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A comparative study on various methods for SiPM gain calibration

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Two methods for the SiPM gain calibration are studied as alternatives to the standard method using single photoelectron charge. One method is based on a statistical analysis of the charge distribution obtained for a Poisson distributed light source. This method is found to be consistent with the single photoelectron method. The other method is based on a statistical analysis of waveforms, which is applied to the SiPMs for the first time. It is found that a relative gain can be evaluated with a reasonable precision by this method.

1. Single photoelectron method

In the single photoelectron method, which is commonly used as the standard method, the gain is calculated from the interval between neighboring photoelectron peaks (Fig. 1). Though it shows high reproducibility and high accuracy, this method can only be used when the photoelectron peaks are clearly separated, namely when the signal to noise ratio is good enough. The light intensity also has to be adjusted to observe several photoelectron peaks.

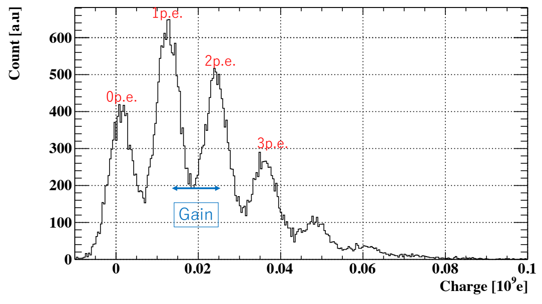


Fig. 1. Typical charge distribution of SiPM illuminated by pulsed LED light.

2. Statistical method

In the statistical method, the gain is calculated from the relation between the mean and the variance of charge distribution using a Poisson distributed light source [1]. As is well known, prompt

Geiger discharges initiated by photons can produce secondary Geiger discharges by the optical crosstalk. In the following, the number of prompt Geiger discharges is assumed to follow Poisson distribution with mean μ and the number of crosstalks generated from a discharge is assumed to follow Borel distribution with the Borel-branching parameter λ. As a result, the statistics of the number of Geiger discharges (N_{pulse}) is modified to generalized Poisson distribution

$$P_{\mu,\lambda}(N_{pulse} = k) = \frac{\mu \cdot (\mu + k \cdot \lambda)^{k-1} \cdot e^{-(\mu + k\cdot\lambda)}}{k!}. \tag{1}$$

In Eq.(1), the mean and the variance of N_{pulse} are $\mu/(1-\lambda)$ and $\mu/(1-\lambda)^3$, respectively. This leads to the following equation

$$mean = Gain \times \frac{\mu}{1-\lambda} \qquad var = Gain^2 \times \frac{\mu}{(1-\lambda)^3}, \tag{2}$$

where *mean* and *var* are the mean and the variance of the detected charge distribution. Therefore, we obtain the gain from the following equation,

$$Gain = (1-\lambda)^2 \times \frac{var}{mean} = \frac{1}{ENF^2} \times \frac{var}{mean}, \tag{3}$$

where ENF is the excess noise factor defined by the following equation

$$ENF = \mu\left(\frac{var}{mean^2}\right). \tag{4}$$

μ in the above equation can be estimated as follows

$$\mu = -\log(f_0), \tag{5}$$

where f_0 is the fraction of the zero-photoelectron events. In Ref. [1], ENF is assumed to be constant as long as the gain is the same. The after-pulse is not considered in this discussion, but according to Ref. [1], the afterpulse effect is negligible when the afterpulse probability is below 25%, which is the case when the overvoltage of a SiPM is at a reasonable level. The advantage of this method is that *mean* and *var* can be measured even when the noise level is relatively high. The disadvantage is that ENF must be determined independently, which still requires measurement with low intensity light.

Fig.2 shows the relation between *var* and *mean* obtained by varying intensity of low-level light. Non-linear relation is found between *mean* and *var*, as opposed to the suggestion in Ref. [1]. However, using the slope around the origin in Fig. 2 to evaluate $var/mean$ and dividing it by ENF^2, we successfully obtained gain consistent with the single photoelectron method in a wide range of the gain as shown in Fig. 3. ENF used in this calculation was determined from measurements where μ lies between 2 to 5.

