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A PROTON SYNCHROTRON FOR 300 GEV

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I Introduction

Until 1959 it was stated, and was generally accepted, that the achievement by artificial acceleration of very high interaction energies in the center-of-momentum frame of two nucleons would be practical in the immediate future only with colliding-beam accelerator systems. Proposals were advanced for colliding-beam accelerators which would provide intensities of protons at about 10 Gev sufficient to obtain significant rates of proton-proton interactions. (Center-of-momentum energies of about 20 Gev.) Detailed consideration of colliding-beam devices at the MURA laboratories has shown that these devices are indeed feasible. It is, however, clear that they will be both complex and costly.

Protons with laboratory energies of several hundred Gev or more would provide interaction energies of 20 Gev or more in the C.M. frame. The availability of hundred-Gev proton beams would have advantages other than that of achieving the desired interaction energies. The interactions would occur in experimentally controllable, and, in general, more favorable conditions (i.e., in targets, or in visual detectors). Interactions other than with the proton could also be observed. The interaction rates would in all likelihood be larger. And, perhaps most important, beams of secondary particles -- mesons, and baryons -- of high energy could be produced and used for the study of a wide variety of interactions.

Budker, Veksler, and others[1] have offered concepts which might, they suggested, make feasible the construction of accelerators for exceedingly high energies. Their suggestions invoked novel principles and configurations of plasmas. Their hopes that such considerations would yield practical possibilities for accelerators have, so far, not been fulfilled.

[1] CERN Symposium on High-Energy Accelerators in 1956.

An examination[2] of the possibility of constructing a more conventional synchrotron capable of accelerating protons to energies of several hundred Gev indicates that such an accelerator is not only very likely feasible, but that it may indeed be more straightforward, and more economical, than the proposed colliding beam devices.

It is the purpose of this note to present a concept of a proton synchrotron with a maximum energy of several hundred Gev. In the next sections (Sections II and III), a description of the accelerator and a set of parameters of a tentative design are given. In the later sections some detailed considerations are given which provide some justifications for the choices of the parameters. It is not considered that the concept, or the particular parameters given, are either optimum or final. It is rather the intention that the description given here may provide a basis for a more detailed study of the feasibility and a detailed design of such an accelerator. The note will also delineate the feasibility considerations which have not been completed until now. The rather elementary considerations which have been completed so far, have uncovered no basic difficulties with the suggested scheme. It is possible, of course, that more detailed analyses will uncover unresolvable problems. It appears, however, that the basic concepts are sufficiently well founded that further study is warranted. The potential utility of an accelerator such as that envisaged certainly makes such studies desirable.

The "feasibility" of a large accelerator rests not only on technical questions, but also on economic ones. The magnitude of effort involved in the construction of the accelerator envisaged here can be roughly ascertained by an examination of such factors as the weight of iron, the energy requirements, the total accelerating voltage, etc. All of these quantities appear to be within a factor of 2 or 3 of those involved in

2 Internal MURA Report No. 465 by Matthew Sands. A report of considerations developed at the MURA Conference on Accelerators in June 1959.

other accelerators recently constructed or under construction, and are no larger than those of accelerators which have been proposed. An attempt has been made to make a rough estimate of the costs involved in executing a Cascade Synchrotron. The details of this estimate are given in Section IX. They indicate that an accelerator of the type suggested here could be constructed at a cost significantly less than $100 million.

The preliminary considerations of this Report support the conclusion that a proton synchrotron with a maximum energy of 300 Gev is both technically and economically feasible.

II <u>The Cascade Synchrotron</u>

An attempt to extrapolate the designs of existing (30 Gev) proton synchrotrons to energies of hundreds of Gev quickly encounters two disturbing features -- the magnet and magnet-power cost, and the low injection field. Consideration of these problems led to the consideration of a <u>Cascade Synchrotron</u> in which two stages of synchrotron acceleration would be used.[4] The advantage of such a system appeared to be that the first stage could have a large aperture and a normal injection field and would provide a significant adiabatic damping of the transverse oscillations. The second stage synchrotron would then require a smaller aperture and would be operated at reasonably high injection fields (which would be particularly desirable in a machine with a small aperture.)

We consider in this report the design of an accelerator, the high energy part of which consists of an alternating-gradient, guide-field

[4] This idea was first suggested to the author several years ago by Robert R. Wilson, and has no doubt occurred to others. M.L. Oliphant, and T.A. Welton have both proposed a cascade system of a cyclotron and a synchrotron.

magnet of large diameter, of small aperture, and with conventional[3] focussing parameters. This main accelerator appears in all of its essential features to be a reasonable extrapolation from the existing 30 Gev machines. We call this accelerator the Main Ring.

