

IMPRINTS OF FUNDAMENTAL PHYSICS
ON THE PRIMORDIAL COSMIC PLASMA

by

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PREVIEW

DEDICATION

To my mother and father, who let me be in this Universe.

To my wife, *Hyupjung*, who let me enjoy being in this Universe.

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Both this thesis work and my personal well-being during my PhD studies here in New York are indebted to countless people. If I enumerated every single meaningful moment with each person I am grateful to, this acknowledgement would be longer than the rest of this thesis. Hence, I will keep it short and neat here.

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ABSTRACT

The primordial cosmic plasma holds a wealth of information about the origins and fundamental laws of the Universe. By refining our theoretical understanding and observational techniques, cosmologists have successfully uncovered and verified key principles of the Universe through the study of this plasma. Investigating the imprints of fundamental physics on the primordial cosmic plasma remains one of the most effective methods in cosmological research today. This thesis demonstrates how such approaches extend our understanding of the Universe.

Significant portion of this thesis is related to cosmological recombination, the process during which electrons and protons combined for the first time, allowing light to decouple from the plasma and propagate freely. The thesis details advancements in computations of recombination history, which are critical for analyzing cosmic microwave background (CMB) anisotropies. Furthermore, it explores how cosmological recombination can be applied to probe the Universe's initial conditions, and address tensions in modern cosmological data particularly by studying the role of fundamental constants such as electron mass in the primordial plasma.

Beyond recombination, this thesis covers a diverse range of cosmological topics, including cosmic birefringence, the polarized Sunyaev-Zel'dovich effect, 21-cm fluctuations from cosmic dawn, and magnetic fields generated by primordial perturbations. Each of these topics offers a unique lens through which the imprints of fundamental physics on the primordial cosmic plasma can be observed and analyzed, contributing to a deeper understanding of the early Universe.

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1 | INTRODUCTION

Cosmology, the study of the Universe's origin, evolution, and large-scale structure, continuously advances our understanding of fundamental physics and the cosmos. A deeper understanding of the primordial cosmic plasma has been pivotal in this progress, as it carries imprints from the early Universe, offering profound insights into its underlying nature.

The journey into modern cosmology began with the formulation of the Big Bang theory in the early 20th century. Edwin Hubble's observations of the expanding Universe [195] provided critical evidence supporting the notion that the Universe originated from a hot, dense state and has been expanding ever since. This theory was further reinforced by the discovery of cosmic microwave background (CMB) radiation, the light freed from the primordial plasma, by Arno Penzias and Robert Wilson in 1965 [290], which provided crucial observational support for the Big Bang model and laid the groundwork for our understanding of the early Universe. In the decades following, the Λ Cold Dark Matter (Λ CDM) model emerged as the most successful framework to describe the Universe's evolution, combining both dark matter and dark energy [287; 311; 292]. The precision measurements of the CMB by the Planck satellite, launched in 2009, further refined this model, providing a detailed understanding of the early Universe [14]. The significant strides made in cosmology have largely been enabled by the study of the primordial cosmic plasma and the imprints of fundamental physics on this plasma, which continue to provide new avenues to probe the Universe.

While the concordance Λ CDM model of cosmology provides accurate predictions for various

cosmological observables, it is essential to acknowledge that it may not represent the ultimate theory we cosmologists seek as it still has some unknowns in itself. Within the realm of this elegant model lie unresolved mysteries such as properties of dark matter and dark energy, and significant discrepancies in the analysis of observational data. The dark matter, a kind of mass not emitting light, was first introduced to explain observational discrepancy between the total mass of galaxy cluster and the luminous matter in the cluster [380], and also observed rotation curves of galaxies [319]. Its existence has been further supported by its gravitational lensing effect and the observations of CMB anisotropies. While the evidences supporting its existence are robust, we have very little understanding about its properties, which has led efforts to build theoretical models for dark matter. This includes weakly interacting massive particles (WIMPs), axions, ultra-light dark matter, and primordial back holes. Their effects on predictions for various cosmological observables have been studied with currently available cosmological data and also upcoming future surveys. On the other hand, the dark energy, which has negative pressure, was proposed to contribute to two thirds of the energy density in the Universe in order to explain the observed acceleration of the Universe. While there have been observational efforts to unveil properties of the dark energy since then, we do not know its detailed properties yet. These two components are what gave the name of current standard cosmological model (Λ implies the cosmological constant which is a certain type of dark energy, and CDM stands for cold dark matter). In addition to these two mysterious components, this standard cosmological model is currently facing a severe challenge of explaining discrepancies in measured values of a very basic parameter of the model from different cosmological data, the Hubble constant H_0 which is the expansion rate of the Universe today. This so-called Hubble tension notably stands as one of the most pressing issues in modern cosmology.

