

Combination of Single Top Quark Production Results from CDF

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Recently, the CDF experiment at the Fermilab Tevatron has used complementary methods to make multiple measurements of the singly produced top quark cross section. All analyses use the same dataset with more than 2 fb^{-1} of CDF data and event selection based on $W + 2$ or $W + 3$ jet events with at least one b -tagged jet. However, due to differences in analysis techniques these results are not fully correlated and a combination provides improved experimental precision. Two independent methods are used to combine the results. This combination results in an improved measurement of the single top production cross section and also the CKM matrix element V_{tb} .

1. Single Top Results from CDF

Measuring the properties of single top production at the Tevatron is challenging. The production cross section is smaller than top pair production and the final state is harder to distinguish from major backgrounds. The background rates are large; in fact, the error on the background prediction is larger than the predicted single top signal. In order to improve separation of signal from background three multivariate techniques were used at CDF: a likelihood function (LF), a matrix element (ME), and a neural network (NN) [1].

In each analysis the multivariate discriminant was used to build template distributions for signal and background expectations and the data was fit to extract the signal component. Although the analyses use the same event selection, they rely on different observables to discriminate signal from background which means that combination of results should provide additional sensitivity to single top quark production. Two very different strategies were developed to combine the CDF single top analyses.

2. NEAT Combination

The NEAT combination method takes the discriminating variable from each analysis and combines them into one “super-analysis”. The new super-discriminant is a neural network in which the weights and topology are optimized for sensitivity by using a technique known as neuro-evolution of augmenting topologies (NEAT) [2]. A separate discriminant is optimized for events with two or three jets and events with one or two b -tags. As in the individual analyses, templates based on the event-by-event output of NEAT are built for signal and background expectations and the data is fit to these to extract the most likely single top component. Figure 1 shows the expected composition of signal and background as well as the CDF data for all tag and jet channels combined.

3. BLUE Combination

As a cross check to the NEAT combination, the method of forming a Best Linear Unbiased Estimate, BLUE, is used to make a weighted average of the three single top cross section measurements [3]. BLUE is based on a simple

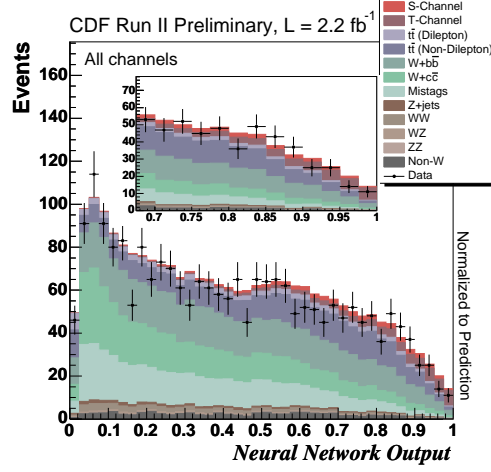


Figure 1: NEAT output of the background prediction and CDF data.

χ^2 method:

$$\chi^2(\mu) = \sum_i^N \left(\frac{m_i - \mu}{\sigma_i} \right)^2 \quad \text{or} \quad \chi^2(\mu) = \delta^T(\mu) \cdot \mathbf{S}^{-1} \cdot \delta(\mu) \quad (1)$$

where m_i and σ_i are the measured value and error of the i th analysis, μ is the average of the measurements, $\delta(\mu)$ is the column vector $\delta_i = m_i - \mu$, and \mathbf{S} is the covariance matrix. In this case, \mathbf{S} is diagonal with $\mathbf{S}_{(i,i)} = \sigma_i^2$. BLUE is based upon the formula above, but in order to combine N measurements which each have E different types of errors, \mathbf{S} is defined as

$$\mathbf{S} = \sum_e^E \mathbf{S}_e \quad \text{and} \quad \mathbf{S}_{e(i,j)} = \sigma_{ei} \cdot \sigma_{ej} \cdot \rho_{e(i,j)} \quad (2)$$

where σ_{ei} is the e th type of error on the i th measurement and $\rho_{e(i,j)}$ is the correlation of the e th error between the i th and j th measurement.