A high injection energy and a small beam diameter are secured by injecting into the Main Ring guide field from a lower energy synchrotron also of conventional design. This injection accelerator we shall call the Booster Ring.

Ejection from the Booster Ring is to be achieved by the techniques which have been proposed and developed for beam extraction from conventional accelerators and for the injection into storage rings.[5] Injection into the Main Ring corresponds to conventional injection schemes.

Protons are to be injected into the Booster Ring from a linear accelerator as for the existing 30 Gev synchrotrons.

III Suggested Parameters for a Cascade Synchrotron

In this section are presented a set of parameters which might constitute the basis for a design of a 300 Gev Cascade Synchrotron. The considerations which led to the particular values chosen are given in later sections. It is evident that all of the parameters are exceedingly tentative (if not qualitative!). The final decisions on any one must be based on extensive consideration of both technical and economic factors.

A glossary of the quantities and symbols used is given in Table I. The symbols and nomenclature are, in general, those adopted by Courant and

[3] By a "conventional" A.G. accelerator we refer to the Brookhaven A.G.S. or the CERN P.S. machines which accelerate protons to an energy of about 30 Gev. These synchrotrons are described by Green and Courant in Handbuch der Physik (S. Flugge) Band XLIV (1959).

[5] See e.g. G. O'Niell, Report of International Conference on High Energy Physics and Instrumentation, CERN, 1959, page 125.

Snyder[6], which may be consulted for more detailed definitions.

In Table II are given the parameters suggested for the guide fields of the Booster Ring and of the Main Ring. For comparison, the parameters of the Brookhaven A.G.S. are also given.

In Table III are listed some quantities relevant to the Accelerating system.

Table IV contains very rough estimates of the gross properties of the guide magnets.

6 E. Courant and H. Snyder: "Theory of the Alternating Gradient Synchrotron", Annals of Physics, Vol. 3, p. 1 (1958).

Table I. Glossary

E_{max}, B_{max},	-- Maximum energy, and maximum guide field induction on the equilibrium orbit.
E_{inj}, p_{inj}, B_{inj}	-- Energy, momentum, and guide field induction at injection.
ρ, R, D	-- Magnetic radius, mean radius and ring diameter.
a, b	-- Radial and vertical useful aperture.
ν	-- Number of betatron oscillations per revolution.
$\lambda\!\!\!^{-} = R/\nu$	-- Reduced wavelength of betatron oscillations. ($\bar{\beta}$ in some treatments).
n	-- Field gradient index.
M	-- Total number of magnet sectors.
T_{acc}	-- The acceleration time; it is presumed that the repetition period will be about 3 times as long.
eV	-- Energy gain per turn.
f_f/f_i	-- Ratio of final frequency to initial frequency.
U_{mag}	-- Energy stored in guide field magnets (rough estimate).
W_{mag}	-- Total weight of magnet (rough estimate).

Table II

Tentative parameters of the guide field of the Cascade Synchrotron, with the parameters of the Brookhaven A.G.S. for comparison.

	Booster Ring	Main Ring	Brookhaven A.G.S.
E_{max}	10 Gev	300 Gev	30 Gev
B_{max}	10 Kg	12 Kg	12 Kg
E_{inj}	50 Mev	10 Gev	50 Mev
p_{inj}	0.3 Mev/c	10 Mev/c	0.3 Mev/c
B_{inj}	300 g	360 g	120 g
ρ	33 m	860 m	85 m
R	50 m	1300 m	130 m
D		1.6 miles	0.16 miles
a	12 cm	5 cm	12 cm
b	5 cm	2 cm	5 cm
a/R	2.4×10^{-3}	3.8×10^{-5}	9×10^{-4}
ν	5.25	43.25	8.75
λ	9.5 m	30 m	15 m
a/λ	1.3×10^{-2}	1.7×10^{-3}	8×10^{-3}
n	140	8,600	360
M	140	1,200	240

Table III

Parameters of the accelerating system.

	Booster Ring	Main Ring	Brookhaven A.G.S.
T_{acc}	1 sec	1 sec	1 sec
eV	7 kev	8 Mev	
f_f/f_i	3.0	1.005	

Table IV

Rough estimates of the gross properties of the guide magnets.

