

IMPRINTS OF FUNDAMENTAL PHYSICS
ON THE PRIMORDIAL COSMIC PLASMA

by

Nanoom Lee

A DISSERTATION SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
DEPARTMENT OF PHYSICS
NEW YORK UNIVERSITY
SEPTEMBER, 2024

Prof. Yacine Ali-Haïmoud

Prof. Roman Scoccimarro

© NANOOM LEE

ALL RIGHTS RESERVED, 2024

PREVIEW

DEDICATION

To my mother and father, who let me be in this Universe.

To my wife, *Hyupjung*, who let me enjoy being in this Universe.

ACKNOWLEDGEMENTS

Both this thesis work and my personal well-being during my PhD studies here in New York are indebted to countless people. If I enumerated every single meaningful moment with each person I am grateful to, this acknowledgement would be longer than the rest of this thesis. Hence, I will keep it short and neat here.

In my academic life, I have been influenced by many great people. First of all, no words can fully express how grateful and fortunate I am to have had two exceptional scholars, *Prof. Yacine Ali-Haïmoud* and *Prof. Roman Scoccimarro*, as my PhD advisors. Their consistent support and patience with me are what enabled me to endure and enjoy the challenges of my PhD journey and to successfully complete the program in the end. They are, needless to say, notable and influential physicists in cosmology, from whom I have learned immensely. Moreover, they are kind and pleasant individuals, making every conversation with them both enjoyable and intellectually stimulating. I always felt respected and supported, even in the face of difficulties in my academic or personal life. They have truly set an example for me of what good advisors should be. I am deeply honored to carry their names as my advisors throughout my career.

I also wish to express my gratitude to *Prof. Marilena LoVerde*, who gave me valuable opportunities to begin my studies in cosmology while I was a master's student at Stony Brook. Additionally, I extend my thanks to *Prof. Eunil Won* at Korea University, who welcomed me into his lab as an undergraduate, allowing me to pursue my dream of studying in the United States. My sincere thanks also go to my middle school science teacher, *Eun Yong Jung*, who inspired and supported

me to follow my dreams, spending time with me even after school. It is no exaggeration to say that everything in my academic career stems from these moments.

Further, I am also deeply grateful to my research collaborators, *Dr. Selim Hotinli, Prof. Marc Kamionkowski, Prof. Vivian Poulin, Dr. Nils Schöneberg, Benjamin Camacho-Quevedo, Dr. Martin Crocce, Dr. Alexander Eggemeier, Dr. Andrea Pezzotta, and Dr. Ariel Sánchez*. A special thanks to *Prof. Marc Kamionkowski* for the opportunities to visit his research group at Johns Hopkins. The early stages of my PhD were also enriched by a group of wonderful classmates, for which I am very thankful.

Beyond physics, many memorable and joyful moments were shared with wonderful people I met in New York. These moments allowed me to enjoy life, find relief, and keep moving forward. If you are reading this acknowledgement as a non-physicist, then you are undoubtedly one of them, and I sincerely thank you.

To my parents and two brothers, who have always supported my dream from the opposite side of the world—thank you. I am forever grateful for your unwavering encouragement.

Lastly, I save this final and most important paragraph for you, my love, *Hyupjung*. We have gone through so much together since I first left our country to pursue my dreams in the United States. You have always been by my side, supporting me even under difficult circumstances that would have tested most people. I am immensely indebted to you. Thank you for everything.

ABSTRACT

The primordial cosmic plasma holds a wealth of information about the origins and fundamental laws of the Universe. By refining our theoretical understanding and observational techniques, cosmologists have successfully uncovered and verified key principles of the Universe through the study of this plasma. Investigating the imprints of fundamental physics on the primordial cosmic plasma remains one of the most effective methods in cosmological research today. This thesis demonstrates how such approaches extend our understanding of the Universe.

