Obtaining the sensitivity of a calibrator for interferometric gravitational wave

A. R. C. Prado[1] , F S Bortoli[1], N. S. Magalhaes[2], R N Duarte[1], C Frajuca[3] and R. C. Souza[1]

[1] Instituto Federal de Educação, Ciência e Tecnologia de Sao Paulo
Rua Pedro Vicente, 625, Caninde – São Paulo – SP – Brazil BR
CEP: 01109-010 www.ifsp.edu.br
[2] Physics Department - Federal University of Sao Paulo
Diadema - SP - Brazil
[3] IMEF Institute - Federal University of Rio Grande
Rio Grande - RS - Brazil

E-mail: frajuca@gmail.com

Abstract: Interferometric gravitational wave detectors (IGWD) are a very complex detector, the need to lock the detector in a dark fringe condition besides the vibrations that affect the mirrors, creates the necessity of using active suspension systems. These active systems make the system reach the desired sensitivity but make the calibration of such detectors much more difficult. To solve this problem a calibrator is proposed, a resonant mass gravitational wave detector could be used to detect the same signal in a narrower band and use the measured amplitude to calibrate the IGWD, as resonant mass gravitational wave detectors are easily calibrated. This work aims to obtain the expected sensitivity of such a calibrator by using lumped modelling in such mechanical detectors. The calibrator is modelled as a spring mass system and the sensitivity curve is presented calculated in by a matlab program. The curve shows that using state of art parameters for the detector the final sensitivity is close to the quantum limit and can be used to calibrate the IGWDs.

1. Introduction

Soon after Albert Einstein developed his theory of General Relativity, there was many developments, one of such developments was the prediction of gravitational waves, as space time can be deformed and dictade how matter should move, if this medium is perturbed, such perturbations should propaged with the speed equal to the speed of light. So after one hundred years experiments to measure such speed are being proposed [1,2].

The authors are part of a Brazilian research group called GRAVITON devoted to the study of gravity. In the search to understand, GRAVITON group members make many contributions to the field of gravitational waves; such efforts can be seen from reference [3] to [26] covering all sorts of aspects in gravitational wave detection.

This expertise gave the author capability to develop a way to calibrate the big detectors that are claimed to have achieved the first detection of these waves. Section 2 will explain how such detectors work and why it is difficult to make its calibration as a whole, section 3 will show how to calculate the sensitivity, section 4 will present the results and section 5 is the conclusion.

2. The Laser Interferometric Gravitational Wave Detectors

Fig. 1 shows the experimental setup: it works as an interferometer, there is a laser source that emits a laser that is divided in two in a beam splitter then the laser enters in two optical cavities called arms, the laser is storage in both arms then they returns to the beam splitter and one fourth of the original laser power goes to the photodetector. The experiment is set in a dark fringe interference at the photodetector. When a gravitational wave arrives perpendicular to the apparatus it changes the length of the arms changing the interference pattern. The problem of the experiment arises when the vibration is taken into account. Vibration of any kind (seismic, thermal, etc) also changes the interference condition of the experiment. To avoid that, clever suspension systems were designed including active systems. The problem is that the active system could respond to the gravitational wave system as well making the calibration of the system quite difficult.

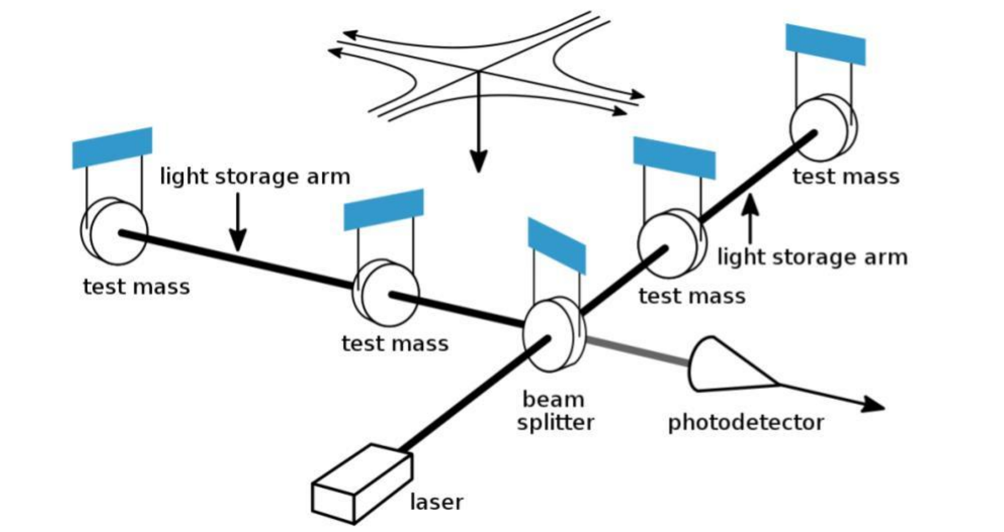

Figure 1: Design experiment of a laser interferometric gravitational wave detector. Caltech/MIT/LIGO Lab

Nevertheless the experiments are taking place and many transient signals were detected as can be seen in Fig. 2. The figure shows the signal in both domains: time and frequency, vertical for the frequency and horizontal for the time. In the figure can be observed that the highest intensity of the signal happens around the frequency of 125 Hz.