U_{mag} (approx) joule	4×10^6	15×10^6	13×10^6
W_{mag} (approx) tons	1200	5500	4200
W/M (approx) tons	8.5	4.5	17
W/2πR (approx) tons/meter	3.8	0.7	5.1

IV Scaling Relations for Alternating-Gradient Synchrotrons

The geometrical characteristics of particle orbits in an A-G guide field are determined by the azimuthal dependence of the magnetic field and the field gradient at the orbit. It is usual in A-G synchrotrons (as distinct from FFAG machines) for the guide field to consist of a sequence of segments in which the gradient alternates in sign, but with a uniform absolute value of the gradient. Spaces of zero field are interspersed along the orbit to provide spaces for accelerating cavities, for measuring apparatus, and for beam injection and extraction equipment.

Considerable attention was given at Brookhaven and at CERN to the choice of the field gradient and of the "lattice" structure of the field, i.e. the details of the arrangement of the positive and the negative gradient sections and of the field free sections, so as to achieve desirable forms for the particle orbits and for the efficient use of magnetic field energy. We assume for our preliminary investigations that it is reasonable to adopt the same "orbit geometry" as that used at Brookhaven. By a given "orbit geometry" we refer to the details of the shape of the betatron oscillations, with, however, one free parameter, which determines the azimuthal scale. We take as a measure of azimuthal scale $\lambda\!\!\!^{-}$, the reduced effective wavelength of the betatron oscillations. In terms of the quantities usually employed in A.G. theory* we define $\lambda\!\!\!^{-}$ to be $\langle 1/\beta \rangle_{Av}^{-1}$ which is not too different from β_{Av}, the average of the variable, local wavelength. Our $\lambda\!\!\!^{-}$ is also given by $C/2\pi\nu$ where C is the circumference of the orbit, and ν is the number of betatron oscillations in one revolution.

According to our assumptions, the specification of $\lambda\!\!\!^{-}$ also determines the azimuthal scale of the magnet sectors, so that the ratio of a sector length to $\lambda\!\!\!^{-}$ is kept always the same as that of A.G.S. If the guide magnet has M sectors, the M/ν is held constant and equal to the value for the AGS.

* See e.g. Reference 3 or 6.

A second scaling assumption has been made for the present considerations: that the design compromises which led to the choice of the pole profiles of the Brookhaven and CERN magnets are not sensitive to the lateral* scale of the magnet. We take, then, that the pole geometry of our magnets is to be geometrically similar (in the Euclidean sense) to that of the A.G.S., i.e. that the pole profiles can be brought to congruence by a change in scale. This scaling rule will also provide the property that the change in the magnetic field across the aperture divided by the field at the center of the aperture is independent of the magnet scale.

Our scaling assumption can be expressed by

$$\frac{\partial B}{\partial r} = k_1 \frac{B}{a} \qquad (4.1)$$

where a is the useful radial aperture and k_1 is determined for each magnet sector from our prototype (the A.G.S.). (There are in the A.G.S. two values of k_1, differing only in sign, for positive and negative gradient magnets).

The field index n has the dependence

$$n = -\frac{r}{B}\frac{\partial B}{\partial r} = -k_1 \frac{r}{a} \quad . \qquad (4.2)$$

The phase change of the lateral oscillations in a magnet section depends only on nL^2/r^2 where L is the length of the sector. Since L is proportional to r and inversely proportional to M, the number of magnet sections, the phase change depends on n/M^2 only. (This dependence could have been assumed directly for the simple magnet structure considered originally by Courant, Livingston, and Snyder.[7] It is also true for any magnet structure under our scaling assumptions.) We must choose, therefor, to have M proportional to $\sqrt{n}$.

* By "lateral" we mean both radial and axial.

7 Courant, Livingston, and Snyder, Phys. Rev. 88, 1190 (1952).

Since our assumptions also provide that ν is proportional to M, and that the mean radius R is proportional to the instantaneous radius r we may take the following relationships as our scaling rules:

$$n = k_2 \frac{R}{a} ,$$

$$\nu = k_3 \sqrt{\frac{R}{a}} ,$$

$$M = k_4 \nu ,$$

and

$$\bar{\lambda} = \frac{R}{\nu} = \frac{1}{k_3} \sqrt{Ra} .$$

The relevant parameters of the A.G.S. and the constants derived from them are:

R = 128 m	k_2 = 42.1
a = 15 cm	k_3 = 3.0
n = 360	k_4 = 27.4
M = 240	
ν = 8.75	

It is necessary for arguments given below to have some scaling for the cost of synchrotron magnets. It is certain that no simple cost relationship can be given, but some rough estimates of the dependence of cost on the magnet scale can be given.