In light of these challenges, a key approach in modern cosmology is the detailed study of the primordial cosmic plasma, which carries imprints of fundamental physics. These imprints offer invaluable insights into the early Universe, allowing cosmologists to probe its conditions and

test theoretical predictions. This thesis explores the intricate relationship between fundamental physics and cosmological observables, focusing on how the imprints left on the primordial plasma can be detected and studied to advance our understanding of the Universe.

The rest of this thesis consists of six chapters. Each chapter is based on a single publication. One significant research area of this thesis is cosmological recombination, a pivotal process that occurred when the Universe cooled enough for electrons and protons to form neutral hydrogen. This epoch significantly impacts the CMB, influencing its anisotropies and providing insights into early Universe conditions. In Ch. 2 we describe the new recombination code **HYREC-2**, which holds the same accuracy as the state-of-the-art codes **HYREC** and **COSMOREC** and at the same time surpasses the computational speed of the code **RECFAST** commonly used for CMB anisotropy data analyses. Ch. 3 shows how perturbed recombination on small scales due to baryon density fluctuations can be formulated and used to probe the initial conditions of the Universe using CMB anisotropy data. We modify the standard recombination history to resolve the Hubble tension and discuss its implications in Ch. 5. In addition to cosmological recombination, this thesis explores various different sub-fields in cosmology. In Ch. 4, we show how CMB anisotropy and galaxy survey data in near future can be used to probe particle physics models (e.g., axion-like particles) proposed to describe possible forms of dark matter and dark energy, through their effect on CMB polarization, so-called cosmic birefringence. Another promising cosmological observable, 21-cm fluctuations, is considered in Ch. 6 focusing on prospects of such observations in near future to provide additional information constraining the effective number of relativistic species, hence probing various light particle models. In Ch. 7, we provide the calculations of the power spectrum of magnetic fields produced by the Biermann-battery mechanism, and show that these can be used to probe the initial conditions of the Universe.

2 | A HIGHLY ACCURATE SUB-MILLISECOND RECOMBINATION CODE: HYREC-2

This chapter is based on N. Lee and Y. Ali-Haïmoud “HYREC-2: a highly accurate sub-millisecond recombination code” Phys.Rev.D 102, 083517 (2020) [227].

2.1 INTRODUCTION

The recombination history of the Universe is a key part of the physics of Cosmic Microwave Background (CMB) anisotropies, the epoch of the dark ages leading to the formation of the first stars, as well as the formation of cosmic structure. When exactly free electrons got bound in the first helium and hydrogen atoms determines, first, the epoch of photon last scattering, thus the sound horizon. This scale is imprinted into CMB power spectra and the correlation function of galaxies, and serves as a standard ruler [124]. The abundance of free electrons also sets the photon diffusion scale, hence the damping of small-scale CMB anisotropies [341]. Lastly, the free-electron fraction determines the epochs of kinematic and kinetic decoupling of baryons from photons, hence the thermal history of the gas, as well as the formation of the first stars and structures.