Significant portion of this thesis is related to cosmological recombination, the process during which electrons and protons combined for the first time, allowing light to decouple from the plasma and propagate freely. The thesis details advancements in computations of recombination history, which are critical for analyzing cosmic microwave background (CMB) anisotropies. Furthermore, it explores how cosmological recombination can be applied to probe the Universe's initial conditions, and address tensions in modern cosmological data particularly by studying the role of fundamental constants such as electron mass in the primordial plasma.

Beyond recombination, this thesis covers a diverse range of cosmological topics, including cosmic birefringence, the polarized Sunyaev-Zel'dovich effect, 21-cm fluctuations from cosmic dawn, and magnetic fields generated by primordial perturbations. Each of these topics offers a unique lens through which the imprints of fundamental physics on the primordial cosmic plasma can be observed and analyzed, contributing to a deeper understanding of the early Universe.

CONTENTS

Dedication	iii
Acknowledgments	iv
Abstract	vi
List of Figures	xiii
List of Tables	xvii
List of Appendices	xviii
1 Introduction	1
2 A Highly Accurate Sub-Millisecond Recombination Code: HYREC-2	4
2.1 Introduction	4
2.2 Hydrogen Recombination Physics	8
2.2.1 Recombination Phenomenology	8
2.2.2 General Recombination Equations	10
2.2.3 Exact Effective Four-Level Equations	11
2.3 HYREC-2 Equations	12
2.3.1 The Base Approximate Model	12

2.3.2	Correction Function	13
2.3.3	Cosmology Dependence	15
2.3.4	Numerical Integrator and Runtime	18
2.4	Accuracy of HYREC-2	19
2.4.1	Sample Cosmologies	19
2.4.2	Accuracy of the Free-Electron Fraction	20
2.4.3	Bias of Cosmological Parameters	21
2.5	Conclusion	26
3	Probing Small-Scale Isocurvature Perturbations with CMB Anisotropies	28
3.1	Introduction	28
3.2	Modified Recombination with Small-Scale Baryon Fluctuations	32
3.2.1	Basic Idea	32
3.2.2	Recombination with a Local Time-Dependent Perturbed Baryon Density	33
3.2.3	Nonlinear Recombination Response Function	35
3.3	Application to Small-Scale Isocurvature Perturbations	38
3.3.1	Isocurvature Modes Considered	38
3.3.2	Baryon Transfer Functions	39
3.3.3	Recombination Perturbations	43
3.4	Constraints from CMB Anisotropies	46
3.4.1	Implementation and Planck Analysis Setup	46
3.4.2	Generalized Fisher Analysis for a CMB Stage-4 Experiment	47
3.4.3	Results	52
3.4.4	Comparison with Other Constraints on Small-Scale Perturbations	55
3.5	Conclusion	59

4	Probing Cosmic Birefringence with Polarized Sunyaev-Zel'dovich Tomography	63
4.1	Introduction	63
4.2	Cosmic Birefringence	66
4.2.1	Phenomenology	66
4.2.2	Polarization Parity-Violation	67
4.2.3	CMB Calibration Angle	68
4.3	Polarized Sunyaev Zel'dovich Tomography	69
4.3.1	The Polarized SZ Effect	69
4.3.2	Effect of Birefringence on the Remote Quadrupole	71
4.4	Forecast	72
4.4.1	Setup	72
4.5	Results	75
4.5.1	Intuition	75
4.5.2	Probing Axion-Like Dark Energy	82
4.6	Discussion	84
5	Resolving the Hubble Tension through Modifications of Cosmological Recombination	86
5.1	Introduction	86
5.2	Setting Up the Problem	88
5.3	Introducing new physics	91
5.4	Result I: Application to Planck CMB Data	93
5.5	Result II: Application to Planck CMB + BAO or Planck CMB + BAO + PantheonPlus	96
5.6	Conclusions	97
6	Probing Light Relics through 21-cm Fluctuations from Cosmic Dawn	98
6.1	Introduction	98