One convenient rule of thumb is that the cost of magnets is in proportion to the magnetic energy stored. This rule would give that the cost is proportional to Ra^2. One may, however, question whether this rule is justified under the scaling constraints which have already been imposed.

If one assumes that the current density in the magnet coils is a fixed quantity, one can argue that the weight of the magnet iron will vary as $Ra^{3/2}$ and the weight of the copper as Ra. The cost of the power supply will vary perhaps as the stored energy, or as Ra^2. For our arguments we shall take as approximately true that the cost of an accelerator magnet (including power supply) will vary with the magnet dimensions as Ra^m where m is about 2.

V. The Parameters of the Booster Ring

In this section we shall consider arguments which lead to a selection of the parameters -- in particular the aperture and radius -- of the Booster Ring. The results will depend parametrically on the characteristics of the Main Ring and on the energy of the Linac injector. The choice of these magnitudes is postponed to Section VI.

We give, first, some simple arguments for the choice of the dimensions of the Booster Ring. More sophisticated arguments given later arrive at similar results.

(a) Elementary Argument for Parameters of Booster Ring.

A primitive argument for the choice of the final energy of the Booster Ring would be that the magnetic field at injection should be the same for both the Booster and Main Rings. One would hope then to minimize difficulties with field errors at injection in both Rings.

We define the following symbols:

p_o : momentum of protons from the Linac.

p_t : momentum of protons at transfer.

p_2 : final momentum in Main Ring.

B_1 (max) : Max. magnetic field in Booster Ring.

B_2 (max) : Max. magnetic field in Main Ring.

The requirement of equal injection momenta for the two rings gives the relationship

$$p_t = \left[p_o p_2 B_1(\max)/B_2(\max)\right]^{1/2} \tag{5.1}$$

Assuming that the maximum magnetic field in the two rings is taken equal, we obtain that the transfer momentum is the geometric mean of the initial and final momenta.

$$p_t = p_o p_2^{1/2} \qquad (5.2)$$

The aperture of the Booster Ring is simply related to that of the Main Ring if we assume that the complete aperture of the Booster Ring is filled by lateral oscillations at injection, and that the aperture of the Main Ring is just large enough to accommodate the lateral oscillations which exist at the time of transfer.

We define the acceptance phase area for radial oscillations of the Booster Ring at injection as a_1^2/λ_1, where a_1 is the radial aperture and λ_1 is the reduced wavelength defined in Section IV. The phase area occupied by the residual oscillations at the end of the Booster acceleration period will be reduced by adiabatic damping by the factor (p_o/p_t). This damped phase area is to be matched to the acceptance of the main ring, a_2^2/λ_2 with suitable matching lenses. We should then arrange that

$$\frac{p_o}{p_t}\,\frac{a_1^2}{\lambda_1} = \frac{a_2^2}{\lambda_2} \qquad (5.3)$$

Using the scaling relation obtained in Section IV which gives

$$\frac{\lambda_2}{\lambda_1} = \left[\frac{R_1 a_1}{R_2 a_2}\right]^{1/2} \qquad (5.4)$$

and assuming equal maximum magnetic fields for the two rings so that

$$\frac{p_2}{p_t} = \frac{R_2}{R_1} \qquad (5.5)$$

we have

$$\frac{a_1}{a_2} = \frac{p_t}{\left[p_o^{\,2} p_2\right]^{1/3}} \tag{5.6}$$

If we use Eq. (5.2) we obtain

$$\frac{a_1}{a_2} = \left(\frac{p_2}{p_o}\right)^{1/6} \tag{5.7}$$

Relations 5.2 and 5.6 or 5.7 serve to determine the parameters of the Booster Ring in terms of the properties of the injector and Main Ring.

The discussion above can be applied mutatis mutandi to the axial aperture which is, in accord with our assumptions, in a constant ratio to the radial aperture.

(b) Refined Argument for Parameters of the Booster Ring.*

The argument used above that the injection fields of the two rings should be equal is of questionable validity. The injection problems themselves depend on the size of the aperture and other factors, and are likely to be most serious for the Main Ring. It is proposed, therefor, that the aperture of the Main Ring is to be chosen in terms of the requirements at injection into it. The detailed arguments are given in Section VI. The dimensions of the Booster Ring are then to be determined by economic arguments. In particular we adopt the criterion that the design shall be that which minimizes the cost of the system per particle accelerated. (It is worth noting that one cannot simply minimize the cost, because the minimum cost is easily achieved by building no accelerator at all!)