The basic physics of hydrogen recombination were laid out in the late sixties in the seminal works of Peebles [286] and Zeldovich, Kurt, and Sunyaev [378]. Their simple but physically accurate effective 3-level model was largely unchanged until the late nineties (see Ref. [194] for

an overview of recombination studies till then), except for improvements in the atomic-physics calculations of case-B recombination coefficients [291]. In 1999, motivated by the approval of the WMAP [57] and *Planck* [359] satellites, Seager, Sasselov & Scott conducted the first modern, detailed recombination calculation [329], explicitly accounting for the non-equilibrium of highly excited hydrogen energy levels (but assuming equilibrium amongst angular momentum substates). They found that the result of their 300-level calculation could be accurately reproduced by an effective 3-level atom model, with the case-B recombination coefficient multiplied by a “fudge factor” $F = 1.14$ [328]. This model was implemented in the code `RECFAST`, which was used for cosmological analyses of WMAP data [173], for which it was sufficiently accurate.

It was realized in the mid-2000’s that the `RECFAST` model for hydrogen recombination would not be sufficiently accurate for the analysis of *Planck* data, as it neglected a variety of physical effects that matter at the required sub-percent level of accuracy (see Ref. [316] for an overview of progress by the end of that decade). On the one hand, the angular momentum substates of the excited states of hydrogen are out of equilibrium, which leads to an overall slow-down of recombination [317; 84; 164; 91]. On the other hand, a variety of radiative transfer effects have to be accounted for, such as feedback from higher-order Lyman transitions, frequency diffusion due to resonant scattering, and two photon transition from higher levels [97; 215; 96; 117; 87; 158; 175; 86].

While conceptually straightforward, the inclusion of hydrogen’s angular momentum substates presented a considerable computational challenge with the standard multilevel method. Indeed, Refs. [164; 91] showed that a recombination history converged at the level needed for *Planck* requires accounting for at least 100 shells of hydrogen energy levels, corresponding to about 5000 separate states. The standard multilevel approach required solving large linear systems at each time step, and even the fastest codes took several hours per recombination history on a standard single-processor machine [91]. This aspect of the recombination problem was solved conclusively a decade ago in Ref. [28], where it was shown that the non-equilibrium dynamics of

the excited states can be accounted for *exactly* with an effective few-level atom model (in practice, 4 levels are enough), with effective recombination coefficients to the lowest excited states accounting for intermediate transitions through the highly excited states (see also [73; 74] for an independent discovery of the method). In contrast with RECFast’s fudged case-B coefficient, these effective rates are exact, temperature-dependent atomic physics functions. Once this computational hurdle was cleared, efficient methods to solve the radiative transfer problem were developed shortly after, leading to the fast state-of-the-art public recombination codes HYREC [29] and COSMOREC [90], in excellent agreement with one another despite their different radiative transfer algorithms. The residual theoretical uncertainty of these codes is estimated at the level of a few times 10^{-4} during hydrogen recombination, due to the neglect of subtle radiative transfer effects [27] and collisional transitions [91], whose rates are uncertain.

The accuracy requirement for helium recombination is not as stringent as it is for hydrogen, given that it recombines well before the time at which most CMB photons last scattered. Still, a variety of important radiative transfer effects must be accounted for at the level of accuracy required for *Planck*, such as the photoionization of neutral hydrogen atoms by resonant 584 Å photons and the emission of intercombination-line photons at 591 Å [354; 176; 355; 318; 214]. These effects are included numerically in COSMOREC and through fast analytic approximations in HYREC, accurate within 0.3%, which is sufficient for *Planck*. In the rest of this work, we focus on hydrogen recombination. We defer the task of extending our approach to helium to future work.

Both HYREC and COSMOREC are interfaced with the commonly used Boltzmann codes CAMB [239; 190] and CLASS [233], and are able to compute a recombination history in about half a second on a standard laptop. Still, the default code for the cosmological analysis of *Planck* data has remained RECFast [14], further modified to approximately reproduce the output of HYREC and COSMOREC. The non-equilibrium of angular momentum substates is approximately accounted for by lowering the case-B coefficient fudge factor from 1.14 to 1.125. Radiative transfer physics are approximately mimicked by introducing a double-Gaussian “fudge function”, correcting the net