3. Waveform method

The basic idea of the waveform method proposed in Ref. [2] is to count the number of Geiger discharges (N_{pulse}) by analyzing the waveform statistically under the assumption that the height of the waveform is proportional to the gain. In this method, individual pulses coming from each Geiger discharge are separated by taking the time derivative (Fig. 4). After separation, $Noisepower$ is calculated as

$$Noisepower := \int \left(\frac{\mathrm{d}^n V(t)}{\mathrm{d}t^n}\right)^2 \mathrm{d}t, \tag{6}$$

where $V(t)$ is the pulse height of the signal at time t. Since the height of each pulse is assumed to be

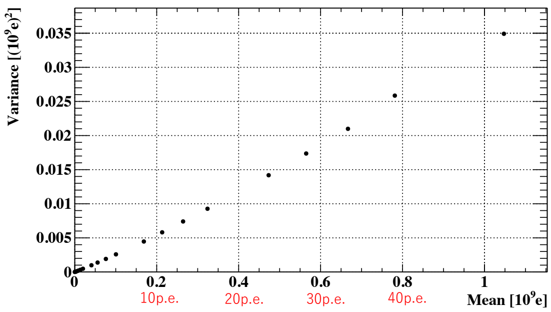


Fig. 2. *var* v.s. *mean* plot is obtained at various light intensity. Non-linear relation can be seen.

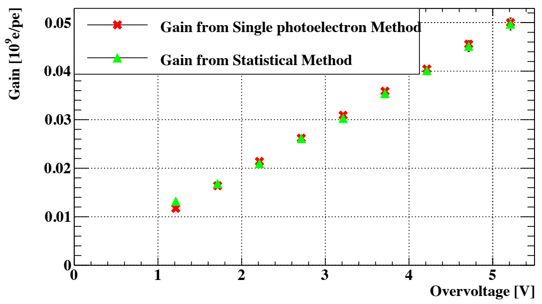
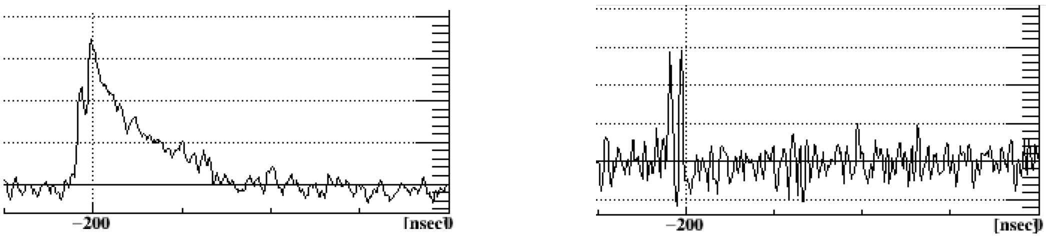


Fig. 3. Gain calculated from single photoelectron method (red cross marker) and statistical method (green trianglular marker).



(a) raw waveform

(b) 1st differentiated waveform

Fig. 4. An example of clearly separated pulse. Two partly overlapping signals found in the raw waveform (a), are clearly separated by taking the time derivative (b).

proportional to the gain, the following relations are satisfied when individual pulses are well separated

$$\int \left(\frac{\mathrm{d}^n V(t)}{\mathrm{d}t^n} \right)^2 \mathrm{d}t \propto Gain^2 \cdot N_{pulse} \tag{7a}$$

$$\int V(t)\,\mathrm{d}t = Gain \cdot N_{pulse}, \tag{7b}$$

where Eq.(7b) corresponds to the charge. By dividing Eq.(7a) by Eq.(7b), a quantity proportional to the gain can be obtained. In Ref. [2], this method has been successfully demonstrated for PMTs in an environment where the noise and the overlapping of the pulses are well suppressed. The advantage of this method is that no dedicated data-taking for calibration is required because the constant intensity light is not necessary in contrast to the statistical method. This advantage allows us to monitor the gain using any data taken in the experiment. The disadvantage is that only relative gain can be obtained in this method.