It is clear that one must still impose the condition that the acceptance of the Main Ring should be matched to the phase area occupied

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* Note added after printing: The detailed arguments of this section are faulty, but the conclusions are basically correct. The subject will be treated fully in a later report.

by the protons after acceleration in the Booster Ring.* We assume, of course, that the full phase area of the Booster Ring has been filled at injection (by multiple turn injection if necessary), and that transfer can be accomplished via a suitable lens system so that the phase areas of the two Rings are matched. With these assumptions, the ratio of the lateral apertures obtained in Eq. (5.6) is still valid.

The number of protons accelerated in the Booster Ring can be written

$$N = \eta \left(\frac{a_1^2}{\lambda_1} \right)^2 \tag{5.8}$$

where η is proportional to the density in lateral phase space at the output of the injector, and to the efficiency for capture into synchronous orbits.** N is also the number of protons accelerated to the peak energy.

It is apparent from Eqs. (5.6) and (5.8) that for a given aperture a_2 of the Main Ring, the larger the transfer momentum, the larger will be the number of protons accelerated. The cost of the Booster Ring, however, rises rapidly with its size. It seems most reasonable, therefor, to select the transfer momentum to maximize the ratio of the number of protons accelerated to the cost of the whole accelerator.

* The approach adopted here is subject to the criticism that only phase areas in the transverse coordinates are matched, and that a large mismatch exists between the phase areas in azimuthal angle-energy (synchrotron oscillation coordinates). A few remarks on this problem will be found in Appendix A.

** The present arguments ignore possible limitations due to space charge effects in the beam. It has been pointed out by Courant and Snyder (private conversation) that such effects may dominate. This section should, therefor, be reworked with this in mind.

A detailed investigation of the costs of the various accelerator components for various values of their parameters is an exceedingly complex and tedious task and cannot be attempted for this report, although a final design should probably be based on such considerations. For our present purposes, we make the gross assumption, which is often adequate for preliminary estimates, that the cost of a guide magnet (of a given type) is proportional to the maximum magnetic energy stored in the guide field. For simplicity we also consider only the magnet cost, omitting the costs of all other components from the analysis. (Many of which should be included.) We adopt, then, as our criterion for the choice of the transition energy, that the ratio of W, the sum of the peak stored energies of the two rings, to N, should be a minimum.

The stored energy is proportional to the magnet radius and the pole gap area. We take for the total stored energy

$$W = K\left[a_1^2 p_t + a_2^2 p_2\right] \qquad (5.9)$$

Using Eqs. (5.8), (5.6) and (5.3) we obtain

$$\frac{W}{N} = \frac{K}{\eta}\left(\frac{\lambda_2}{a_2}\right)^2\left(\frac{p_o}{p_2}\right)^{2/3}\left[p_t + \frac{p_o^{4/3}\, p_2^{5/3}}{p_t^2}\right] \qquad (5.10)$$

The choice of transfer momentum p_t which minimizes the ratio of Eq. (5.10) is

$$p_t = {2^{1/3}\, p_o^{4}\, p_2^{5}}^{\;1/9} \qquad (5.11)$$

which is not too different from the geometric mean encountered earlier.

With this value of p_t one finds that the stored energy in the Main Ring is only <u>one-half</u> that of the Booster Ring! The Booster Ring is in this sense larger than the accelerator proper. The optimum transfer momentum gives for the ratio of stored-energy to proton number the form

$$\left(\frac{W}{N}\right)_{opt} = \frac{3}{2^{2/3}} \frac{K}{\eta} \left(\frac{\lambda_2}{a_2}\right)^2 \frac{p_o^{10/9}}{p_2^{1/9}} \quad (5.12)$$

This relation gives the interesting result that the cost-per-proton figure depends universely on a_2, arguing for the larger apertures; and increases nearly linearly with the ultimate proton energy p_2. The approximate linear dependence on p_o is surprising and, in fact, misleading, as we have not included the dependence of either the injector cost or the injector emittance coefficient η on the injection momentum. The investigation of this dependence is deferred for the time being.

