Matter without matter: pure gravitational creation

[a,b,c]Hideki Maeda[1] and [d]Naresh Dadhich[2]

[a]*Graduate School of Science and Engineering, Waseda University, Tokyo 169-8555, Japan*
[b]*Department of Physics, Rikkyo University, Tokyo 171-8501, Japan*
[c]*Department of Physics, International Christian University, 3-10-2 Osawa, Mitaka-shi, Tokyo 181-8585, Japan*
[d]*Inter-University Centre for Astronomy & Astrophysics, Post Bag 4, Pune 411 007, India*

Abstract

We obtain a new exact black-hole solution in Einstein-Gauss-Bonnet gravity with a cosmological constant which bears a specific relation to the Gauss-Bonnet coupling constant. The spacetime is a product of the usual 4-dimensional manifold with a $(n-4)$-dimensional space of constant negative curvature, i.e., its topology is locally $\mathcal{M}^n \approx \mathcal{M}^4 \times \mathcal{H}^{n-4}$. The solution has two parameters and asymptotically approximates to the field of a charged black hole in anti-de Sitter spacetime. The most interesting and remarkable feature is that the Gauss-Bonnet term acts like a Maxwell source for large r while at the other end it regularizes the metric and weakens the central singularity. It is a pure gravitational creation including Maxwell field in four-dimensional vacuum spacetime. The solution has been generalized to make it radially radiate null radiation representing gravitational creation of charged null dust. This paper is based on the results in [1].

1 Model and basic equation

Throughout this paper we use units such that $c = 1$. The Greek indices run $\mu = 0, 1, \cdots, n-1$. We write action of Einstein-Gauss-Bonnet gravity with a cosmological constant for $n \ge 5$,

$$S = \int d^n x\sqrt{-g}\biggl[\frac{1}{2\kappa_n^2}(R - 2\Lambda + \alpha L_{GB})\biggr] + S_{\rm matter}, \tag{1}$$

where α is the Gauss-Bonnet (GB) coupling constant and all other symbols having their usual meaning. The GB Lagrangian is given by

$$L_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}. \tag{2}$$

This form of action follows from low-energy limit of heterotic superstring theory [2]. In that case, α is identified with the inverse string tension and is positive definite, so we assume $\alpha \ge 0$ in this paper. It should be noted that L_{GB} makes no contribution in the field equations for $n \le 4$.

The gravitational equation following from the action (1) is given by

$$\mathcal{G}^\mu{}_\nu \equiv G^\mu{}_\nu + \alpha H^\mu{}_\nu + \Lambda\delta^\mu{}_\nu = \kappa_n^2 T^\mu{}_\nu, \tag{3}$$

where

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R, \tag{4}$$

$$H_{\mu\nu} \equiv 2\Bigl[RR_{\mu\nu} - 2R_{\mu\alpha}R^\alpha_{\ \nu} - 2R^{\alpha\beta}R_{\mu\alpha\nu\beta} + R_\mu^{\ \alpha\beta\gamma}R_{\nu\alpha\beta\gamma}\Bigr] - \frac{1}{2}g_{\mu\nu}L_{GB}. \tag{5}$$

We consider the n-dimensional spacetime locally homeomorphic to $\mathcal{M}^4 \times \mathcal{K}^{n-4}$ with the metric, $g_{\mu\nu} = {\rm diag}(g_{AB}, r_0^2\gamma_{ab})$, $A, B = 0, \cdots, 3;\ a, b = 4, \cdots, n-1$. Here g_{AB} is an arbitrary Lorentz metric on

[1]E-mail:hideki@gravity.phys.waseda.ac.jp
[2]E-mail:nkd@iucaa.ernet.in

$\mathcal{M}^4$, r_0 is a constant and γ_{ab} is the unit metric on the $(n-4)$-dimensional space of constant curvature $\mathcal{K}^{n-4}$ with its curvature $\bar{k}=\pm1,0$. Then $\mathcal{G}^\mu{}_\nu$ gets decomposed as follows:

$$\begin{aligned} \mathcal{G}^A{}_B &= \left[1+\frac{2\bar{k}\alpha(n-4)(n-5)}{r_0^2}\right]{}^{(4)}G^A{}_B \\ &+\left[\Lambda-\frac{\bar{k}(n-4)(n-5)}{2r_0^2}-\frac{\bar{k}^2\alpha(n-4)(n-5)(n-6)(n-7)}{2r_0^4}\right]\delta^A_B, \end{aligned} \tag{6}$$

$$\begin{aligned} \mathcal{G}^a{}_b &= \delta^a{}_b\left[-\frac{1}{2}{}^{(4)}R+\Lambda-\frac{(n-5)(n-6)\bar{k}}{2r_0^2}\right. \\ &\left.-\alpha\left\{\frac{\bar{k}(n-5)(n-6)}{r_0^2}{}^{(4)}R+\frac{1}{2}{}^{(4)}L_{GB}+\frac{(n-5)(n-6)(n-7)(n-8)\bar{k}^2}{2r_0^4}\right\}\right], \end{aligned} \tag{7}$$

where the superscript (4) means the geometrical quantity on $\mathcal{M}^4$.