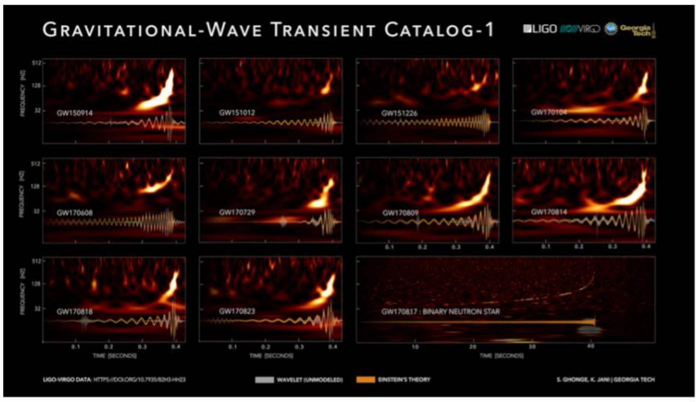

Figure 2: Catalogue including various signals of GW detections. Figure credits: LIGO.

These were not the only gravitational wave detectors. There are the resonant mass gravitational wave detectors, they worked for many decades. They consist of a mass that, when the gravitational wave passes through it, vibrates if the system is in the same frequency of the gravitational wave (because of that the name resonant) [10]. There were many detectors working in coincidence around the world, an example of one of these detectors can be seen in Fig. 3. This detector was called Allegro and operated for more than two decades in the Louisiana State University.

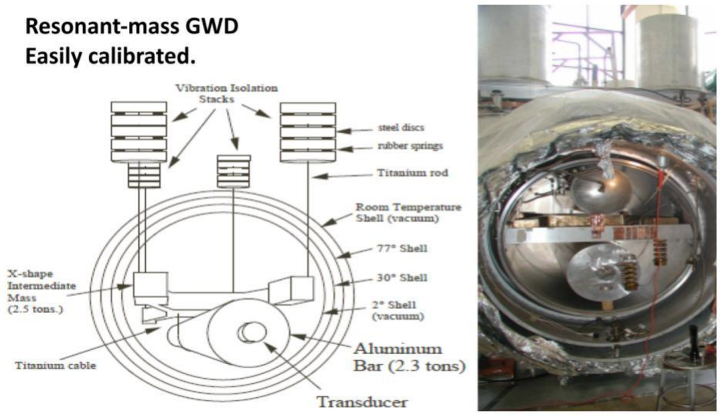

Figure 3: Example of a resonant mass gravitational wave detector. The authors.

No gravitational wave was detected with this kind of detector probably because the operational frequency was in a different range, they operated close to 1 kHz frequency, and as can be seen in Fig. 2 a good detection window will be close to 100 Hz. The frequency of 1 kHz came for historical reasons, the idea was to use a bar as a detector, and a bar that vibrates at 1 kHz is impractical to be constructed. Nevertheless thi kind of detectors are easily calibrated.

It has been proposed a new kind of resonant mass antenna to build a gravitational wave detector and use the signal detected by such a detector to calibrate the laser interferometric gravitational wave detectors. This resonant mass is called the antenna of the detector.

There are two main options for the construction of such antenna: Sapphire with the properties: specific mass of 3.98 g.cm^{-3}, sound velocity of 9,4 km.s^{-1} , mechanical quality factor (Q_M) of 3×10^9 , Young modulus of 431.80 GPa, shear modulus of 170 GPa, Poisson ratio of 0.27, tensile yield strength of 15.5 GPa and bulk modulus of 312.9 GPa; and Niobium, with the properties: specific mass of 8.57 g.cm^{-3}, sound velocity of 3.48,4 km.s^{-1} ,elasticity modulus of 98.6 GPa, ultimate tensile strength of 172 MPa, yield strength of 103.0 MPa, QM of 2 x 10^8, poisson ratio of 0,38 and hardness (nano identification) of 2.5GPa Q_M = 2 x 10^8.

Sapphire presents the best properties to build such an antenna but is impractical to get a very massive detector using this material because of manufacturing limitations, then the Niobium is the next possible choice.