We applied this method to SiPMs for the first time as far as we know. To apply this method to SiPMs, the discussion in the above must be extended because there are noises specific to SiPMs. When white noise is added to $V(t)$, the Eq.(7a) is modified as follows:

$$\int \left(\frac{\mathrm{d}^n V(t)}{\mathrm{d}t^n} \right)^2 \mathrm{d}t \propto Gain^2 \cdot N_{pulse} + offset, \tag{8}$$

where $offset$ depends on the strength of the noise. Moreover, the overlapping of pulses also has to be kept in mind because prompt crosstalk pulses fully overlap with the primary pulse. When the pulses are fully overlapped, the following equation holds:

$$\int \left(\frac{\mathrm{d}^n V(t)}{\mathrm{d}t^n} \right)^2 \mathrm{d}t \propto Gain^2 \cdot N_{pulse}^2, \tag{9}$$

which is expected to contribute partly to the $Noisepower$ as a quadratic term of N_{pulse}.

The relation between $charge$ and $Noisepower$ measured with SiPM is shown in Fig. 5. The second-order derivative was taken in the waveform analysis, namely with $n = 2$ in Eq.(6). In general,

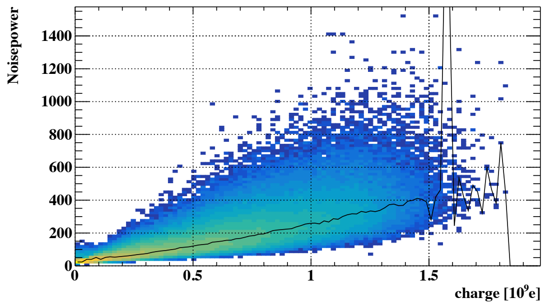


Fig. 5. Two dimensional histogram of $charge$ and $Noisepower$. $n = 2$ was used for calculation in Eq(6). The black line describes the ”ridge” of the distribution.

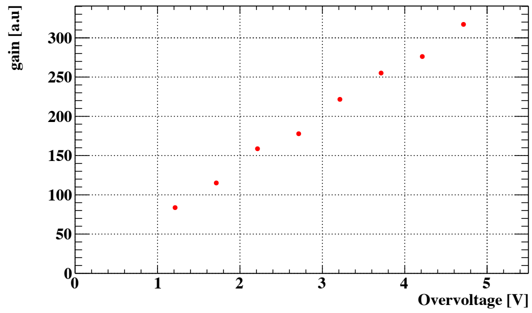


Fig. 6. Comparison between the overvoltage (X axis) and the gain in this method (Y axis), where a clear linear correlation can be seen.

n must be chosen so as to optimize the pulse separation and the signal to noise ratio. In Fig. 5, non-linearity between $Noisepower$ and $charge$ is seen in the strong light region, which appears to be the effect of overlapping ($c.f.$ Eq.9). To evaluate $Noisepower/charge$ (which corresponds to the gain in this method), the slope in Fig. 5 around the origin is used, where the effect of overlapping is expected to be minimized. Fig.6 shows the relation between overvoltage and the slope at the origin. It indicates that the performance of this method is good enough to monitor the SiPM gain.

4. Conclusion

The statistical method and the waveform method are studied as SiPM gain calibration methods alternative to the single photoelectron method. Though non-linearity between the mean and the variance of detected charge was observed as opposed to the discussion in Ref. [1], the gain obtained in the statistical method was found to be consistent with the single photoelectron method. The waveform method [2] has been applied to SiPM for the first time in this study and it was found to work for SiPMs by taking into account the effects of white noise and crosstalk. It is concluded that this method can be used to monitor the SiPM gain.

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