The beauty of BLUE lies in the fact that you don't have to run a minimization routine on this χ^2 to get the mean. After defining \mathbf{H} to be the inverse of the covariance matrix and sum_H as below, a weight for each measurement, w_i , can then be calculated:

$$\text{Define } \mathbf{H} \equiv \mathbf{S}^{-1} \quad \text{and} \quad sum_H \equiv \sum_{i=1}^N \sum_{j=1}^N \mathbf{H}_{i,j} \quad \text{so that} \quad w_i \equiv \sum_{j=1}^N \frac{\mathbf{H}_{i,j}}{sum_H} \quad (3)$$

BLUE then predicts

$$\mu_{\text{best}} = \sum_{i=1}^N w_i \cdot m_i \quad \text{and} \quad \sigma = \frac{1}{\sqrt{sum_H}} \quad (4)$$

where μ_{best} is the minimum of the χ^2 and σ is the total error.

BLUE needs the measured values, uncertainties, and correlations between each analysis before the combined result can be calculated. Correlated pseudo-experiments are thrown for each analysis and the correlations of the resulting cross sections measured by each analysis are summarized in Table I.

In general, errors may have dependence on the value measured. Analyses which measure lower than expected could have a smaller error and this may bias the combination. In order to avoid this bias we run BLUE iteratively. Another complication is that errors need not be symmetric. Since BLUE is based on a Gaussian approximation it is

	LF	ME	NN
LF (1.79 pb)	1.0	0.599	0.741
ME (2.17 pb)	—	1.0	0.609
NN (1.97 pb)	—	—	1.0

Table I: Correlations coefficients between pairs of cross section values.

not able to treat this directly. To handle this we use Asymmetric Iterative BLUE (AIB). AIB is a set of three BLUE combinations which uses average errors (for central value), upper errors, or lower errors as follows:

$$\mathcal{R}_{\text{upper}} = \frac{\sigma_{\text{upper BLUE}}}{\sigma_{\text{upper BLUE}} + \sigma_{\text{lower BLUE}}} \quad (5)$$

$$\sigma_{\text{upper}} = 2 \cdot \mathcal{R}_{\text{upper}} \cdot \sigma_{\text{center BLUE}} \quad \text{and} \quad \sigma_{\text{lower}} = 2 \cdot (1 - \mathcal{R}_{\text{upper}}) \cdot \sigma_{\text{center BLUE}} \quad (6)$$

3.1. Results

The results of the the combination and each individual analysis are summarized in Figure 2. Fitting for the combined single-top cross section NEAT measures $\sigma_{s+t} = 2.2 \pm 0.7$ pb which corresponds to a measurement of $|V_{tb}| = 0.88 \pm 0.14(\text{exp}) \pm 0.07(\text{theory})$. In addition the NEAT combination improves expected sensitivity (pValue [4]) by about 9 %. The BLUE cross check measures a consistent cross section with neat and gains about 7 % in expected sensitivity. In addition, BLUE was used to study the consistency of the three analyses and finds that the combined measurement had a χ^2 which was better than 87 % of pseudo-experiments. Also, about 15 % of pseudo-experiments thrown with the standard model expectation of single top events measured a cross section below 2.1 pb. So, BLUE confirms that the three CDF measurements are highly compatible with one another and that the combined result represents a deviation from the standard model expectation of about 1σ .

In summary, the three CDF analyses have been combined with two different techniques resulting in an improved sensitivity and measurement of the single top production cross section.

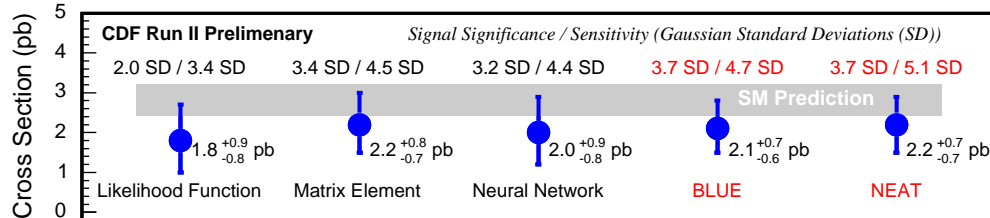


Figure 2: Cross section measurements and sensitivity results for each analysis and the BLUE and NEAT combinations using 2.2 fb^{-1} of CDF data.

References

- [1] T. Aaltonen *et al.* [CDF Collaboration], arXiv:0809.2581 [hep-ex].
- [2] Kenneth O. Stanley and Risto Miikkulainen, *Evolutionary Computation* **10** (2) 99-127, 2002.
- [3] L. Lyons, D. Gibaut, and P. Clifford, *NIM* **A270**, 110-117 (1988), L. Lyons, A. Martin, and D. Saxon, *Phys. Rev. D* **41**, 3 (1990)
- [4] W.M. Yao *et al.* (Particle Data Group), *J. Phys. G* **33**, 1 (2006).