Density dependent effective interactions and recollections of the Rutgers-Princeton years

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Abstract. The density-dependent effective interactions given by the new Brown-Rho (new-BR) scalings and chiral three-nucleon force V_{3N} are compared with the empirical density-dependent force of the Skyrme interactions. The new-BR scaling is based on a Skyrmion-half-Skyrmion model where nuclear matter is treated as a Skyrmion matter for density smaller than a transition density $n_{1/2} \simeq 0.32 fm^{-3}$, while a half-Skyrmion matter for density greater. In this model, the meson mass, nucleon mass and meson-nucleon coupling are all scaled with density, making the resulting two-nucleon interaction density dependent. By integrating out a participating nucleon over the Fermi sea, Holt, Kaiser and Weise have obtained an effective three-nucleon force $\bar{V}_{3N}$ which is also a density-dependent two-nucleon interaction. The equation of state for symmetric nuclear matter given by the new-BR-scaled V_{2N}, $\bar{V}_{3N}$, and (unscaled-V_{2N} + a Skyrme-type density-dependent force) are all found to be closely similar to each other.

1. Introduction

I have known Aldo for a very long time, starting from 1964 when he was a postdoc at the Rutgers University and I was one at the Princeton University. Rutgers and Princeton are located at nearby townships, separated by merely about 20 miles. Their nuclear physics groups were both very active and worked closely together. We had two joint seminars every week: Monday afternoon seminar at Rutgers and Thursday night Bull Session (which lasted typically more than 3 hours) at Princeton. The faculty members in these groups were A. Arima, Ben Bayman, Gerry Brown, J. de Boer, Ruby Sherr, Igal Talmi, George Temmer,... and among the young postdocs were Joe Ginocchio, Tony Green, G. Sartoris, Chun Wa Wong, Larry Zamick,... in addition to Aldo and me. We were having very good times together, including the well-remembered Rutgers-Princeton nuclear-physics soccer games.

At one of the joint seminars, the then newly developed Skyrme effective interaction was discussed. This interaction is of the form (see e.g. [1])

$$V_{sky} = \sum_{i<j} V(i,j) + \sum_{i<j<k} V_{3b}(i,j,k),$$
$$V_{3b} = t_3 \delta(\vec{r}_i - \vec{r}_j)\delta(\vec{r}_j - \vec{r}_k),$$
$$\rightarrow \frac{t_3}{6}(1 + x_3 P_\sigma)\delta(\vec{r}_1 - \vec{r}_2)\rho(\vec{r}_{av}) \equiv D_{sky} \tag{1}$$

where the last term D_{sky} is a 'density-dependent' zero-range nucleon-nucleon interaction. (Note that the strength of D_{sky} is typically rather strong such as $t_3 \simeq 14000$ MeV fm^6.[1]) Where does

D_{sky} come from **?** There were many discussions about this question at the seminar. At that time, I think nobody had an answer, except that it was empirically needed for nuclear saturation. (In other words, without this term the nuclear matter calculated with V_{sky} can not reproduce empirical nuclear matter saturation energy of $E_0/A \simeq -16$ MeV and density $n_0 \simeq 0.16$ fm^{-3}.)

After so many years, I think we are making progress toward answering the above question. As to be described later, density-dependent effective interactions are generated by both the Brown-Rho scalings [2–7] and/or chiral three-nucleon force V_{3N}[8, 9]. It is of interest to study if the effects of such interactions can be reproduced, to certain extents, by a Skyrme-type empirical density dependent force. We shall do so in this report. The organization of the present report is as follows. In Section 2 we shall describe the new-BR scaling [7] and briefly discuss its density dependent effects to nuclear matter and neutron stars. In Section 3 a similar description will be presented for the chiral three-nucleon force. We shall compare the density-dependent effects from new-BR scaling, chiral three-nucleon force, and an empirical Skyrme-type density dependent force. A summary will also be included in this section.