6.2	Cosmic Dawn	100
6.3	Forecasts	102
6.4	Discussion	108
7	Magnetic Fields from Small-Scale Primordial Perturbations	111
7.1	Introduction	111
7.2	Evolution of Perturbations	114
7.2.1	Implementation	114
7.2.2	Initial Conditions and Results	117
7.3	Magnetic Field from the Biermann-Battery Mechanism	118
7.3.1	General Expressions	118
7.3.2	Application to Nearly Scale-Invariant Adiabatic Initial Conditions	122
7.4	Magnetic Field as a Probe of Small-Scale Initial Conditions	125
7.4.1	Setup	125
7.4.2	Application to Enhanced Small-Scale Adiabatic Perturbations	126
7.4.3	Application to Small-Scale Isocurvature Perturbations	127
7.5	Conclusion	130
A	Appendix to Ch. 2	132
A.1	Explicit Expression for the Correction Function	132
A.2	Equations for RECFast in Our Notation	133
A.2.1	Peebles' Effective Three-Level Model	133
A.2.2	Fudge Factors and Functions	134
B	Appendix to Ch. 3	136
B.1	BBN Limits to Small-Scale Baryon Inhomogeneities	136

C	Appendix to Ch. 4	138
C.1	Rotated Spectra	138
C.2	Covariance Matrix	139
C.3	The Remote Quadrupole Moment Seen by Electrons	140
D	Appendix to Ch. 5	143
D.1	Data	143
D.1.1	CMB – Planck	143
D.1.2	BAO – BOSS DR12	144
D.1.3	Uncalibrated SNIa – PantheonPlus	145
D.2	Numerical Techniques	146
D.3	Changes in Planck’s Best-Fit and Chi-Squared due to Time-Varying Electron Mass $m_e(z)$	147
D.4	Comparison with PCAs	151
D.5	Validation Test: Time-Varying Electron Mass $m_e(z)$ from Planck CMB	152
D.6	Time-Varying Electron Mass $m_e(z)$ from Planck + BAO or Planck + BAO + Pan- theonPlus	154
D.7	Estimating Errors from the Approximations taken in the Formalism	154
D.8	Time-Varying Fine Structure Constant $\alpha(z)$	156
E	Appendix to Ch. 7	161
E.1	Derivation of the Biermann-Battery Mechanism	161
E.1.1	Electron-Proton Slip and Current Density	161
E.1.2	Combining with Maxwell’s Equations	163
E.2	Numerical Implementation Details	165
E.3	Comparison with Flitter et al. 2023 [140]	166

PREVIEW

LIST OF FIGURES

2.1	Fractional difference in the rate of change of the free-electron fraction as a function of redshift	14
2.2	Correction to the Lyman- α decay rate $\Delta(z)$ for the fiducial cosmology	19
2.3	First derivatives of the correction function	20
2.4	Ten thousands sample cosmologies used to check the accuracy of HYREC-2	21
2.5	Fractional differences in the free-electron fraction	22
2.6	Fractional error in C_ℓ^{TT}	23
2.7	Bias in cosmological parameters when using HYREC-2 and RECFAST	27
3.1	Baryon density transfer functions for the four different initial conditions	39
3.2	Ratio $n_e^{(2)}(z; k)/n_e^{(0)}(z)$ with four different initial conditions	44
3.3	Fractional change of the free-electron abundance n_e with four different isocurvature initial conditions	45
3.4	Marginalized 68% and 95% confidence intervals for the Λ CDM + small-scale isocurvature models	53
3.5	Means and 68% confidence intervals of H_0	54
3.6	95% CL upper limits on (sensitivities to) the amplitude of the four isocurvature modes as a function of wavenumber, for a Dirac-delta spike	55