The decomposition leads to a general result in terms of the following no-go theorem on $\mathcal{M}^4$:

Theorem 1 *If (i) $r_0^2=-2\bar{k}\alpha(n-4)(n-5)$ and (ii) $\alpha\Lambda=-(n^2-5n-2)/[8(n-4)(n-5)]$, then $\mathcal{G}^A{}_B=0$ for $n\geq 6$ and $\bar{k}$ and Λ being non-zero.*

The proof simply follows from substitution of the conditions (i) and (ii) in Eq. (6).

These conditions also imply for $\alpha>0$, $\bar{k}=-1$ and $\Lambda<0$. Hereafter we set $\bar{k}=-1$, i.e., the local topology of the extra dimensions is $\mathcal{H}^{n-4}$, and obtain the vacuum solution ($T^\mu{}_\nu\equiv0$) satisfying the conditions (i) and (ii). The governing equation is then a single scalar equation on $\mathcal{M}^4$, $\mathcal{G}^a{}_b=0$, which is given by

$$\frac{1}{n-4}{}^{(4)}R+\frac{\alpha}{2}{}^{(4)}L_{GB}+\frac{2n-11}{\alpha(n-4)^2(n-5)}=0. \tag{8}$$

2 Exact solutions

2.1 Schwarzschild-like solution

We seek a static solution with the metric on $\mathcal{M}^4$ reading as:

$$g_{AB}dx^Adx^B=-f(r)dt^2+\frac{1}{f(r)}dr^2+r^2d\Sigma^2_{2(k)}, \tag{9}$$

where $d\Sigma^2_{2(k)}$ is the unit metric on $\mathcal{K}^2$ and $k=\pm1,0$. Then, Eq. (8) yields the general solution for the function $f(r)$:

$$f(r)=k+\frac{r^2}{2(n-4)\alpha}\left[1\mp\sqrt{1-\frac{2n-11}{3(n-5)}+\frac{4(n-4)^2\alpha^{3/2}\mu}{r^3}-\frac{4(n-4)^2\alpha^2q}{r^4}}\right], \tag{10}$$

where μ and q are arbitrary dimensionless constants. The solution does not have the general relativistic limit $\alpha\to0$. There are two branches of the solution indicated by sign in front of the square root in Eq. (10), which we call the minus- and plus-branches.

There exists a central curvature singularity at $r=0$ as well as the branch singularity at $r=r_\mathrm{b}>0$ where the term inside the square root in Eq. (10) is zero. This solution can represent a black hole depending on the parameters. The n-dimensional black hole with $(n-4)$-dimensional compact extra dimensions is called the Kaluza-Klein black hole. The warp-factor of the submanifold r_0^2 is proportional to GB parameter α which is supposed to be very small. Thus, compactifying $\mathcal{H}^{n-4}$ by appropriate identifications, we obtain the Kaluza-Klein black-hole spacetime with small and compact extra dimensions.

The function $f(r)$ is expanded for $r\to\infty$ as

$$f(r)\approx k\mp\frac{\alpha^{1/2}\mu\sqrt{3(n-4)(n-5)}}{r}\pm\frac{\alpha q\sqrt{3(n-4)(n-5)}}{r^2}+\frac{r^2}{2(n-4)\alpha}\left(1\mp\sqrt{\frac{n-4}{3(n-5)}}\right). \tag{11}$$

This is the same as the Reissner-Nordström-anti-de Sitter (AdS) spacetime for $k = 1$ in spite of the absence of the Maxwell field. This suggests that μ is the mass of the central object and q is the charge-like new parameter.

Further, the solution (10) agrees with the solution in the Einstein-GB-Maxwell-Λ system having the topology of $\mathcal{M}^n \approx \mathcal{M}^2 \times \mathcal{K}^{n-2}$ although it does not admit $n = 4$. The solution is given for $n \geq 5$ by

$$ds^2 = -g(r)dt^2 + \frac{1}{g(r)}dr^2 + r^2 d\Sigma^2_{n-2(k)} \tag{12}$$

with

$$g(r) = k + \frac{r^2}{2(n-3)(n-4)\alpha}\left[1 \mp \sqrt{1 + \frac{8(n-3)(n-4)\alpha\Lambda}{(n-1)(n-2)} + \frac{8(n-3)(n-4)\kappa_n^2\alpha M}{(n-2)V^{\bar{k}}_{n-2}r^{n-1}} - \frac{(n-4)\alpha\kappa_n^2 Q^2}{(n-2)\pi g_c^2 r^{2(n-2)}}}\right], \tag{13}$$

where g_c is the coupling constant of the Maxwell field, and M and Q are mass and charge respectively [3]. k is the curvature of $\mathcal{K}^{n-2}$ and a constant V^k_{n-2} is its surface area on compactifications. The non-zero component of the Maxwell field reads as

$$F_{rt} = \frac{Q}{r^{n-2}} \tag{14}$$

representing the coulomb force of a central charge in n-dimensional spacetime.