2. New Brown-Rho scaling
In the early version of the Brown-Rho scaling [2, 6] (to be referred to as the old-BR scaling), the scaling function $\Phi(n)$ of the form

$$\Phi(n) = \frac{m^*}{m} = 1 - C\frac{n}{n_0}, \quad C \simeq 0.15 \tag{2}$$

is employed. Here m^* denotes the meson mass in nuclear medium of density n, and m in free space. n_0 is the nuclear-matter saturation density ($0.16 fm^{-3}$). This scaling has had important and desirable effects in density region near n_0. With its inclusion, nuclear matter calculations have given satisfactory saturation properties [10, 11]. It has also played a key role in shell-model calculations for the extra-long life time (~ 5000yrs) of the $^{14}C - ^{14}N$ β-decay [12].

The above scaling is clearly meant for low densities only. (For instance, the above $\Phi(n)$ is undefined at high n.) In fact the nuclear equation of state (EOS) at densities considerably higher than n_0 is still, by far, largely uncertain. The new-BR scaling is an attempt of studying the high-density nuclear EOS, which is needed for describing neutron-star properties. It is based on a Skyrmion-half-Skyrmion lattice model [13, 14] where nuclear matter is found as composed of Skyrmions at densities below the transition density $n_{1/2}$, while as half-Skyrmions for densities above. It has been estimated that $n_{1/2}$ lies typically between 1.3 and $2n_0$ [15].

A first application of this Skyrmion-half-Skyrmion model to nuclear matter and neutron stars was carried out in [7]. In this model we have Skyrmions for $n \leq n_{1/2}$ (Region I) and half-Skyrmions for $n < n_{1/2} < n_\chi$ (Region II), where the chiral restoration density n_χ is set to be $10n_0$. The scaling functions for these two regions are different. We have for region-I

$$\frac{m_M^*}{m_M} = \frac{m_N^*}{m_N} = \Phi_I(n), \quad \Phi_I(n) = \frac{1}{1 + c_I\frac{n}{n_0}} \ (mesons, \ nucleons),$$

$$\frac{g^*}{g} = 1 \ (coupling \ constants \ unscaled), \tag{3}$$

and for region-II

$$\frac{m_M^*}{m_M} = \Phi_{II}(n), \quad \Phi_{II}(n) = \frac{1}{1 + c_{II}\frac{n}{n_0}} \ (mesons),$$

$$\frac{m_N^*}{m_N} = y(n) \simeq 0.8 \ (nucleons), \quad \frac{g^*}{g} = \Phi_{II}(n) \ (coupling \ constant \ g_{N\rho}). \tag{4}$$

Some differences between the above scaling and that of the old-BR scaling [10–12] may be mentioned. In the old-BR scaling, only the mass and cut-off parameter of the ρ, σ and ω mesons are scaled. In addition to these scalings, the nucleon mass and the nucleon-ρ coupling constant $g_{N\rho}$ are also scaled in new-BR as indicated above. Furthermore, in new-BR the scalings in regions I and II, are different, having parameters c_I and c_{II} for the two regions.
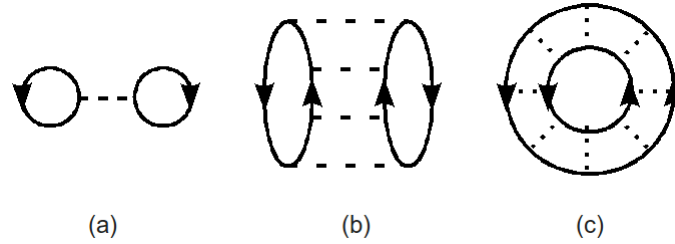

Figure 1. Diagrams included in the all-order $pphh$ ring-diagram summation.