3.7	95% CL upper limits on (sensitivities to) the amplitude of the four isocurvature modes as a function of spectral index n_I	56
4.1	Small-scale fluctuations of CMB polarization magnitude induced by galaxy (or electron-density) fluctuations	65
4.2	The evolution of the ϕ field as a function of redshift z	67
4.3	The scalar and tensor remote E - and B - parity quadrupole signals	71
4.4	The sensitivities on each $\Delta\alpha_i$	75
4.5	With prior on the calibration angle	77
4.6	A deeper analysis of the prospects of breaking the calibration-angle degeneracy with pSZ-tomography	78
4.7	Model 1	80
4.8	Model 2	81
4.9	The sensitivities to m_ϕ	83
4.10	The same plot as Fig. 4.9 but the rotation angle is normalized to have the total angle 0.5 degrees.	83
5.1	Solutions for $\frac{\Delta m_e}{m_e}(z)$ given target values of the CMB-only best-fit Hubble constant H_0 , using Planck data [16] alone.	93
5.2	Posteriors of H_0 (top), S_8 (middle), and Ω_m (bottom) inferred from Planck full likelihood	94
6.1	Left: Dimensionless power spectrum of $\delta_{v^2} \equiv \sqrt{3/2} \left(v_{bc}^2/v_{\text{rms}}^2 - 1 \right)$, right: 21-cm power spectra at $z = 15, 20, 25, 30$	102
6.2	2σ sensitivities to N_{eff} for $\Lambda\text{CDM} + N_{\text{eff}}$ (Y_p is varied to be consistent with BBN) .	105
6.3	Sensitivities to $\Lambda\text{CDM} + N_{\text{eff}} + Y_{\text{He}}$. For 21-cm signals, Foreground II and regular feedback are assumed.	107

7.1	Transfer functions for δ_{n_e} (left column) and $\delta_{T_{\text{gas}}}$ (right column), for adiabatic initial conditions (top panels) and CIPs (bottom panels)	119
7.2	Power spectrum of the magnetic field produced by the Biermann-battery mechanism with primordial adiabatic perturbations, extrapolating the Planck CMB best-fit cosmology [16] to small scales, at $z = 20$	123
7.3	The rms of velocity-averaged magnetic field produced by the Biermann-battery mechanism with primordial adiabatic perturbations, extrapolating the Planck CMB best-fit cosmology [16] to small scales, as a function of redshift	124
7.4	Velocity-averaged magnetic field power spectrum at $z = 20$ produced by adiabatic primordial perturbations with a Dirac-Delta spike at $k = k_0 \in \{10, 10^4\} \text{ Mpc}^{-1}$ whose amplitude saturates CMB spectral distortion upper limits, Eq. (7.25)	128
7.5	The rms magnetic field at $z = 20$ produced by the Biermann-battery mechanism with a Dirac-delta primordial isocurvature power spectrum peaked at $k = k_0$. . .	129
D.1	68% and 95% confidence level constraints from BOSS DR12 [22] at three effective redshifts $z_{\text{eff}} = 0.38, 0.51, 0.61$	145
D.2	Functional derivatives of best-fit parameters with three data sets (Planck CMB, Planck CMB + BOSS DR12 BAO, Planck CMB + BOSS DR12 BAO + PantheonPlus)	147
D.3	Functional derivatives of best-fit chi-squared and quadratic response of a change in best-fit chi-squared with respect to logarithmic change in electron mass $m_e(z)$ at each redshift	148
D.4	Functional derivatives of the best-fit parameters with respect to the fractional changes in electron mass $m_e(z)$ translated to the basis of $\{\Omega_m, \Omega_m h^3, h\}$	150
D.5	Comparison with PCAs	151
D.6	Contour plot with two models, ΛCDM and $\Lambda\text{CDM} + m_e(z)$	153

