial $U(1)$ symmetry which has a big impact on the order of chiral phase transition. Studies using various discretization schemes have been carried out recently^[29–41] and detailed investigations in the chiral, thermodynamic and continuum limit will be crucially needed.

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利用格点量子色动力学研究手征平滑过渡温度和手征相变温度

李胜泰¹, 丁亨通^{2,†}

(1. 中国科学院近代物理研究所, 兰州 730000;

2. 华中师范大学“夸克与轻子”教育部重点实验室&粒子物理研究所, 武汉 430079)

摘要: 回顾了最近关于手征平滑过渡温度和手征相变温度的研究结果。首先给出了在零重子化学势能下的手征平滑过渡温度为 156.5(1.5) MeV, 其次, 给出了在非零重子化学势能下手征相转变曲线的二阶及四阶曲率分别为 0.012(4) 和 0.000(4)。接着讨论了在格点 QCD 中第一次得到的量子色动力学的手征相变温度。在热力学极限、连续极限及手征极限下, 我们得到手征相变温度为 132^{+3}_{-6} MeV。

关键词: 手征平滑过渡温度; 手征相变温度; QCD 临界点; 格点量子色动力学

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† 通信作者: 丁亨通, E-mail: hengtong.ding@mail.ccnu.edu.cn。