We have used a V_{low-k} ring-diagram method [7, 10, 11] to calculate the nuclear EOS with the new-BR scaling. In this method the $pphh$ ring diagrams as shown in figure 1 are summed to all orders. There diagram (a), (b) and (c) are respectively a 1st-, 4-th- and 8-th-order $pphh$ ring diagram. Our method reduces to a V_{lowk} Hartree-Fock (HF) method if we include only diagram (a). Each vertex of the diagrams is a V_{lowk} interaction [16–19] derived from the Bonn potential [20] with its parameters scaled with density according to the new-BR scaling. (We have employed the Bonn potential as its nucleon-mass parameter can be conveniently scaled.)
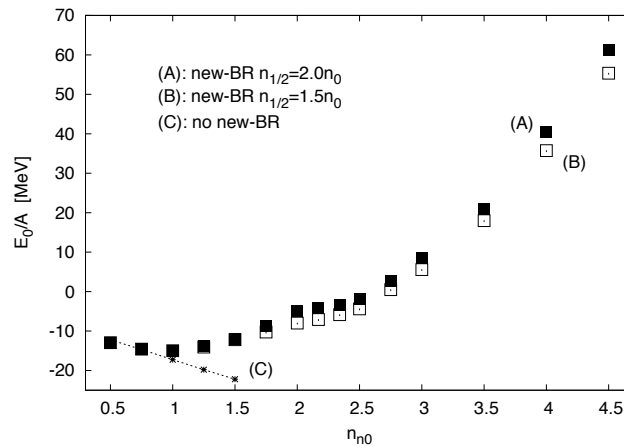

Figure 2. The EOS for symmetric nuclear matter calculated with new-BR scaling.

In figure 2 we present two EOSs so calculated for symmetric nuclear matter. Two choices for the transition density are used, $n_{1/2} = 2n_0$ (A) and $= 1.5n_0$ (B). As seen, the calculation without new-BR scaling (C) is unable to describe the nuclear matter saturation properties (it saturates at at density $\sim 2.5n_0$ and $E_0/A \simeq -22 MeV$). The region-I scaling parameters of $c_I(\rho) = c_I(N) = 0.130$, $c_I(\sigma) = 0.121$ and $c_I(\omega) = 0.139$ were used; these values are determined

by fitting the empirical nuclear matter properties. The coupling constant $g_{N\rho}$ is not scaled in region I, while it is scaled in region II. The EOS of figure 2 gives saturation properties E_0/A=-15.0 MeV, saturation density $n_{sat} = 0.93n_0$ and compression modulus $K = 206$MeV.

In the $n > n_{1/2}$ region, the meson scaling parameters are taken to be the same as in region I, namely $c_{II} = c_I$. But a different scaling is used for nucleons: in region II we use $m_N^*/m_N = y(n) = 0.77$ (0.78) respectively for $n_{1/2} = 2n_0$ (1.5n_0). These values are used to make the values of E_0/A just below and above $n_{1/2}$ approximately equivalent. But their slopes are clearly discontinuous at $n_{1/2}$, implying a Skyrmion half-Skyrmion phase transition there. In figure 3 we display the pressure-density EOS calculated with new-BR. ($p(n) = n[d\epsilon(n)/dn] - \epsilon(n)$, ϵ being the energy density.) As seen, there is discontinuity at $n_{1/2}$. In fact from the results of figure 3, the Skyrmion half-Skyrmion coexistance region can be determined. For instance, this coexistence region is $\sim 1.7n_0 < n < \sim 2.4n_0$ for the $n_{1/2} = 2n_0$ case.
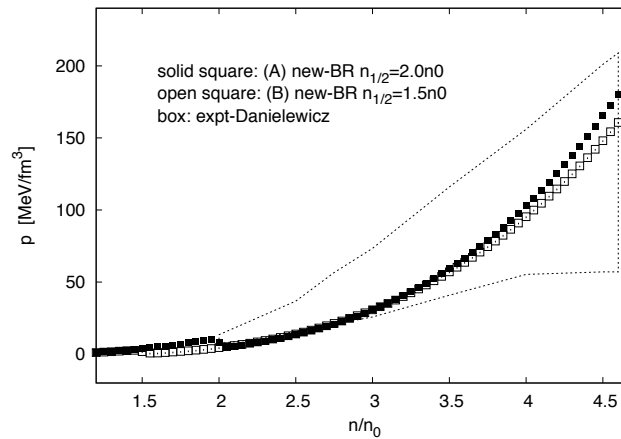

Figure 3. Pressure in symmetric nuclear matter calcualted with new-BR scaling.