D.7	Solutions for $\frac{\Delta m_e}{m_e}(z)$ given a required shift in the CMB + BAO (left) and CMB + BAO + PantheonPlus (right) best-fit Hubble constant H_0	154
D.8	The same plots as Fig. D.2 but with $f(z) = \ln \alpha(z)$ instead of $\ln m_e(z)$	157
D.9	The same plots as Fig. D.3 but with $f(z) = \ln \alpha(z)$ instead of $\ln m_e(z)$	157
D.10	The same plots as Fig. 5.1 and D.7 but with $f(z) = \ln \alpha(z)$ instead of $\ln m_e(z)$. With CMB + BAO (CMB + BAO + PantheonPlus) data, we could lower the Hubble tension down to $\sim 1.4\sigma$ (2.4σ).	158
D.11	The same plots as Fig. 5.2 but together with Λ CDM + $\alpha(z)$ model	158
D.12	Contour plot with two models, Λ CDM and Λ CDM + $\alpha(z)$	159
E.1	Comparison of our results with those of F+23, assuming a scale-invariant primordial power spectrum for CIPs	168

LIST OF TABLES

2.1	Cosmological parameters relevant to hydrogen recombination, along with the adopted fiducial values	18
2.2	Default run time of each code	19
2.3	Noise parameters and the fraction of sky adopted in the Fisher matrix estimates. .	25
4.1	Survey details used in forecasts	74
6.1	Forecasts with CMB S4 + DESI BAO + 21cm SKA1-low for two different foregrounds	108
6.2	Improvement factors $\frac{\sigma_{\text{CMB+BAO}}}{\sigma_{\text{CMB+BAO+21cm}}}$ of CMB + DESI BAO + 21cm SKA1-low for three different CMB surveys with Foreground II and regular feedback	108
6.3	1σ sensitivities of CMB S4 + DESI BAO + 21cm SKA1-low to N_{eff} with three feedback levels of 21cmvFAST and two foreground settings	109
D.1	The estimated error in chi-squared due to each approximation for all three cases .	156

LIST OF APPENDICES

Appendix to Ch. 2	132
Appendix to Ch. 3	136
Appendix to Ch. 4	138
Appendix to Ch. 5	143
Appendix to Ch. 7	161

1 | INTRODUCTION

Cosmology, the study of the Universe's origin, evolution, and large-scale structure, continuously advances our understanding of fundamental physics and the cosmos. A deeper understanding of the primordial cosmic plasma has been pivotal in this progress, as it carries imprints from the early Universe, offering profound insights into its underlying nature.

The journey into modern cosmology began with the formulation of the Big Bang theory in the early 20th century. Edwin Hubble's observations of the expanding Universe [195] provided critical evidence supporting the notion that the Universe originated from a hot, dense state and has been expanding ever since. This theory was further reinforced by the discovery of cosmic microwave background (CMB) radiation, the light freed from the primordial plasma, by Arno Penzias and Robert Wilson in 1965 [290], which provided crucial observational support for the Big Bang model and laid the groundwork for our understanding of the early Universe. In the decades following, the Λ Cold Dark Matter (Λ CDM) model emerged as the most successful framework to describe the Universe's evolution, combining both dark matter and dark energy [287; 311; 292]. The precision measurements of the CMB by the Planck satellite, launched in 2009, further refined this model, providing a detailed understanding of the early Universe [14]. The significant strides made in cosmology have largely been enabled by the study of the primordial cosmic plasma and the imprints of fundamental physics on this plasma, which continue to provide new avenues to probe the Universe.

While the concordance Λ CDM model of cosmology provides accurate predictions for various

cosmological observables, it is essential to acknowledge that it may not represent the ultimate theory we cosmologists seek as it still has some unknowns in itself. Within the realm of this elegant model lie unresolved mysteries such as properties of dark matter and dark energy, and significant discrepancies in the analysis of observational data. The dark matter, a kind of mass not emitting light, was first introduced to explain observational discrepancy between the total mass of galaxy cluster and the luminous matter in the cluster [380], and also observed rotation curves of galaxies [319]. Its existence has been further supported by its gravitational lensing effect and the observations of CMB anisotropies. While the evidences supporting its existence are robust, we have very little understanding about its properties, which has led efforts to build theoretical models for dark matter. This includes weakly interacting massive particles (WIMPs), axions, ultra-light dark matter, and primordial black holes. Their effects on predictions for various cosmological observables have been studied with currently available cosmological data and also upcoming future surveys. On the other hand, the dark energy, which has negative pressure, was proposed to contribute to two thirds of the energy density in the Universe in order to explain the observed acceleration of the Universe. While there have been observational efforts to unveil properties of the dark energy since then, we do not know its detailed properties yet. These two components are what gave the name of current standard cosmological model (Λ implies the cosmological constant which is a certain type of dark energy, and CDM stands for cold dark matter). In addition to these two mysterious components, this standard cosmological model is currently facing a severe challenge of explaining discrepancies in measured values of a very basic parameter of the model from different cosmological data, the Hubble constant H_0 which is the expansion rate of the Universe today. This so-called Hubble tension notably stands as one of the most pressing issues in modern cosmology.