The proposed geometry for the new antenna can be seen in Fig. 4. This figure shows a normal mode simulation of the vibration mode of interesse. It's a color figure with colors showing the displacement from the central position of the mass, the redder the higher the displacement the more blue the smaller it is.

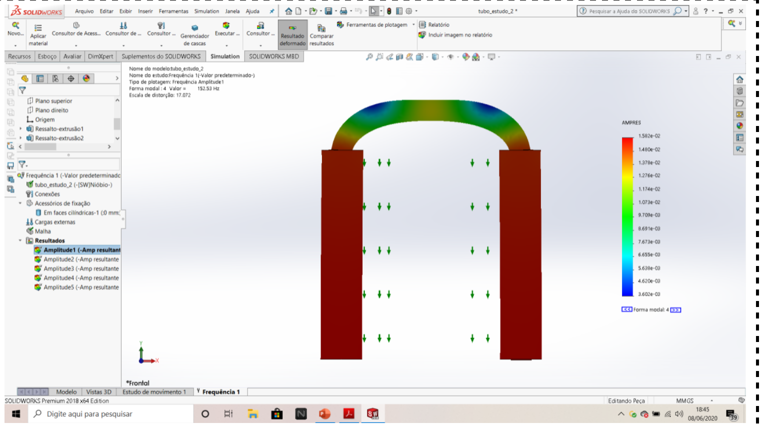

Figure 4: Chosen geometry for the mechanical antenna, Here the figure shows the antenna on its highest displacement as it vibrates. The authors.

3. Calculation of the sensitivity

The following sections are the result of [4]. The calculations are made considering the system as a spring mass model. To make possible measuring the displacement of the side masses of the system, one of those will be connected to a system of smaller resonators that mechanically amplifies the vibration. It will be a set of two resonators whose masses will diminish with the same proportion from the mass of the initial mass. This will lead us to three coupled equations of motion that can be solved numerically in the frequency domain, in the same way that was done in [4].

To the calculation was incorporated thermal noise, back action noise (from the microwave source to be used to measure the displacement of the last mass), phase noise (from the microwave source) and the electronic noise from the amplifier.

In order to calculate the sensitivity, the follow characteristics were implemented:

- M_b = 10,000 kg;
- M_1 = 10 kg;
- M_2 = 0.01 Kg;
- Q_{Mb} = 250 10^6; (mechanical quality factor of the side bars);

- $Q_1 = 500\ 10^6$;
- $Q_2 = 500\ 10^6$;
- $Q_e = 10^6$; (electrical quality factor of the microwave cavity);
- T = 10 mK; (temperature of the detector);
- $T_{Amp} = 4$ K; (temperature of the amplifier);
- $df/dx = 3\ 10^{14}$ Hz m^{-1}; (frequency displacement sensitivity of the cavity);
- $S_P = -185$ dBc @ 100Hz; (microwave phase noise);
- $S_A = -165$ dBc @ 100Hz; (microwave amplitude noise).

4. Result
Fig.5 shows the displacement sensitivity of such a detector, the unit is in strain divided by the square root of Hertz because the sensitivity should be squared before making the integration and obtain the total sensitivity which is equal to 1.31×10^{-21}. This result is admensional because it represents the proportion of same length that will deform in the presence of a gravitational wave, if the length is one kilometer the displacement will be of the order of one thousandth of an atomic nucleus. The quantum limit for this measurement is equal to 1.29×10^{-21}. It means the experiment will be measuring on phonon of the vibration of the mechanical antenna.

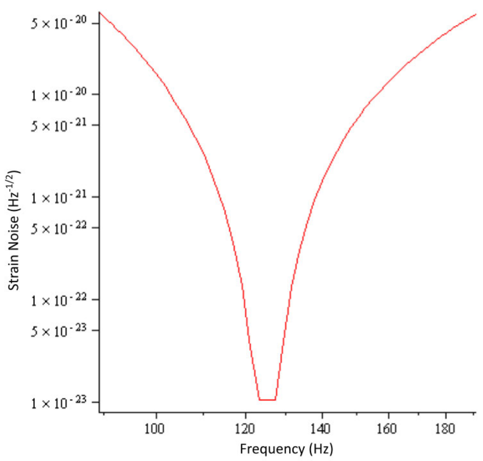

Figure 5: Displacement sensitivity of the calibration detector.
The authors.

5. Conclusion
The calculation for the strain sensitivity shows that the measurement of a gravitational wave is possible with such a detector, as the first gravitational wave ever detected had a strain of 10^{-18}. Nevertheless it was a rare event, but other events were detected and the strain sensitivity of this new detector reaches the strain sensitivity of the laser interferometric gravitational wave detectors, meaning that the wave detected by then will be detected by this calibrator. Then the signal will be used to calibrate these detectors.

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