In figure 3 our calculated pressure is compared with the empirical constraint of Danielewicz et al. [21]. It is encouraging that our calculated pressure is generally within the range allowed by the constraint. It would be very useful to check experimentally the Skyrmion half-Skyrmion phase transition. This may require much effort, as to locate the pressure 'kink' near $n_{1/2}$ would need a highly precise, and difficult, experimental determination of the nuclear matter pressure there . As another check of our EOS, we have also performed neutron star calcualtions using the new-BR EOS [7]. The calculated mass and radius of the maximum-mass neutron star are ($M = 2.39M_\odot$, $R = 10.9km$) for $n_{1/2} = 2.0n_0$, and ($M = 2.38M_\odot$, $R = 10.9km$) for $n_{1/2} = 1.5n_0$. These masses are considerably larger than the observed masses of $1.97 \pm 0.04M_\odot$ [22] and $2.01 \pm 0.04M_\odot$ [23], indicating that our new-BR neutron EOS is probably too stiff.

3. Chiral three-nucleon force and summary
As we see from the preceeding section, nuclear matter calculations with the new-BR scaled V_{2N} can satisfactorily describe nuclear matter saturation properties but not so with the unscaled V_{2N}. Can the calculations using V_{2N}(unscaled) plus V_{3N} also give satisfactory nuclear saturation

properties? In this section we shall address this question. We employ the lowest-order (NNLO) chiral V_{3N} of the form $V_{3N} = V_{3N}^{2\pi} + V_{3N}^{1\pi} + V_{3N}^{ct}$ where

$$V_{3N}^{(2\pi)} = \sum_{i\neq j\neq k} \frac{g_A^2}{8 f_\pi^4} \frac{\vec{\sigma}_i \cdot \vec{q}_i \, \vec{\sigma}_j \cdot \vec{q}_j}{(\vec{q}_i^{\,2} + m_\pi^2)(\vec{q}_j^{\,2} + m_\pi^2)} F_{ijk}^{\alpha\beta} \tau_i^\alpha \tau_j^\beta, \tag{5}$$

$$V_{3N}^{(1\pi)} = -\sum_{i\neq j\neq k} \frac{g_A c_D}{8 f_\pi^4 \Lambda_\chi} \frac{\vec{\sigma}_j \cdot \vec{q}_j}{\vec{q}_j^{\,2} + m_\pi^2} \vec{\sigma}_i \cdot \vec{q}_j \, \vec{\tau}_i \cdot \vec{\tau}_j, \tag{6}$$

$$V_{3N}^{(\mathrm{ct})} = \sum_{i\neq j\neq k} \frac{c_E}{2 f_\pi^4 \Lambda_\chi} \vec{\tau}_i \cdot \vec{\tau}_j, \tag{7}$$

where $g_A = 1.29$, $f_\pi = 92.4$ MeV, $\Lambda_\chi = 700$ MeV, $m_\pi = 138.04$ MeV/c^2, $\vec{q}_i = \vec{p}_i\,' - \vec{p}_i$ is the difference between the final and initial momentum of nucleon i and

$$\begin{aligned} F_{ijk}^{\alpha\beta} &= \delta^{\alpha\beta}\left(-4c_1 m_\pi^2 + 2c_3 \vec{q}_i \cdot \vec{q}_j\right) \\ &+ c_4 \epsilon^{\alpha\beta\gamma} \tau_k^\gamma \vec{\sigma}_k \cdot (\vec{q}_i \times \vec{q}_j). \end{aligned} \tag{8}$$

The parameters $c_1 = -0.76\,\mathrm{GeV}^{-1}$, $c_3 = -4.78\,\mathrm{GeV}^{-1}$, $c_4 = 3.96\,\mathrm{GeV}^{-1}$ are well known, constrained by low-energy NN phase shifts.[24] But the parameters c_D and c_E are not well determined. A range of c_D and c_E values on the Navratil $c_D - c_E$ curve [25] can all fit well the binding energies of ^3H and ^3He.