In light of these challenges, a key approach in modern cosmology is the detailed study of the primordial cosmic plasma, which carries imprints of fundamental physics. These imprints offer invaluable insights into the early Universe, allowing cosmologists to probe its conditions and

test theoretical predictions. This thesis explores the intricate relationship between fundamental physics and cosmological observables, focusing on how the imprints left on the primordial plasma can be detected and studied to advance our understanding of the Universe.

The rest of this thesis consists of six chapters. Each chapter is based on a single publication. One significant research area of this thesis is cosmological recombination, a pivotal process that occurred when the Universe cooled enough for electrons and protons to form neutral hydrogen. This epoch significantly impacts the CMB, influencing its anisotropies and providing insights into early Universe conditions. In Ch. 2 we describe the new recombination code HYREC-2, which holds the same accuracy as the state-of-the-art codes HYREC and COSMOREC and at the same time surpasses the computational speed of the code RECFAST commonly used for CMB anisotropy data analyses. Ch. 3 shows how perturbed recombination on small scales due to baryon density fluctuations can be formulated and used to probe the initial conditions of the Universe using CMB anisotropy data. We modify the standard recombination history to resolve the Hubble tension and discuss its implications in Ch. 5. In addition to cosmological recombination, this thesis explores various different sub-fields in cosmology. In Ch. 4, we show how CMB anisotropy and galaxy survey data in near future can be used to probe particle physics models (e.g., axion-like particles) proposed to describe possible forms of dark matter and dark energy, through their effect on CMB polarization, so-called cosmic birefringence. Another promising cosmological observable, 21-cm fluctuations, is considered in Ch. 6 focusing on prospects of such observations in near future to provide additional information constraining the effective number of relativistic species, hence probing various light particle models. In Ch. 7, we provide the calculations of the power spectrum of magnetic fields produced by the Biermann-battery mechanism, and show that these can be used to probe the initial conditions of the Universe.

2 | A HIGHLY ACCURATE SUB-MILLISECOND RECOMBINATION CODE: HYREC-2

This chapter is based on N. Lee and Y. Ali-Haïmoud “HYREC-2: a highly accurate sub-millisecond recombination code” Phys.Rev.D 102, 083517 (2020) [227].

2.1 INTRODUCTION

The recombination history of the Universe is a key part of the physics of Cosmic Microwave Background (CMB) anisotropies, the epoch of the dark ages leading to the formation of the first stars, as well as the formation of cosmic structure. When exactly free electrons got bound in the first helium and hydrogen atoms determines, first, the epoch of photon last scattering, thus the sound horizon. This scale is imprinted into CMB power spectra and the correlation function of galaxies, and serves as a standard ruler [124]. The abundance of free electrons also sets the photon diffusion scale, hence the damping of small-scale CMB anisotropies [341]. Lastly, the free-electron fraction determines the epochs of kinematic and kinetic decoupling of baryons from photons, hence the thermal history of the gas, as well as the formation of the first stars and structures.