Adopting arbitrarily an injection energy of 50 Mev (p_o = 0.3 Bev/c) and a final energy of 300 Bev (p_2 = 300 Bev/c) one obtains for the optimm transfer momentum

$$p_t = \underline{17.7 \text{ Bev/c}} \,. \quad (5.15)$$

The ratio of the apertures of the two rings is found from Eq. (5.6) to be

$$\frac{a_1}{a_2} = 2^{1/3} \left(\frac{p_2}{p_o}\right)^{2/9} \quad (5.16)$$

For our proposed injection and final energies we have

$$\frac{a_1}{a_2} = 5.85. \quad (5.17)$$

VI Choice of the Parameters of the Main Ring

In the previous Section we have derived some relations between the parameters of the Main and Booster Rings. In this Section we attempt to determine suitable values for the parameters of the Main Ring.

The final energy of the Main Ring has been chosen to be 300 Gev. No sharp justification can be given for any particular value of the final energy. There are no known or expected thresholds in the hundred Gev region. The particular value chosen provides the same interaction energy in the c.m. System as two colliding 12 Gev protons (the energy which has been proposed for colliding beam devices). It is also a factor of 10 above the highest existing machine energy (for U.S. accelerators). Since typical c.m. energies vary as the square-root of the laboratory projectile energy, a 300 Gev machine will provide interaction and secondary particle energies about a factor of 3 above the CERN and Brookhaven accelerators. Since an energy of 300 Gev appears to be feasible, it has seemed worthwhile to consider initially a synchrotron of this energy. It may be worthwhile later to consider designs for other energies so that some feeling can be obtained about how the costs and difficulty depend on energy in the several hundred Gev region.

The only remaining parameter which is left to be chosen is the aperture of the Main Ring. It is clear that the cost of the whole accelerator may depend critically on the aperture chosen. For large apertures the space-charge limited intensity will vary approximately as the area of the aperture, as will also the cost. For small apertures, a significant fraction of the aperture is ineffective, due to the imperfections of the magnet (wanderings of the closed orbit), so that the intensity will vary as some higher power of the aperture area and will fall to zero for apertures below some value. Also fixed (aperture independent) costs may dominate. It seems reasonable for a first estimate to consider an aperture near the cross-over between "large" and "small".

The criterion is adopted here that the aperture shall be chosen so that the closed orbit at injection shall, with 98 percent probability, lie within the useful aperture. It is assumed that provisions would be incorporated in the accelerator design for the measurement and adjustment of the errors of the closed orbit. In the unlikely case that the closed orbit did not lie within the aperture, these adjustments would be called upon to move the orbit into the aperture. In any case, these adjustments would be relied upon to provide a closed orbit which did not vary about the center of the aperture by more than 1/4 of the aperture itself. It is clear that the reproducibility of the magnet from pulse to pulse must be held to closer tolerances (by say a factor of 4) than those required for the initial construction of the magnet.

The expected value for the amplitude of the excursions of the equilibrium orbit about the design center of the aperture has been computed in Ref. 6. We consider here only those effects due to random placement errors of the individual magnets. If X represents the displacement of the closed orbit and ϵ the r.m.s. placement error of the magnets, we have from Ref. 6 (Eq. 4.20) that

$$X = P\,\epsilon \qquad (6.1)$$

with

$$P = \frac{2\pi}{|\sin \pi\nu|}\;\frac{R}{\rho}\;\frac{|n|}{\nu}\left(\frac{F}{M}\right)^{1/2} . \qquad (6.2)$$

$\underline{F}$ is the form factor for betatron oscillations; the other symbols have been defined earlier.

According to our assumption, we wish to place* $2X = a$. But our scaling assumptions of Section IV make $\underline{P}$ also dependent upon $\underline{a}$. In fact, the scaling relations provide that

$$P \propto \left(\frac{R}{a}\right)^{1/4} \qquad (6.3)$$

* For this discussion $a = a_2$, the aperture of the Main Ring.

Denoting the values which apply to the A.G.S. by the subscript 0, we may write:

$$P = P_o \left(\frac{R}{R_o} \frac{a_o}{a} \right)^{1/4} \qquad (6.4)$$

Combining (1) and (4) with 2X = a we have

$$a = \left(\frac{Ra_o}{R_o} \right)^{1/5} \left(2P_o \epsilon \right)^{4/5} \qquad (6.5)$$

Taking the A.G.S. values P = 45, a_o = 12 cm, and R/R_o = 10, we have

$$a = 96 \text{ cm}^{1/5} \epsilon^{4/5} \qquad (6.6)$$

Experience with the CERN and Brookhaven synchrotrons has shown that r.m.s. errors in magnet placement of 0.005 inches can be achieved.* The smaller magnet cross-sections of this machine may make smaller errors realizable (one might even argue that at least a part of the error is proportional to a). Other problems may make these tolerances harder to realize. We adopt at this stage an expected r.m.s. magnet placement error of 0.005 inch, or

$$\epsilon = 0.013 \text{ cm} \qquad (6.7)$$

We obtain

$$a = 3 \text{ cm} \qquad (6.8)$$

* It is a property of A.G. guide fields that the close tolerances are required only on the relative positions of the magnets within a distance comparable with the betatron wavelength. We are thus contemplating magnet placement errors of 0.005 inches in a few hundred feet. This requirement will place stringent requirements on the Magnet foundation.