By integrating out one participating nucleon over the Fermi sea, Holt, Kaiser and Weise [8, 9] have reduced V_{3N} to a density-dependent 2-body force $\bar{V}_{3N}$. Comparing with V_{3N}, $\bar{V}_{3N}$ is much more convenient for nuclear matter calculations. Briefly speaking, they are related by

$$V_{3N} = \frac{1}{36} \Sigma \langle 123|V_{3N}|456\rangle a_3^+ a_2^+ a_1^+ a_4 a_5 a_6,$$

$$\bar{V}_{3N} = \frac{1}{4} \Sigma \langle 12|D_{2N}|45\rangle \, a_2^+ a_1^+ a_4 a_5,$$

$$\langle ab|D_{2N}|cd\rangle = \sum_{h\leq k_F} \langle abh|V_{3N}|cdh\rangle. \tag{9}$$

It is seen that $\bar{V}_{3N}$ is a density (k_F) dependent 2-body interaction. Note well there is a n-body counting factor $C(nb)$ to be included in calculations: We have $C(nb) = (1, 1/2,$ and $1/3)$ respectively for (2-, 1-, 0-)body vertices. Thus the vertex in (a) of figure 1 is $(V_{2N} + \bar{V}_{3N}/3)$, and each vertex in (b) and (c) is $(V_{2N} + \bar{V}_{3N})$.

We have performed ring-diagram nuclear matter calculations with $(V_{2N} + \bar{V}_{3N})$. The parameters c_D and c_E of V_{3N} (see Eqs.(6-7)) are not well known; they can have a range of values as allowed by the Navratil curve [25]. We have used several sets of allowed c_D and c_E parameters in our ring-diagram nuclear matter calculations and found that the allowed parameters c_D=3.0 and c_E=0.2 are a 'best-fit' choice (among the several sets calculations we have made) in reproducing nuclear saturation properties. We have used these parameters in the present calculation, and with them the saturation propertes given by the $(V_{2N} + \bar{V}_{3N})$ EOS are $(E_0/A = -15.6$ MeV, $n_{sat} = 0.98 n_0$, $K = 132$ MeV). Neutron star calculations using the $(V_{2N} + \bar{V}_{3N})$ EOS have also been carried out. The mass and radius of the maximum-mass neutron star so obtained are $(1.99 M_\odot, 9.85$ km); that these values are both smaller than the new-BR results mentioned earlier (near figure 3) indicates the new-BR neutron EOS is stiffer than the corresponding $(V_{2N} + \bar{V}_{3N})$ EOS especially at high densities.

As a summary, let us compare the three EOSs in figure 4. The results from new-BR$(n_{1/2} = 2n_0)$ and $(V_{2N} + V_{3N})$ are remarkably similar to each other, especially for densities
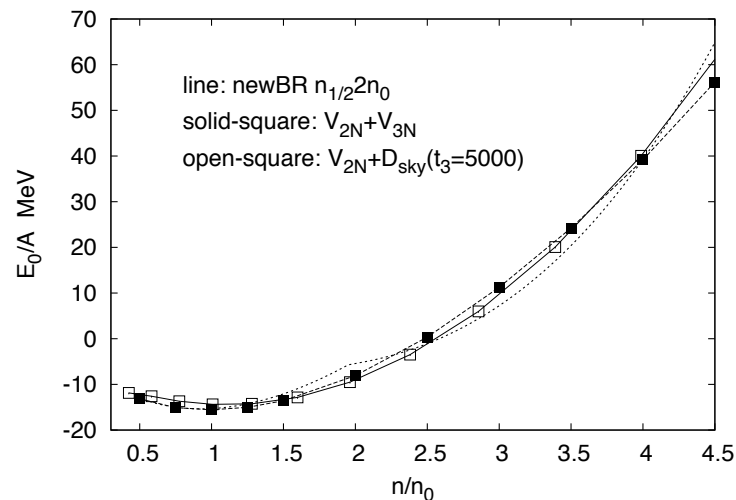
Figure 4. Comparision of ring-diagram nuclear matter calculations.