Thus the parameters μ and q act as mass and "charge" respectively in spite of the absence of the Maxwell field. The new "gravitational charge" q is generated by our choice of the topology of spacetime, splitting it into a product of the usual 4-spacetime and a space of constant curvature. Thus, the solution (10) manifests gravitational creation of the Maxwell field, i.e., "matter without matter".

Clearly the global structure of our solution (10) will be similar to that of the solution (13). Note that $f(0) = k \mp \sqrt{-q}$, which produces a solid angle deficit and it represents a spacetime of global monopole. This means that at $r = 0$ curvatures will diverge only as $1/r^2$ and so would be density which on integration over volume will go as r and would therefore vanish. This indicates that singularity is weak as curvatures do not diverge strongly enough.

2.2 Vaidya-like solution

It is well known that Schwarzschild spacetime could be made to radiate null (Vaidya) radiation by transforming the metric into retarded/advanced time coordinate and then making mass parameter function of the time coordinate. It is interesting to note that the same procedure also works here. This solution (10) can thus be generalized to include Vaidya radiation and it would be given by

$$g_{AB}dx^A dx^B = -\tilde{f}(v,r)dv^2 + 2dvdr + r^2 d\Sigma^2_{2(k)}, \tag{15}$$

$$\tilde{f}(v,r) \equiv k + \frac{r^2}{2(n-4)\alpha}\left[1 \mp \left\{1 - \frac{2n-11}{3(n-5)} + \frac{4(n-4)^2\alpha^{3/2}\tilde{M}(v)}{r^3} - \frac{4(n-4)^2\alpha^2\tilde{q}(v)}{r^4}\right\}^{1/2}\right], \tag{16}$$

where $\tilde{M}(v)$ and $\tilde{q}(v)$ are arbitrary functions. As expected, this solution is quite similar to the null dust solution with the topology of $\mathcal{M}^2 \times \mathcal{K}^{n-2}$ [4].

This solution manifests gravitational creation of an ingoing charged null dust as another complete example of "matter without matter". Using this solution and the solution (10), we can construct completely vacuum spacetime representing the formation of a black hole from an AdS spacetime by gravitational collapse of a gravitationally created charged null dust.

3 Discussions and conclusion

In this paper, we obtained new exact solutions in Einstein-Gauss-Bonnet gravity which offer direct and purely classical examples of curvature manifesting as matter, i.e., "matter without matter". The origins of the Maxwell field and a null dust fluid have been proposed.

We have found a new Kaluza-Klein vacuum black hole solution (10) of Einstein-Gauss-Bonnet gravity with topology of product of the usual 4-spacetime with a negative constant curvature space. In this solution we have brought the GB effects down on four dimensional black hole. This solution manifests gravitational creation of the Maxwell field and asymptotically resembles a charged black hole in AdS background. What really happens is that GB term regularizes the metric and weakens the singularity while the presence of extra dimensional hyperboloid space generates the Kaluza-Klein modes giving rise to the Weyl charge. This is indeed the most interesting and remarkable feature of the new solution which needs to be probed further for greater insight and application. The global structure of the solution (10) depending on the parameters will be shown in the forthcoming paper. Also, we have successfully generalized this solution into Vaidya-like metric on $\mathcal{M}^4$. That solution manifests gravitational creation of an ingoing charged null dust.

Now we explain the origin of "matter without matter". For the metric in the form of Eq. (9), one just requires one second-order differential equation (8) to determine the metric fully and it will in general have two constants of integration. On the other hand, the trace of the Einstein-Gauss-Bonnet equation (3) is given by

$$-\frac{n-2}{2}R - \frac{(n-4)\alpha}{2}L_{GB} + n\Lambda = \kappa_n^2 T. \tag{17}$$

The basic equation (8) for g_{AB} resembles this equation with $T = 0$ and $\Lambda = \Lambda_{\rm eff}$ defined by

$$\Lambda_{\rm eff} \equiv -\frac{C(2n-11)}{\alpha(n-4)^2(n-5)}, \tag{18}$$

where C is some positive constant. Thus, Eq. (8) will generate a Maxwell-like charge as well as a null dust because vanishing trace is characteristic of a null dust and the Maxwell field in four dimensions. That is why it is not surprising that there occur Maxwell-like additional gravitational charge or a gravitationally created null dust in our solution. It is noted that this happens only in four dimensions because electromagnetic stress tensor is not trace-free in other dimensions.

In the original Kaluza-Klein theory, the origin of the Maxwell field is the extra-dimensional component of the five-dimensional metric with which the five-dimensional vacuum Einstein equation is decomposed into the four-dimensional Einstein-Maxwell equation [5]. Here on the other hand, we have given completely different and novel generation of Maxwell field as well as of null dust fluid in the framework of Einstein-Gauss-Bonnet gravity. This is a partial success to explain origin of all matter in our four-dimensional universe. Since our mechanism works only for trace free matter fields, creation of other matter, especially with non-zero trace remains a very important open problem. Undoubtedly its solution will have a great bearing on our understanding of spacetime and matter.

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