Using the relation (5.17) we find for the two rings

$$a_1 \text{ (Booster Ring)} = 17 \text{ cm}$$
$$a_2 \text{ (Main Ring)} = 3 \text{ cm} \tag{6.9}$$

The results obtained in the above treatment give a startlingly small aperture for the Main Ring. It also appears somewhat unreasonable that the stored energy in the Main Ring should be only one-half that of the Booster Ring, and that the cost of the Main Ring magnet should be much smaller than the cost of the tunnel which houses it. Such intuitive and subjective feelings have led to the tentative proposal of an aperture for the Main Ring about 1.5 times larger and for the Booster Ring about 2/3 as large as the values derived above.

Another crucial problem in the guide magnet design is that of the width of the stop-bands at the resonances. Reference 6 gives for the width of the stop-band

$$\delta\nu = \frac{R}{\rho}\,\frac{n}{\sqrt{M}}\left(\frac{\Delta n}{n}\right)_{rms} \tag{6.10}$$

The factors which depend on scale are just those which appear in Eq.(6.2) and accordingly vary as does $\underline{P}$, namely, as $\left(\frac{R}{a}\right)^{1/4}$. For 300 Gev ring with a 3 cm aperture we find

$$\delta\nu = 10\left(\frac{\Delta n}{n}\right)_{rms} \tag{6.11}$$

Experience indicates that it is possible to maintain $\underline{n}$ within 1 percent on all magnets. Such a tolerance gives a stop-band width of 0.1 which is probably barely tolerable. It does not appear impossible to achieve a precision of a factor of 2 or 3 greater if this is required. The error in ν is expected to be about 1/2 the width of the stop band and thus appears to be acceptable.

The values computed refer to a 3 cm aperture, the somewhat larger aperture adopted in Section II makes the situation somewhat more favorable with regard to gradient errors.

VII Acceleration

Operating convenience and the desire for the largest achievable average intensities both argue for an acceleration time as short as is practicable. Since the stored energy in the rings is comparable to that of the A.G.S. it seems reasonable to suggest an acceleration time of 1 sec for both rings. Such an acceleration time requires an average energy gain per turn of about 8 Mev, or a total gap voltage around the ring of about 12 million volts.

The A.G.S. has 24 long (10 ft.) field-free sections in the magnet ring. The scaling relations provide that the number of such sections increases in proportion to ν and the length in proportion to λ. The Main Ring will, therefor, have about 120 field-free sections, each 20 ft. long. If only 100 were used for R.F. stations the voltage per cavity would be about 120 kilovolts.

The modulation in frequency required is only one part in 200, so it appears feasible to use relatively high Q cavities tuned either by a small mechanical deformation or by ferromagnetic or ferroelectric materials.

The Princeton-Penn accelerator group has designed a cavity with 100 KV which operates at 10 KW power input. Such a system repeated 120 times would provide the necessary accelerating voltage with a total R.F. power of 1.2 Megawatts. The C.E.A. synchrotron is to provide for an energy gain of 6 Mev per turn with the expenditure of 250 KW. Even allowing for the small frequency shift required for the Main Ring, the R.F. problems do not appear overly difficult.

A brief examination of the phase oscillations has not revealed any expectation of difficulty. The increase in phase oscillation amplitude at the transition energy (at about 70 Gev) is easily accommodated in the aperture.

It is evident that the frequency tolerances required are quite exceptional. The relative aperture a/R of about 3×10^{-5} requires that frequency errors be significantly less than this at injection and impossibly precise at the transition energy. There seems to be no reason to expect that the beam controlled acceleration techniques which have been so successful at CERN and Brookhaven cannot also be applied to the Main Ring. These problems evidently need a detailed examination.

No consideration has been given to the acceleration problems of the Booster Ring, as it would seem that they are quite similar to those of existing accelerators.