near n_0. Nuclear matter calculations with V_{2N} alone are unable to give satisfactory saturation properties. and this shortcoming can be amended by including either new-BR or V_{3N}. There may be some underlying equivalence between new-BR and V_{3N}, and its further study will be interesting as well as useful. In figure4, we also compare the above EOSs with the EOS given by (unscaled-V_{2N} + D_{sky}) with t_3=5000 MeV (see equation(1)). It is a 'surprise' that all three are in good qualitative agreement. That the density dependent effects on symmetric nuclear matter from new-BR and/or V_{3N} may be well reproduced by an empirical density-dependent force of the Skyrme type is an encouraging result, indicating that new-BR and/or V_{3N} may provide a microscopic foundation for the empirical Skyrme force.

4. Acknowledgements

References
[1] Ring P and Schuck P, *The Nuclear Many-Body Problem* (Springer-Verlag, New York, 1980)
[2] Brown G E, Rho M 1991 *Phys. Rev. Lett* **66** 2720
[3] Hatsuda T and Lee S H 1992 *Phys. Rev. C* **46** R34
[4] Brown G E, Rho M 1996 *Phys. Rep.* **269** 333
[5] Brown G E, Rho M 2002 *Phys. Rep.* **363** 85
[6] Brown G E, Rho M,2004 *Phys. Rep.* **396** 1
[7] Dong H, Kuo T T S, Lee H K, Machleidt R, Rho M 2013 *Phys. Rev. C* **87**, 054332
[8] Holt J W, Kaiser N and Weise W 2009 *Phys. Rev. C* **79**, 054331

[9] Holt J W, Kaiser N and Weise W 2010 *Phys. Rev. C* **81**, 0240021
[10] Siu L W, Holt J W, Kuo T T S and Brown G E 2009 *Phys. Rev. C* **79**, 0540004
[11] Dong H, Kuo T T S and Machleidt R 2009 *Phys. Rev. C* **80**, 065803
[12] Holt J W, Brown G E, Kuo T T S, Holt J D and Machleidt R 2008 *Phys. Rev. Lett* **100**, 062501
[13] Golhaber A S and Manton N S 1987 *Phys. Lett.* **B198**, 231
[14] Lee H K, Park B Y and Rho M 2011 *Phys. Rev. C* **83**, 025206
[15] Park B Y, Min D P, Rho M and Vento V 2002 *Nucl. Phys. A* **707**, 381; Lee H J*et al.* 2003 *Nucl. Phys. A*
     723, 427; Rho M, arXiv:0711.3895 [nucl-th]
[16] Bogner S K, Kuo T T S and Coraggio L 2001 *Nucl. Phys. A* **684** 432
[17] Bogner S K, Kuo T T S, Coraggio L, Covello A and Itaco N 2002 *Phys. Rev. C* **65** 051301R
[18] Bogner S K, Kuo T T S and Schwenk A 2003 *Phys. Rep.* **386** 1
[19] Holt J D, Kuo T T S and Brown G E 2004 *Phys. Rev. C* **69** 034329
[20] Machleidt R *Computational Nuclear Physics 2–Nuclear Reactions* (Langanke, Maruhn, Koonin, eds., Springer
     NY 1993), Chap.1, p.1
[21] Danielewicz P, Lacey R, and Lynch W A 2002 *Science* **298**, 1592
[22] Demorest P B *et. al* 2010 *Nature* **Vol. 467** 1081
[23] Antoniadis J *et. al* 2013 *Science* **340** 1223232
[24] Rentmeester M C M, Timmermans R G E and de Swart J J 2003 *Phys. Rev. C* **67** 044001
[25] Navratil P, Gueorguiev V G, Vary J P, Ormand W E and Nogga A 2007 *Phys. Rev. Lett* **99**, 042501