The basic physics of hydrogen recombination were laid out in the late sixties in the seminal works of Peebles [286] and Zeldovich, Kurt, and Sunyaev [378]. Their simple but physically accurate effective 3-level model was largely unchanged until the late nineties (see Ref. [194] for

an overview of recombination studies till then), except for improvements in the atomic-physics calculations of case-B recombination coefficients [291]. In 1999, motivated by the approval of the WMAP [57] and *Planck* [359] satellites, Seager, Sasselov & Scott conducted the first modern, detailed recombination calculation [329], explicitly accounting for the non-equilibrium of highly excited hydrogen energy levels (but assuming equilibrium amongst angular momentum substates). They found that the result of their 300-level calculation could be accurately reproduced by an effective 3-level atom model, with the case-B recombination coefficient multiplied by a “fudge factor” $F = 1.14$ [328]. This model was implemented in the code RECFast, which was used for cosmological analyses of WMAP data [173], for which it was sufficiently accurate.

It was realized in the mid-2000’s that the RECFast model for hydrogen recombination would not be sufficiently accurate for the analysis of *Planck* data, as it neglected a variety of physical effects that matter at the required sub-percent level of accuracy (see Ref. [316] for an overview of progress by the end of that decade). On the one hand, the angular momentum substates of the excited states of hydrogen are out of equilibrium, which leads to an overall slow-down of recombination [317; 84; 164; 91]. On the other hand, a variety of radiative transfer effects have to be accounted for, such as feedback from higher-order Lyman transitions, frequency diffusion due to resonant scattering, and two photon transition from higher levels [97; 215; 96; 117; 87; 158; 175; 86].

While conceptually straightforward, the inclusion of hydrogen’s angular momentum substates presented a considerable computational challenge with the standard multilevel method. Indeed, Refs. [164; 91] showed that a recombination history converged at the level needed for *Planck* requires accounting for at least 100 shells of hydrogen energy levels, corresponding to about 5000 separate states. The standard multilevel approach required solving large linear systems at each time step, and even the fastest codes took several hours per recombination history on a standard single-processor machine [91]. This aspect of the recombination problem was solved conclusively a decade ago in Ref. [28], where it was shown that the non-equilibrium dynamics of

the excited states can be accounted for *exactly* with an effective few-level atom model (in practice, 4 levels are enough), with effective recombination coefficients to the lowest excited states accounting for intermediate transitions through the highly excited states (see also [73; 74] for an independent discovery of the method). In contrast with RECFAST’s fudged case-B coefficient, these effective rates are exact, temperature-dependent atomic physics functions. Once this computational hurdle was cleared, efficient methods to solve the radiative transfer problem were developed shortly after, leading to the fast state-of-the-art public recombination codes HYREC [29] and COSMOREC [90], in excellent agreement with one another despite their different radiative transfer algorithms. The residual theoretical uncertainty of these codes is estimated at the level of a few times 10^{-4} during hydrogen recombination, due to the neglect of subtle radiative transfer effects [27] and collisional transitions [91], whose rates are uncertain.

The accuracy requirement for helium recombination is not as stringent as it is for hydrogen, given that it recombines well before the time at which most CMB photons last scattered. Still, a variety of important radiative transfer effects must be accounted for at the level of accuracy required for *Planck*, such as the photoionization of neutral hydrogen atoms by resonant 584 Å photons and the emission of intercombination-line photons at 591 Å [354; 176; 355; 318; 214]. These effects are included numerically in COSMOREC and through fast analytic approximations in HYREC, accurate within 0.3%, which is sufficient for *Planck*. In the rest of this work, we focus on hydrogen recombination. We defer the task of extending our approach to helium to future work.

Both HYREC and COSMOREC are interfaced with the commonly used Boltzmann codes CAMB [239; 190] and CLASS [233], and are able to compute a recombination history in about half a second on a standard laptop. Still, the default code for the cosmological analysis of *Planck* data has remained RECFAST [14], further modified to approximately reproduce the output of HYREC and COSMOREC. The non-equilibrium of angular momentum substates is approximately accounted for by lowering the case-B coefficient fudge factor from 1.14 to 1.125. Radiative transfer physics are approximately mimicked by introducing a double-Gaussian “fudge function”, correcting the net