VIII Transfer

The problem of transferring an accelerated beam from one circular guide field to another has not been encountered in existing synchrotrons. It is, however, quite analogous to the problem of transferring particles from an accelerator into a storage ring. No consideration has been given to the technical details of the transfer problem. The justification for this neglect is based on the following two statements: (1) The problem of ejecting a beam of good optical quality from the Booster Ring is the same as the problem of injecting into a high-field storage ring. (2) The problem of injecting into the Main Ring at an injection field of 300 gauss is the same as the injection problem of any AG synchrotron. Problem (1) above has been solved by O'Niell (Ref. 5); problem (2) has a standard solution.

It may be remarked that at both Brookhaven and CERN plans are under way for the ejection of good quality high-energy beams for neutrino experiments. These schemes appear quite reasonable in execution. It appears that they have not been adopted heretofor only because there has been no demand for such beams.

IX Estimate of Costs

It is, of course, not possible to give any completely justified cost estimate for an accelerator in such an early stage of study as now obtains for the Cascade Synchrotron. It is, however, useful to attempt to make some estimates which can serve two useful purposes: (1) to indicate whether the general magnitude of the cost reasonably warrants further study of the general design; and (2) to focus attention on the major cost items both for the design considerations and for later efforts to obtain more precise cost estimates.

The problem of obtaining a reasonable cost estimate for the Cascade Synchrotron is greatly aided by the fact that this accelerator is rather conventional in its details. Cost figures based on experience with recently constructed accelerators are applicable with only moderate interpolations or extrapolations. It is believed that the estimates presented here are <u>real</u>.

The detailed cost estimates are given in Table IV, which was prepared with the assistance of M.H. Blewett.

It should be emphasized that the estimated costs refer <u>only</u> to the accelerator proper. No estimate has been included of the funds which would be required for the initial instrumentation for the experimental program or of the continuing costs of an adequate experimental program.

Table IV

Estimate of Costs

General Expenses		
Experimental Building; Shielding		$7 million
Offices and Laboratories		3
Wells and Cooling		2
Utilities and Roads		3
		$15 million
Salaries; Administration; Architect		$12 million
Accelerator Components		
	Booster	**Main Ring**
Tunnels and Foundation	$2.0 million	$13 million
Cores and hardware	1.5	5
Coils and bus	1.0	3
Tests and survey	0.5	2
Injector	2.0	
Vacuum system	0.5	4
R-F, electronics and ejector	1.0	6
Controls and wiring	0.5	3
Design and tests	1.0	4
	$10 million	$40 million
Total		$77 million

Appendix A

The Azimuthal Filling Problem

A disturbing feature of the configuration of the Cascade Synchrotron suggested in this Report is that after their transfer from the Booster Ring to the Main Ring the accelerated protons occupy only a small fraction ($\approx 1/26$) of the circumference of the Main Ring. One has the feeling that the Main Ring is not being used efficiently; one may also criticise the relatively low duty-ratio of the high-energy beam.

It should be pointed out that the low azimuthal filling factor does not imply a low overall intensity of accelerated particles. The acceptance (proportional to $a^4 R/\lambda^2$) of the Booster Ring is nearly the same (0.9) as the acceptance of the Brookhaven A.G.S. If the limiting intensity is not space charge limited (but by the emittance of the injector) the 300 Gev intensity would be about that of the A.G.S. If the same linac were to inject directly into the Main Ring,(ignoring the problem of the low injection field) the acceptance would be down by the factor $(a_2/a_1)^4(\lambda_1/\lambda_2)^2(R_2/R_1) = 1/10$. The factor of 26 in the circumference is more than compensated by the factor of 260 gained in lateral phase area. In fact, a larger gain is more likely, since the fraction of the aperture available for lateral oscillations would undoubtedly be less in the Main Ring than in the Booster Ring.

The Booster Ring provides both a higher injection field and a higher maximum theoretical intensity.

Similar arguments apply to space charge limited intensities so long as the same injection energy is considered. Detailed studies of other possible injection schemes should, however, be made to ascertain whether other injection methods which provide more efficient azimuthal filling, and at higher energies, might not be more satisfactory (Linac, Cyclotron,etc.)

It is possible in principle to consider a booster of larger aperture and the use of a "beam splitter" which would peel off a fraction of the protons on each of several revolutions of the Booster Ring. No serious thought has as yet been given to this possibility.

Courant, Snyder, and Walker have proposed that high intensities could be achieved by operating the Booster Ring at a high repetition rate, by injecting many pulses into the Main Ring at successive azimuthal positions, and by then accelerating the many groups of protons to high energy. A preliminary investigation of this possibility looks promising and it will be investigated further.